** Eigen values and vectors :

$\rightarrow$ Eigen value (E.V) ($\lambda = ?$)

ex: Find characteristic roots of matrix -

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

Sol $\Rightarrow$

characteristic matrix of 'A'.

$$A - \lambda I = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & 1 & 2 \\ 1 & -\lambda & -1 \\ 2 & -1 & -\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 2 \\ 1 & -\lambda & -1 \\ 2 & -1 & -\lambda \end{vmatrix}$$

$$= \lambda^3 - 6\lambda + 4 = 0$$

2

$8 - 12 + 4$

$$\Rightarrow (\lambda - 2) \; (\lambda^2 + 2\lambda - 2) = 0$$

$\lambda = ?$

$\cancel{\lambda = 2}$

$\therefore$ characteristic root's are $2, -1 \pm \sqrt{3}$

$\rightarrow$ Eigen values.

$ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex: 5) given matrix ~~find~~ $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ find

eigen value & eigen vector.

$\Rightarrow |A - \lambda I| = 0$

$$= \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix}$$

$= (6-\lambda)\left((3-\lambda)^2 - 1\right) + 2\left(-2(3-\lambda)+2\right) + 2\left(2 - 2(3-\lambda)\right) = 0$

$= (6-\lambda)\ (3-\lambda+1)(3-\lambda-1) - 4(3-\lambda-1) + 4(1-3+\lambda) = 0$

$= (6-\lambda)\ (4-\lambda)(2-\lambda) - 4(2-\lambda) - 4(2-\lambda) = 0$

$= (6-\lambda)(4-\lambda)(2-\lambda) - 8(2-\lambda) = 0$

$= (2-\lambda)\ \left[(6-\lambda)(4-\lambda) - 8\right] = 0$

$= (2-\lambda)\ (24 - 6\lambda - 4\lambda + \lambda^2 - 8) = 0$

$= (2-\lambda)\ (\lambda^2 - 10\lambda + 16) = 0$

$\lambda^2 - 8\lambda - 2\lambda + 16 = 0$

$\lambda(\lambda-8) - 2(\lambda-8) = 0$

$= (2-\lambda)\ (\lambda-2)(\lambda-8) = 0$

$$\begin{array}{r|l} 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

$\lambda = (8, 2, 2) \rightarrow$ eigen values.

The eigen vectors of 'A' corresponding to the eigen value 8 :

$(A-8I)X = 0$

$$\begin{bmatrix} 6-8 & -2 & 2 \\ -2 & 3-8 & -1 \\ 2 & -1 & 3-8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix}$$

$$\Rightarrow \underbrace{\begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix}}_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$6+6=12$

rank $= 2$, $n = 3$

$\boxed{n > r}$ $n - r = 1 \rightarrow$ 1 linear Independent solution exist.

$-2x_1 - 2x_2 + 2x_3 = 0 \Rightarrow x_1 + x_2 - x_3 = 0$

$-3x_2 - 3x_3 = 0 \Rightarrow x_2 + x_3 = 0$

Let, $x_3 = K$

$x_2 = -x_3 = -K$ | $x_1 = -(-K) + K = K + K = 2K$ | $x_3 = K$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2K \\ -K \\ K \end{bmatrix} \rightarrow \text{eigen vector.}$$

The eigen vector's of A corresponding to the given value 2.

$$(A - 2I)X = 0$$

$$\begin{bmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ 2 & -1 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$+2-2$ $\quad r = 1, n = 3 \mid n > r \mid n - r = 2$

$\cancel{-2x_2 + 2x_3 = 0 \Rightarrow x_2 - x_3 = 0}$

$\cancel{x_2 - x_3 = 0 \Rightarrow x}$

$$4x_1 - 2x_2 + 2x_3 = 0$$

$$2x_1 - x_2 + x_3 = 0 \qquad \Big| \text{ let, } x_1 = n, \; x_2 = m$$

$$2n - m + x_3 = 0$$

$$x_3 = m - 2n$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} n \\ m \\ m-2n \end{bmatrix} \rightarrow \text{eigen vectors.}$$

** $\Rightarrow$ if eigen values of A are $\lambda_1, \lambda_2, \lambda_3 \cdots \lambda_n$,
then, A^2 are $\lambda_1^2, \lambda_2^2, \cdots \lambda_n^2$.