ICSE 2025 EXAMINATION

Sample Question Paper - 7

Mathematics

Time: 2 Hours **Max. Marks: 80**

General Instructions:

1. Answer to this Paper must be written on the paper provided separately
2. You will not be allowed to write during first 15 minutes.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
7. Omission of essential working will result in loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables are provided.

SECTION-A

(Attempt all questions from this Section.)

QUESTION 1.

Choose the correct answers to the questions from the given options.

(Do not copy the questions, write the correct answer only.)

(i) The money required to buy 100, ₹50 shares quoted at ₹48.50 is :

(a) ₹5000 (b) ₹2425

(c) ₹2425 (d) ₹4850

Answer: (d) ₹4850

(ii) Lavanya deposited ₹2000 per month for 24 months in HDFC bank's recurring deposit account. If the bank pays an interest of 10% p.a., then the amount she gets on maturity is:

(a) ₹48,000 (b) ₹45,000

(c) ₹53,000 (d) ₹5,000

Answer: (c) ₹53,000

(iii) If $\frac{5-2x}{3} < \frac{x}{6} - 5$ then

(a) $x > 8$ (b) $x < 8$

(c) $x \leq 8$ (d) $x \geq 8$

Answer: (d) $x \geq 8$

(iv) The roots of the quadratic equation x2 – 0.04 = 0 are

(a) ±0 2 (b) ±0.02

(c) 0.4 (d) 2

Answer: (a) ±0 2

(v) The ratios 2 : 3, 8 : 15, 11 : 12, 7 : 16 in their ascending order of magnitude are:

(a) $\frac{2}{3} < \frac{7}{16} < \frac{8}{15} < \frac{11}{12}$ (b) $\frac{7}{16} < \frac{8}{15} < \frac{2}{3} < \frac{11}{12}$

(c) $\frac{7}{16} < \frac{8}{15} < \frac{7}{12} < \frac{2}{3}$ (d) $\frac{2}{3} < \frac{11}{12} < \frac{8}{15} < \frac{7}{16}$

Answer:

(b) $\frac{7}{16} < \frac{8}{15} < \frac{2}{3} < \frac{11}{12}$

(vi) If $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$, then the value of p is:

(a) 6 (b) 4

(c) 5 (d) 8

Answer: (c) 5

(vii) The simplified form of $\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix}\begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Answer:

(b) $\begin{bmatrix} 1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$

(viii) **Statement 1 :** If sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term is $a_n = 6n - 7$.

Statement 2 : n^{th} term of an AP, whose sum to n terms is S_n, is given by $a_n = S_{n-1} - S_n$

(a) Both the statement are true. (b) Both the statement are false.

(c) Statement 1 is true and statement 2 is false. (d) Statement 1 is false and statement 2 is true.

Answer: (c) Statement 1 is true and statement 2 is false.

(ix) C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is $(-2, 4)$, then x and y respectively are

(a) -6 and 1 (b) -6 and 2

(c) 6 and -1 (d) 6 and -2

Answer: (a) -6 and 1

(x) The gradient of the line joining $P(6,\ k)$ and $Q(1-3k,\ 3)$ is $\frac{1}{2}$. What is the value of k ?

(a) -11 (b) 7

(c) 7 (d) -3

Answer: (a) –11

(xi) **Assertion :** In the $\triangle ABC$, $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm, then $\triangle ABC$ is a right angle triangle.

Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

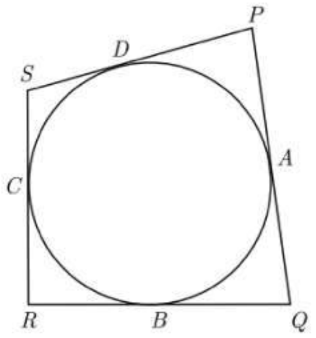
Answer:

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(xii) In the given figure if sides PQ, QR, RS and SP of a quadrilateral PQRS touch a circle at point A, B, C and D respectively, then PD + BQ is equal to

(a) PQ (b) QR

(c) PS (d) SR



Answer: (a) PQ

(xiii) During conversion of a solid from one shape to another, the volume of the new shape will

(a) Increase
(b) Decrease
(c) Remain unaltered
(d) Be doubled

Answer: (c) Remain unaltered

(xiv) If $4\tan\theta = 3$, then $\left(\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$
(d) $\frac{3}{4}$

Answer: (c) 1/2

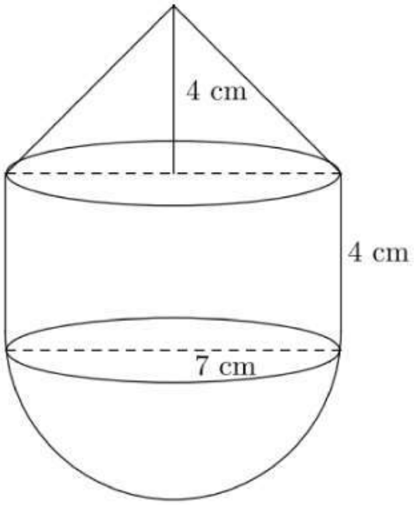
(xv) A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(a) 40
(b) 240
(c) 480
(d) 750

Answer: (c) 480

QUESTION 2.

(i) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid.



Answer:

The volume of solid = Volume of cone + Volume of cylinder + Volume of the hemisphere.

Volume of cone = $\frac{\pi r^2 h}{3} = \frac{22 \times 7 \times 7 \times 4}{7 \times 3} = \frac{616}{3} cm^3$

Volume of cylinder = $\pi r^2 h = \frac{22 \times 7 \times 7 \times 4}{7} = 616 cm^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3 = \frac{2 \times 22 \times 7 \times 7 \times 7}{3 \times 7} = \frac{2156}{3} cm^3$

Total volume = $\frac{616}{3} + 616 + \frac{2156}{3} = 1540\ cm^3$

(ii) Reeta gets ₹ 48528 after 36 months. How much money she should invest per month in a R.D. account to get required amount when the bank pays the rate of interest 8% per annum?

