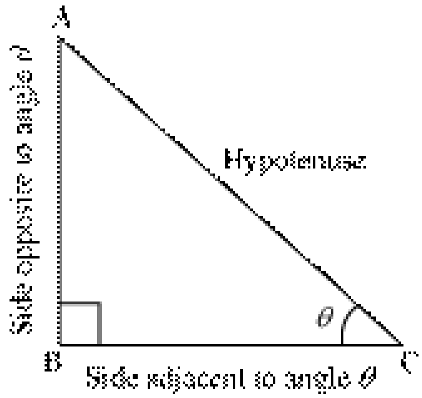
Trigonometrical Identities

- **Trigonometric Ratio**



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{AB}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{BC}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{BC}{AB}$$

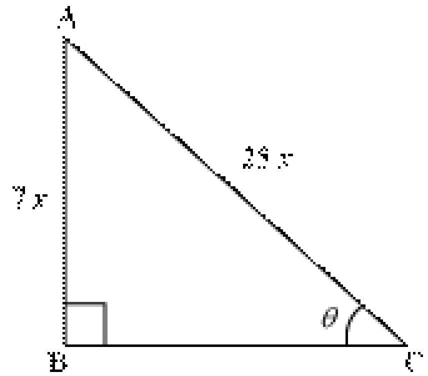
$$\text{Also, } \tan\theta = \frac{\sin\theta}{\cos\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

If one of the trigonometric ratios of an acute angle is known, then the remaining trigonometric ratios of the angle can be calculated.

Example:

If $\sin\theta = \frac{7}{25}$, then find the value of sec θ(1 + tan θ).

Solution:



It is given that $\sin\theta = \frac{7}{25}$

$$\sin\theta = \frac{AB}{AC} = \frac{7}{25}$$

⇒ AB = 7*x* and AC = 25*x*, where *x* is some positive integer

By applying Pythagoras theorem in ΔABC, we get:

$$AB^2 + BC^2 = AC^2$$
$$\Rightarrow (7x)^2 + BC^2 = (25x)^2$$
$$\Rightarrow 49x^2 + BC^2 = 625x^2$$
$$\Rightarrow BC^2 = 625x^2 - 49x^2$$
$$\Rightarrow BC = \sqrt{576}\,x = 24x$$

$$\therefore \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{25}{24}$$
$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{7}{24}$$
$$\therefore \sec\theta(1+\tan\theta) = \frac{25}{24}\left(1+\frac{7}{24}\right) = \frac{25}{24}\times\frac{31}{24} = \frac{775}{576}$$

- **Trigonometric Identities**
  1. $\cos^2 A + \sin^2 A = 1$
  2. $1 + \tan^2 A = \sec^2 A$
  3. $1 + \cot^2 A = \text{cosec}^2 A$

Example:
If $\cos\theta = \frac{5}{7}$, find the value of cot θ + cosec θ

Solution:
We have, $\cos\theta = \frac{5}{7}$

Now, $\sin^2\theta + \cos^2\theta = 1$

$$\therefore \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(\frac{5}{7}\right)^2}$$

$$= \sqrt{\frac{49-25}{49}} = \frac{2\sqrt{6}}{7}$$

$$\therefore \operatorname{cosec}\theta = \frac{7}{2\sqrt{6}}$$

Also, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$= \frac{\frac{5}{7}}{\frac{2\sqrt{6}}{7}} = \frac{5}{2\sqrt{6}}$$

$$\therefore \cot\theta + \operatorname{cosec}\theta = \frac{5}{2\sqrt{6}} + \frac{7}{2\sqrt{6}}$$

$$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \sqrt{6}$$

- **Trigonometric Ratios of Complementary Angles**

$$\sin(90^\circ-\theta)=\cos\theta \qquad \cos(90^\circ-\theta)=\sin\theta$$
$$\tan(90^\circ-\theta)=\cot\theta \qquad \cot(90^\circ-\theta)=\tan\theta$$
$$\operatorname{cosec}(90^\circ-\theta)=\sec\theta \qquad \sec(90^\circ-\theta)=\operatorname{cosec}\theta$$

Where θ is an acute angle.

Example 1: Evaluate the expression

$$\sin 28^\circ \sin 30^\circ \sin 54^\circ \sec 36^\circ \sec 62^\circ$$

Solution:

$$\begin{aligned}
&\sin 28^\circ \sin 30^\circ \sin 54^\circ \sec 36^\circ \sec 62^\circ\\
&=(\sin 28^\circ \sec 62^\circ)(\sin 54^\circ \sec 36^\circ)\sin 30^\circ\\
&=\{\sin 28^\circ \operatorname{cosec}(90^\circ-62^\circ)\}\{\sin 54^\circ \operatorname{cosec}(90^\circ-36^\circ)\}\sin 30^\circ\\
&=(\sin 28^\circ \operatorname{cosec} 28^\circ)(\sin 54^\circ \operatorname{cosec} 54^\circ)\sin 30^\circ\\
&=\left(\sin 28^\circ \frac{1}{\sin 28^\circ}\right)\left(\sin 54^\circ \frac{1}{\sin 54^\circ}\right)\times\frac{1}{2}\\
&=\frac{1}{2}
\end{aligned}$$

Example 2: Evaluate the expression

$$4\sqrt{3}(\sin 40^\circ \sec 30^\circ \sec 50^\circ)+\frac{\sin^2 34^\circ+\sin^2 56^\circ}{\sec^2 31^\circ-\cot^2 59^\circ}$$

Solution:

$$4\sqrt{3}\left(\sin 40^\circ \sec 30^\circ \sec 50^\circ\right)+\frac{\sin^2 34^\circ+\sin^2 56^\circ}{\sec^2 31^\circ-\cot^2 59^\circ}$$

$$=4\sqrt{3}\left[\sec 30^\circ\left(\sin 40^\circ \sec 50^\circ\right)\right]+\frac{\sin^2 34^\circ+\sin^2\left(90-56^\circ\right)}{\sec^2 31^\circ-\tan^2\left(90-59^\circ\right)}$$

$$\left[\because \cos\left(90^\circ-\theta\right)=\sin\theta, \tan\left(90^\circ-\theta\right)=\cot\theta\right]$$

$$=4\sqrt{3}\left[\sec 30^\circ \sin 40^\circ \operatorname{cosec}\left(90-50^\circ\right)\right]+\frac{\sin 34^\circ+\cos^2 34^\circ}{\sec^2 31^\circ-\tan^2 31^\circ}$$

$$=4\sqrt{3}\left[\frac{2}{\sqrt{3}}\sin 40^\circ \operatorname{cosec} 40^\circ\right]+\frac{1}{1}$$

$$=8+1=9$$

- If we know the sine, cosine and tangent values for θ lying between 0° and 90°, then we can find the trigonometric ratio values for all other angles using trigonometric identities and complementary angle identities.
- To find the sine, cosine and tangent values of all angles between 0° and 90°, we use the trigonometric tables.

For example, to find the value of $\sin 29^\circ 44'$, we read the table of natural sines in the horizontal line, which begins with 29°, and in the vertical column headed by $42'$.

sin 29°42' = 0.4955

Now, we read, in the same horizontal line, the value written in the mean difference column headed by $2'\left(44'=42'+2'\right)$.

It is found to be 5.

As θ increases, sine value increases.

∴ The mean difference value is to be added.

$$\therefore \sin 29^\circ 44'=\sin\left(29^\circ 42'+2'\right)$$

$$=0.4955+0.0005=0.4960$$