Chapter - 10

Alternating Current

Whenever a voltage source is connected to a circuit, then free electrons in conductors have a motion in a particular direction, along with random motion. The rate of flow of charge at any point of the circuit is called current. In direct current circuits, the current source is a cell or a battery and a resistor R is used to control the current. Generally, the electrical energy is generated as alternating current due low cost of production and conveniance in transmission to long distances. It can be converted to direct current easily when required. Generally alternating current and voltage varies sinusoidal with time. To control it capacitor C and inductor L are also used, together with R. In a R-L-C circuit it is not necessary that current and voltage are in phase, i.e. it is not necessary that the current to be maximum when voltage is maximum. A transformer is used to step-up or step down the AC voltage, so that its transmission over a long distance is possible economically and at low energy loss.

In this chapter we will study phase relationship between AC voltage and current in different circuits, power, watt less current, transformer etc.

10.1 Direct Current

The current/voltage whose direction of flow does not change with time is called Direct Current. This current is produced by the such voltage sources whose terminals have constant polarity with time. If this current is ploted with time we get a straight line parallel to the time axis. This current (or voltage) is called unidirectional or direct current (or voltage). Its frequency f is zero. (fig. 10.1)

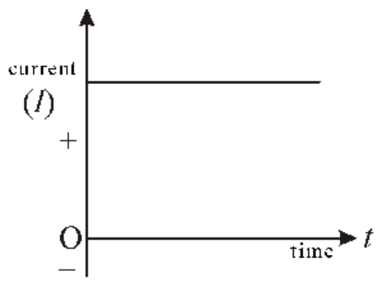


Fig 10.1 : Direct current

If the current from some special devices like rectifier is studied than it is found that it has a defenite direction but its value has a small peroidic/ pulsatingchange such currents are called direct current (or voltage) of unequal fluctuations or pulsating DC.

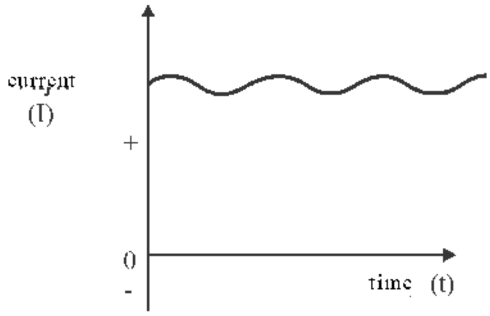


Fig 10.2 : Nonuniform DC

10.2 Alternating Current

The current (voltage) which changes its direction peoriodically with time and alternatively becomes positive and negative in each half cycle is called alternating current. It is obtained from the sources whose terminals change their polarity periodally with time.

Alternating current may be of many types according to their wave forms-few of them are following-

10.2.1 Square Wave Current

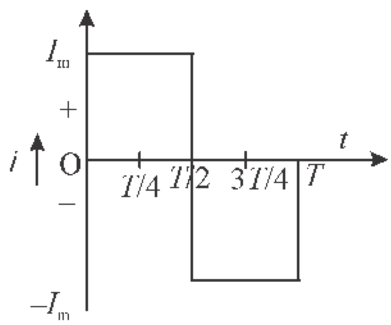


Fig 10.3 : Square ac wave

In this type of current, current will remain I_m (maximum) from t = 0 to t = T/2 and at T/2 it suddenly becomes - I_m, (minimum) which remains same up to t = T, again becomes zero at time T.

Thus for $0 \le t \le T/2$, $I = I_m$

for $T/2 \le t \le T$, $I = -I_m$

10.2.2 Triangular ac Wave Current

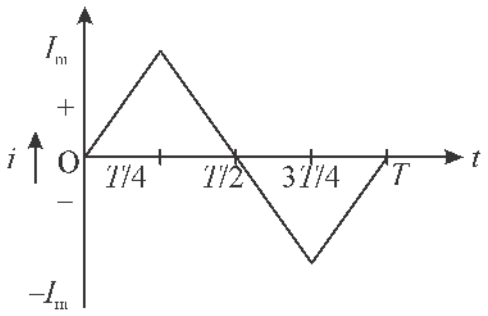


Fig 10.4 : Triangular ac wave current

In this type of current, the current linearly increases from 0 to I_m, from t = 0 to t = T/4: then linarly decreases to 0 at T/2 and becomes - I_m at t = 3T/4. Ultimately at t = T it becomes zero. (Fig 10.4)

at	$t = 0$	$I = 0$
at	$t = T/4$	$I = I_m$
at	$t = T/2$	$I = 0$
at	$t = 3T/4$	$I = -I_m$
at	$t = T$	$I = 0$

10.2.3 Sinusoidal Wave ac Current

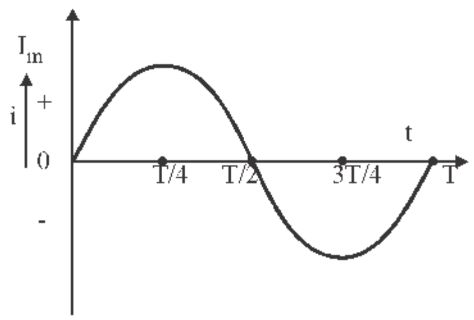


Fig 10.5 : Sinusoidal wave ac current

This is the simplest and basic form of ac It varies as sine or cosine functions. (Fig 10.5), hence are called sinusoidal current.

In this chapter, we will study sinusoidal ac.It is to be mentioned that all forms of ac i.e. square or triangle all are mathematically produced by super position of sinusoidal waves of different amplitude and frequencies.

The frequecny of ac used for domestic use is 50 Hz in India. In U.S.A it is 60 Hz. At any instant t the ac current and voltage are given by the equation :-

$$I = I_m \sin(\omega t + \phi) \qquad \ldots (10.1)$$

$$V = V_m \sin(\omega t) \qquad \ldots (10.2)$$

here I_m and V_m are the maximum value of ac current and voltage and are also called peak values of current and voltage.

The symbol of a.c. voltage is ~

This type of ac can be produced by a rotating coil in uniform magnetic field or by electronic oscillatory circuits.

10.3 Instantaneous, Peak, Average and Root Mean Square of Alternating Voltage and Current

10.3.1 Instantaneous Value

The value of current or voltage at any instant in an ac circuit is called instantaneous value.It can be zero, positive or negative. Equation (10.1) and (10.2) gives instantaneous values in a simple periodic form. Here ϕ is the phase difference in voltage and current at any instant t.

10.3.2 Peak Value

The maximum value of ac voltage or current in a complete cycle is called its peak value. It also represents the amplitude of alternating change. In equations (10.1) and (10.2) I_m and V_m are the peak values of alternating current and voltage respectively.

10.3.3 Average Value

In an ac circuit the magnitude and direction of voltage/current changes periodically with time. The average of all these values for a complete cycle is called average value of AC. For a complete cycle the average is -

$$I_{av} \text{(For a complete cycle)} = \frac{\int_0^T I\,dt}{\int_0^T dt} = \frac{I_m}{T}\left[\int_0^T \sin \omega t\, dt\right]$$

$$= \frac{I_m}{T}\left(\frac{-\cos \omega t}{\omega}\right)_0^T = \frac{-I_m}{\omega T}(\cos \omega T - \cos 0)$$

I_{av} (for a complete cycle) $= \frac{-I_m}{\omega T}(0)$

$(\because \omega T = 2\pi \;\text{ and }\; \cos 2\pi = 1\ )$

I_{av} (for complete cycle) = 0

Hence the average of AC for a complete cycle is always zero.

Average value for first positive half cycle-

$$I_{av} = \frac{\int_0^{T/2} I_m \sin \omega t\, dt}{\int_0^{T/2} dt} = \frac{I_m}{T/2}\left(-\frac{\cos \omega t}{\omega}\right)_0^{T/2}$$

$$= -\frac{2I_m}{\omega T}\left(\cos\frac{\omega T}{2} - \cos 0\right) = \frac{2I_m}{\pi} = 0.636\, I_m$$

similarly, for second half cycle the value will be

$$I_{av} = -0.636\, I_m$$

10.3.4 Root Mean Square Value

In an ac circuit for a complete cycle the square root of average of squares of current and voltage is called root mean square value of current and voltage current I_{rms} and voltage V_{rms} .

$$I_{rms} = \sqrt{I_{av}^2} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t\, dt}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T\left(\frac{1-\cos 2\omega t}{2}\right)dt} = \sqrt{\frac{I_m^2}{2T}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^T}$$

$$= \sqrt{\frac{I_m^2}{2T}\left(T - \frac{\sin 2\omega T}{2\omega}\right)} = \sqrt{\frac{I_m^2}{2T}(T-0)} = \sqrt{\frac{I_m^2}{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m \qquad \ldots (10.3)$$

Similarly $\quad V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m \qquad \ldots (10.4)$

The values shown by all ac meters in a circuit are the RMS values of these quantities.

