CHAPTER

4

Simplification of Fractions

The process of solving a fraction in Algebra is same as that of Arithmetic. First of all we find LCM (Lowest Common Multiple) of algebraic expressions and then simplify.

Rule of $\overrightarrow{\text{BODMAS}}$ is always taken into consideration.

SOLVED OBJECTIVE TYPE QUESTIOS

1. The simplified form of

$$\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12} \text{ is :}$$

(1) $\frac{2(x+4)}{x+3}$ (2) $\frac{x+3}{2(x+4)}$

(3) $\frac{x+4}{x+3}$ (4) $\frac{x+3}{x+4}$

2. The simplified form of $\frac{x+1}{x-1}+\frac{x^2-1}{x+1}$ is :

(1) $\frac{x^2+x+2}{x-1}$ (2) $\frac{x^2-x+2}{x-1}$

(3) $\frac{x+1}{x^2-2x+1}$ (4) $\frac{x+2}{x^2-2x+1}$

3. The simplified form of $\frac{a^2+2a+3}{a^2-1}+\frac{a-4}{a+1}$ is :

(1) $\frac{2a^2+3a+7}{a^2+1}$ (2) $\frac{2a^2-3a+7}{a^2+1}$

(3) $\frac{2a^2-3a+7}{a^2-1}$ (4) $\frac{2a^2-3a+9}{a^2-1}$

4. The simplified form of $\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}$ is :

(1) $\frac{2x^3-2}{x^2+2}$ (2) $\frac{2x^3+1}{x^2+2}$

(3) $\frac{2x^3-1}{(x^2+2)^2}$ (4) $\frac{2x^3+1}{(x^2+2)^2}$

5. The simplified form of

$$\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4+1} \text{ is :}$$

(1) $\frac{8}{1-x^8}$ (2) $\frac{8}{x^8-1}$

(3) $\frac{6}{x^6-1}$ (4) $\frac{6}{1-x^6}$

6. The simplified form of $\frac{x}{x-y}+\frac{y}{x+y}+\frac{2xy}{y^2-x^2}$ is :

(1) 1 (2) – 1

(3) $x+y$ (4) $\frac{x^2+y^2}{x^2-y^2}$

7. The simplified form of

$$\frac{1+x}{1-x}-\frac{1-x}{1+x}+\frac{4x}{1+x^2}+\frac{8x^3}{1-x^4} \text{ is :}$$

(1) $\frac{8}{1-x^2}$ (2) $\frac{8}{1-x^3}$

(3) $\frac{8x}{1-x^2}$ (4) $\frac{6x}{1-x^2}$

8. The simplified form of

$$\frac{1}{x+a}+\frac{1}{x+b}+\frac{1}{x+c}+\frac{ax}{x^3+ax^2}+\frac{bx}{x^3+bx^2}+\frac{cx}{x^3+cx^2}$$
is :

(1) $\frac{1}{x}$ (2) $\frac{1}{x^2}$

(3) $\frac{2}{x}$ (4) $\frac{3}{x}$

9. $\frac{1}{1-a+a^2}-\frac{1}{1+a^2+a}-\frac{2a}{1+a^2+a^4}=?$

(1) 0 (2) –1

(3) 1 (4) 2

10. $\frac{1}{a^2-3a+2}+\frac{2}{a^2-5a+6}-\frac{3}{a^2-4a+3}=?$

(1) $\frac{1}{(a-1)(a-2)}$

(2) $\frac{1}{(a-2)(a-3)}$

(3) $\frac{1}{(a-1)(a-2)(a-3)}$

(4) $\frac{1}{(a+1)(a-2)(a-3)}$

11. $\dfrac{\dfrac{a}{a-b}-\dfrac{a}{a+b}}{\dfrac{b}{a-b}-\dfrac{b}{a+b}}+\dfrac{\dfrac{a+b}{a+b}+\dfrac{a-b}{a+b}}{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}\times\dfrac{a^2}{a^2+b^2}=?$

(1) $\frac{3a}{2b}$ (2) $\frac{2b}{3a}$

(3) $\frac{2a}{3b}$ (4) $\frac{3b}{2a}$

12. $\left(\frac{x-y}{x+y}-\frac{x+y}{x-y}-\frac{4y^2}{x^2-y^2}\right)\frac{x-y}{2y}=?$

(1) −2 (2) 2

(3) 0 (4) 4

13. $\frac{1}{1-x}-\frac{1}{1+x}-\frac{2x}{1+x^2}-\frac{4x^3}{1+x^4}-\frac{8x^7}{1+x^8}-\frac{16x^{15}}{1-x^{16}}=?$

(1) 0 (2) −1

(3) 1 (4) − 2

14. If a and b are positive quantities and

$x=\frac{a+b}{2}$, $y=\sqrt{ab}$ and $z=\frac{2ab}{a+b}$, which of the following is correct ?