Answer:

The formula for the maturity value (MV) of a Recurring Deposit (RD) account is:

$$MV = P \times n + P \times \frac{n(n+1)}{2} \times \frac{r}{12 \times 100},$$

where:

- MV = Maturity Value = ₹48,528,
- P = Monthly investment (to be found),
- n = Number of months = 36,
- r = Rate of interest per annum = 8%.

$$48,528 = P \times 36 + P \times \frac{36(36+1)}{2} \times \frac{8}{12 \times 100}.$$

1. **Simplify the second term (interest part):**
   First, calculate $\frac{36(36+1)}{2}$:

$$\frac{36(36+1)}{2} = \frac{36 \times 37}{2} = 666.$$

Now calculate the interest factor:

$$\frac{8}{12 \times 100} = \frac{8}{1200} = \frac{1}{150}.$$

So:

$$\frac{666 \times 1}{150} = \frac{666}{150} = 4.44.$$

$$48,528 = P \times 36 + P \times 4.44.$$

Combine terms:

$$48,528 = P(36 + 4.44) = P(40.44).$$

$$P = \frac{48,528}{40.44}.$$

Simplify:

$$P \approx 1200.$$

Reeta should invest **₹1200 per month** in the RD account.

(iii) Prove that : $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\,\text{cosec}\,\theta.$

Answer:

$$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}}$$

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= 2 \times \frac{1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta \qquad \text{Hence Proved}$$

QUESTION 3.

(i) Share of Vegii Foods is sold at premium of ₹ 25. The face value of a share is ₹100. Now answer the following:

(i) Find the amount required by Vedant to purchase 5000 shares.

(ii) What would be the gain of the original share holder from Vedant if he had bought each share at 10% discount?

Answer:

Share of face value ₹ 100 is sold at premium of ₹ 25, it means that market price of share is ₹ 125.

Market value of 5000 shares $= 125 \times 5000$

$= ₹\, 625000$

The original share holder bought shares at 10% discount i.e. he had bought each share for ₹ 90.

Amount paid by the original share holder

$= 90 \times 5000 = ₹\, 450000$

Gain to the original share holder

$= 625000 - 450000$

$= ₹\, 175000$

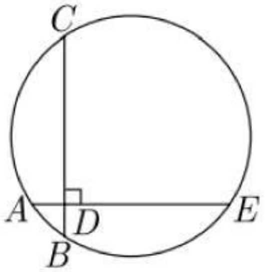
Also, Gain percent $= \frac{175000}{450000} \times 100$

$= 38.88$

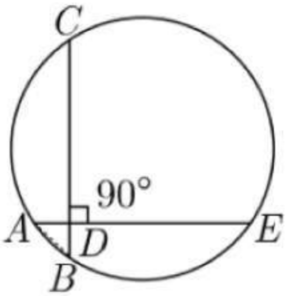
(i) Thus, amount paid by Vedant to buy 5000 shares is ₹ 156250.

(ii) Gain is 38.33 %.

(ii) In the given figure, AE and BC intersect each other at point D. ∠CDE = 90°, AB = 5cm, BD = 4cm and CD = 9cm, find DE.



Answer: We have redrawn the given figure as below.



From figure,

Since, ∠CDE = 90° so, ∠ADB = 90° (∵ vertically opposite angles are equal.)

In right angle triangle △ADB, by pythagoras theorem,

$\Rightarrow AB^2 = AD^2 + BD^2$

$\Rightarrow 5^2 = AD^2 + 4^2$

$\Rightarrow 25 = AD^2 + 16$

$\Rightarrow AD^2 = 25 - 16$

$\Rightarrow AD^2 = 9$

$\Rightarrow AD = 3$ cm.

Chords AE and CB intersect each other at D.

In △ADB and △CDE,

∠BAD = ∠DCE (∵ angles in same segment are equal.)

∠ADB = ∠CDE (∵ vertically opposite angles are equal.)

△ADB ~ △CDE. (By AA axiom)

Since △ADB ~ △CDE, Hence, the ratio of corresponding sides are equal.

$$\therefore \frac{AD}{CD} = \frac{BD}{DE}$$

$\therefore$ AD × DE = CD × BD

$\Rightarrow$ 3 × DE = 9 × 4

$\Rightarrow \text{DE} = \frac{36}{3}$

$\Rightarrow$ DE = 12 cm.

Hence, the length of DE = 12 cm.

(iii) The marks obtained by 120 students in a test are given below

Marks	Number of students
0-10	5
10-20	9
20-30	16
30-40	22
40-50	26
50-60	18
60-70	11
70-80	6
80-90	4
90-100	3

Draw an ogive for the given distribution on a graph sheet.

Use suitable scale for ogive to estimate the following

(i) The median.

(ii) The number of students who obtained more than 75% marks in the test.

(iii) The number of students who did not pass the test, if minimum marks required to pass is 40.

Answer: The Cumulative frequency table for the given continuous distribution is given below.

Marks	Frequency	Cumulative frequency cf
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120

Step 2: Draw the **ogive**:

- Plot the cumulative frequency values on the **y-axis**.
- Plot the upper class boundaries $(10, 20, 30, \ldots, 100)$ on the **x-axis**.
- Mark the points $(10, 5), (20, 14), (30, 30), \ldots, (100, 120)$.
- Join the points smoothly to draw the ogive curve.

Step 3: Use the ogive to estimate:

(i) **The median**:

- Total number of students = 120.
- The median corresponds to the cumulative frequency at:

$$\text{Median Position} = \frac{120}{2} = 60.$$

- Locate 60 on the y-axis, move horizontally to the ogive curve, and then vertically downward to the x-axis.
- From the ogive graph, the approximate **median** is **40-50 marks** (closer to 42).