The rms value is also called virtual or effective value which means that rate of heating effect in a resistor is same as that of equivalent value of dc and I_{rms}. The rate of heat generated by ac in a complete cycle is -

$$H_{av} = \frac{\int_0^T I^2 R\, dt}{\int_0^T dt} = I_{rms}^2 \times R$$

from a dc the rate current the raate of heat produced is $= I_{dc}^2 R$

Hence both rates are equal - $I_{dc} = I_{rms}$

Thus $\quad I \qquad = I \qquad = I_{rms} = \frac{I_m}{\sqrt{2}} \qquad \ldots (10.5)$

If we want to measure the ac voltage and current using dc moving coil meters, they will give zero reading, since the torque on the coil changes so rapidly, that it will not respond due to inertia of the coil.

To measure ac we use hot wire meters, which are based on heating effect. The heat produced depends on V_{rms}^2 or I_{rms}^2, hence the scale of meters is nor linearly marked. The relative distances between the making will be in the ratio of 1 : 4 : 9 : . . . etc. for the currents I, 2I, 3I, ... etc.

The domestic supply of ac in India has $V_{rms} = 220\text{ V}$. Hence its peak value will be

$$V_m = \sqrt{2}\, V_{rms} = \sqrt{2} \times 220 = 311\text{ V}$$

10.3.5 Properties of ac

Merits : -

(i) Alternating voltage can be steped up or steped down, so the transmission of electri power is possible at high voltage and low current with very small power loss for long distance transmission.

(ii) It can be easily converted to dc by a rectifier.

(iii) Alternating current generators and motors are more rugged, and convenient in operation, also their cost is less than dc generators and motors.

Demerits : -

(i) Alternating current voltage of any value is more dangerous than its equivalent dc voltage, because its peak value is $\sqrt{2}$ times the rms value.

(ii) Skin effect :- High frequency ac current does not uniformly pass through the whole cross-section of the conductor. It prefers the surface layer (skin) of the conductor. So a thick wire in replaced by a bunch of thin wires to reduce this effect.

(iii) It can't be used directly for elecrolysis, electro plating and making electro magnet.

Example 10.1 : Find RMS value of the AC current given by $I = I_1 \cos\omega t + I_2 \sin\omega t$.

Solution : $I = I_1 \cos\omega t + I_2 \sin\omega t$

hence

$$I^2 = I_1^2 \cos^2\omega t + I_2^2 \sin^2\omega t + 2I_1 I_2 \sin\omega t \cos\omega t$$

$$\overline{I^2} = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{\int_0^T (I_1^2 \cos^2\omega t + I_2^2 \sin^2\omega t) + 2I_1 I_2 \sin\omega t \cos\omega t}{\int_0^T dt} dt$$

$$\overline{I^2} = I_1^2 \times \frac{1}{2} + I_2^2 \times \frac{1}{2} + 0$$

since the average value of $\sin^2\omega t$ & $\cos^2\omega t$ for a complete cycle is 1/2 and that of $\sin 2\omega t = 0$.

$$I_{rms} = \sqrt{\overline{I^2}} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2}} = \frac{1}{\sqrt{2}}\left(I_1^2 + I_2^2\right)^{\frac{1}{2}}$$

Example 10.2 : The rms value of a sinusoidal ac of frequecny 50 Hz is $200\sqrt{2}$ V . Write down the equation for its instantaneous value at time t.

Solution : Given

$V_{rms} = 200\sqrt{2}$ V and $f = 50 Hz$

So $V_m = \sqrt{2}\, V_{rms} = \sqrt{2} \times 200\sqrt{2} = 400$ V

$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314$ rad/s

hence $V = V_m \sin\omega t$

$V = 400 \sin 314t$ V

Example 10.3 : Find the frequency of ac voltage given by $V = 400 \sin 100\pi t$

Solution : General equation of AC voltage is

$V = V_m \sin\omega t = V_m \sin 2\pi f t$

and the given equation of voltage

$V = 400 \sin 100\pi t$

comparing the two equations we get

$2f = 100$

Thus

$f = 50$ Hz

Example 10.4 : The peak value of ac current in a circuit is 5 A. What will be the value of current given by (i) ac ammeter (ii) dc ammeter.

Soution : (i) Since ac ammeter always meausres rms value

then $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 2.5 \times \sqrt{2} = 3.535$ A

(ii) dc ammeter meausres averages value for one cycle hence $I = 0$.

Example 10.5 : The rms voltage in a circuit is 220 V then find peak value of voltage.

Solution : $V_m = \sqrt{2} \times V_{rms}$

Given $V_{rms} = 220$ V

$V_m = 220 \times \sqrt{2} = 311.08$ V

Example 10.6 : The ac current is given by $I = 3 \sin 2\pi t$ A , then find (i) rms value of current (ii) instantaneous value of current at $t = \frac{1}{2} s$.

Solution : (i) $I_{rms} = \frac{I_m}{\sqrt{2}}$, then

Given $I_m = 3$ A, $t = \frac{1}{2} s$

$I_{rms} = \frac{3}{\sqrt{2}} = 2.12$ A

(ii) $I = 3 \sin 2\pi \times \frac{1}{2} = 3 \sin\pi = 0$

Example 10.7 : Find the time to reach from zero to its maximum value of ac current of frequency 50 Hz .

Solution : Time taken by current to reach from zero to its maximum value is

$$t = \frac{T}{4}$$

thus $t = \frac{1}{4f}$

given $f = 50$ Hz

thus $t = \frac{1}{4 \times 50} = 0.005\, s$

10.4 Phase Relation between alternating voltage and alternating current in different types of ac circuits and phasor diagram

10.4.1 Pure resistive ac Circuit

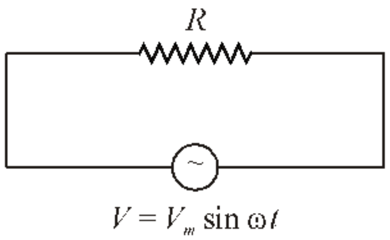


Fig 10.6 : Pure resistor in ac circuit

In fig 10.6 a pure resistor R is connected to an ac source voltage $V = V_m \sin \omega t$. If the current in the circuit be I, then using Kirchhoff's loop law, the voltage developed across R is equal to voltage applied.

$$V_m \sin \omega t = IR$$

$$I = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \ldots (10.6)$$

here $I_m = \frac{V_m}{R}$ is the peak value of ac current. It is clear from equation (10.6) that on applying sinusoidal voltage, we get sinusoidal current, and both are in same phase. It means that value of V and I will be zero simultaniously and maximum simultaniously.

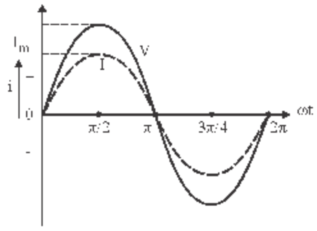


Fig 10.7 : Sinusoidal nature of ac voltage and current and their phase relation in pure resistive ac circuit

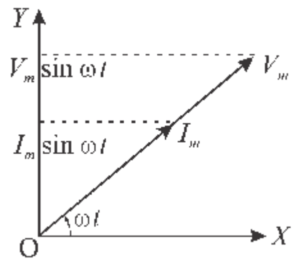


Fig 10.8 : Phasor diagram in pure resistive ac circuit

Fig 10.7 represent the sinusoidal nature of ac voltage and current. As mentioned earlier it is not always necessary that V and I are in same phase in ac circuits. For such ac circuits the concept of phasor makes the analysis simple. To represent ac quantities (V, I, R, X, Z etc), a rotating vector is used whose magnitude equals to the peak value and rotational frequency is equal to the frequency of ac voltage or current. Such a rotating vector is called phasor, and the related diagram is called phasor diagram. If the tail of rotating vector is at origin, and vector coincide with X axis at t = 0, then at instant t it makes an angle $\theta = \omega t$ with X-axis, and the y component gives the instantaneous value at instant t. Fig 10.8 shows the values of ac voltage and ac current at an instant t, represented by their respective phasors. These phasor are shown as making an angle ωt with x-axis. Both phasor rotates in anticlock-wise direction with frequency ω. The vertical components of the phasor represents instantaneous values of AC voltage or current that's why they are taken in Y-axis. Fig 10.8 shows the phasor diagram for a pure resistive circuit, V and I are in same direction and phase difference between them is zero. But as you will see later, they will not be in same phase for inductive or capacitive circuits.

A pure resistance obstructs the flows of current which is independent of the frequency of applied ac.

