CHAPTER - 9 - DIFFERENTIAL EQUATIONS - EXERCISE : 9.2

In each of exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding diff. eqn. :

Q No 1. $y = e^x + 1$; $y'' - y' = 0$

Sol. Given $y = e^x + 1$

$\Rightarrow y' = \frac{dy}{dx} = e^x + 0$

$y'' = \frac{d}{dx}(e^x) = e^x$

$\therefore$ L.H.S $= y'' - y' = e^x - e^x = 0 =$ R.H.S.

Hence $y = e^x + 1$ is soln. of given diff. eqn.

Q No. 2. $y = x^2 + 2x + C$; $y' - 2x - 2 = 0$

Sol. Given $y = x^2 + 2x + C$

$\Rightarrow y' = \frac{d}{dx}(x^2 + 2x + C) = 2x + 2$

$\therefore$ LHS $= y' - 2x - 2$

$= 2x + 2 - 2x - 2 = 0 =$ RHS.

$\therefore y = x^2 + 2x + C$ is soln of given diff. eqn.

Q No. 3 $y = \cos x + C$; $y' + \sin x = 0$

Sol. Given $y = \cos x + C$

$\therefore y' = \frac{dy}{dx} = \frac{d}{dx}(\cos x + C) = -\sin x$

$\therefore$ LHS $= y' + \sin x = -\sin x + \sin x = 0 =$ RHS.

$\therefore y = \cos x + C$ is solution of given diff. eqn.

Q No 4 $y = \sqrt{1 + x^2}$: $y' = \frac{xy}{1 + x^2}$

Sol. Given $y = \sqrt{1 + x^2}$

Squaring both sides.

$y^2 = 1 + x^2$.

Differentiating both sides w.r.t x

$$2y\cdot\frac{dy}{dx} = 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{2y} = \frac{x}{y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{xy}{y^2} = \frac{xy}{1+x^2}$$

Hence $y = \sqrt{1+x^2}$ is soln. of given differential equation.

QNo.5 $y = Ax$; $xy' = y$ $(x \neq 0)$

Sol. Given $y = Ax$

$\Rightarrow y' = A \Rightarrow xy' = Ax \Rightarrow xy' = y$

$\therefore y = Ax$ is Soln. of given diff. eqn.

QNo 6 $y = x\sin x$: $xy' = y + x\sqrt{x^2-y^2}$ ($x \neq 0$ and $x > y$ or $x < -y$)

Sol. Given $y = x\sin x$

$\Rightarrow y' = x\cdot\cos x + \sin x$

$\Rightarrow xy' = x^2\cos x + x\sin x$

$\Rightarrow xy' = x^2\sqrt{\cos^2 x} + y$ $[\because y = x\sin x]$

$= x^2\sqrt{1-\sin^2 x} + y.$

$= x^2\sqrt{1-(y/x)^2} + y$ $[\because y = x\sin x \Rightarrow \sin x = y/x]$

$= x^2\sqrt{\frac{x^2-y^2}{x^2}} + y$

$= x\sqrt{x^2-y^2} + y.$

$\therefore y = x\sin x$ is Soln of given diff eqn.

QNo7. $xy = \log y + C$ ...(1) ; $y' = \frac{y^2}{1-xy}$ $(xy \neq 1)$

Sol. $xy = \log y + C$

$\Rightarrow x\cdot\frac{dy}{dx} + y\cdot 1 = \frac{1}{y}\cdot\frac{dy}{dx} + 0$

$\Rightarrow xy\cdot y' + y^2 = y'$

$\Rightarrow (1-xy)y' = y^2$

$\Rightarrow y' = \frac{y^2}{1-xy}$ $(xy \neq 0)$

$\therefore$ (1) is soln of given diff. eqn.

QNo. 8. $y - \cos y = x$ $(y\sin y + \cos y + x)y' = y$.

Sol.. Given $y - \cos y = x$ ....(i)

$\Rightarrow y' - (-\sin y)y' = 1$

$\rightarrow y'(1+\sin y) = 1$. $\Rightarrow y \cdot y'(1+\sin y) = y$.

$\Rightarrow y'(y + y\sin y) = y$.

$\Rightarrow y'(x + \cos y + y\sin y) = y$ $[\because y - \cos y = x \Rightarrow y = x + \cos y]$

$\therefore$ (1) is soln. of given diff. eqn.

QNo.9. $x + y = \tan^{-1} y$: $y^2 y' + y^2 + 1 = 0$

Sol. $x + y = \tan^{-1} y$. ...(1)

$\Rightarrow 1 + \frac{dy}{dx} = \frac{1}{1+y^2} \cdot \frac{dy}{dx}$ ie $1 + y' = \frac{1}{1+y^2} \cdot y'$

$\Rightarrow (1+y')(1+y^2) = y'$

$\Rightarrow (1+y^2) + y'(1+y^2) = y'$

$\Rightarrow 1 + y^2 + y' + y'y^2 = y'$ $\Rightarrow y^2 y' + y' + 1 = 0$

$\therefore$ (1) is soln. of given diff. eqn.

QNo 10 $y = \sqrt{a^2 - x^2}$; $x \in (-a, a)$: $x + y\frac{dy}{dx} = 0$ $(y \neq 0)$

Sol. Given $y = \sqrt{a^2 - x^2}$

$\Rightarrow y^2 = a^2 - x^2$

$\Rightarrow 2y \cdot \frac{dy}{dx} = 0 - 2x$ $\Rightarrow y\frac{dy}{dx} = -x$

$\Rightarrow x + y \cdot \frac{dy}{dx} = 0$

$\therefore y = \sqrt{a^2 - x^2}$ is soln. of given diff. eqn.

Q No. 11 The number of arbitrary constants in the general solution of differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4

Sol.. Since the number of arbitrary constants in the general solution of differentian equation of n^{th} order is 'n'.

In this case (D) is correct option.

Q No 12 The number of arbitrary constants in the particular solution of diffl. eqn of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

Sol. In particular solution of differential equation of 'n^{th} order there is no arbitrary constants

$\therefore$ (D) is correct option

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