(ii) Number of students who obtained more than 75% marks:

- 75% of the total marks = 75% of $100 = 75$.
- From the graph, identify the cumulative frequency at 75 marks.
- The cumulative frequency at 75 marks is approximately **113**.
- Number of students who scored more than 75 marks:

$$\text{Students} = 120 - 113 = 7.$$

(iii) Number of students who did not pass the test:

- Minimum marks to pass = 40.
- From the graph, identify the cumulative frequency at 40 marks.
- The cumulative frequency at 40 marks is approximately **52**.
- Number of students who did not pass:

$$\text{Students} = 52.$$

SECTION-B

(Attempt any four questions.)

QUESTION 4.

(i) If $A = \begin{bmatrix} 2 & 3 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -8 \\ 8 \end{bmatrix}$, find matrix X such that $2AX = B$.

Answer:

Given:

1. $A = \begin{bmatrix} 2 & 3 \\ 0 & -2 \end{bmatrix}$
2. $B = \begin{bmatrix} -8 \\ 8 \end{bmatrix}$.

We need to solve for X in the equation:

$$2AX = B$$

Step 1: Simplify the equation

First, divide both sides of the equation by 2 to simplify:

$$AX = \frac{B}{2}$$

Simplify $B/2$:

$$\frac{B}{2} = \frac{1}{2}\begin{bmatrix} -8 \\ 8 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}.$$

Thus, the equation becomes:

$$AX = \begin{bmatrix} -4 \\ 4 \end{bmatrix}.$$

Step 2: Write the equation in matrix form

Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where x_1 and x_2 are the unknowns to solve for. Then the equation $AX = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$ becomes:

$$\begin{bmatrix} 2 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}.$$

Perform matrix multiplication on the left-hand side:

$$\begin{bmatrix} 2x_1 + 3x_2 \\ -2x_2 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}.$$

Step 3: Solve for x_1 and x_2

From the matrix equation, equate the corresponding elements:

1. From the first row:

$$2x_1 + 3x_2 = -4.$$

2. From the second row:

$$-2x_2 = 4.$$

Solve for x_2 from the second equation:

$$x_2 = -2.$$

Substitute $x_2 = -2$ into the first equation:

$$2x_1 + 3(-2) = -4.$$

Simplify:

$$2x_1 - 6 = -4.$$

Add 6 to both sides:

$$2x_1 = 2.$$

Divide by 2:

$$x_1 = 1.$$

Step 4: Write the solution for X

The matrix X is:

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Final Answer:

The matrix X is:

$$X = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

(ii) In ΔABC, $AP : PB = 2 : 3$, PO is parallel to BC and is extended to Q, so that CQ is parallel to BA. Find

(i) area of ΔAPO : area of ΔABC.

(ii) area of ΔAPO : area of ΔCQO.

Answer:

Given:

1. $AP : PB = 2 : 3$.
2. $PO \parallel BC$, and PO is extended to Q such that $CQ \parallel BA$.

We need to find:

1. **Area of $\triangle APO$: Area of $\triangle ABC$.**
2. **Area of $\triangle APO$: Area of $\triangle CQO$.**

Step 1: Understanding the problem and geometry

1. The ratio $AP : PB = 2 : 3$ implies that P divides AB in the ratio $2 : 3$. Hence, $AP = \frac{2}{5}AB$ and $PB = \frac{3}{5}AB$.
2. Since $PO \parallel BC$, the line PO divides $\triangle ABC$ into two **similar triangles**:
   - $\triangle APO \sim \triangle ABC$ (by **Basic Proportionality Theorem** or Thales' theorem).
3. Similarly, $CQ \parallel BA$, so CQ divides $\triangle ABC$ into two smaller triangles.

Step 2: Solving Part (i)

Find the ratio of the area of $\triangle APO$ to the area of $\triangle ABC$:

- Since $PO \parallel BC$, $\triangle APO \sim \triangle ABC$.
- The ratio of the areas of similar triangles is equal to the **square of the ratio of their corresponding sides**.

The side ratio $AP : AB = \frac{2}{5}$.

Therefore:

$$\frac{\text{Area of } \triangle APO}{\text{Area of } \triangle ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}.$$

Answer for Part (i):

$$\text{Area of } \triangle APO : \text{Area of } \triangle ABC = 4 : 25.$$

Step 3: Solving Part (ii)

Find the ratio of the area of $\triangle APO$ to the area of $\triangle CQO$:

1. Since $CQ \parallel BA$, the triangles $\triangle CQO$ and $\triangle ABC$ are also similar.
    - The side ratio $CQ : CB = \frac{3}{5}$ (because $PB : AB = \frac{3}{5}$).

Thus:

$$\frac{\text{Area of } \triangle CQO}{\text{Area of } \triangle ABC} = \left(\frac{CQ}{CB}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}.$$

2. To find the ratio $\text{Area of } \triangle APO : \text{Area of } \triangle CQO$, divide the ratios:

$$\frac{\text{Area of } \triangle APO}{\text{Area of } \triangle CQO} = \frac{\frac{4}{25}}{\frac{9}{25}} = \frac{4}{9}.$$

Answer for Part (ii):

$$\text{Area of } \triangle APO : \text{Area of } \triangle CQO = 4 : 9.$$

Final Answers:

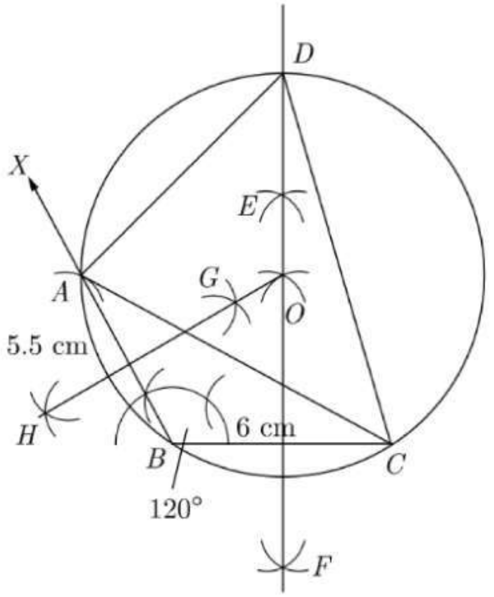
1. $\text{Area of } \triangle APO : \text{Area of } \triangle ABC = 4 : 25.$
2. $\text{Area of } \triangle APO : \text{Area of } \triangle CQO = 4 : 9.$

(iii) Construct a ΔABC in which base $BC = 6$ cm, $AB = 5.5$ cm and $\angle ABC = 120^\circ$.

