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166

(I)

Total No. of Questions : **24**
Total No. of Printed Pages : **3**

Regd. No.

Part - III

MATHEMATICS - PAPER - I (A)

(English Version)

Time : 3 Hours **Max. Marks : 75**

Note : *This question paper consists of Three Sections - A, B and C.*

SECTION - A

I. Very Short Answer Type Questions. **10x2=20**

(i) Answer *all* the questions.

(ii) Each question carries *two marks.*

1. If f : R\{0} → R is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$.

2. Determine whether the following function is even or odd :
$f(x) = a^x - a^{-x} + \sin x$

3. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, find A^2.

4. Find the rank of the following matrix :

$$\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

5. $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors, then find m and n.

6. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.

7. For what values of λ, the vectors $\bar{i} - \lambda\bar{j} + 2\bar{k}$ and $8\bar{i} + 6\bar{j} - \bar{k}$ are at right angles ?

8. Find a sine function whose period is $\frac{2}{3}$.

9. Express $\dfrac{(\sqrt{3}\cos 25^\circ + \sin 25^\circ)}{2}$ as a sine of an angle.

10. If $\sinh x = 3$, then show that $x = \log_e(3 + \sqrt{10})$.

SECTION - B

II. Short Answer Type Questions. **5x4=20**

(i) Answer *any five* questions.

(ii) Each question carries *four marks.*

11. If $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ then find AA'. Do A and A' commute with respect to multiplication of matrices.

12. $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. Prove that the following four points are coplanar :

$-\bar{a} + 4\bar{b} - 3\bar{c},\ 3\bar{a} + 2\bar{b} - 5\bar{c},\ -3\bar{a} + 8\bar{b} - 5\bar{c},\ -3\bar{a} + 2\bar{b} + \bar{c}$

13. Find a vector of magnitude 3 and perpendicular to both the vectors.

$\bar{b} = 2\bar{i} - 2\bar{j} + \bar{k}$ and $\bar{c} = 2\bar{i} + 2\bar{j} + 3\bar{k}$

14. If $3\sin\theta + 4\cos\theta = 5$, then find the value of $4\sin\theta - 3\cos\theta$.

15. Show that :

$$\cos^2\left(\frac{\pi}{10}\right) + \cos^2\left(\frac{2\pi}{5}\right) + \cos^2\left(\frac{3\pi}{5}\right) + \cos^2\left(\frac{9\pi}{10}\right) = 2$$

16. Show that, for any $\theta \in R$,

$$4\sin\frac{5\theta}{2}\cos\frac{3\theta}{2}\cos 3\theta = \sin\theta - \sin 2\theta + \sin 4\theta + \sin 7\theta.$$

17. Prove that :

$$\text{Tan}\frac{A}{2} + \text{Tan}\frac{B}{2} + \text{Tan}\frac{C}{2} = \frac{bc + ca + ab - s^2}{\Delta}$$

SECTION - C

III. Long Answer Type Questions. **5x7=35**

(i) Answer *any five* questions.

(ii) Each question carries *seven marks.*

18. If $f(x)=x^2$ and $g(x)=|x|$, find the following functions.

(i) $f+g$ (ii) $f-g$ (iii) fg

(iv) $2f$ (v) f^2 (vi) $f+3$

19. Solve $3x+4y+5z=18$, $2x-y+8z=13$ and $5x-2y+7z=20$ by using 'Matrix Inversion Method'.

20. Solve $x-y+3z=5$, $4x+2y-z=0$, $-x+3y+z=5$ by using Cramer's Rule.

21. Show that the line joining the pair of points $6\bar{a}-4\bar{b}+4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a}-2\bar{b}-3\bar{c}$, $\bar{a}+2\bar{b}-5\bar{c}$ intersect at the point $-4\bar{c}$ when $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors.

22. If $\bar{a}=7\bar{i}-2\bar{j}+3\bar{k}$, $\bar{b}=2\bar{i}+8\bar{k}$ and $\bar{c}=\bar{i}+\bar{j}+\bar{k}$, then compute $\bar{a}\times\bar{b}$, $\bar{a}\times\bar{c}$ and $\bar{a}\times(\bar{b}+\bar{c})$. Verify whether the cross product is distributive over vector addition.

23. If A, B, C are angles of a triangle, then prove that :

$$\sin^2\frac{A}{2}+\sin^2\frac{B}{2}-\sin^2\frac{C}{2}=1-2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

24. Show that :

$$r+r_3+r_1-r_2=4R\cos B$$