(1) $x<y>z$ (2) $x>y>z$

(3) $x<y<z$ (4) $x>y<z$

15. If $(x-a)$ $(x-b-1] = a-b$, find the value of x.

(1) $(1+a)$ (2) b

(3) $(1+a)$, b (4) $(1-a)$, b

16. If $\frac{x+a}{x+b}=\frac{x+3a}{x+a+b}$ find the value of x.

(1) $a+2b$ (2) $a-2b$

(3) $a+b$ (4) $a-b$

17. If $x+y+z=0$ then

$\frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\frac{1}{z^2+x^2-y^2}=?$

(1) 1 (2) −1

(3) 0 (4) 2

18. $\frac{a^2-(b-c)^2}{(a+c)^2-b^2}+\frac{b^2-(a-c)^2}{(a+b)^2-c^2}+\frac{c^2-(a-b)^2}{(b+c)^2-a^2}=?$

(1) 1 (2) −1

(3) −4 (4) −2

19. If $x+\frac{1}{y}=1$ and $y+\frac{1}{z}=1$ then $z+\frac{1}{x}=?$

(1) −1 (2) x

(3) x^2 (4) 1

20. If $\frac{p}{b-c}=\frac{q}{c-a}=\frac{r}{a-b}$, then $p+q+r=?$

(1) 0 (2) 1

(3) −1 (4) −2

21. $\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-c)(b-a)}+\frac{c^2}{(c-a)(c-b)}=?$

(1) 1 (2) −1

(3) $-x+a$ (4) 2

22. $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}=?$

(1) abc (2) $\frac{1}{abc}$

(3) $a^2b^2c^2$ (4) $\frac{1}{a^2b^2c^2}$

ANSWERS

1. (1)	**2.** (2)	**3.** (3)	**4.** (4)	**5.** (2)
6. (1)	**7.** (3)	**8.** (4)	**9.** (1)	**10.** (3)
11. (1)	**12.** (1)	**13.** (1)	**14.** (2)	**15.** (3)
16. (2)	**17.** (3),	**18.** (1)	**19.** (4)	**20.** (1)
21. (1)	**22.** (2)			