(i) Construct a circle circumscribing the ΔABC.

(ii) Draw a cyclic quadrilateral $ABCD$ so that D is equidistant from B and C.

Answer:



Steps of Construction :

Step 1. Draw a line segment $BC = 6$ cm.

Step 2. At B, draw a ray BX making an angle of 120° with BC.

Step 3. Taking radius 5.5 cm, with B as centre cut $BA = 5.5$ cm.

Step 4. Join A to C, which is the required ΔABC.

Step 5. Draw perpendicular bisector EF of BC and GH of AB. They both intersect at point O.

Step 6. Draw a circle, with O as centre and OA or OC or OB as radius. Then, we get the required circumcircle.

Step 7. Extend FE line in the direction of E, which meet the circle at point D, which is equidistant from B and C.

Step 8. Join AD and CD. Hence, $ABCD$ is the required cyclic quadrilateral.

QUESTION 5.

(i) Find the mean for the following data :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	16	36	34	6

Answer:

We prepare following cumulative frequency table to find median class.

Class	x_i (class marks)	f_i	$f_i x_i$	c.f.
0-10	5	8	40	8
10-20	15	16	240	24
20-30	25	36	900	60
30-40	35	34	1190	94
40-50	45	6	270	100
		$\sum f_i = 100$	$\sum f_i x_i = 2640$	

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{2640}{100} = 26.4$

(ii) Rate of GST for different categories are given below :

Category	Rate of GST
Cosmetics	18%
Automobiles	28%
Dry Fruits	5%
Edible Oils	5%
Processed food	12%
Spices	5%

(a) Mrs. Sharma bought a pack of face cream for ₹500 at 20% discount. Calculate amount of GST paid by Mrs. Sharma.

(b) On Jiya's birthday his father bought a cake and some biscuits for Rs.1800 at a discount of Rs 300. Calculate amount of GST paid by Jiya's father.

(c) Mrs. Mathur bought turmeric (curcuma) for Rs.1200 at 10% discount. Calculate amount of GST paid by Mrs. Mathur.

Answer:

(a) Mrs. Sharma bought a face cream

- Cost of face cream = ₹500
- Discount = 20%
- GST Rate for Cosmetics = 18%

Step 1: Calculate the price after discount

$$\text{Discount} = 20\% \text{ of } 500 = \frac{20}{100} \times 500 = 100$$

$$\text{Price after discount} = 500 - 100 = ₹400$$

Step 2: Calculate GST

GST is applied on the discounted price:

$$\text{GST} = 18\% \text{ of } 400 = \frac{18}{100} \times 400 = 72$$

Amount of GST paid by Mrs. Sharma = ₹72.

(b) Jiya's father bought a cake and biscuits

- **Total cost** = ₹1800
- **Discount** = ₹300
- **GST Rate for Processed Food = 12%**

Step 1: Calculate the price after discount

$$\text{Price after discount} = 1800 - 300 = ₹1500$$

Step 2: Calculate GST

GST is applied on the discounted price:

$$\text{GST} = 12\% \text{ of } 1500 = \frac{12}{100} \times 1500 = 180$$

Amount of GST paid by Jiya's father = ₹180.

(c) Mrs. Mathur bought turmeric

- **Cost of turmeric** = ₹1200
- **Discount** = 10%
- **GST Rate for Spices = 5%**

Step 1: Calculate the price after discount

$$\text{Discount} = 10\% \text{ of } 1200 = \frac{10}{100} \times 1200 = 120$$

$$\text{Price after discount} = 1200 - 120 = ₹1080$$

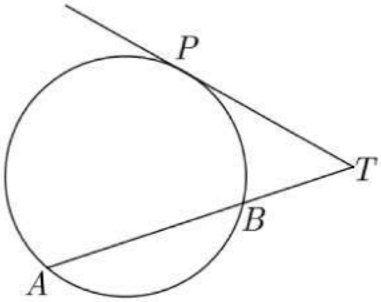
Step 2: Calculate GST

GST is applied on the discounted price:

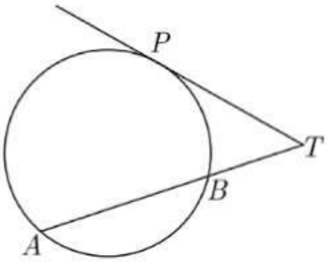
$$\text{GST} = 5\% \text{ of } 1080 = \frac{5}{100} \times 1080 = 54$$

Amount of GST paid by Mrs. Mathur = ₹54.

(iii) In the given figure, PT is a tangent to the circle. Find PT, if AT = 16cm and AB = 12cm.



Answer: We have redrawn the given figure as below.



PT is tangent.

Hence by theorem,

PT^2 = AT x BT

PT^2 = 16 x (AT - AB)

PT^2 = 16 x (16 - 12)

PT^2 = 16 x 4 = 64

PT = 8 cm.

QUESTION 6.
(i) If p, q, r, are in A.P., show that p^{th}, q^{th} and r^{th} terms of any G.P. are themselves in G.P.