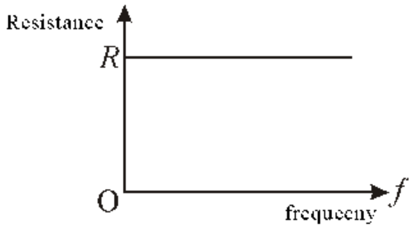


Fig 10.9 : Dependence of R on frequecny

10.4.2 A Pure Inductive ac Circuit

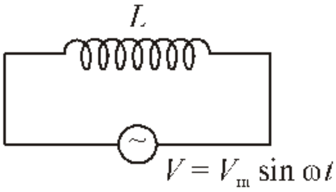


Fig 10.10 : Pure inductive ac circuit

Fig 10.10 represents pure inductor (coil of thick copper wire of resistance, R=O) is joined to an ac voltage $V = V_m \sin \omega t$. Inductance of the coil is L. Since the voltage changes with time, the current will also change. The voltage developed

across the inductor $\left(-L\dfrac{dI}{dt}\right)$. From Kirchhoff's

law $V_m \sin \omega t = \dfrac{LdI}{dt}$,

$$dI = \frac{V_m}{L}\sin \omega t\, dt \quad \ldots (10.7)$$

The current $\int dI = \int \dfrac{V_m}{L}\sin \omega t\, dt$

$$I = \frac{V_m}{L}\left(-\frac{\cos \omega t}{\omega}\right)$$

$$I = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \ldots (10.8)$$

here $I_m = \dfrac{V_m}{L\omega}$. The Lω has the same dimensions as resistance and called Inductive reactance, represented by X_L.

$$X_L = L\omega \quad \ldots (10.10)$$

and $$I_m = \frac{V_m}{X_L} \quad \ldots (10.11)$$

X_L controlls current in ac circuit in the same way as a resistor, (but with a different mechanism given later). Unit of X_L is ohm.

From equation (10.8) it is evident that the current in pure inductive circuit is also sinusoidal with same frequency that of applied voltage, but lags behind the applied voltage by a phase of $\pi/2$. Which means that the current gets its maximum value compared to voltage after a time interval of T/4 ; (Fig 10.11).

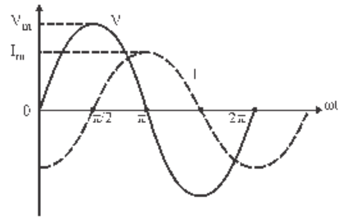


Fig 10.11 : Graph of V and I with ωt

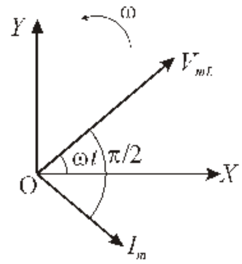


Fig 10.12 : Phasor diagram for pure inductive circuit

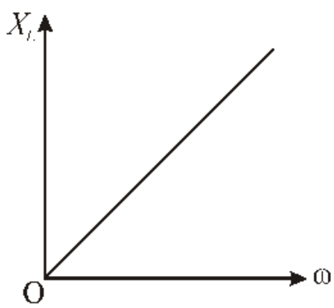


Fig 10.13 : Variation of X_L with frequency

From equation 10.10 $X_L = L\omega$

$$X_L = L \times 2\pi f \quad \ldots (10.12)$$

Fig 10.13 represents graph between X_L and ω. Slope of this graph $\tan\phi$, represents cofficient of self inductance of the coil.

For direct current, $f = 0$, hence $X_L = 0$, so a pure inductor short circuits the dc circuit, but opposes the flow of ac current.

In a pure inductive circuit alternative voltage is $V = V_m \sin\omega t$ and the current is given by

$I = I_m \sin(\omega t - \frac{\pi}{2})$ so voltage leads the current by $\pi/2$ or 90^0.

The magnetic flux $\phi = LI$; $\phi \propto I$ and the power P = VI.

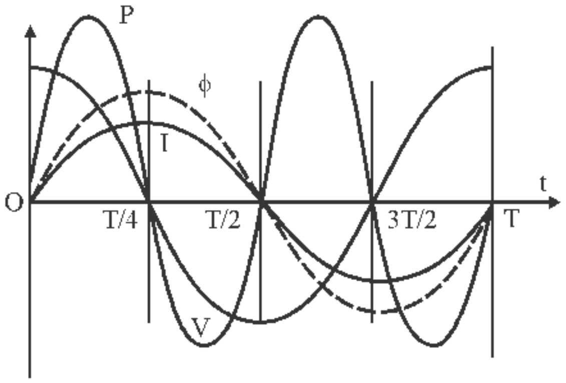


Fig 10.14 : Graph of Power in pure inductive ac circuit

Fig 10.14 shows all the four quantities in one complete cycle of ac, for an inductor.

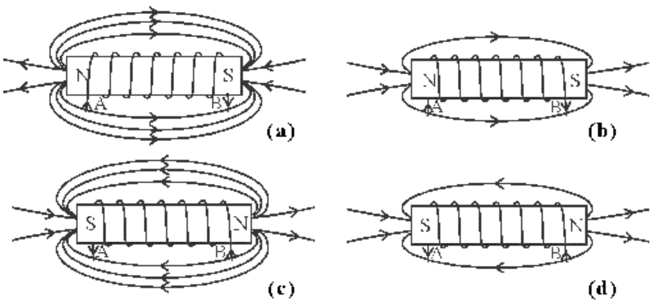


Fig 10.15 : Change in and for an inductor for one complete cycle. (a) 0 to T/4 (b) T/4 to T/2 (c) T/2 to 3T/4 (d) 3T4/4 to T

To understand the power and flux change in an inductive circuit. Let us consider the fig 10.15 in which change in current is considered with its respective change in flux.

In fig (A) the current enters at A and reach the maximum value, so the flux. The core is magnetized, voltage and current are both positive hence their product, power is also positive. It means the circuit absorbs energy from source.

Fig (B) current decreases from T/4 to T/2. At T/2 the core is demagnetized and the total flux becomes zero. Voltage is negative and current is positive, which means that power is negative, it implies that circuit returns rhe absorbed energy to the source.

Fig (C) the current is increasing in opposite direction during T/2 to 3T/4 and flux also, core is magnetized in opposite direction. Both voltage and current are negative, their product power is positive. The circuit absorbs energy from source.

Fig (D) from 3T/4 to T current decreases to zero, V is positive while I is negative, Power is negative, core is demagnetized and the circuit returns the absorbed energy to the source.

For a complete cycle, average power in an inductor is zero. We will prove this in section 10.7.

10.4.3 Pure Capacitive ac Circuit

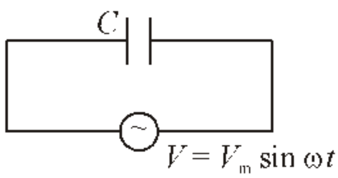


Fig 10.16 : A capacitor connected to ac source

Fig 10.16 shows a capacitor of capacity C is connected to ac voltage $V = V_m \sin\omega t$. If the current in the circuit is I, and voltage developed across the capacitor is V_C, then from Kirchhoff's law $V - V_C = 0$; If change on capacitor in q at time t, then instantaneous voltage accross it is

$$V_C = \frac{q}{C}$$

so $\frac{q}{C} = V_m \sin\omega t$ or $q = V_m C \sin\omega t$

Thus, current in the circuit is

$$I = \frac{dq}{dt} = \frac{d}{dt}(V_m C \sin\omega t)$$

$$I = V_m C\omega \cos \omega t$$

since $\quad \cos \omega t = \sin(\omega t + \frac{\pi}{2})$

hence $\quad I = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots (10.13)$

where $\quad I_m = V_m C\omega$

or $\quad I_m = \frac{V_m}{1/C\omega} \qquad \ldots (10.14)$

The dimension of $1/C\omega$ is that of a resistor and its unit is ohm, and it is the measure of obstacle produced by a capacitor in the ac circuit. It is called capacitive reactance, and expressed as X_C.

$$X_C = \frac{1}{C\omega}$$

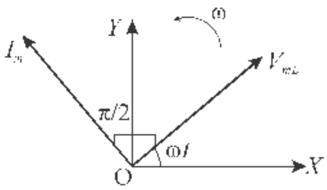


Fig 10.17 (A) Phasor diagram for pure capacitive ac circuit

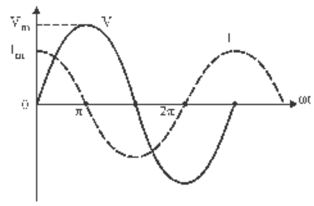


Fig 10.17 (B) V and I plotted against ωt

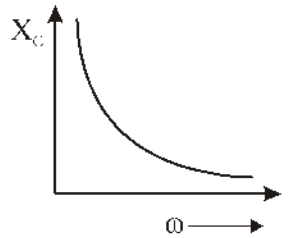


Fig 10.17 (C) Variation of X_C with ω

As it is evident from the equation 10.13, that if a sinusoidal voltage is applied to a pure capacitor the current will also be sinusoidal, but leads the voltage by $\pi/2$. It means that current reach the maximum value earlier by time T/4 compared to the voltage.

From equation (10.15) $X_C = \frac{1}{C\omega}$ and graph (10.17) (C) show the variation of X_C with ω. For dc, $f = 0$ ($\omega = 2\pi f = 0$) $X_C = \infty$ so dc is not allowed by a capacitor. But for ac, X_C has some finite value, so a capacitor allows it to pass through.

For a pure capacitive circuit

$V = V_m \sin \omega t$; and $I = I_m \sin(\omega t + \frac{\pi}{2})$,

hence the voltage lags the current by $\pi/2$ radian.

q = CV and $P = VI$. All the four quantities are plotted against time in fig (10.18) for a complete cycle.

