CBSE Class 10 Mathematics Basic

Sample Paper - 01 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

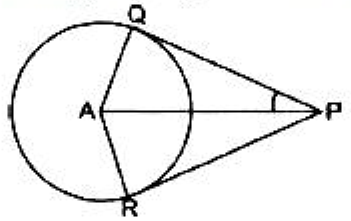
1. Show that 12^n cannot end with digit 0 or 5 for any natural number n.

OR

Without actually performing the long division, Check whether $\frac{64}{455}$ will have the terminating decimal expansion or non-terminating repeating decimal expansion.

2. Write the discriminant of the quadratic equation $x^2 + 4x + q = 0$
3. Does the pair of the linear equation have no solution? Justify your answer.
$3x + y - 3 = 0, 2x + \frac{2}{3}y = 2$

4. In figure, PQ and PR are tangents to circle with centre A. If $\angle QPA = 27°$, then find $\angle QAR$.



5. Find the Arithmetic Mean of (a - b) and (a + b).

OR

Find the 10^{th} term of AP: 10.0,10.5,11.0,11.5,....

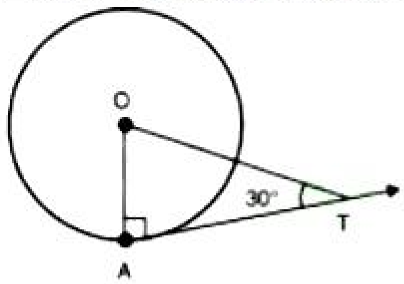
6. Find the Arithmetic Mean of 13 and 19.
7. If $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$. Prove that $\frac{x}{a} = \frac{y}{b}$.

OR

State whether the following equation is quadratic equation in x?

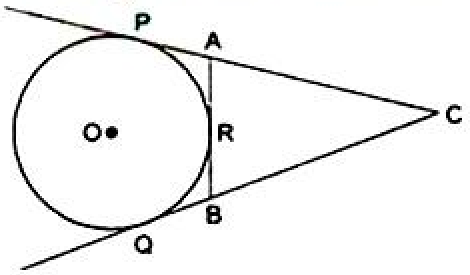
$2x^2 + \frac{5}{2}x - \sqrt{3} = 0$

8. What is the distance between two parallel tangents of a circle of radius 4 cm?
9. In given figure, if AT is a tangent to the circle with centre O, such that OT = 4 cm and $\angle OTA = 30^o$, then find the length of AT (in cm).



OR

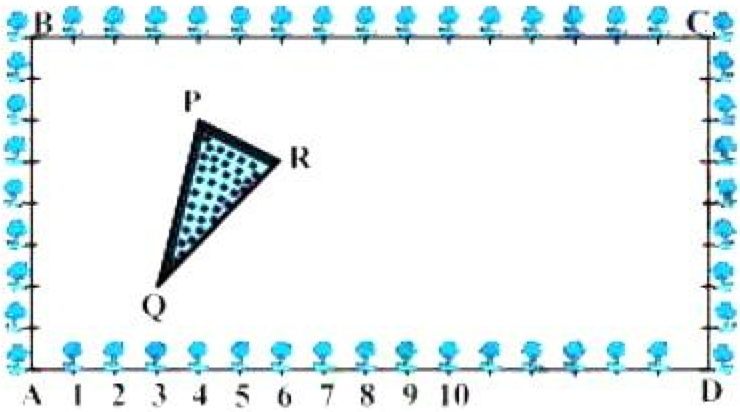
In figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR.



10. A ladder is placed in such a way that its foot is at a distance of 5 m from a wall and its tip

reaches a window 12 m above the ground. Determine the length of the ladder.

11. Find the 21st term of the A.P: $-4\frac{1}{2}, -3, -1\frac{1}{2}, \ldots$
12. Evaluate: $\sin^2 30° \cos^2 45° + 4 \tan^2 30° + \frac{1}{2}\sin^2 90° - 2\cos^2 90° + \frac{1}{24}\cos^2 0°$.
13. Prove that : $\cos^2\theta\,(1 + \tan^2\theta) = 1$.
14. If the lateral surface area of a cylinder is 94.2 cm^2 and its height is 5 cm, then find radius of its base.
15. For what value of k will k+9, 2k-1, and 2k+7 are consecutive terms of an AP.
16. A number is chosen at random from the numbers - 3, - 2, - 1, 0,1, 2, 3. What will be the probability that the square of this number is less than or equal to 1?
17. The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar is planted on the boundary of the plot at a distance of 1m from each other. There is a triangular grassy lawn inside the plot as shown in Fig. The students have to sow seeds of flowering plants on the remaining area of the plot.



i. Considering A as the origin, what are the coordinates of A?
   a. (0, 1)
   b. (1, 0)
   c. (0, 0)
   d. (-1, -1)

ii. What are the coordinates of P?
   a. (4, 6)
   b. (6, 4)
   c. (4, 5)
   d. (5, 4)

iii. What are the coordinates of R?

a. (6, 5)
b. (5, 6)
c. (6, 0)
d. (7, 4)