Answer:

Proof

1. Given:
    - p, q, and r are in arithmetic progression (A.P.)
    - We need to consider any geometric progression (G.P.)
2. Properties of A.P. and G.P.:

- For p, q, r in A.P.: q - p = r - q
- Let's denote this common difference as d: q = p + d and r = q + d = p + 2d
- For a G.P. with first term a and common ratio x: nth term = ax^{n-1}

3. Let's consider the pth, qth, and rth terms of this G.P.:
   - pth term: $T_p = ax^{p-1}$
   - qth term: $T_q = ax^{q-1} = ax^{(p+d)-1} = ax^{p-1} \cdot x^d = T_p \cdot x^d$
   - rth term: $T_r = ax^{r-1} = ax^{(p+2d)-1} = ax^{p-1} \cdot x^{2d} = T_p \cdot (x^d)^2$
4. To prove that T_p, T_q, and T_r are in G.P., we need to show that:
   $\frac{T_q}{T_p} = \frac{T_r}{T_q}$
5. Let's verify this:
   $\frac{T_q}{T_p} = \frac{T_p \cdot x^d}{T_p} = x^d$
   $\frac{T_r}{T_q} = \frac{T_p \cdot (x^d)^2}{T_p \cdot x^d} = x^d$
6. Since both ratios are equal to x^d, we have proven that:
   $\frac{T_q}{T_p} = \frac{T_r}{T_q} = x^d$

(ii) Eleven years ago an investment earned Rs 7,000 for the year. Last year the investment earned Rs 14,000. If the earnings from the investment have increased the same amount each year, what is the yearly increase and how much income has accrued from the investment over the past 11 years?

Answer: To solve this problem, we'll break it down into two parts: finding the yearly increase and calculating the total income over 11 years.

Part 1: Yearly Increase

Let's set up an equation to find the yearly increase:

Initial earnings: Rs 7,000
Final earnings after 10 years: Rs 14,000
Let x be the yearly increase

Equation: 7000 + 10x = 14000

Solving for x:
10x = 14000 - 7000
10x = 7000
x = 700

Therefore, the yearly increase is Rs 700 [2].

Part 2: Total Income Over 11 Years

To calculate the total income, we need to sum up the earnings for each year. We can use the arithmetic sequence formula:

$S_n = \frac{n}{2}(a_1 + a_n)$

Where:

- S_n is the sum of the sequence
- n is the number of terms (11 years)
- a_1 is the first term (Rs 7,000)
- a_n is the last term (Rs 14,000)

Plugging in the values:

$$S_{11} = \frac{11}{2}(7000 + 14000)$$
$$S_{11} = \frac{11}{2}(21000)$$
$$S_{11} = 11 \times 10500$$
$$S_{11} = 115500$$

Therefore, the total income accrued from the investment over the past 11 years is Rs 115,500.

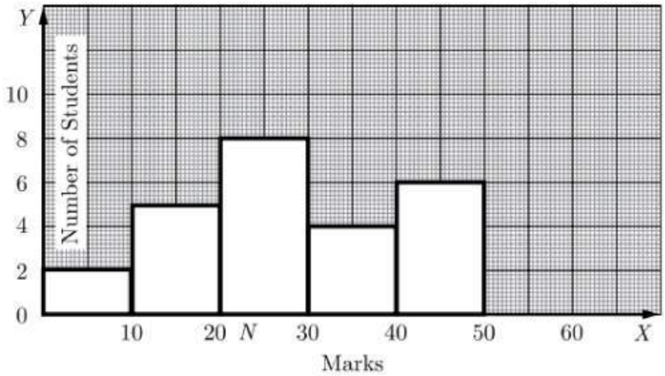
(iii) The histogram adjacent represents the scores obtained by 25 students in a Mathematics mental test.

Use the data to

(i) Frame a frequency table

(ii) To calculate mean

(iii) To determine the modal class



Answer: (i) Frequency Table:

From given histogram we can get following frequency table

Scores	Number of students
0-10	2
10-20	5
20-30	8
30-40	4
40-50	6
Total	25

(ii) Mean:

Class interval	Frequency (f)	Mean value (x)	fx
0 – 10	2	5	10
10 – 20	5	15	75
20 – 30	8	25	200
30 – 40	4	35	140
40 – 50	6	45	270
	$\sum f = 25$		$\sum f = 695$

$\therefore$ Mean = $\frac{\sum fx}{\sum f}$

= $\frac{695}{25}$

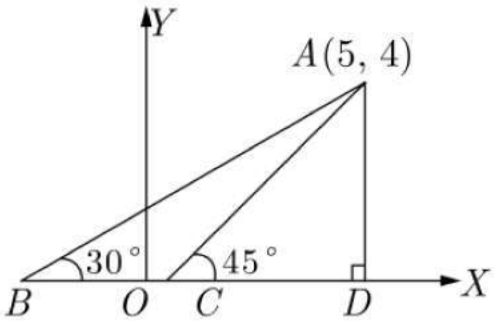
= 27.8

(iii) Modal Class

In frequency distribution, the class which have maximum frequency is called model class. Here class 20-30 has frequency 8, which is maximum amongst all given frequencies. Thus class 20-30 is modal class.

QUESTION 7.

(i) Find the equation of lines AB, AC and AD in the given figure where A(5, 4). Also find the coordinates of C.



Answer:

Slope of AB $\quad m_1 = \tan 30^\circ$

$$= \frac{1}{\sqrt{3}}$$

Equation of line AB, having slope $\frac{1}{\sqrt{3}}$ and passing through point $A(5, 4)$,

$$(y - y_1) = m(x - x_1)$$

$$y - 4 = \frac{1}{\sqrt{3}}(x - 5)$$

$$\sqrt{3}\,y = x - 5 + 4\sqrt{3}$$

$$x - \sqrt{3y} - 5 + 4\sqrt{3} = 0 \qquad L_1:\ AB$$

Slope of AC $\quad m_2 = \tan 45^\circ$

$$= 1$$

Equation of line AC, having slope 1 and passing through point $A(5, 4)$,

$$(y - y_1) = m(x - x_1)$$

$$y - 4 = 1(x - 5)$$

$$y = x - 1$$

$$x - y - 1 = 0 \qquad L_2:\ AC$$

Since AD is perpendicular to x-axis, its slope is not defined. This line is parallel to y-axis and x coordinate of every point is 5. Thus equation of this line is $x = 5$.