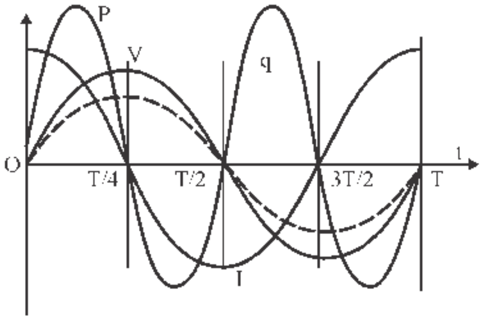


Fig 10.18 : Graph for power in a capacitor

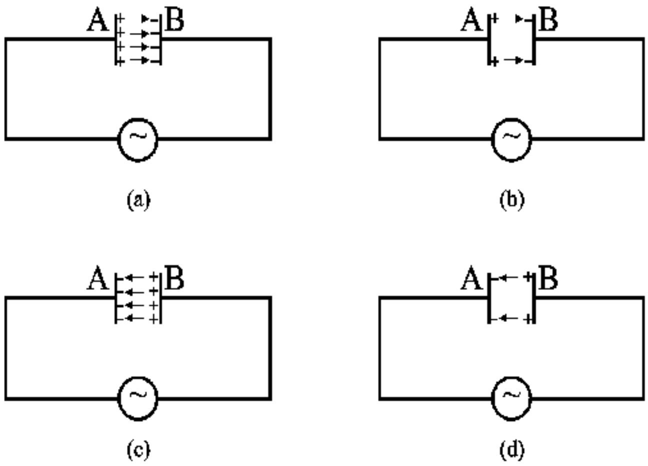


Fig 10.19

Fig 10.19 : (a) shows that from t = 0 to T/4 current increases from zero to maximum, hence the charge on the plates and the potential. Plate a becomes positive while B is negative. Circuit absorbs energy from the source.

Fig 10.19 (b) the current is in opposite direction, the charge on the plate decreases to zero hence the potential. Circuit returned energy to the source from $T/4$ to $T/2$. Fig 10.19 (c) the charge on the plates rises to its maximum value, hence the potential and field between the plates. The circuit absorbs energy from the source from $T/2$ to $3T/4$. Plate A becomes negative while the plate B is positive. Voltage and current both are negative. So power is positive.

Fig 10.19 (d) again shows that during $3T/4$ to T, the stored charges decreases, as the voltage. At T,the capacitor is completely discharged. Current is positive while potential is negative, hence power is negative. It means that during this interval. Circuit returns the abrobed energy to the source.

In this way the net energy absorbed during one complete cycle by a capacitor is always zero.

Example 10.8: A resistanceless coil of inductance $L = \frac{5}{\pi}$ m H is connected to an ac source of frequency 50 Hz. Find the inductive reactance of the coil. If the current in the circuit is 0.5 A. Find the voltage across the coil.

Solution : Inductive reactance $X_L = L\omega$

or $\quad X_L = L \times 2\pi f$

given $\quad f = 50\,\text{Hz}, L = 5/\pi\,\text{mH}, I = 0.5\text{ A}$

so $\quad X_L = \frac{5}{\pi} \times 2\pi \times 50 \times 10^{-3} = 0.5\,\Omega$

Voltage developed across inductor is

$$V_L = I \times L = 0.5 \times 0.5 = 0.25\text{ V}$$

Example10.9 : Capacity of a capacitor is 50 pF. Find its capacitive reactance at 5 kHz.

Solution : Capacitive reactance

$$X_C = \frac{1}{C\omega} = \frac{1}{C \times 2\pi f}$$

given $\quad C = 50\text{ pF} = 50 \times 10^{-12}\text{ F}, f = 5 \times 10^3\text{ Hz}$

so $\quad X_C = \frac{1}{50 \times 10^{-12} \times 2 \times 3.14 \times 5 \times 10^3}$

$$= 6.37 \times 10^4\,\Omega$$

Example 10.10 : A capacitor of capacity 1 μF is connected to a source $V = 200\sqrt{2}\sin 100t$ V. Find the currrent in the circuit.

Solution : $V_{rms} = \frac{V_m}{\sqrt{2}}$

given $\quad V_m = 200\sqrt{2}\text{ V},\ \omega = 100\text{ rad/s}$

$C = 10^{-6}$ F

Now $\quad V_{rms} = \frac{200\sqrt{2}}{\sqrt{2}} = 200\text{ V}$

and $\quad X_C = \frac{1}{C\omega} = \frac{1}{100 \times 10^{-6}} = 10^4\,\Omega$

hence $\quad I_{rms} = \frac{V_{rms}}{X_C} = \frac{200}{10^4} = 0.02\text{ A}$

Example 10.11: A coil is used with a 50 Hz ac source. What will be value of inductance to obtain a reactance of 100 Ω?

Solution : Inductive reactance $X_L = L \times 2\pi f$

given $\quad X_L = 100\Omega, f = 50\,Hz$

$$L = \frac{X_L}{2\pi f} = \frac{100}{2 \times 3.14 \times 50} = 0.318\text{ H}$$

10.4.4 L-R Series ac Circuit

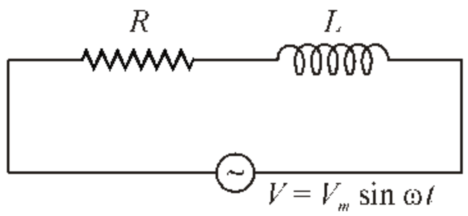


Fig 10.20 : R-L series ac circuit

Alternating voltage $V = V_m \sin \omega t$ is applied to the R-L series circuit. At any instant t the current in the circuit is I, V_L and V_R are the potential at L and R. The net potential developed across R and L is V_{RL}. Then from Kirchhoff's law :-

$$V - V_{RL} = 0$$

From phasor diagram 10.21 (A) we find that V_{mR} and I_m are in phase. But V_{mR} and V_{mL} have a phase difference of $\pi/2$. They are normal to each other. The resultant voltage. $V_{mRL} = \sqrt{V_{mR}^2 + V_{mL}^2}$

But $\quad V_{mR} = I_{mR}R$ and $V_{mL} = I_m X_L$

hence $\quad V_{mRL} = \sqrt{I_m^2 R^2 + I_m^2 X_L^2}$

and $\quad I_m = \dfrac{V_{mRL}}{\sqrt{R^2 + X_L^2}} \quad \ldots (10.16)$

here $\sqrt{R^2 + X_L^2}$, is the effective obstacle of the L-R combintion for ac current; which is given by Z and called impedance of the circuit.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (L\omega)^2} \quad \ldots (10.17)$$

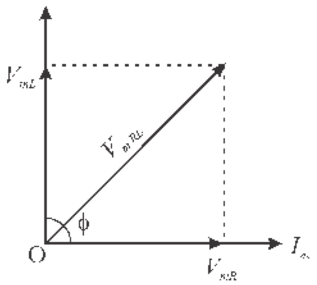


Fig 10.21 (A) Phasor diagram

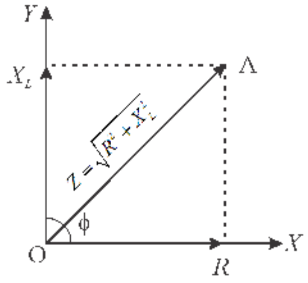


Fig 10.21 (B) Inpedance diagram

From Fig 10.21 (A) it is clear that the current in L-R series circuit is leading the applied voltage by ϕ.

Hence $I = I_m \sin(\omega t - \phi) \quad \ldots (10.18)$

Similarly, from fig 10.21 (B) we find the impedance Z of the circuit. R and X_L are in X and Y direction. The resultant Z makes an angle ϕ with X-axis.

So $\tan\phi = \dfrac{V_{mL}}{V_{mR}} = \dfrac{I_m X_L}{I_m R} = \dfrac{X_L}{R}$

hence $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) \quad \ldots (10.19)$

In R-L series ac circuit, the graph between ac voltage and current with ωt is given by fig10.22 (A) where the current lags behind the applied voltage by an angle ϕ. Fig 10.22 (B) represents phasor diagram of ac voltage and current.

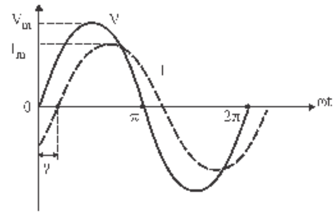


Fig 10.22 (A) Alternating voltage and current in a series R-L circuit

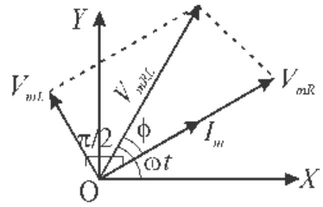


Fig 10.22 (B) Series R-L circuit phasor diagram

Example 10.12 : A 0.5 H inductor gives 0.5 A in a 100 V dc source. If the current in the circuit.

Solution : For dc source $X_L = 0$,

thus $R = \dfrac{V}{I}$

given is $L = 0.5$ H, $V = 100$ V

$f = 50$ Hz,

$$R = \frac{100}{0.5} = 200\,\Omega$$

In ac circuit, $Z = \sqrt{R^2 + L^2\omega^2}$

where $\quad \omega = 2\pi f = 2 \times 3.14 \times 50 = 314$ rad/s

so $\quad Z = \sqrt{(200)^2 + (0.5 \times 314)^2} = 254.26\,\Omega$

$$I = \frac{V}{Z} = \frac{100}{254.26} = 0.39\text{ A}$$

Example 10.13 : An electric bulb has rating 100V, 10 A. If it is used at 200 V; 50 Hz ac circuit, then find the inductance of the choke coil used in series.