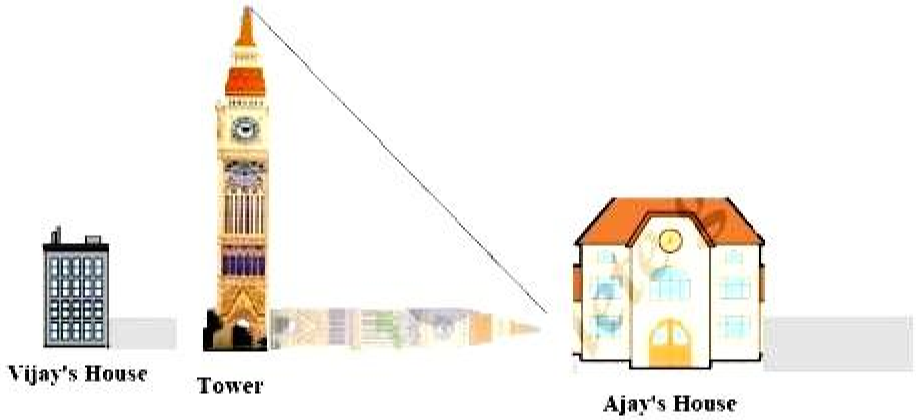
iv. What are the coordinates of D?

a. (16, 0)
b. (0, 0)
c. (0, 16)
d. (16, 1)

v. What are the coordinates of P if D is taken as the origin?

a. (12, 2)
b. (-12, 6)
c. (12, 3)
d. (6, 10)

18.



Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house is 20 m when Vijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground. At the same time, the house of Ajay casts 20 m shadow on the ground.

i. What is the height of the tower?

a. 20 m
b. 50 m
c. 100 m
d. 200 m

ii. What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m?

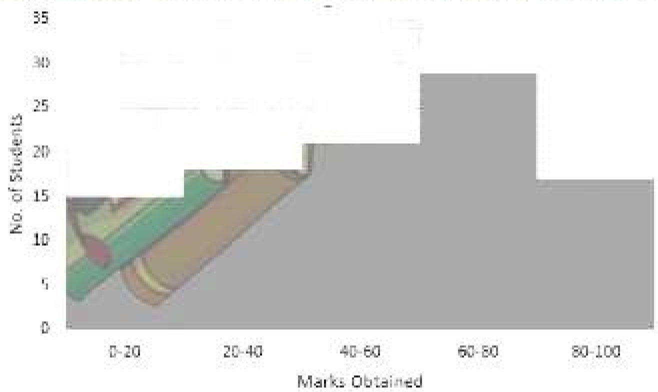
a. 75 m
b. 50 m
c. 45 m
d. 60 m

iii. What is the height of Ajay's house?
a. 30 m
b. 40 m
c. 50 m
d. 20 m

iv. When the tower cast shadow of 40 m, Same time what will be the length of the shadow of Ajay's house?
a. 16 m
b. 32 m
c. 20 m
d. 8 m

v. When the tower cast shadow of 40 m, Same time what will be the length of the shadow of Vijay's house?
a. 15 m
b. 32 m
c. 16 m
d. 8 m

19. Recently the half-yearly examination was conducted in DAV public school. The mathematics teacher maintains a record of the marks of 100 students. On the basis of the recorded data of the marks obtained in Mathematics, the histogram is given below:



On the basis of the above histogram, answer the following questions:

i. Identify the modal class from the given graph.

a. 80 - 100
b. 20 - 40
c. 60 - 80
d. 40 - 60

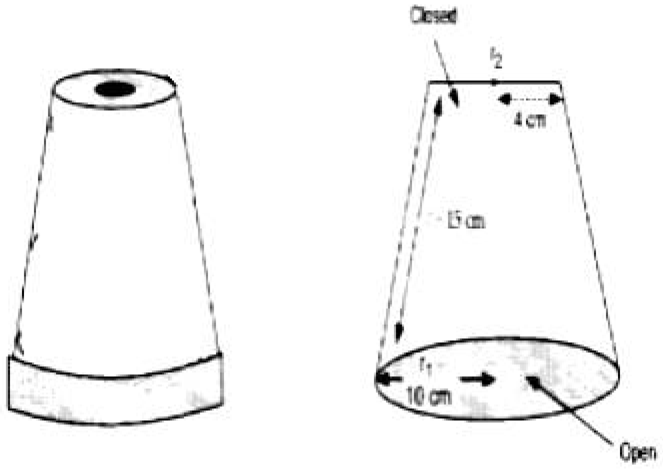
ii. Find the mode of the distribution of marks obtained by the students in an examination.
a. 78
b. 68
c. 48
d. 58

iii. Given the mean of the above distribution is 53, using empirical relationship estimate the value of its median.
a. 78
b. 68
c. 48
d. 58

iv. The construction of the cumulative frequency table is useful in determining the
a. Median
b. Mean
c. Mode
d. All of these

v. What will be the upper limit of the modal class?
a. 100
b. 80
c. 40
d. 60

20. During the battle of Turks against the Rajputs of India, the Turk soldiers wore a costume with a metallic shield-like knee pads, buckler (elbow shield) and cap to save themselves from injuries. The headgear cap (a fez) used by these soldiers is shaped like the frustum of a cone with its radius on the open side 10 cm, and radius at the upper base as 4 cm and its slant height as 15 cm.



