CLASS : XI , CHAPTER 3 - TRIGONOMETRIC FUNCTIONS

EXERCISE 3.3

Prove that

Q No 1 : $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{6} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$

LHS $= \left(\sin\frac{\pi}{6}\right)^2 + \left(\cos\frac{\pi}{6}\right)^2 - \left(\tan\frac{\pi}{6}\right)^2$ $\left[\because \sin^2\theta = (\sin\theta)^2 \text{ and so on.}\right]$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2 = \frac{1}{4} + \frac{1}{4} - 1 = \frac{1+1-4}{4} = -\frac{2}{4} = -\frac{1}{2} = \text{RHS}$

Q No 2 $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

LHS $= 2\left(\sin\frac{\pi}{6}\right)^2 + \left[\operatorname{cosec}\left(\pi+\frac{\pi}{6}\right)\right]^2\left[\cos\frac{\pi}{3}\right]^2$

$= 2\left(\sin\frac{\pi}{6}\right)^2 + \left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\cos\frac{\pi}{3}\right)^2$

$= 2\left(\frac{1}{2}\right)^2 + (-2)^2\left(\frac{1}{2}\right)^2 = 2\times\frac{1}{4} + 4\times\frac{1}{4} = \frac{1}{2} + 1 = \frac{3}{2}$

Q No 3 $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

LHS $= (\cot 30)^2 + \operatorname{cosec}(150^\circ) + 3(\tan 30)^2$

$= (\cot 30)^2 + \sec 60^\circ + 3(\tan 30)^2$

$= (\sqrt{3})^2 + 2 + 3\times\left(\frac{1}{\sqrt{3}}\right)^2 = 3+2+1 = 6$

Q No 4 . $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

LHS $= 2\left(\sin\frac{3\pi}{4}\right)^2 + 2\left(\cos\frac{\pi}{4}\right)^2 + 2\left(\sec\pi/3\right)^2$

$= 2(\sin 135^\circ)^2 + 2(\cos 45)^2 + 2(\sec 60)^2$

$= 2(\cos 45^\circ)^2 + 2(\cos 45)^2 + 2(\sec 60^\circ)^2$

$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$

$= 1+1+8 = 10$

Q No 5 Find the values of

(i) $\sin 75^\circ$ (ii) $\tan 15^\circ$

Sol (i) $\sin 75^\circ = \sin(45^\circ+30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\tan 15^\circ = \tan(45^\circ - 30^\circ)$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2} = \frac{3+1-2\sqrt{3}}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

Prove the following:

QNO 6 $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) = \sin(x+y)$

$$LHS = \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$= \cos\left[\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right] \quad \left[\because \cos A\cos B - \sin A \sin B = \cos(A+B)\right.$$

$$= \cos\left[\frac{\pi}{2}-(x+y)\right] = \sin(x+y) = RHS \quad \left[\because \cos(\pi/2-\theta) = \sin\theta\right]$$

QNo7 : $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{(1+\tan)^2}{(1-\tan x)^2}$

$$LHS : \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}.\tan x.}}{\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}} = \frac{\frac{1+\tan x}{1-1.\tan x}}{\frac{1-\tan x}{1+1.\tan x}} \quad \left[\because \tan\frac{\pi}{4} = 1\right.$$

$$= \frac{1+\tan x}{1-\tan x} \times \frac{1+\tan x}{1-\tan x}$$

$$= \frac{(1+\tan x)^2}{(1-\tan x)^2} = RHS.$$

Q No 8: $$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

$$\text{LHS} = \frac{\cos(180^\circ+x)\cos x}{\sin(180^\circ+(-x))\cos(90^\circ+x)} = \frac{\cos(2\times 90+x)\cos x}{\sin[2\times 90^\circ+(-x)]\cos[1\times 90^\circ+x]}$$

$$= \frac{-\cos x\cos x}{-\sin(-x)(-\sin x)} = \frac{-\cos^2 x}{-\sin^2 x} = +\cot^2 x = \text{RHS}$$

Q No 9. $$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right] = 1$$

Sol. $$\text{LHS} = \cos(3\times 90^\circ+x)\cos(4\times 90^\circ+x)\left[\cot(3\times 90^\circ+(-x))\cot(4\times 90^\circ+x)\right]$$

$$= (+\sin x)(\cos x)\left[-\tan(-x)+\cot x\right]$$

$$= \sin x\cos x\left[\tan x+\cot x\right] = \sin x\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$$

$$= \sin x\cos x\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cdot\cos x}\right] \because \sin^2 x+\cos^2 x = 1 = \text{RHS.}$$

