Logarithms

Olympiad Comprehensive Book

NOTES

Earlier you have studied that if two powers of the same base are equal (when base in not equal to -1, 0 or 1) then the indices are equal. For example if $3^x = 3^y$ then $x = y$. But we cannot calculate the value of x when $3^x = 7^3$. The necessity of the concept of logarithms arises for such problems. Logarithms are useful in long calculations of multiplication and division.

Logarithms are mathematical statements which are used to answer a slightly different question for exponents whose base is a positive real number. Thus if a is a positive real number other than 1 and $a^x = n$, then x is called the logarithm of n to the base a, and written as log n.

Note: If no base is mentioned in a logarithm then it is taken as 10.

Logarithms to the base 10 are known as common logarithms.

Properties of Logarithms

- $\log_a(xy) = \log_a x + \log_a y$
- $log_x x = 1$
- $log_{b^n} a^m = \frac{m}{n} \log_b a$
- $\log_a x^n = n \log_a x$
- $\log_a x = \frac{1}{\log_x a}$
- $a^{\log_a x} = x$; a $\neq 0, \neq \pm 1$, x > 0
- $\log_a \left(\frac{x}{y}\right) = \log_{a^x} - \log_{a^y}$
- $\log_a x = \frac{\log_b x}{\log_b a}$

Thus last formula given above is also known as base changing formula.

Antilogarithms

The logarithm of a number always contains which are characteristic and mantissa. The integral part is known as characteristic and the decimal part is known as mantissa. Mantissa is always kept positive. The number whose logarithm is x is called the antilogarithm of x and is denoted by antilog x.

- **Example:**

The value of $\log_{343}7$ is:

(a) 0 (b) 7

(c) $\frac{1}{3}$ (d) $\frac{1}{7}$

(e) None of these

Ans. (c)

Explanation: let $\log_{343}7 = x$, then $343^x = 7$

$$\Rightarrow (7^3)^x = 7 \Rightarrow 7^{3x} = 7 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$$

- **Example:**

If $\log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y = \log(x+y)$, then:

(a) $x = y$ (b) $x + y = 1$

(c) $x = 2y$ (d) $x - y = 1$

(e) None of these

Ans. (a)

Explanation: $log2 \ + \frac{1}{2}logx \ + \frac{1}{2}logy \ = log(x+y)$

$$\Rightarrow \log(2x\sqrt{x}xy) = \log(x+y) \Rightarrow (x-y)^2 = 0 \Rightarrow x = y$$