By using the above information, find the following:

i. The curved surface area of the cap is:

a. 650 cm^2

b. 660 cm^2

c. 606 cm^2

d. 666 cm^2

ii. Area of the closed base is:

a. 55.285

b. 50.285

c. 52.285

d. 56.285

iii. The area of the material used for making it.

a. 701.28 cm^2

b. 720.28 cm^2

c. 710.28 cm^2

d. 717.28 cm^2

iv. During the conversion of a solid from one shape to another the volume of the new shape will:

a. increase

b. remain unaltered

c. double

d. decrease

v. The formula to find the volume of the frustum of a cone is:

a. $\frac{2}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$
b. $\frac{1}{3}\pi h(r_1^2 + r_2^2)$
c. $\frac{1}{3}\pi h(r_1^3 + r_2^3 + r_1 r_2)$
d. $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Part-B

21. The decimal expansion of the rational number $\frac{79}{2^3 \times 5^4}$ will terminate after how many places of decimal?
22. Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin.

OR

If P (2, 1), Q (4, 2), R(5, 4) and S(3, 3) are vertices of a quadrilateral, find the area of the quadrilateral PQRS.

23. Find the zeroes of the quadratic polynomial given as: $x^2 + 7x + 10$, and also verify the relationship between the zeroes and the coefficients.
24. Draw a circle of radius 3 cm. Take a point P outside the circle at a distance of 5.8 cm from its centre. Draw tangents from P to the circle.
25. If sin (A + B) = sin Acos B + cos A sin B and cos (A - B) = cos A cos B + sin A sin B, find the values of
   i. sin 75°
   ii. cos 15°

OR

Prove that: $\left(\sin^8\theta - \cos^8\theta\right) = \left(\sin^2\theta - \cos^2\theta\right)\left(1 - 2\sin^2\theta\cos^2\theta\right)$

26. A point P is 25 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 24 cm. Find the radius of the circle.
27. Prove that $\sqrt{2}$ is an irrational number.
28. Solve: $x^2 + 5x - (a^2 + a - 6) = 0$

OR

The numerator of a fraction is one less than its denominator. If three is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction.

29. If α and β are the zeros of the quadratic polynomial f(x) = $kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2$ = 24, find the values of k.

30. E is a point on side AD produced of a parallelogram ABCD and BE intersects CD at F. Prove that $\Delta ABE \sim \Delta CFB.$

OR

ABCD is a trapezium in which AB||DC. P and Q are points on sides AD and BC such that PQ||AB. If PD= 18, BQ = 35 and QC= 15, find AD.

31. Two dice are thrown at the same time. Find the probability of getting:
    i. same number on both dice
    ii. sum of two numbers appearing on both the dice is 8.

32. As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships.

33. Find the mean of the following frequency distribution:

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	6	8	10	9	7

34. Find the area of a rhombus each side of which measures 20 cm and one of whose diagonals is 24 cm.

35. On selling a tea set at 5% loss and a lemon set at 15% gain, a crockery seller gains ₹ 7. If he sells the tea set at 5% gain and the lemon set at 10% gain, he gains ₹ 13. Find the actual price of each of the tea set and the lemon set.

36. A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flagstaff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower.

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Solution

Part-A

1. $12 = 2^2 \times 3$

   $\therefore 12^n = (2^2 \times 3)^n = (2^2)^n \times 3^n$

   So, only primes in the factorisation of 12^n are 2 and 3 and, not 5.

   Hence, 12^n cannot end with digit 0 or 5.

OR

   The number is $\frac{64}{455}$

   Factorize the denominator we get,

   $455 = 5 \times 7 \times 13$

   Since, the denominator is not in the form of $2^m \times 5^n$, and it also contains 7 and 13 as its factors,

   Its decimal expansion will be non-terminating repeating.

2. $x^2 + 4x + q = 0$

   Here, a = 1, b = 4, c = 1

   $D = b^2 - 4ac = 4^2 - 4(1)(q)$

   $= 16 - 4q$

   Hence, discriminant is 16-4q.

3. No

   The Condition for no solution is : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (parallel lines)

   Given pair of equations,

   $3x + y - 3 = 0$

   and $2x + \frac{2}{3}y = 2$

   Comparing with ax + by + c = 0;

   Here, $a_1 = 3$, $b_1 = 1$, $c_1 = -3$;

   And $a_2 = 2$, $b_2 = 2/3$, $c_2 = -2$;

$a_1/a_2 = 2/6 = 3/2$

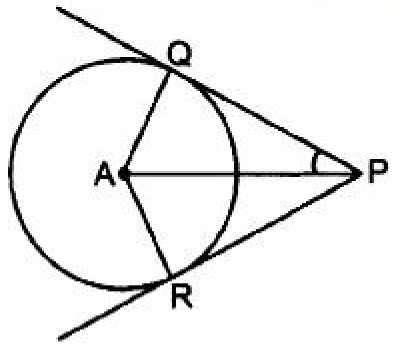
$b_1/b_2 = 4/12 = 3/2$

$c_1/c_2 = -3/-2 = 3/2$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e coincident lines

Hence, the given pair of linear equations is coincident and having infinitely many solutions.