Solution : Impedance $Z = \sqrt{R^2 + (L\omega)^2}$ for bulb

$V = 100$ V, $\ I = 10$ A

$f = 50\,\text{Hz}$, $V = 200$ V

Resistance of the bulb R = V/I

$$R = \frac{100}{10} = 10\Omega$$

$$Z = \frac{\text{ac voltage}}{\text{current}} = \frac{200}{10} = 20\text{W}$$

$$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314\,\text{rad/s}$$

using $Z = \sqrt{R^2 + (2\pi fL)^2}$

$$20 = \sqrt{(10)^2 + (314 \times L)^2}$$

$$L^2 = \frac{300}{314 \times 314}$$

$$L = \frac{\sqrt{300}}{314} = 0.055\,\text{H}$$

Example 10.14 : A coil of self inductance $1/\pi$ H is in series with a 300 Ω resistor. A 200 V, 200 Hz ac is applied to the combination. Find phase difference between voltage and current.

Solution : $\tan\phi = \frac{L\omega}{R} = \frac{2\pi fL}{R}$

$$L = \frac{1}{\pi}\,\text{H}, \; f = 200\,\text{Hz}, \; R = 300\,\Omega$$

hence $\tan\phi = \frac{2\pi \times 200 \times 1}{300 \times \pi} = \frac{4}{3}$

so $\phi = \tan^{-1}\left(\frac{4}{3}\right)$

Example 10.15 : A coil is connected to a 220 V and 50 H_z alternating source develops 200 watt power it draws a current of 2 A. Find the resistance and inductance of the coil.

Solution : Power $P = I_{rms}^2 R$

given $V_{rms} = 220\,\text{V}$, $f = 50\,\text{Hz}$, , $I_{rms} = 2\,\text{A}$

$P = 200$ W

$$R = \frac{P}{I_{rms}^2} = \frac{200}{(2)^2} = 50\Omega$$

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{220}{2} = 110\Omega$$

From $Z^2 = R^2 + X_L^2$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(110)^2 - (50)^2} = 98\Omega$$

$$X_L = L \times 2\pi f$$

$$L = \frac{X_L}{2\pi f} = \frac{98}{2 \times 3.14 \times 50}$$

$$L = 0.312\,\text{H}$$

Example 10.16 : A coil of inductance 0.4 H and negligible resistance is in series with 120 Ω resistor. If it is connected to 200 / π Hz; 100 V ac source, then find total impedance, phase angle and current in the circuit.

Solution : Impedance $Z = \sqrt{R^2 + (L\omega)^2}$

given is $L = 0.4\,\text{H}, R = 120\,\Omega, f = \frac{200}{\pi}\text{Hz}$

$$V_{rms} = 100\,\text{V}$$

$$Z = \sqrt{(120)^2 + (400 \times 0.4)^2}$$

$$Q = 200\,\Omega$$

$$\omega = 2 \times 3.14 \times \frac{200}{\pi} = 400\,\text{rad/s}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{400 \times 0.4}{120}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{200} = 0.5\,\text{A}$$

10.4.5 R-C Series ac Circuit

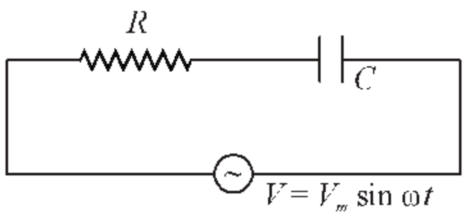


Fig 10.23 A series RC ac circuit

In fig 10.23 an ac voltage $V = V_m \sin \omega t$ is applied to a series combination capacitance C and resistance R. At any instant t, if the voltage across R and C are V_R and V_C and the current is I the resultant voltage developed across the R-C combination is V_{RC}. From Kirchhoff's law we get $V - V_{RC} = 0$.

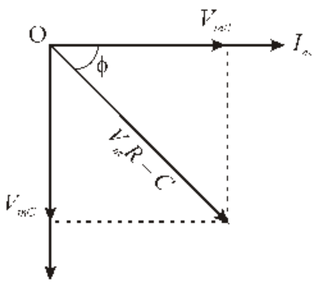


Fig 10.24 Phasor diagram of RC series ac circuit

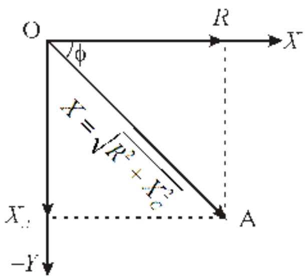


Fig 10.25 Impedance diagram

From phasor diagram 10.24, it is evident that V_{mR} is in phase with I_m, But V_{mC} lags behind the current by $\pi/2$. The resultant potential across the combination is

$V_{mRC} = \sqrt{V_{mR}^2 + V_{mC}^2}$. But $V_{mR} = I_m R$ and $V_{mc} = I_m X_C$

$$V_{mRC} = \sqrt{(I_m R)^2 + (I_m X_C)^2}$$

$$V_{mRC} = I_m \sqrt{R^2 + X_C^2}$$

$$= \frac{V_{mRC}}{I_m} \qquad \ldots (10.20)$$

or $$\sqrt{R^2 + X_C^2}, = \frac{V_{mRC}}{I_m} = Z$$

here $\sqrt{R^2 + X_C^2}$, is the effective obstacle in the circuit and is called impedance Z of the circuit. Hence

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2} \qquad \ldots (10.21)$$

From fig 10.24, it is clear that on applying sinusoidal voltage to the RC series combination sinusoidal current is obtained. But the current leads the voltage by an angle ϕ.

$$I = I_m \sin(\omega t + \phi) \qquad \ldots (10.22)$$

From fig 10.25 we get the imdepance of the circuit. Resistance is shown on X-axis, while X_C on Y-axis. The resultant Z is represented by OA, which makes an angle ϕ with X-axis.

$$\tan\phi = \frac{V_{mC}}{V_{mR}} = \frac{I_m X_c}{I_m R} = \frac{X_c}{R}$$

and $$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) \qquad \ldots (10.23)$$

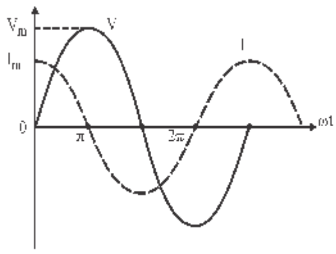


Fig 10.26 : V-I graph for series ac circuit

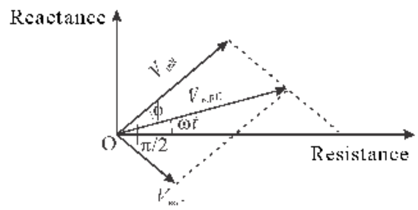


Fig 10.27 Phasor diagram for the circuit

The variation of ac voltage and current in series R-C circuit with ωt is as shown in Fig 10.26. It is clear that both the voltage and current have same freqency, but current leads the potential by an angle ϕ. Fig 10.27 shows the phasor diagram for the combination.

Example 10.17: A series combination of 100 μF capacitor and 40 Ω resistor are joined to 110 V, 60 Hz ac source. Find maximum current in the circuit.

Solution : $I_m = \dfrac{V}{\sqrt{R^2 + \dfrac{1}{C^2\omega^2}}}$

given $V = 110$ V, $C = 100 \times 10^{-6}$ F

$R = 40\ \Omega$, $f = 60$ Hz

$$I_m = \frac{110}{\sqrt{(40)^2 + \frac{1}{(100\times10^{-6}\times2\times3.14\times60)^2}}}$$

$$= \frac{110}{\sqrt{(40)^2 + \frac{1}{(376.8\times10^{-4})^2}}} = \frac{110}{\sqrt{(40)^2 + (26.54)^2}}$$

$$= 2.29 \text{ A}$$

10.4.6 L-C-R Series ac Circuit

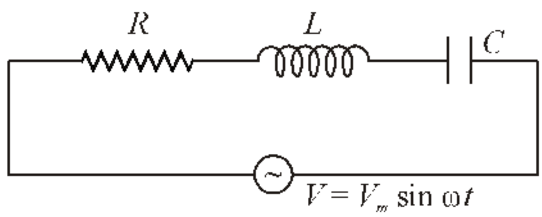


Fig 10.28 : L-C-R series ac circuit

The AC voltage $V = V_m \sin \omega t$ is applied to the series L-C-R circuit. At any instant t, the cutrrent in the cuircuit is I. The voltage developed across the elements is V_R, V_L and V_C. $V_R = IR$, $V_L = IXL$ and $V_C = IX_C$. The net voltage developed across the series combination is V_{LCR}. From Kirchhoff's loop law we get $V - V_{LCR} = 0$

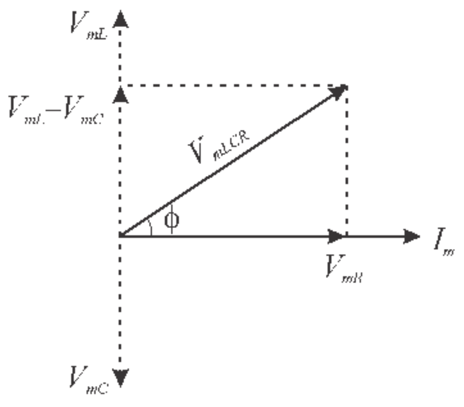


Fig 10.29 : Phasor diagram for $X_L > X_C$

From the phasor diagram (10.29) we see that V_{mR} is in phase with I_m while V_{mL} leads the current by $+\pi/2$ and V_{mC} lags the current by $-\pi/2$. Voltage across L and C are opposite to each other, hence V_{LC} = (V_{mL} - V_{mC}); which is perpendicular to V_{mR}.