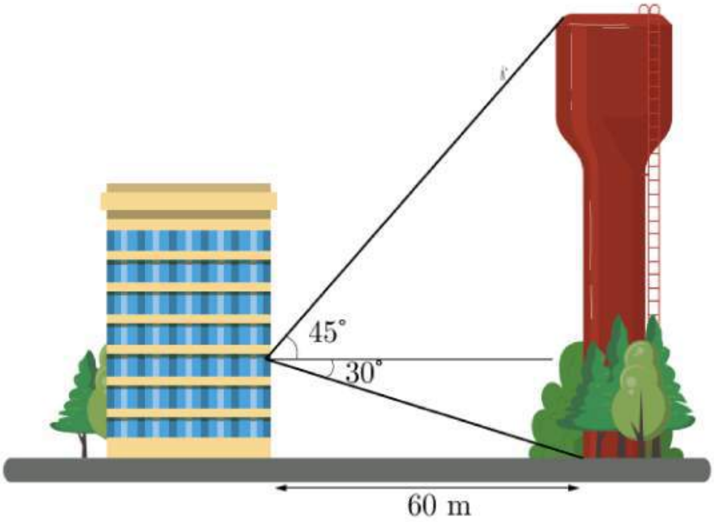
Point C is intersection of line AC and x. Thus y coordinate of point C must be zero. Substituting $y = 0$ in equation of line AC,

$$x - 0 - 1 = 0$$

$$x = 1$$

Thus we get point $C(1, 0)$.

(ii) Water Tower : A water tower is a building that is used to hold and give out water. It is almost always built on a high place. It works because a pump gives water to the tower, and gravity makes the saved water go out to the places that need water. Those places are connected to the tower by pipes.



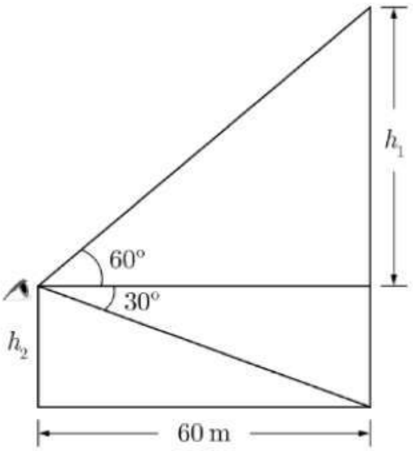
A water tower is located 60 meter from a building (see the figure). From a window in the building, an observer notes that the angle of elevation to the top of the tower is 60° and that the angle of depression to the bottom of the tower is 30°.

(i) How tall is the tower?

(ii) How high is the window?

Answer:

Let h_1 be the height of top of tower from window and h_2 be height of window from ground. We draw a diagram of the situation as shown below.



Now $$\tan 60^\circ = \frac{h_1}{60}$$

$$\sqrt{3} = \frac{h_1}{60}$$

$$h_1 = 60 \times \sqrt{3}$$

$$= 60 \times 1.732 \text{ m}$$

$$= 103.56 \text{ m}$$

Now $$\tan 30^\circ = \frac{h_2}{60}$$

$$\frac{1}{\sqrt{3}} = \frac{h_2}{60}$$

$$h_2 = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

$$= 20 \times 1.732 \text{ m}$$

$$= 34.64 \text{ m}$$

Height of tower, $$h = h_1 + h_2$$

$$= 103.56 + 34.64$$

$$= 138.56 \text{ m}$$

(i) Tower is 138.56 m tall.
(ii) Height of the window is 103.56 meter.

QUESTION 8.

(i) Find the value of x, which satisfies the in equation

$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2,\ x \in W$. Graph the solution set on the number line.

Answer:

Step 1: Solve the compound inequality step by step

The compound inequality is:

$$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$$

First, isolate the middle term step by step.

1. **Break the compound inequality into two parts:**

$$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \quad \text{and} \quad \frac{1}{2} - \frac{2x}{3} \leq 2$$

Solve the first part: $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3}$

1. Convert $-2\frac{5}{6}$ to an improper fraction:

$$-2\frac{5}{6} = -\frac{17}{6}$$

The inequality becomes:

$$-\frac{17}{6} < \frac{1}{2} - \frac{2x}{3}$$

2. Eliminate the fractions by multiplying through by the **LCM of 6** (which is 6). Remember, multiplying by a positive number does not change the inequality:

$$6 \cdot \left(-\frac{17}{6}\right) < 6 \cdot \frac{1}{2} - 6 \cdot \frac{2x}{3}$$

Simplify:

$$-17 < 3 - 4x$$

3. Rearrange to isolate x. Subtract 3 from both sides:

$$-17 - 3 < -4x$$

$$-20 < -4x$$

4. Divide through by -4. **Note:** Dividing by a negative number reverses the inequality:

$$\frac{-20}{-4} > x$$

Simplify:

$$5 > x \quad \text{or equivalently} \quad x < 5$$

Solve the second part: $\frac{1}{2} - \frac{2x}{3} \leq 2$

1. Start with the inequality:

$$\frac{1}{2} - \frac{2x}{3} \leq 2$$

2. Eliminate the fractions by multiplying through by the **LCM of 6**:

$$6 \cdot \frac{1}{2} - 6 \cdot \frac{2x}{3} \leq 6 \cdot 2$$

Simplify:

$$3 - 4x \leq 12$$

3. Rearrange to isolate x. Subtract 3 from both sides:

$$-4x \leq 12 - 3$$

$$-4x \leq 9$$

4. Divide through by -4. **Note:** Dividing by a negative number reverses the inequality:

$$x \geq \frac{9}{-4}$$

Simplify:

$$x \geq -\frac{9}{4}$$

Since $x \in \mathbb{W}$ (whole numbers), we only consider $x \geq 0$.

Combine the Results

From the two parts, we have:

1. $x < 5$
2. $x \geq 0$ (since x is a whole number).