4. PQ and PR are tangents to circle with centre A.



$\angle QPA = \angle RPA$
$\Rightarrow \angle RPA = 27^\circ$
$\angle QPR = \angle QPA + \angle RPA$
$= 27^\circ + 27^\circ = 54^\circ$
Now, $\angle QAR + \angle QPR = 180^\circ$
$\Rightarrow \angle QAR = 180^\circ - 54^\circ = 126^\circ$

5. A.M of (a - b) and (a + b) = $\frac{1}{2}[(a-b)+(a+b)]$ = a.

OR

a = 10, d = 10.5 - 10 = 0.5

$a_{10} = a + 9d = 10 + 9 \times 0.5 = 14.5$

6. A.M between $13\ of\ 19$ = $\frac{1}{2}(13+19) = 16.$
7. Given, $\left(x^2+y^2\right)\left(a^2+b^2\right) = (ax+by)^2$
or, $x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$
or, $x^2b^2 + y^2a^2 - 2abxy = 0$
or, $(xb-ya)^2 = 0\ \left[\because (a-b)^2 = a^2+b^2-2ab\right]$
or, xb = ya
$\therefore\ \frac{x}{a} = \frac{y}{b}$ Hence Proved.

OR

We have, $2x^2 + \frac{5}{2}x - \sqrt{3} = 0$

$\Rightarrow 4x^2 + 5x - 2\sqrt{3} = 0$

Clearly, it is in the form of $ax^2 + bx + c = 0$

$2x^2 + \frac{5}{2}x - \sqrt{3} = 0$ is a quadratic equation.

8. Two parallel tangents can exist at the two ends of the diameter of the circle. Therefore, the distance between the two parallel tangents will be equal to the diameter of the circle.

   In the problem the radius of the circle is given as 4 cm.

   Therefore,

   Diameter = 4×2

   Diameter = 8 cm

   Hence, the distance between the two parallel tangents is 8 cm.

9. In given figure, AT is a tangent to the circle with centre O, such that OT = 4 cm and $\angle$OTA = 30°, then we have to find the length of AT (in cm).

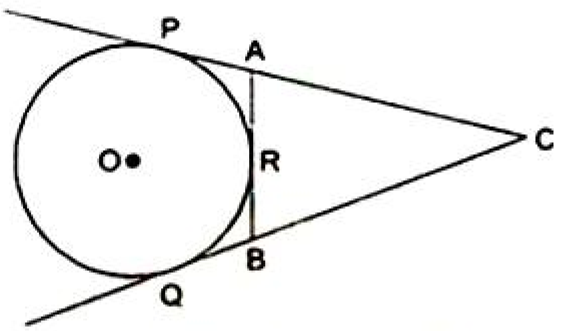
   $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

   $\frac{AT}{OT} = \cos 30^\circ$

   $\therefore \text{AT} = \text{OT} \cos 30^\circ$

   or, $AT = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}\text{cm}$

OR



Since, CP = CQ = 11cm [Length of the two tangents from same external point]

$CQ = CB + BQ$

But, $BQ = BR$

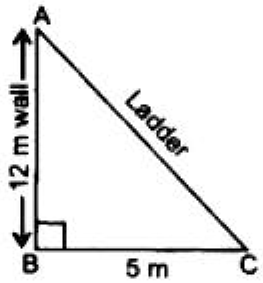
$11 = 7 + BR$

$\Rightarrow$ BR = 4 cm

10. Let AC be the ladder, AB be the wall and BC be the distance of ladder from the foot of the

wall.

In right $\triangle ABC$,



$AC^2 = AB^2 + BC^2$ { using Pythagoras theorm for right-angled triangle}

$\Rightarrow AC^2 = (12)^2 + 5^2$

$\Rightarrow AC^2 = 144 + 25$

$\Rightarrow AC = 13$ m

11. Given A.P is: $-4\frac{1}{2}, -3, -1\frac{1}{2}$

Here, $a = -4\frac{1}{2}$, $d = 1\frac{1}{2}$

21st term is given by

$$a_{21} = a + 20d$$
$$= \frac{-9}{2} + 20 \times \frac{3}{2}$$
$$= \frac{-9+60}{2}$$
$$= \frac{51}{2}$$
$$= 25\frac{1}{2}$$

12. We know that, Sin30°=(1/2), Cos45°=($1/\sqrt{2}$), Sin90°=1, Cos90°=0, Cos0°=1 & tan30°=($1/\sqrt{2}$),

putting these values in the given expression, we get:-

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}\cos^2 0^\circ$$
$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2} \times (1)^2 - 2 \times (0)^2 + \frac{1}{24}(1)^2$$
$$= \frac{1}{4} \times \frac{1}{2} + 4 \times \frac{1}{3} + \frac{1}{2} \times 1 - 2 \times 0 + \frac{1}{24} \times 1$$
$$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} - 0 + \frac{1}{24}$$
$$= \frac{3+32+12-0+1}{24}$$
$$= \frac{48}{24} = 2$$

13. We have,

LHS = $\cos^2\theta(1 + \tan^2\theta)$

= $\cos^2\theta\sec^2\theta$ [$\because 1 + \tan^2\theta = \sec^2\theta$]

= $\cos^2\theta\left(\frac{1}{\cos^2\theta}\right)$ [$\because \sec\theta = \frac{1}{\cos\theta}$].

= 1 =R.H.S, Hence proved.