EXPLANATIONS

1. (1) $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

$=\frac{(x-3)(x+2)}{(x-3)(x+3)}+\frac{(x+6)(x-4)}{(x-4)(x+3)}$

$=\frac{x+2}{x+3}+\frac{(x+6)}{x+3}=\frac{(x+2)+(x+6)}{x+3}$

$=\frac{2x+8}{x+3}=\frac{2(x+4)}{x+3}$

2. (2) $\frac{x+1}{x-1}+\frac{x^2-1}{x+1}$

$=\frac{(x+1)(x+1)+(x-1)(x^2-1)}{(x-1)(x+1)}$

$$= \frac{(x+1)^2+(x-1)^2(x+1)}{(x-1)(x+1)}$$

$$= \frac{(x+1)\{(x+1)+(x-1)^2\}}{(x-1)(x+1)} = \frac{(x+1)+(x-1)^2}{x-1}$$

$$= \frac{x+1+x^2-2x+1}{x-1} = \frac{x^2-x+2}{x-1}$$

OR

$$\frac{x+1}{x-1}+\frac{x^2-1}{x+1} = \frac{x+1}{x-1}+\frac{(x+1)(x-1)}{x+1}$$

$$= \frac{x+1}{x-1}+(x-1) = \frac{(x+1)+(x-1)^2}{(x-1)}$$

$$= \frac{x+1+x^2-2x+1}{x-1} = \frac{x^2-x+2}{x-1}$$

3. (3) $\frac{a^2+2a+3}{a^2-1}+\frac{a-4}{a+1}$

$$= \frac{(a^2+2a+3)(a+1)+(a-4)(a^2-1)}{(a^2-1)(a+1)}$$

$$= \frac{(a^2+2a+3)(a+1)+(a-4)(a-1)(a+1)}{(a-1)(a+1)(a+1)}$$

$$= \frac{(a+1)\left[a^2+2a+3+(a-4)(a-1)\right]}{(a-1)(a+1)(a+1)}$$

$$= \frac{a^2+2a+3+a^2-5a+4}{(a-1)(a+1)} = \frac{2a^2-3a+7}{a^2-1}$$

4. (4) $\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}$

$$= \frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{x^2+2}{(x^2+2)^2}$$

$$= \frac{(2x^3+x^2+3)-(x^2+2)}{(x^2+2)^2} = \frac{2x^3+1}{(x^2+2)^2}$$

5. (2) $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4+1}$

$$= \left[\frac{1}{x-1}-\frac{1}{x+1}\right]-\frac{2}{x^2+1}-\frac{4}{x^4+1}$$

$$= \frac{(x+1)-(x-1)}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4+1}$$

$$= \frac{2}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4+1}$$

$$= \left[\frac{2}{x^2-1}-\frac{2}{x^2+1}\right]-\frac{4}{x^4+1}$$

$$= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)}-\frac{4}{x^4+1}$$

$$= \frac{2x^2+2-2x^2+2}{(x^2-1)(x^2+1)}-\frac{4}{x^4+1} = \frac{4}{x^4-1}-\frac{4}{x^4+1}$$

$$= \frac{4(x^4+1)-4(x^4-1)}{(x^4-1)(x^4+1)} = \frac{4x^4+4-4x^4+4}{x^8-1} = \frac{8}{x^8-1}$$

6. (1) $\frac{x}{x-y}+\frac{y}{x+y}+\frac{2xy}{y^2-x^2}$

$$= \frac{x(x+y)+y(x-y)}{(x-y)(x+y)}+\frac{2xy}{(x^2-y^2)}$$

$$= \frac{x^2+xy+yx-y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$= \frac{x^2+2xy-y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$= \frac{x^2+2xy-y^2-2xy}{x^2-y^2} = \frac{x^2-y^2}{x^2-y^2} = 1$$

7. (3) $\frac{1+x}{1-x}-\frac{1-x}{1+x}+\frac{4x}{1+x^2}+\frac{8x^3}{1-x^4}$

$$= \left[\frac{1+x}{1-x}-\frac{1-x}{1+x}\right]+\frac{4x}{1+x^2}+\frac{8x^3}{1-x^4}$$

$$= \frac{(1+x)^2-(1-x)^2}{(1-x)(1+x)}+\frac{4x}{1+x^2}+\frac{8x^3}{1-x^4}$$

$$= \frac{(1+2x+x^2)-(1-2x+x^2)}{1-x^2}+\frac{4x}{1+x^2}+\frac{8x^3}{1-x^4}$$

$$= \left[\frac{4x}{1-x^2}+\frac{4x}{1+x^2}\right]+\frac{8x^3}{1-x^4}$$

$$= \frac{4x(1+x^2)+4x(1-x^2)}{(1-x^2)(1+x^2)}+\frac{8x^3}{1-x^4}$$

$$= \frac{4x+4x^3+4x-4x^3}{1-x^4}+\frac{8x^3}{1-x^4}$$

$$= \frac{8x}{1-x^4} + \frac{8x^3}{1-x^4}$$

$$= \frac{8x+8x^3}{1-x^4} = \frac{8x(1+x^2)}{(1-x^2)(1+x^2)} = \frac{8x}{1-x^2}$$

8. (4)

$$\frac{1}{x+a} + \frac{1}{x+b} + \frac{1}{x+c} + \frac{ax}{x^3+ax^2} + \frac{bx}{x^3+bx^2} + \frac{cx}{x^3+cx^2}$$

$$= \left[\frac{1}{x+a} + \frac{ax}{x^2(x+a)}\right] + \left[\frac{1}{x+b} + \frac{bx}{x^2(x+b)}\right] + \left[\frac{1}{x+c} + \frac{cx}{x^2(x+c)}\right]$$

$$= \left[\frac{1}{x+a} + \frac{a}{x(x+a)}\right] + \left[\frac{1}{x+b} + \frac{b}{x(x+b)}\right] + \left[\frac{1}{x+c} + \frac{c}{x(x+c)}\right]$$

$$= \frac{x+a}{x(x+a)} + \frac{x+b}{x(x+b)} + \frac{x+c}{x(x+c)} = \frac{1}{x} + \frac{1}{x} + \frac{1}{x} = \frac{3}{x}$$