Combining these, the solution is:

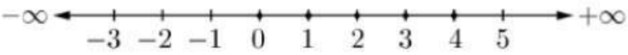
$$0 \leq x < 5$$

Final Solution

The whole numbers that satisfy the inequality are:

$$x = 0, 1, 2, 3, 4$$

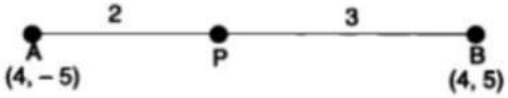
Required set is $\{0,1,2,3,4\}$. [as $x \in W$]
The graph of the solution set on the number line
is shown by dotted points.



(ii) If the line joining the points A(4, –5) and B(4, 5) is divided by the point P, such that AP/AB = 2/5, then find the coordinates of P.

Answer:

From the given



$$\frac{AP}{AB} = \frac{2}{5}$$

$$\Rightarrow \quad \frac{AB}{AP} = \frac{5}{2}$$

$$\frac{AB}{AP} - 1 = \frac{5}{2} - 1$$

$$\Rightarrow \quad \frac{PB}{AP} = \frac{3}{2}$$

$\therefore$ **Coordinates of P**

$$= \left(\frac{mx_2 + nx_1}{m + x}, \frac{my_2 + xy_1}{m + x}\right)$$

$$= \left(\frac{2 \times 4 + 3 \times 4}{2 + 3}, \frac{2 \times 5 + 3 \times (-5)}{2 + 3}\right)$$

$$= \left(\frac{8 + 12}{5}, \frac{10 - 15}{5}\right)$$

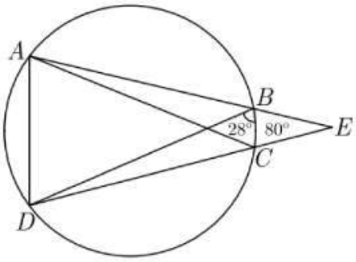
$$= (4, -1)$$

(iii) In the given figure $ABCD$ is a cyclic quadrilateral. AB and DC are produced to meet at E. Given that $BC = BE$, $\angle CBE = 80^{\circ}$ and $\angle DBC = 28^{\circ}$.

(i) Find $\angle BEC$

(ii) Find $\angle DAC$

(iii) Find $\angle BAC$.



Answer:

(i) $\angle BEC$

Given that $BC = BE$

Equal side make equal angle in any triangle.

Thus $\angle BCE = \angle BEC$

$$\angle BCE + \angle BEC + \angle CBE = 180^{\circ}$$

$$\angle BEC + \angle BEC + 80^{\circ} = 180^{\circ}$$

$$2\angle BEC = 100^{\circ}$$

$$\angle BEC = 50^{\circ}$$

Now $\angle DBE = \angle DBC + \angle CBE$

$$= 28^{\circ} + 80^{\circ}$$

$$= 108^{\circ}$$

(ii) Finding ∠DAC

In cyclic quadrilateral ABCD:

- Given that ∠DBC = 28°
- In a cyclic quadrilateral, angles in the same segment are equal
- Therefore, ∠DAC = ∠DBC = 28°

(iii)

$\angle BAC$

Due to angles in the same segment,

$$\angle BAC = \angle BDC$$

QUESTION 9.

(i) If $\frac{a^3+3ab^2}{3a^2b+b^3} = \frac{x^3+3xy^2}{3x^2y+y^3}$, prove that $\frac{x}{a} = \frac{y}{b}$.

Answer:

Step 1: Observe the given equation

The equation provided can be written as:

$$\frac{a^3+3ab^2}{3a^2b+b^3} = \frac{x^3+3xy^2}{3x^2y+y^3}.$$

Notice that both the numerator and denominator have a similar structure. Each term can be factored using the identity for **symmetric polynomials.**

Step 2: Factor the numerator and denominator

1. **Numerator:**
   a^3+3ab^2 can be factored as:

   $$a^3+3ab^2 = a(a^2+3b^2).$$

   Similarly, x^3+3xy^2 can be factored as:

   $$x^3+3xy^2 = x(x^2+3y^2).$$

2. **Denominator:**
   $3a^2b+b^3$ can be factored as:

   $$3a^2b+b^3 = b(3a^2+b^2).$$

   Similarly, $3x^2y+y^3$ can be factored as:

   $$3x^2y+y^3 = y(3x^2+y^2).$$

Step 3: Substitute back into the equation

Now substitute the factored forms into the original equation:

$$\frac{a(a^2+3b^2)}{b(3a^2+b^2)} = \frac{x(x^2+3y^2)}{y(3x^2+y^2)}.$$

Step 4: Eliminate common terms

To simplify, compare the **numerators** and **denominators** on both sides. We observe:

1. The term a in the numerator and b in the denominator on the left-hand side.
2. The term x in the numerator and y in the denominator on the right-hand side.

By symmetry of the given equation, for the equality to hold:

$$\frac{a}{b} = \frac{x}{y}.$$

Step 5: Rearrange to prove $\frac{x}{a} = \frac{y}{b}$

From $\frac{a}{b} = \frac{x}{y}$, taking reciprocals:

$$\frac{b}{a} = \frac{y}{x}.$$

Cross-multiply:

$$\frac{x}{a} = \frac{y}{b}.$$

Final Answer:

Hence, we have proved:

$$\frac{x}{a} = \frac{y}{b}.$$

(ii) A fast train takes 3 hours less than a slow train for a journey of 60 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train.

Answer:

Total distance of a journey $= 600$ km
Let x be the speed of fast train then speed of slow will be $(x-10)$.
Time taken by fast train, $t_1 = \frac{60}{x}$
Time taken by slow train, $t_2 = \frac{60}{x-10}$
According to questions, we have

$$t_2 - t_1 = 3$$

$$\frac{600}{x-10} - \frac{600}{x} = 3 \qquad \left[t = \frac{d}{s}\right]$$

$$600\left[\frac{x-x+10}{(x-10)\,x}\right] = 3$$

$$\frac{6000}{x^2-10x} = 3$$

$$x^2 - 10x - 2000 = 0$$

$$x^2 - 50x + 40x - 2000 = 0$$

$$x(x-50) + 40(x-50) = 0$$

$$(x-50)(x+40) = 0 \quad x = 50, -40$$

But negative speed can not be possible. Thus, the speed of fast train is 50 km/hr, and the speed of slow train is $50 - 10 = 40$ km.