14. Lateral surface area of a cylinder = 94.2 cm^2

    h = 5 cm

    $2\pi rh = 94.2$

    $\Rightarrow 2 \times 3.14 \times r \times 5$= 94.2

    $\Rightarrow r = \frac{94.2}{2\times3.14\times5}$ = 3 cm

15. We know that difference between any two consecutive terms of an AP is equal

    $\therefore a_2 - a_1 = a_3 - a_2$

    let $a_1 = (k+9), a_2 = (2k-1), a_3 = (2k+7)$

    $\Rightarrow$ (2k - 1) - (k + 9) = (2k + 7) - (2k - 1)

    $\Rightarrow$ k - 10 = 8

    $\Rightarrow$ k = 18

16. No. of all possible outcomes = 7

    No. of favourable outcomes = -1, 0 ,1 = 3

    $probability \ = \frac{Number\ of\ outcome\ favorable}{Total\ number\ of\ outcome}$

    $\therefore$ required probability = $\frac{3}{7}$

17. It can be observed that the coordinates of point P, Q and R are (4, 6), (3, 2), and (6, 5) respectively.

    i. (c) (0, 0)

    ii. (a) (4, 6)

    iii. (a) (6, 5)

    iv. (a) (16, 0)

    v. (b) (-12, 6)

18. i. (c) 100 m

    ii. (d) 60 m

    iii. (b) 40 m

    iv. (a) 16 m

    v. (d) 8 m

19. First, we will convert the graph into the tabular form as shown below:

Marks obtained	0 - 20	20 - 40	40 - 60	60 - 80	80 -100
Number of students	15	18	21	29	17

    i. (c) Modal class is the class having maximum number of frequency.

Here, maximum frequency is 29 and it belongs to class 60-80, so Modal class = 60-80

ii. (b) Mode = $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

Here, l = 60, f_1 = 29, f_0 = 21, f_2 = 17 and h = 20

Mode = $60 + \frac{29-21}{2\times29-21-17} \times 20$

$= 60 + \frac{8}{58-38} \times 20$ = 68

iii. (d) Mode = 3 median - 2 mean

Mode = 68 and mean = 53 (given)

∴ 3 median = mode + 2 mean

3 median = 68 + 2 × 53

Median = $\frac{174}{3}$ = 58

Hence, Median = 58

iv. (a) Median

v. (b) 80

20. Clearly, the fez is in the shape of a frustum of a cone with radii of one base as r_1 = 10 cm and radii of another base as r_2 = 4 cm and slant height l = 15 cm. Then,

i. (b) The curved surface area of cap = $\pi(r_1 + r_2)l$

$\Rightarrow \quad A = \frac{22}{7} \times (10+4) \times 15$ = 660 cm^2

ii. (b) 50.285

iii. (a) Let A be the area of the material used:

A = Curved surface area + Area of the closed base

$\Rightarrow A = 660 + \pi r_2^2$

$\Rightarrow A = 660 + \frac{22}{7} \times 4^2$

= 710.28 cm^2

iv. (b) remains unaltered

v. (d) $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Part-B

21. The decimal expansion of the rational number $\frac{p}{2^m \times 5^n}$ where m and n are non-negative integers will terminate after

i. m places of decimal if m > n

ii. n places of decimal if n > m.

Here the denominator $2^3 \times 5^4$ is of the form $2^m \times 5^n$ where m and n are non-negative integers.

Here 4> 3

means n > m.

So, n places of decimal.

So, the given rational number will terminate after 4 places.

22. Let the coordinates of the third vertex be (x, y). Then by centroid formula, coordinates of centroid of given triangle are,

$\left(\frac{x-3+0}{3}, \frac{y+1-2}{3}\right) = \left(\frac{x-3}{2}, \frac{y-1}{3}\right)$

We have centorid is at origin (0, 0)

$\therefore \frac{x-3}{3} = 0 \quad \text{and} \ \frac{y-1}{3} = 0$

$\Rightarrow$ x - 3 = 0 $\Rightarrow$ y - 1 = 0

$\Rightarrow$ x = 3 $\Rightarrow$ y = 1

Hence, the coordinates of the third vertex are (3, 1).

OR

Area of $\triangle$PQR = $\frac{1}{2}$ [2(2 - 4) + 4(4 - 1) + 5(1 - 2)

$= \frac{1}{2}|2 \times -2 + 4 \times 3 + 5 \times -1|$

$= \frac{1}{2}|-4 + 12 - 5| = \frac{3}{2}$

$= \frac{3}{2}$ sq. unit.

Area of $\triangle$PRS $= \frac{1}{2}|2(4-3) + 5(3-1) + 3(1-4)|$

$= \frac{1}{2}|2 + 10 - 9|$

$= \frac{3}{2}$ sq. units

$\therefore$ Area of quadrialteral = Area of $\triangle$PQR + Area of $\triangle$PRS = $\frac{3}{2} + \frac{3}{2}$

= 3 sq. units

23. We have,

$x^2 + 7x + 10 = (x + 2)(x + 5)$

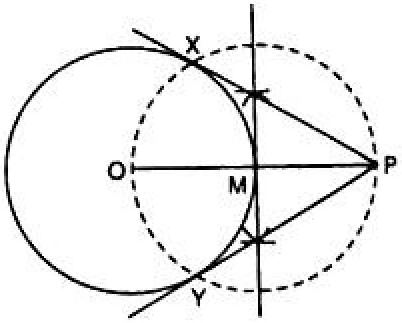
So, the value of $x^2 + 7x + 10$ is zero when x + 2 = 0 or x + 5 = 0, i.e., when x = – 2 or x = –5.