9. (1) $\frac{1}{1+a^2-a} - \frac{1}{1+a^2+a} - \frac{2a}{1+a^4+a^2}$

$$= \frac{1+a^2+a-(1+a^2-a)}{(1+a^2-a)(1+a^2+a)} - \frac{2a}{1+a^4+a^2}$$

$$= \frac{1+a^2+a-1-a^2+a}{(1+a^2)^2-a^2} - \frac{2a}{1+a^4+a^2}$$

$$= \frac{2a}{1+a^4+2a^2-a^2} - \frac{2a}{1+a^4+a^2}$$

$$= \frac{2a}{1+a^4+a^2} - \frac{2a}{1+a^4+a^2} = 0$$

10. (3) $\frac{1}{a^2-3a+2} + \frac{2}{a^2-5a+6} - \frac{3}{a^2-4a+3}$

$$= \frac{1}{a^2-2a-a+2} + \frac{2}{a^2-2a-3a+6} - \frac{3}{a^2-3a-a+3}$$

$$= \frac{1}{a(a-2)-1(a-2)} + \frac{2}{a(a-2)-3(a-2)} - \frac{3}{a(a-3)-1(a-3)}$$

$$= \frac{1}{(a-2)(a-1)} + \frac{2}{(a-2)(a-3)} - \frac{3}{(a-3)(a-1)}$$

$$= \frac{1(a-3)+2(a-1)-3(a-2)}{(a-1)(a-2)(a-3)}$$

$$= \frac{a-3+2a-2-3a+6}{(a-1)(a-2)(a-3)}$$

$$= \frac{1}{(a-1)(a-2)(a-3)}$$

11. (1)

$$\frac{\frac{a(a+b)-a(a-b)}{(a-b)(a+b)}}{\frac{b(a+b)-b(a-b)}{(a-b)(a+b)}} + \frac{\frac{(a+b)^2+(a-b)^2}{(a-b)(a+b)}}{\frac{(a+b)^2-(a-b)^2}{(a-b)(a+b)}} \times \frac{a^2}{a^2+b^2}$$

$$= \frac{a^2+ab-a^2+ab}{ab+b^2-ab+b^2} + \frac{a^2+b^2+2ab+a^2+b^2-2ab}{a^2+b^2+2ab-a^2-b^2+2ab} \times \frac{a^2}{a^2+b^2}$$

$$= \frac{2ab}{2b^2} + \frac{2(a^2+b^2)}{4ab} \times \frac{a^2}{a^2+b^2}$$

$$= \frac{2ab}{2b^2} + \frac{2a}{4b} = \frac{a}{b} + \frac{a}{2b} = \frac{2a+a}{2b} = \frac{3a}{2b}$$

12. (1) $\left[\frac{(x-y)^2-(x+y)^2}{(x+y)(x-y)} - \frac{4y^2}{x^2-y^2}\right]\frac{x-y}{2y}$

$$= \left[\frac{x^2+y^2-2xy-x^2-y^2-2xy}{x^2-y^2} - \frac{4y^2}{x^2-y^2}\right]\frac{x-y}{2y}$$

$$= \left(\frac{-4xy}{x^2-y^2} - \frac{4y^2}{x^2-y^2}\right)\frac{x-y}{2y}$$

$$= \frac{-4xy-4y^2}{x^2-y^2} \times \frac{x-y}{2y} = \frac{-4y(x+y)}{(x-y)(x+y)} \times \frac{x-y}{2y} = -2$$