(For $X_L > X_C$)

So $V_{mLCR} = \sqrt{(V_{mR})^2 + (V_{mL} - V_{mC})^2}$ and

$V_{mR} = I_m R$, $V_{mL} = I_m X_L$ and $V_{mC} = I_m X_C$

$$V_{mLCR} = \sqrt{(I_m R)^2 + (I_m X_L - I_m X_C)^2}$$

$$= I_m \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_m = \frac{V_{mLCR}}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad \ldots (10.24)$$

$\sqrt{R^2 + (X_L - X_C)^2}$ = Z is the effective resistance of the series combination, and is called impedance of the circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \ldots (10.25)$$

$$\tan \phi = \frac{X_L - X_C}{R}$$

$$Z = \sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2} \qquad \ldots (10.26)$$

The current I_m lags the voltages V_{CR} by an angle ϕ.

If $X_C > X_L$; $V_{mC} > V_{mL}$ the current will lead the voltage.

$$\tan \phi = \frac{V_{mL} - V_{mC}}{V_{mR}} = \frac{I_m X_L - I_m X_C}{I_m R}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad \ldots (10.27)$$

From eq. (10.26) and (10.27) it is clear that both Z and ϕ depends on the three element R, X_L and X_C.

Special Conditions -

(i) If $V_{mL} > V_{mC}$;k $X_L > X_C$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ From eq. (10.25)

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ From eq. (10.27)

The phasor diagram is fig. 10.29. The value of ϕ

will be positive and have the value between 0 to $\pi/2$; the circuit will behave like L-R circuit and current lags the potential. In this condition Z is given by fig. (10.30); ac voltage and currents are given by phasor diagram fig.(10.31). The current in the circuit is given by

$$I = I_m \sin(\omega t - \phi) \quad \ldots (10.28)$$

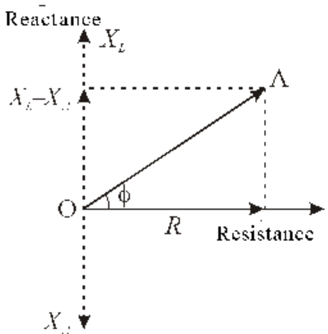


Fig 10.30 ($X_L > X_C$) impedance diagram

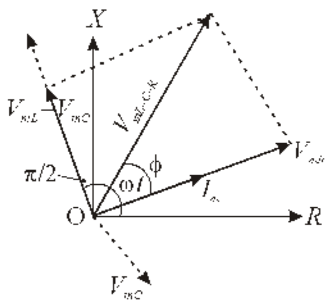


Fig 10.31 Phasor diagram ($X_L > X_C$)

(ii) If $X_C > X_L$ and $V_{mc} > V_{ML}$ then from equation (10.25) $Z = \sqrt{R^2 + (X_C - X_L)^2}$

and From (10.27) $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

In this current leads the applied voltage and value of ϕ will be between 0 to $\pi/2$. And the circuit will behave like R-C circuit. And the current in the circuit is given by

$$I = I_m \sin(\omega t + \phi)$$

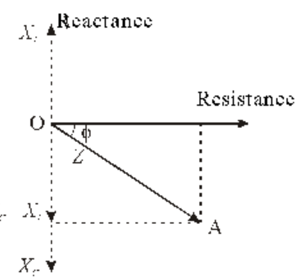


Fig 10.32 ($X_C > X_L$) impedance diagram

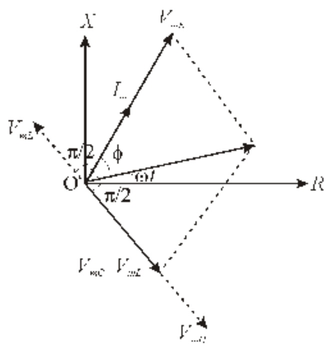


Fig 10.33 Phasor diagram for ($X_C > X_L$)

(iii) If $X_L = X_C$ or $V_{mL} = V_{mC}$ then $\phi = 0$, and the resultant voltage and current are in phase i.e they have the some phase. This condition is called resonance.

10.5 L-C-R Series Resonance Circuit

If an ac circuit contains L,C and R in series, then normally a phase diffence exists between voltage and current due to imdepance of these elements.

If the frequency of the applied ac voltage is increased ωL get increased and $1/\omega C$ decreased whereas R is unaffected. By decreasing frequency, ωL is decreases $1/\omega C$ increases. A condition will reach when $X_L = X_C$; so that the resultant reactance $X_L - X_C = 0$, $\phi = 0$. The current in the circuit is maximum. This condition is called resonance. And circuit is called series resonant circuit.

For resonance $X_L = X_C$ $\quad \ldots (10.29)$

and $X = X_L - X_C = 0$ $\quad$ From equ. (10.25)

$Z = Z_{min} = R$ $\quad \ldots (10.30)$

which means the impedance of circuit will be minimum and equal to resistance.

From eq. (10.27) $\phi = \tan^{-1}(0) = 0$ $\quad \ldots (10.31)$

which indicates that the resultant voltage and current are in same phase. Hence from equ. (10.28)

$$I = I_m \sin \omega t \quad \ldots (10.32)$$

At resonance angular frequency ω_r; $X_L = X_C$

$$L\omega_r = \frac{1}{C\omega_r} \text{ or } \omega_r = \frac{1}{\sqrt{LC}} \quad (\omega_r = 2\pi f_r) \ldots (10.33)$$

and the resonant frequency f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \ldots (10.34)$$

The peak value of current is $(I_m)_{max} = \frac{V_{mLCR}}{R}$

or $\quad (I_{rms})_{max} = \frac{V_{rms}}{R}$

For the condition of resonance, impedance and phasor diagrams are given by Fig. (10.34) and Fig. (10.35).

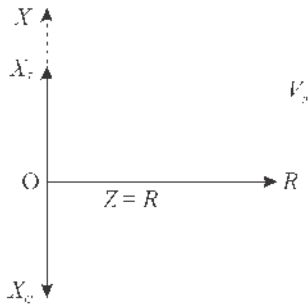


Fig 10.34 Impedance at resonance

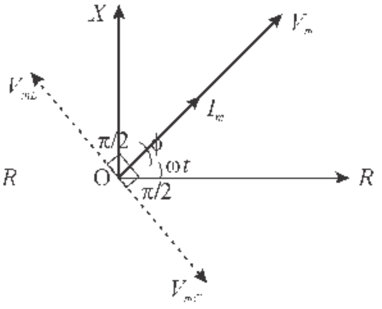


Fig 10.35 Phasor diagram for resonant LCR

The variation of I_m and impedance Z, of he series L-C-R circuit is given by fig. 10.37 and fig. 10.36. At resonant frequency (I_m) will be maximum and Z will be minimum.

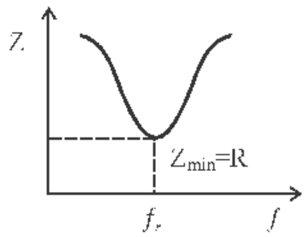


Fig 10.36 Graph between f and Z

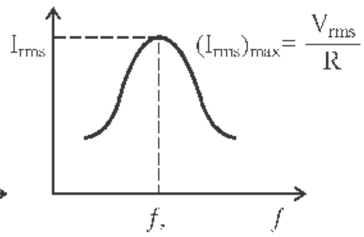


Fig 10.37 Graph between f and I_m

Analytical Solution of L-C-R Series Circuit

For L-C-R series circuit the voltage equation is

so $L\frac{dI}{dt}+IR+\frac{q}{c}=V_m\sin\omega t$ but $I=\frac{dq}{dt}$;

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{c}=V_m\sin\omega t \qquad \text{. . . (i)}$$

This equation is similar to the equatition of forced oscillations, hence its solution is

$$q=q_m\sin(\omega t+\theta) \qquad \text{. . . (ii)}$$

and $\frac{dq}{dt}=q_m\omega\cos(\omega t+\theta)$ . . . (iii)

$$\frac{d^2q}{dt^2}=-q_m\omega^2\sin(\omega t+\theta) \qquad \text{. . . (iv)}$$

Substituting these values in equ. (i)

$$q_m\omega[R\cos(\omega t+\theta)+(X_C-X_L)\sin(\omega t+\theta)]=V_m\sin\omega t$$

$$q_m\omega z\left[\frac{R}{Z}\cos(\omega t+\theta)+\left(\frac{X_C-X_L}{Z}\right)\sin(\omega t+\theta)\right]=V_m\sin\omega t$$