Q No 10 Prove that $\sin(n+1)x\,\sin(n+2)x+\cos(n+1)x\,\cos(n+2)x = \cos x$

Sol Using formula $\cos(A-B) = \cos A\cos B+\sin A\sin B$ on LHS

$$\cos[(n+2)x-(n+1)x] = \cos[nx+2x-nx-x] = \cos x = \text{RHS.}$$

Q No 12: $$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x.$$

Sol Using C-D formula. $\cos C-\cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$ on LHS

$$= -2\sin\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\sin\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}$$

$$= -2\sin\frac{3\pi}{4}\sin x = -2\sin(135^\circ)\sin x$$

$$= -2\sin(90^\circ+45)\sin x = -2\cos 45\sin x = -2\times\frac{1}{\sqrt{2}}\sin x$$

$$= -\sqrt{2}\sin x.$$

Q No 12 $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

LHS $\sin^2 6x - \sin^2 4x = (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$ | $a^2 - b^2 = (a+b)(a-b)$

$= \left(2 \sin \frac{6x+4x}{2} \cos \frac{6x-4x}{2}\right)\left(2 \cos \frac{6x+4x}{2} \sin \frac{6x-4x}{2}\right)$ [using C-D formulae

$= (2 \cdot \sin 5x \sin x)(2 \cos 5x \sin x)$

$= (2 \sin 5x \cos 5x)(2 \sin x \cos x)$ [$\because \sin 2\theta = 2\sin\theta\cos\theta$]

$= \sin 10x \cdot \sin 2x = \text{RHS}$

Q No 13 $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$.

LHS. $\cos^2 2x - \cos^2 6x = (1 - \sin^2 2x) - (1 - \sin^2 6x)$ [$\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$= 1 - \sin^2 2x - 1 + \sin^2 6x = \sin^2 6x - \sin^2 2x$

$= (\sin 6x - \sin 2x)(\sin 6x + \sin 2x)$

$= \left(2 \cos \frac{6x+2x}{2} \sin \frac{6x-2x}{2}\right)\left(2 \sin \frac{6x+2x}{2} \cos \frac{6x-2x}{2}\right)$

$= (2 \cos 4x \sin 2x)(2 \sin 4x \cdot \cos 2x)$

$= (2 \sin 2x \cos 2x)(2 \sin 4x \cdot \cos 4x)$

$= \sin 4x \cdot \sin 8x$

Q No 14 $\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$.

Sol. LHS $= \sin 2x + \sin 4x + \sin 4x + \sin 6x$ [using $2\sin 4x = \sin 4x + \sin 4x$

$= \left(2 \sin \frac{4x+2x}{2} \cos \frac{4x-2x}{2}\right) + \left(2 \sin \frac{6x+4x}{2} \cos \frac{6x-4x}{2}\right)$ | C-D formulae

$= (2 \sin 3x \cos x) + (2 \sin 5x \cdot \cos x)$

$= 2 \cos x [\sin 3x + \sin 5x]$

$= 2 \cos x \left[2 \sin \frac{5x+3x}{2} \cos \frac{5x-3x}{2}\right]$ [Again applying C-D formula

$= 2 \times 2 \cos x \cdot \sin 4x \cdot \cos x$

$= 4 \cos^2 x \sin 4x$.

Q No 15 $\cot 4x\left(\sin 5x+\sin 3x\right)=\cot x\left(\sin 5x-3x\right)$

LHS
$$\cot 4x\left[2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}\right]\qquad\left[\because \sin C+\sin D=2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right.$$

$$=\cot 4x\left[2\sin 4x\cdot\cos x\right]$$

$$=\frac{\cos 4x}{\sin 4x}\times 2\sin 4x\cos x=2\cos 4x\cos x$$

$$=2\cos 4x\cdot\frac{\cos x}{\sin x}\cdot\sin x=\cot x\left(2\cos 4x\sin x\right)$$

$$=\cot 4x\left[\sin(4x+x)-\sin(4x-x)\right]\qquad\left[\because 2\cos A\sin B=\sin(A+B)-\sin(A-B)\right.$$

$$=\cot 4x\left[\sin 5x-\sin 3x\right]=\text{RHS}.$$

Q No 16 $\dfrac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\dfrac{\sin 2x}{\cos 10x}$

Sol: $\text{LHS}=\dfrac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$

$$=\frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}\qquad\left[\text{using } C-D \text{ formulae}\right.$$

$$=-\frac{\sin 7x\sin 2x}{\cos 10x\cdot\sin 7x}=-\frac{\sin 2x}{\cos 10x}=\text{RHS}.$$