13. (1)

$$\frac{1}{1-x} - \frac{1}{1+x} - \frac{2x}{1+x^2} - \frac{4x^3}{1+x^4} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{1+x-1+x}{(1+x)(1-x)} - \frac{2x}{1+x^2} - \frac{4x^3}{1+x^4} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{2x}{1-x^2} - \frac{2x}{1+x^2} - \frac{4x^3}{1+x^4} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{2x(1+x^2)-2x(1-x^2)}{(1-x^2)(1+x^2)} - \frac{4x^3}{1+x^4} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{2x+2x^3-2x+2x^3}{1-x^4} - \frac{4x^3}{1+x^4} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{4x^3}{1-x^4} - \frac{4x^3}{1+x^4} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{4x^3(1+x^4-4x^3(1-x^4)}{(1-x^4)(1+x^4)} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{4x^3+4x^7-4x^3+4x^7}{1-x^8} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{8x^7}{1-x^8} - \frac{8x^7}{1+x^8} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{8x^7(1+x^8) - 8x^7(1-x^8)}{(1-x^8)(1+x^8)} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{8x^7 + 8x^{15} - 8x^7 + 8x^{15}}{1-x^{16}} - \frac{16x^{15}}{1-x^{16}}$$

$$= \frac{16x^{15}}{1-x^{16}} - \frac{16x^{15}}{1-x^{16}} = 0$$

14. (2) $x - y = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$

$$= \frac{(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab}}{2} = \frac{(\sqrt{a}-\sqrt{b})^2}{2}$$

= positive

$\therefore x > y$

Again, $y - z = \sqrt{ab} - \frac{2ab}{a+b} = \frac{\sqrt{ab}(a+b) - 2ab}{a+b}$

$$= \frac{\sqrt{ab}(a+b-2\sqrt{ab})}{a+b}$$

$$= \frac{\sqrt{ab}\left[(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab}\right]}{(a+b)}$$

$$= \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})^2}{a+b}$$ = a positive quantity

$\therefore y > z$

$\therefore x > y > z$

15. (3) $(x-a)(x-b-1) = a-b$

$\Rightarrow x^2 - xb - x - ax + ab + a = a - b$

$\Rightarrow x^2 - xb - x - ax + ab + a - a + b = 0$

$\Rightarrow x^2 - x - ax - bx + b + ab = 0$

$\Rightarrow x(x-1-a) - b(x-1-a) = 0$

$\Rightarrow (x-1-a)(x-b) = 0$

If, $x - 1 - a = 0$ then $x = 1 + a$

$\Rightarrow x - b = 0$ then $x = b$

Hence, $x = (1+a), b$

16. (2) $\frac{x+a}{x+b} = \frac{x+3a}{x+a+b}$

$$\Rightarrow \frac{x+a+b}{x+b} = \frac{x+3a}{x+a}$$

$$\Rightarrow \frac{x+b+a}{x+b} = \frac{x+a+2a}{x+a}$$

$$\Rightarrow \frac{x+b}{x+b} + \frac{a}{x+b} = \frac{x+a}{x+a} + \frac{2a}{x+a}$$

$$\Rightarrow 1 + \frac{a}{x+b} = 1 + \frac{2a}{a+x}$$

$$\Rightarrow \frac{a}{x+b} = \frac{2a}{x+a} \Rightarrow \frac{1}{x+b} = \frac{2}{x+a}$$

$\Rightarrow x + a = 2(x+b) = 2x + 2b$

$\Rightarrow 2x - x = a - 2b$

$\therefore x = a - 2b$

17. (3) $\because x + y + z = 0$

$\therefore x + y = -z,$

$\Rightarrow (x+y)^2 = (-z)^2$

$\Rightarrow x^2 + y^2 + 2xy = z^2$

$\Rightarrow z^2 = x^2 + y^2 + 2xy$

Similarly, $x^2 = y^2 + z^2 + 2yz$

and, $y^2 = z^2 + x^2 + 2xz$

Expression

$$= \frac{1}{x^2+y^2-z^2} + \frac{1}{y^2+z^2-x^2} + \frac{1}{z^2+x^2-y^2}$$

Putting the values of = z^2, x^2 and y^2

$$= \frac{1}{x^2+y^2-(x^2+y^2+2xy)} + \frac{1}{y^2+z^2-(y^2+z^2+2yz)} + \frac{1}{z^2+x^2-(z^2+x^2+2xz)}$$

$$= \frac{1}{-2xy} + \frac{1}{-2yz} + \frac{1}{-2xz}$$

$$= \frac{-1}{2xy} - \frac{1}{2yz} - \frac{1}{2xz} = \frac{-z-x-y}{2xyz}$$

$$= \frac{-(z+x+y)}{2xyz} = \frac{0}{2xyz} \quad (\because x+y+z=0)$$

$= 0$

18. (1) $\frac{a^2-(b-c)^2}{(a+c)^2-b^2} + \frac{b^2-(a-c)^2}{(a+b)^2-c^2} + \frac{c^2-(a-b)^2}{(b+c)^2-a^2}$