Therefore, the zeroes of $x^2 + 7x + 10$ are -2 and -5.

Now,

sum of zeroes = $-2 + (-5) = -(7) = \frac{-(7)}{1} = \frac{-(\text{ Coefficient of } x)}{\text{Coefficient of } x^2}$

product of zeroes = $(-2) \times (-5) = 10 = \frac{10}{1} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$

24. 

Steps of construction:

i. Draw a circle of radius 3 cm, whose centre is O.

ii. Take a point P at a distance of 5.8 cm from its centre.

iii. Join OP.

iv. Draw perpendicular bisector of OP which cuts OP in M.

v. With M as a centre and radius MO, draw a circle which cuts the given circle at X and Y.

vi. Join PX and PY.

PX and PY are the required tangents.

25. i. sin A cos B + cos A sin B

Taking A = 45° and B = 30°, we have

sin(45° + 30°) = sin45°cos30° + cos 45°sin30°

$\therefore \sin 75^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

ii. cos (A - B) = cos A cos B + sin A sin B

Taking A = 45° and B = 30°, we have

cos 15° = $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

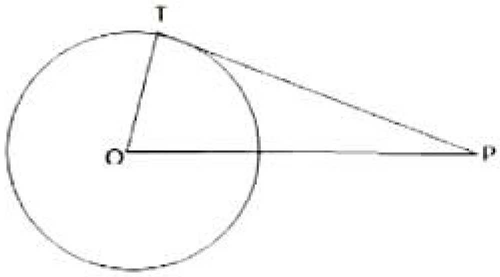
OR

We have,

$$\text{LHS} = \sin^8\theta - \cos^8\theta = \left(\sin^4\theta\right)^2 - \left(\cos^4\theta\right)^2 = \left(\sin^4\theta - \cos^4\theta\right)\left(\sin^4\theta + \cos^4\theta\right)$$

$$\Rightarrow \text{LHS} = \left(\sin^2\theta - \cos^2\theta\right)\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta + \cos^4\theta\right)$$

$$\Rightarrow \text{LHS} = \left(\sin^2\theta - \cos^2\theta\right)\left\{\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta\right\}$$
$$\Rightarrow \text{LHS} = \left(\sin^2\theta - \cos^2\theta\right)\left\{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta\right\}$$
$$\Rightarrow \text{LHS} = \left(\sin^2\theta - \cos^2\theta\right)\left(1 - 2\sin^2\theta\cos^2\theta\right) = \text{RHS}$$

26. 

Given: OP = 25cm.

Let TP be the tangent, so that TP = 24cm

Join OT where OT is radius.

Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

$\therefore OT \perp PT$

In $\triangle OTP$,

By Pythagoras theorem, $OT^2 + TP^2 = OP^2$

$OT^2 + 24^2 = 25^2$

$OT^2 = 625 - 576$

$OT^2 = 49$

OT = 7

The radius of the circle will be 7cm.

27. We have to prove that $\sqrt{2}$ is an irrational number.

Let $\sqrt{2}$ be a rational number.

$\therefore \quad \sqrt{2} = \frac{p}{q}$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get,

or, $2 = \frac{p^2}{q^2}$

or, $p^2 = 2q^2$

$\therefore p^2$ is divisible by 2.

p is divisible by 2........(i)

Let p = 2r for some integer r

or, $p^2 = 4r^2$

$2q^2 = 4r^2$ [$\because p^2 = 2q^2$]

or, $q^2 = 2r^2$

or, q^2 is divisible by 2.

$\therefore$q is divisible by 2..........(ii)

From (i) and (ii)

p and q are divisible by 2, which contradicts the fact that p and q are co-primes.

Hence, our assumption is wrong.

$\therefore \sqrt{2}$ is irrational number.

28. Given, $x^2 + 5x - (a^2 + a - 6) = 0$

splitting $a^2 + a - 6$

$\Rightarrow x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$

$\Rightarrow x^2 + 5x - [\ a(a + 3) - 2\ (a + 3)\ ] = 0$

$\Rightarrow x^2 + 5x - (a + 3)(a - 2) = 0$

Now splitting the middle term

$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a + 3)\ (a - 2) = 0$

$\Rightarrow x\ [x + (\ a + 3)] - (\ a - 2)[\ x + (a + 3)] = 0$

$\Rightarrow [x + (\ a + 3)\ ]\ [\ x - (a - 2)] = 0$

$\Rightarrow x + (\ a + 3) = 0$ or $x - (\ a - 2) = 0$

Therefore, x=-(a + 3) or (a-2)