Q No 17 $\dfrac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x.$

Sol LHS
$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$$

$$=\frac{\sin 4x\cdot\cos x}{\cos 4x\cdot\cos x}=\frac{\sin 4x}{\cos 4x}=\tan 4x=\text{RHS}$$

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Q No 18: $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan\dfrac{x-y}{2}$

Sol LHS $= \dfrac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}}$ [using C-D formulae]

$= \dfrac{\sin\frac{x-y}{2}}{\cos\frac{x-y}{2}} = \tan\dfrac{x-y}{2} = \text{RHS}.$

Q No 19: $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Sol LHS $= \dfrac{2\sin\frac{x+3x}{2}\cos\frac{3x-x}{2}}{2\cos\frac{x+3x}{2}\cos\frac{3x-x}{2}}$ [Applying C-D formula]

$= \dfrac{2\sin 2x\cos x}{2\cos 2x\cos x} = \dfrac{\sin 2x}{\cos 2x} = \tan 2x.$

Q No 20. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Sol. LHS: $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \dfrac{\sin x - \sin 3x}{-(\cos^2 x - \sin^2 x)}$

$= \dfrac{2\cos 2x\sin(-x)}{-\cos 2x} = \dfrac{-2\cos 2x\sin x}{-\cos 2x} = 2\sin x = \text{RHS}$

Q No 21: $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Sol: LHS $\dfrac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x} = \dfrac{2\cos\frac{4x+2x}{2}\cos\frac{4x-2x}{2} + \cos 3x}{2\sin\frac{4x+3x}{2}\cos\frac{4x-2x}{2} + \sin 3x}$

$= \dfrac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x} = \dfrac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)} = \dfrac{\cos 3x}{\sin 3x} = \cot 3x$

$= \text{RHS}$

Q No 22 $\cot x \cos 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Sol :

Now $3x = 2x + x$

$$\Rightarrow \cot 3x = \cot(2x + x)$$

$$\Rightarrow \cot 3x = \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x} \qquad \left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

$$\Rightarrow \cot 3x\left(\cot 2x + \cot x\right) = \cot 2x \cot x - 1$$

$$\Rightarrow \cot 3x \cot 2x + \cot 3x \cot x = \cot 2x \cot x - 1$$

$$\Rightarrow \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

Hence proved.

Q No 23 $\tan 4x = \dfrac{4\tan x(1-\tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$

Sol.

$4x = 2x + 2x$

$$\Rightarrow \tan 4x = \tan(2x + 2x)$$

$$= \frac{\tan 2x + \tan 2x}{1 - \tan 2x \cdot \tan 2x} = \frac{2\tan 2x}{1 - \tan^2 2x}$$

$$= \frac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1 - \left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2} \qquad \left[\text{Again using } \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right.$$

$$= \frac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{(1-\tan^2 x)^2 - (2\tan x)^2}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x}{1-\tan^2 x} \times \frac{(1-\tan^2 x)^2}{1 + \tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

ie $$\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1 + \tan^4 x - 6\tan^2 x}$$

Q No 24 $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

Sol :

$$\cos 4x = \cos 2(2x) = \cos^2 2x - \sin^2 2x$$
$$= (\cos^2 x - \sin^2 x)^2 - (2\sin x \cos x)^2$$
$$= (\cos x + \sin x)^2 (\cos x - \sin x)^2 - 4\sin^2 x \cos^2 x$$
$$= (\underline{\cos^2 x + \sin^2 x} + 2\sin x \cos x)(\underline{\cos^2 x + \sin^2 x} - 2\sin x \cos x) - 4\sin^2 x \cos^2 x$$
$$= (1 + 2\sin x \cos x)(1 - 2\sin x \cos x) - 4\sin^2 x \cos^2 x$$
$$= [1 - (2\sin x \cos x)^2] - 4\sin^2 x \cos^2 x$$
$$= 1 - 4\sin^2 x \cos^2 x - 4\sin^2 x \cos^2 x = 1 - 8\sin^2 x \cos^2 x$$

Hence proved.

Q No 25 $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Sol :

$$\cos 6x = \cos(2 \times 3x)$$
$$= 2\cos^2 3x - 1 \qquad [\because \cos 2\theta = 2\cos^2\theta - 1]$$
$$= 2(\cos 3x)^2 - 1$$
$$= 2[4\cos^3 x - 3\cos x]^2 - 1 \qquad [\cos 3x = 4\cos^3 x - 3\cos x]$$
$$= 2[16\cos^6 x + 9\cos^2 x - 24\cos^4 x] - 1$$
$$= 32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1$$

Hence proved.

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