Let us $\frac{R}{Z}=\cos\phi,\ \frac{X_C-X_L}{Z}=\sin\phi$

$$\tan\phi=\frac{X_C-X_L}{R}$$

$$\phi=\tan^{-1}\left(\frac{X_C-X_L}{R}\right) \quad \text{Or}$$

$$q_m\omega z[\cos(\omega t+\theta)\cos\phi+\sin(\omega t+\theta)\sin\phi]=V_m\sin\omega t$$

$$q_m\omega z\ \cos(\omega t+\theta-\phi)=V_m\sin\omega t$$

Comparing both sides of the above equation

$$V_m=q_m\omega z=I_m z$$

So $I_m=q_m\omega$

and $\theta-\phi=-\frac{\pi}{2}$

$$\theta=-\frac{\pi}{2}+\phi$$

From equatiom (iii)

$$\frac{dq}{dt}=I=I_m\cos(\omega t+\phi-\frac{\pi}{2})$$

Or $I=I_m\sin(\omega t+\phi)$

$$\frac{R}{z}=\cos\phi \text{ and } \frac{X_C-X_L}{Z}=\sin\phi$$

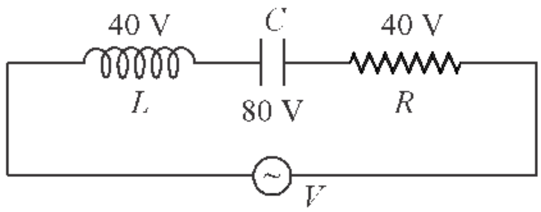
Squaring and adding

$$\frac{R^2}{Z^2}+\frac{(X_C-X_L)^2}{z^2}=1$$

$$Z^2=R^2+(X_C-X_L)^2$$

$$Z=\sqrt{(R^2)+(X_C-X_L)^2}$$

Example 10.18: Find the voltage of the given ac circuit.



Solution : From Phasor

$$V_{rms} = \sqrt{V_R^2 + (V_C - V_L)^2}$$

here $V_R = 40$ V,

$V_L = 40$ V, $V_C = 80$ V

$$= \sqrt{(40)^2 + (80-40)^2}$$

$$= 40\sqrt{2} = 56.56\,\text{V}$$

Example 10.19 : A series L-C-R circuit contains $R = 12\ \Omega$, $X_L = 18\ \Omega$ and $X_C = 23\ \Omega$. Find the impedance of the circuit and phase difference.

Solution : Impedance of the circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$R = 12\Omega, X_C = 23\Omega, X_L = 18\Omega$$

$$Z = \sqrt{(12)^2 + (23-18)^2}$$

$$= \sqrt{144+25} = \sqrt{169} = 13\,\Omega$$

and phase difference is given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{5}{12}$$

$$\phi = \tan^{1}\left(\frac{5}{12}\right)$$

Example 10.20 : A voltage source of 110 V; 50 Hz is connected to series combination of R =10 Ω; L = 2 / π H ; and C = 1 / π µF. Find phase difference between V and I.

Solution : Phase difference is given by

$$\tan\phi = \frac{X_C - X_L}{R}$$

Given $R = 10\Omega, f = 50\,Hz, L = \frac{2}{\pi}H, C = \frac{1}{\pi}\times 10^{6}F$

$$\tan\phi = \frac{\frac{1}{2\ f\text{x}\,C} - L\text{x}2\ f}{R}$$

$$\tan\phi = \frac{\frac{1}{2\pi\times 50\times\frac{1}{\pi}\times 10^{6}} - \frac{2}{\pi}\times 2\pi\times 50}{10}$$

$$\frac{10^4 - 200}{10} = \frac{980}{10} = 980$$

$$\phi = \tan^{1}(980)$$

Example 10.21 : For a L-C-R series circuit, voltage and current are given by $V = 300\sin 100t$ and $I = 6\sin(100t - \phi)$. If the resistance in the circuit is of 40 Ω, find (i) impedance (ii) reactance (iii) phase difference between voltage and current.

Solution : Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

given $V_m = 300\,\text{V}, I_m = 6\text{A}, \omega = 100$ rad/s and R = 40 Ω

(i) $Z = \frac{V_m}{I_m} = \frac{300}{6} = 50\Omega$

(ii) $Z^2 = R^2 + (X_L - X_C)^2$

$\therefore X_{LC} =$ $(X_L - X_C) = \sqrt{Z^2 - R^2}$

$$= \sqrt{(50)^2 - (40)^2} = 30\,\Omega$$

Phase difference is given by

(iii) $\phi = \tan^{1}\left(\frac{X_L - X_C}{R}\right)$

$$\phi = \tan^{1}\left(\frac{30}{40}\right)$$

$$\phi = \tan^{-1}\left(\frac{3}{4}\right)$$

Example 10. 22: Find angular frerquency and frequency for maximum current in an ac circuit containing an inductor of L = 0.5H and a capacitor of C = 8 μF.

Solution : Resosant angular frequency is given by

$$\omega_r = \frac{1}{\sqrt{LC}}$$

because current is maximum at resonant frequency,

so, $\omega_r = 2\pi f_r$

$L = 0.5$ H, $C = 8\times10^{-6}$F

$$\omega_r = \frac{1}{\sqrt{0.5\times8\times10^{-6}}} = \frac{10^3}{2} = 500 \text{ rad/s}$$

$$f_r = \frac{\omega_r}{2\pi} = \frac{500}{2\pi} = \frac{250}{\pi}\text{Hz}$$

Example 10. 23: At resonance the values of R = 20 Ω, L = 0.1 H and C = 200 μF. If the inductor is replaced by L = 100 H, find value of C for same resonance frequency.

Solution : Given for first condition L = 0.1 H, C = 200 μF for second condition L = 100 H, C = ?

Since resonance frequency is same

$$f_r = \frac{1}{2p\sqrt{LC}} = \frac{1}{2p\sqrt{L'C'}}$$

$LC = L'C'$

$0.1 \times 200 \times 10^{-6} = 100 \times C'$

$$C' = \frac{0.1\times200\times10^{-6}}{100} = 0.2\mu\text{F}$$

Example 10.24 : A wave of wavelength 300 m is being transmitted form a center. We have a capacitor of C = 2.4 μF. Find the value of inductor to tune (resonance circuit) the station.

Solution : $f = \frac{1}{2\pi\sqrt{LC}}$

given $\lambda = 300\,\text{m}$, $C = 2.4\ \mu\text{F}$

$$f = v = \frac{c}{\lambda} = \frac{3\times10^8}{300} = 10^6\,Hz$$

$$L = \frac{1}{4\pi^2 f^2 C}$$

hence $L = \frac{1}{4\times(3.14)^2\times(10^6)^2\times(2.4\times10^{-6})} = 10^{-8}\text{ H}$

Example 10.25 : An ac circuit of 220 V, 50 Hz has a resistor of 11 Ω, inductor of $2/\pi^2$ H. For what value of C the circuit will be at resonance? At so find the current in the circuit.

Solution : $f = \frac{1}{2\pi\sqrt{LC}}$

given $V_{rms} = 220\text{ V}, f = 50\text{ Hz}, R = 11\Omega, L = \frac{2}{\pi^2}\text{H}$

$$C = \frac{1}{4\pi^2 L f^2} = \frac{1}{4\pi^2\times\frac{2}{\pi^2}\times50\times50} = 50\ \mu\text{F}$$

The current in resonant circuit

$$I_{rms} = \frac{V_{rms}}{Z_{min}} = \frac{V_{rms}}{R}$$

$$I_{rms} = \frac{220}{11} 20\text{A}$$

10.6 Half Power Point Frequencies, Bandwidth and Quality Factor of a Series Resonance Circuit

10.6.1 Half Power Points of Frequency

Fig 10.38 shows variation of current I_{rms} with freqency in L-C-R series circuit. At resonance frequency the current in the circuit is maximum i.e. $(I_{rms})_{max}$. The power dissipation will be $(I^2_{rms})_{max}R$, and will be maximum.

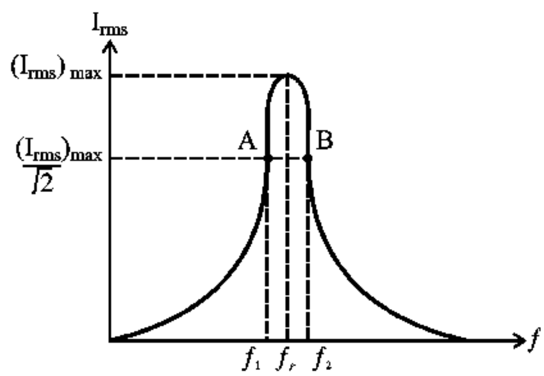


Fig 10.38 : Current I_{rms} and frequency f graph for L-C-R circuit

There exist two frequencies $f_1 < f_r$ and $f_2 > f_r$ at which the current in the circuit will be 1/2 of the maximum value. And the circuit comsumes half the power of its maximum value. These frequencies are thus called half power frequencies. At these freqencies the effective current is given as I_{rms}.

$$I_{rms}^2 R = \frac{1}{2}(I_{rms})_{\max}^2 R$$

$$I_{rms} = \frac{(I_{rms})_{\max}}{\sqrt{2}}$$

$$= 0.707\left(I_{rms}\right)_{rms}$$

Hence the current at half power frequencies will be of the maximum value.