OR

Assume denominator = x then, numerator = x - 1

$\therefore$ Fraction = $\frac{x-1}{x}$

According to given situation, we have

$\frac{x-1+3}{x+3} = \frac{x-1}{x} + \frac{3}{28}$

$\Rightarrow \frac{x+2}{x+3} - \frac{x-1}{x} = \frac{3}{28}$

$\Rightarrow \frac{(x+2)x-(x-1)(x+3)}{(x+3)x} = \frac{3}{28}$

$\Rightarrow \frac{x^2+2x-(x^2+2x-3)}{x^2+3x} = \frac{3}{28}$

$\Rightarrow 3 \times 28 = 3(x^2 + 3x)$

$\Rightarrow x^2 + 3x - 28 = 0$

Factorize the above quadratic equation, we get

$\Rightarrow (x + 7)(x - 4) = 0 \Rightarrow x = -7$ or $x = 4$

Rejecting $x = -7$ $\therefore x = 4$

$\therefore$ Fraction is $\frac{4-1}{4} = \frac{3}{4}$

29. α *and* β *are zeros of* $kx^2 + 4x + 4$

$\therefore \quad \alpha + \beta = -\frac{4}{k}$, and $\alpha\beta = \frac{4}{k}$

Now $\alpha^2 + \beta^2 = 24$

$\Rightarrow \quad (\alpha + \beta)^2 - 2\alpha\beta = 24$

$\Rightarrow \quad \left(-\frac{4}{k}\right)^2 - 2 \times \frac{4}{k} = 24$

$\Rightarrow \quad \frac{16}{k^2} - \frac{8}{k} = 24$

$\Rightarrow 16 - 8k = 24k^2$

$3k^2 + k - 2 = 0$

$3k^2 + 3k - 2k - 2 = 0$

$3k(k + 1) - 2(k + 1) = 0$

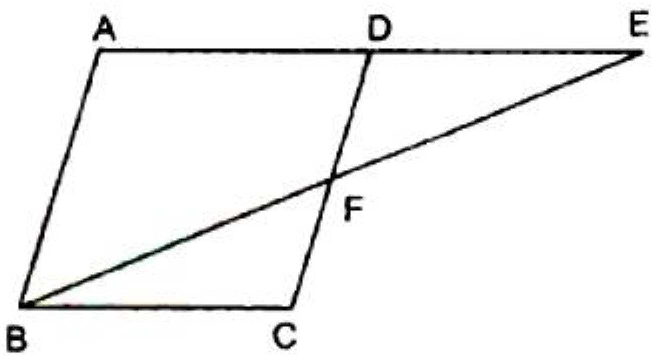
$(k + 1)(3k - 2) = 0$

$k + 1 = 0$ or, $3k - 2 = 0$

$k = -1$ or, $k = \frac{2}{3}$

Hence, $k = -1$ or, $k = \frac{2}{3}$

30. In Δ's ABE and CFB, we have

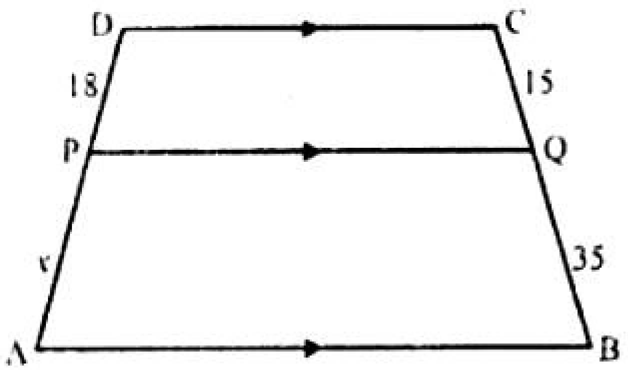


$\angle AEB = \angle CBF$ [Alternate angles]

$\angle A = \angle C$ [Opposite angles of a parallelogram]

Thus, by AA-criterion of similarity, we have, $\Delta ABE \sim \Delta CFB.$

OR



In trapezium ABCD.

$AB \| DC$

P and Q are points on AD and BC respectively such that

$\mathrm{PQ} \| \mathrm{BC}$, PD = 18, BQ = 35, QC = 15

Let PD = x

$\because \mathrm{DC} \| \mathrm{AB} \| \mathrm{PQ}$

$\therefore \frac{\mathrm{DP}}{\mathrm{PA}} = \frac{CQ}{\mathrm{QB}}$

$\Rightarrow \frac{18}{x} = \frac{15}{35}$

$\Rightarrow x = \frac{18 \times 35}{15}$ = 42

$\therefore$ AD = AP + PD = 42 + 18 = 60

31. Total number of possible outcomes = 6× 6 = 36

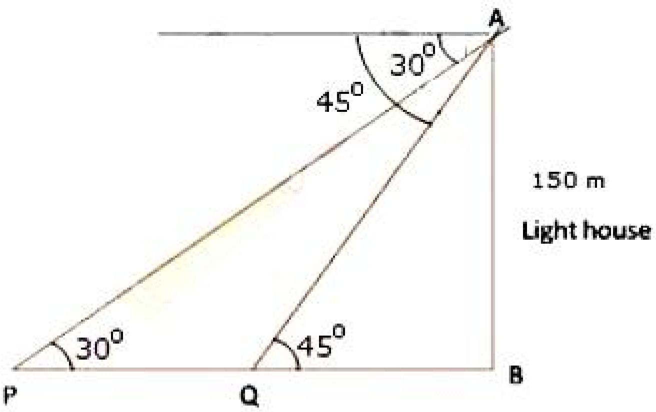
   i. Favourable outcomes = (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

      P(same number on both dice) = $\frac{6}{36} = \frac{1}{6}$

   ii. Favourable outcomes = (2,6), (3,5), (4,4), (6,2), (5,3)

      P(sum is 8) = $\frac{5}{36}$

32.