(iii) By using ruler and compass only, construct a quadrilateral ABCD in which 6.5cm, AB = 6.5cm, AD = 4cm and ∠DAB = 75° C is equidistant from the sides AB and CD, also C is equidistant from the points A and B.

Answer:

Steps of Construction :

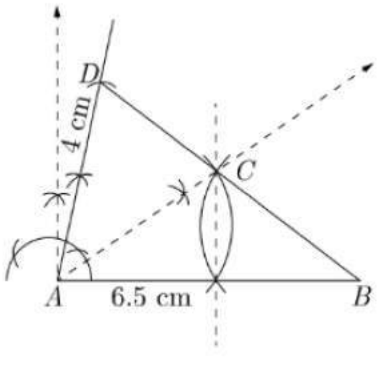
Step 1 : Draw a line segment $AB = 6.5$ cm.

Step 2 : At A, draw a ray making an angle of 75° and cut off $AD = 4$ cm.

Step 3 : Draw the bisector of $\angle DAB$.

Step 4 : Draw perpendicular bisector of AB intersecting the angle bisector at C.

Step 5 : Join CB and CD. $ABCD$ is the required quadrilateral.



QUESTION 10.

(i) The polynomial $p(x) = kx^3 + 9x^2 + 4x - 8$ when divided by $(x+3)$ leaves a remainder $10(1-k)$. Find the value of k.

Answer:

When a polynomial p(x) is divided by (x+3), the remainder equals p(-3) (using the polynomial remainder theorem).

Therefore:
p(-3) = 10(1-k)

Let's evaluate p(-3):

1. p(x) = kx³ + 9x² + 4x - 8
2. Substituting x = -3:
$p(-3) = k(-3)^3 + 9(-3)^2 + 4(-3) - 8$
$= -27k + 81 - 12 - 8$
$= -27k + 61$

Since this equals 10(1-k):
$-27k + 61 = 10(1-k)$
$-27k + 61 = 10 - 10k$
$-27k + 10k = -61 + 10$
$-17k = -51$
$k = 3$

Therefore, the value of k is 3.

(ii) Two unbiased coins are tossed simultaneously. Find the probability of getting :

(i) At least one head,

(ii) Almost one head,

(iii) No head.

Answer:

(i) Probability of getting at least one head

At least one head means **one or more heads.** The favorable outcomes are:

$$F = \{HH, HT, TH\}.$$

The number of favorable outcomes is:

$$n(F) = 3.$$

The probability is given by:

$$P(\text{at least one head}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}.$$

Substitute the values:

$$P(\text{at least one head}) = \frac{3}{4}.$$

(ii) Probability of getting almost one head

"Almost one head" means **exactly one head**. The favorable outcomes are:

$$F = \{HT, TH\}.$$

The number of favorable outcomes is:

$$n(F) = 2.$$

The probability is:

$$P(\text{exactly one head}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}.$$

Substitute the values:

$$P(\text{exactly one head}) = \frac{2}{4} = \frac{1}{2}.$$

(iii) Probability of getting no head

No head means both coins show **tails**. The only favorable outcome is:

$$F = \{TT\}.$$

The number of favorable outcomes is:

$$n(F) = 1.$$

The probability is:

$$P(\text{no head}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}.$$

Substitute the values:

$$P(\text{no head}) = \frac{1}{4}.$$

(iii) Use graph paper for this question. Plot the points O(0, 0), A(– 4, 4), B(–3, 0) and C(0, – 3)

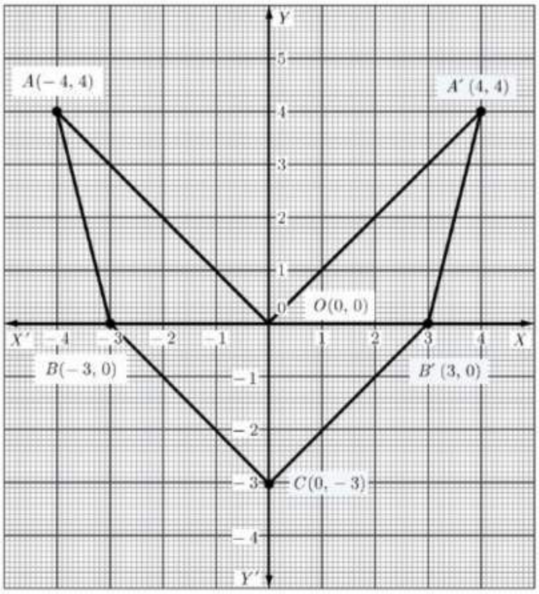
(i) Reflect points A and B in the y-axis and name them A' and B' respectively. Write down their coordinates.

(ii) Name the figure OABCB' A'.

(iii) State the line of symmetry of this figure.

Answer:

Take 2 cm = 1 unit on both the axes.
Plot the points $O(0, 0)$, $A(-4, 4)$, $B(-3, 0)$ and $C(0, -3)$ on the graph paper as shown below.



(i) Reflect points A and B in the y-axis and name them A' and B'. Write down their coordinates.

- **Reflection Rule**: When a point (x, y) is reflected in the y-axis, its x-coordinate changes sign while the y-coordinate remains the same.
  - $A(-4, 4)$ becomes $A'(4, 4)$.
  - $B(-3, 0)$ becomes $B'(3, 0)$.

Coordinates of reflected points:

- $A' = (4, 4)$
- $B' = (3, 0)$

(ii) The figure $OABCB'A'$ is a (concave) hexagon.

(iii) The line of symmetry of the figure $OABCB'A'$ is y-axis.