$$= \frac{(a+b-c)(a-b+c)}{(a+c+b)(a+c-b)} + \frac{(b+a-c)(b-a+c)}{(a+b+c)(a+b-c)} + \frac{(c+a-b)(c-a+b)}{(b+c+a)(b+c-a)}$$

$$= \frac{a+b-c}{a+b+c} + \frac{b-a+c}{a+b+c} + \frac{c+a-b}{a+b+c}$$

$$= \frac{a+b-c+b-a+c+c+a-b}{a+b+c} - \frac{a+b+c}{a+b+c} = 1$$

19. (4) $\because\ x + \frac{1}{y} = 1 \quad \therefore x = 1 - \frac{1}{y} = \frac{y-1}{y}$

$$\Rightarrow \frac{1}{x} = \frac{y}{y-1}$$

and, $y + \frac{1}{z} = 1 \ \therefore\ \frac{1}{z} = 1 - y$

$$\Rightarrow z = \frac{1}{1-y}$$

$$\therefore z + \frac{1}{x} = \frac{1}{1-y} + \frac{y}{y-1}$$

$$= \frac{1}{1-y} - \frac{y}{1-y} = \frac{1-y}{1-y} = 1$$

20. (1) $\because\ \frac{p}{b-c} = \frac{q}{c-a} = \frac{r}{a-b} = k$ (Let)

$\therefore\ p = k(b-c),\ q = k(c-a),\ r = k(a-b)$

Expression $= p + q + r$

$= k(b-c) + k(c-a) + k(a-b)$

$= k(b-c+c-a+a-b)$

$= k \times 0 = 0$

21. (1) $\frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-c)(b-a)} + \frac{c^2}{(c-a)(c-b)}$

$$= -\frac{a^2}{(a-b)(c-a)} - \frac{b^2}{(b-c)(a-b)} - \frac{c^2}{(c-a)(b-c)}$$

(Writing in cyclic order)

$$= \frac{-a^2(b-c) - b^2(c-a) - c^2(a-b)}{(a-b)(b-c)(c-a)}$$

$$= \frac{-a^2b + a^2c - b^2c + b^2a - c^2a + c^2b}{(a-b)(b-c)(c-a)}$$

$$= \frac{-a^2b + b^2a + a^2c - b^2c - c^2a + c^2b}{(a-b)(b-c)(c-a)}$$

$$= \frac{-ab(a-b) + c(a^2-b^2) - c^2(a-b)}{(a-b)(b-c)(c-a)}$$

$$= \frac{-ab(a-b) + c(a-b)(a+b) - c^2(a-b)}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)[-ab + c(a+b) - c^2]}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)[-ab + ca + cb - c^2]}{(a-b)(b-c)(b-a)}$$

$$= \frac{(a-b)[-a(b-c) + c(b-c)]}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)(b-c)(-a+c)}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 1$$

22. (2) $\frac{1}{a(a-b)(a-c)} + \frac{1}{b(b-a)(b-c)} + \frac{1}{c(c-a)(c-b)}$

$$= -\frac{1}{a(a-b)(c-a)} - \frac{1}{b(a-b)(b-c)} - \frac{1}{c(c-a)(b-c)}$$

(writing in cyclic order)

$$= \frac{-bc(b-c) - ac(c-a) - ab(a-b)}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{-bc(b-c) - ac^2 + a^2c - a^2b + ab^2)}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{-bc(b-c) - a^2b + a^2c + ab^2 - ac^2}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{-bc(b-c) - a^2(b-c) + a(b^2-c^2)}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{-bc(b-c) - a^2(b-c) + a(b-c)(b+c)}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)[-bc - a^2 + a(b+c)]}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)[-bc - a^2 + ab + ac]}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)[-a^2 + ab + ac - bc]}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)[-a(a-b) + c(a-b)]}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)[(a-b)(-a+c)]}{abc(a-b)(b-c)(c-a)}$$

$$= \frac{(b-c)(a-b)(c-a)}{abc(a-b)(b-c)(c-a)} = \frac{1}{abc}$$