Height of light house AB = 150m

In ΔABQ

$\tan 45^\circ = \frac{AB}{BQ}$

$\Rightarrow 1 = \frac{150}{BQ}$

BQ = 150m

In ΔABP

$\tan 30^\circ = \frac{AB}{PB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{PB}$

$\Rightarrow PB = 150\sqrt{3} = 150 \times 1.73 = 259.5m$

∴ Distance between two ships

PQ = PB - BQ = 259.5 - 150 = 109.5m

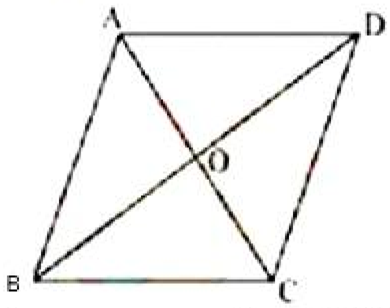
33. Calculation of mean:

Class interval	Mid – value (x_i)	f_i	$f_i x_i$
0 – 6	3	6	18
6 – 12	9	8	72
12 – 18	15	10	150
18 – 24	21	9	189
24 – 30	27	7	189
		Σf_i = 40	$\Sigma f_i x_i$ = 618

We know that, Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{618}{40}$

= 15.45

34. 

Let us suppose that ABCD be the given rhombus.

In this fig, AB = BC = CD = AD = 20 cm and BD = 24 cm

Since, we know that the diagonals of a rhombus bisect each other,

∴ OA = $\frac{1}{2}AC$

$\therefore$ AC = 2 $\times$ OA .... (i)

Also, OB = $\frac{1}{2}BD$

= $\frac{1}{2} \times 24$

= 12 cm

and $\angle$AOB = 90°

Now, in right ΔAOB, using Pythagoras theorem, we get,

$\mathrm{OA}^2 + \mathrm{OB}^2 = \mathrm{AB}^2$

$\Rightarrow OA^2 + 12^2 = 20^2$

$\Rightarrow OA^2 + 144 = 400$

$\Rightarrow OA^2 = 256$

$\Rightarrow$ OA = 16

$\Rightarrow$ AC = 2 $\times$ OA = 32 (from(i))

Thus, the length of the other diagonal is 32 cm.

Area of the given rhombus = $\frac{1}{2} \times AC \times BD$

= $\frac{1}{2} \times 32 \times 24$

= 384 cm^2

Hence, the area of the given rhombus is 384 cm^2

35. Let the cost price of the tea set and the lemon set be ₹ x and ₹ y respectively.

Loss on the tea set = ₹ $\frac{5x}{100}$ = ₹ $\frac{x}{20}$

Gain on the lemon set = ₹ $\frac{15y}{100}$ = ₹ $\frac{3y}{20}$

$\therefore$ Net gain = ₹ $\frac{3y}{20} - \frac{x}{20}$

$\frac{3y}{20} - \frac{x}{20} = 7$

$\Rightarrow 3y - x = 140$ ....(i)

Gain on the tea set = ₹ $\frac{5x}{100}$ = ₹ $\frac{x}{20}$

Loss on the lemon set = ₹ $\frac{10y}{100}$ = ₹ $\frac{y}{10}$

$\therefore$Net gain = ₹ $\left(\frac{x}{20} + \frac{y}{10}\right)$

$\frac{x}{20} + \frac{y}{10} = 13$

$\Rightarrow x + 2y = 260$ ......(ii)

By adding equations (i) and (ii), we get

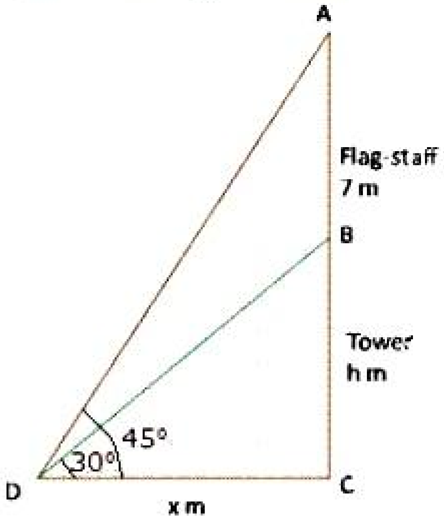
$\Rightarrow 5y = 400$

$\Rightarrow y = 80$

Substituting $y = 80$ in (ii), we get $x = 100$.

∴ the actual price of the tea set is ₹ 100 and that of the lemon set is ₹ 80.

36.



Let us suppose that BC be the height of tower = h m

Suppose DC denotes distance = x m

Now, in ΔBCD

$\tan 30^o = \frac{BC}{DC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = h\sqrt{3}$ ....(i)

In ΔACD

$\tan 45^o = \frac{AC}{DC}$

$\Rightarrow 1 = \frac{7+h}{x}$

$\Rightarrow x = 7 + h$

$\Rightarrow h\sqrt{3} - h = 7$ [By Using (i)]

$\Rightarrow h = \frac{7}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h = \frac{7(\sqrt{3}+1)}{3-1}$

$\Rightarrow h = \frac{7(1.73+1)}{2}$

$= \frac{19.11}{2}$

∴ Height of tower = 9.555 m.

Thus, the height of the tower is 9.552 m