10.6.2 Band Width

L-C-R resonant circuit is able to absorb more energy from the source in the frequency interval (f_2-f_1). This gap between the half power frequencies f_2-f_1 is called band width.

the current at half power frequencies f_1 and f_2 is

$$I_{rms} = \frac{(I_{rms})_{\max}}{\sqrt{2}}$$

$$\frac{V_{rms}}{\sqrt{R^2 + (L\omega - \frac{1}{c\omega})^2}} = \frac{V_{rms}}{\sqrt{2}R}$$

Or $$R^2 + (L\omega - \frac{1}{c\omega})^2 = 2R^2$$

Or $$(L\omega - \frac{1}{c\omega})^2 = R^2$$

Or $$\left(L\omega - \frac{1}{c\omega}\right) = \pm R$$

R.

for f_1 $$L\omega_1 - \frac{1}{c\omega_1} = -R \quad \text{. . . (10.35)}$$

R_2

for f_2 $$L\omega_2 - \frac{1}{c\omega_2} = R \quad \text{. . . (10.36)}$$

Adding the equation 10.35 and equation 10.36 we get

$$L(\omega_1 + \omega_2) - \frac{1}{C}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

Or $$L\left(\omega_1 + \omega_2\right) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1\omega_2}\right)$$

Or $$\omega_1\omega_2 = \frac{1}{LC} \quad \text{. . . (10.37)}$$

Similarly subtracting equ. 10.35 from equ. 10.36

$$L(\omega_2 - \omega_1) + \frac{1}{c}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R$$

Or $$L(\omega_2 - \omega_1) + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1\omega_2}\right) = 2R$$

We get

$$2L(\omega_2 - \omega_1) = 2R$$

$$(\omega_2 - \omega_1) = \frac{R}{L} \quad \text{. . . (10.38)}$$

hence band width $= \omega_2 - \omega_1 = \dfrac{R}{L}$

Or $$f_2 - f_1 = \frac{R}{2\pi L} \quad \text{. . . (10.39)}$$

The above equation gives the expression for band width.

10.6.3 Quality Factor

Behaviour of L-C-R circuit depends on the value of R. At different values of R, we see that as the value of R decreases, sharpness of resonance curve increases. Resonance current will be maximum and band width $f_2 - f_1 = \dfrac{R}{2\pi L}$ will be minimum.

Hence at low values of R, the resonances curve will be more sharp, as given by fig 10.39.

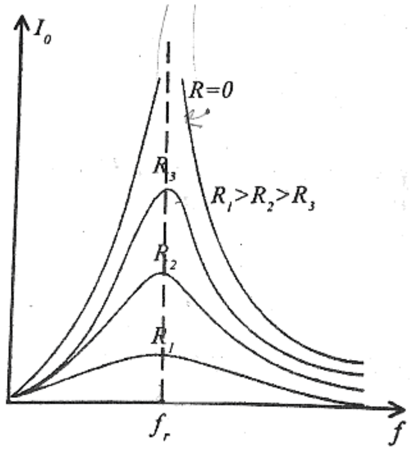


Fig 10.39 Comparision of Band with for $R_1 < R_2 < R_3$

For large value of R, the value of $(I_{rms})_{max}$ decreases, and band width increases.

By changing the frequency below or above f_r, the change in $(I_{rms})_{max}$, sharpens the curve. If the change in I_{rms} is slower, flater the curve. The sharpness of the resonance curve is given by a characterstic factor called qualiy factor Q.

$$Q = \frac{\text{resonant frequency}}{\text{band width}} = \frac{f_r}{f_2 - f_1} = \frac{Lw_r}{R} \quad \ldots (10.40)$$

$$\because \quad \omega_r = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{L\omega_r}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \ldots (10.41)$$

Or $Q = \dfrac{\text{Capaitive or inductive reactance at resonant frequency}}{\text{resistance}}$

Or $Q = \dfrac{\text{Potential on L or C}}{\text{Applied potential}} = \dfrac{V_L}{V} = \dfrac{V_C}{V}$

Normally $Q > I$; hence V_L or $V_C > V$ so there is a voltage amplification and Q is also a measure of voltage amplification. Series resonant circuit is used to tune a radio. It is also called selective circuit. The signal recieved by the antenna work as a source. To select a specific station we tune first by changing inductor L and then change the capacity of the capacitor such that frequency of the circuit is equals to the transmitting frequency of that station. So the signal of this frequency gives maximum current, and the other near by frequencies are supressed by dominance of f_r. The side frequencies are more supressed if Q is large.

Example 10.26 : An L-C-R circuit contains R = 100 Ω, L = 1 mH and C = 1000 μF. Find the resonant frequency and band width.

Solution : $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Given $R = 100\,\Omega, L = 1\,\mu\text{H}, C = 1000\,\mu F$

$$f_r = \frac{1}{2\pi\sqrt{10^{-3} \times 1000 \times 10^{-6}}} = \frac{1000}{2\pi}\text{Hz}$$

Band width $f_2 - f_1 = \dfrac{R}{2\pi L} = \dfrac{100}{2\pi \times 10^{-3}} = \dfrac{5000}{\pi}\text{Hz}$

Example 10.27 : A series L-C-R circuit contains R = 14 Ω and inductor of L = 7 mH. The frequency of the source is equal to resonance frequency. If the quality factor is 1/2 then find (i) Band width (ii) capacitive reactance.

Solution : Band width $\omega_2 - \omega_1 = \dfrac{R}{L}$

given $R = 14\Omega, L = 7 \times 10^{-3} H, Q = \dfrac{1}{2}$

$$\omega_2 - \omega_1 = \frac{14}{7 \times 10^{-3}} = 2 \times 10^3 \text{ m}$$

$$Q = \frac{1}{c\omega \times R}$$

$$\frac{1}{C\omega} = Q \times R = \frac{1}{2} \times 14 = 7\,\Omega$$

Example 10.28 : Resonant frequency of an L-C-R circuit is 600 Hz. At frequencies 570 Hz and 620 Hz the current in the circuit is $1/\sqrt{2}$ of its maximum value at resonance. Find quality factor, X_L, X_C, L and C at resonance. $(R = 3\,\Omega)$.

Solution : $Q = \dfrac{f_r}{f_2 - f_1}$

given $f_r = 600\,\text{Hz},\ f_1 = 570\,\text{Hz},\ f_2 = 620\,\text{Hz}$

$$Q = \frac{600}{620 - 570} = 12$$

$$Q = \frac{L\omega_r}{R}$$

$$L\omega_r = Q \times R$$

$$L\omega_r = \frac{1}{C\omega_r}$$

$$\frac{1}{C\omega_r} = X_C = 36\ \Omega$$

Also at resonance $X_C = X_L$ hence XC = 36 .

$$L\omega_r = 36$$

$$L = \frac{36}{\omega r} = \frac{36}{2\pi fr} = \frac{36}{2 \times 3.14 \times 600} = 9.56\,\text{mH}$$

$$\frac{1}{C\omega_r} = 36$$

$$C = \frac{1}{36 \times 2\pi fr} = \frac{1}{36 \times 2 \times 3.14 \times 600} = 7.37\ \mu F$$

10.7 Average Power in AC Circuit

The rate of absorption of energy is called power of the circuit. It is the product of voltage and current in the circuit. The voltage and current has a phase difference, depending on its elements, hence the power also depends on phase. Let the voltage and current at any instant be

$$V = V_m \sin \omega t$$

$I = I_m \sin(\omega t - \phi)$ here current lags the potential by ϕ

Hence instantaneous power is

$$P = VI = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m \sin \omega t \sin(\omega t - \phi) \quad \ldots (10.42)$$

using $\left[\sin C \sin D = \frac{1}{2}\{\cos(C-D) - \cos(C+D)\}\right]$

$$P = \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$$

$$P = \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi) \ldots (10.43)$$

It shows that instantaneous power has two components. One is constant and other varies with time, periofically. The average power over one complete cycle is -

$\overline{P}_{av} = \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \overline{\cos(2\omega t - \phi)}$ the

since average of cosine function over one complete cycle is zero.

$$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \times 0$$

$$\left[\because \overline{\cos(2\omega t - \phi)} = 0\right]$$

$$P_{av} = \frac{1}{2} V_m I_m \cos\phi$$

$$P_{av} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = V_{rms} I_{rms} \cos\phi \quad \ldots (10.44)$$

$$P_{av} = P_{vir} \cos\phi \quad \ldots (10.45)$$

here P_{av} is called virtual power and $\cos\phi$ is power factor of the circuit.

10.8 Power Factor

The cosine of the phase angle between AC voltage and AC current in the circuit is called power factor of the circuit. It depends on the elements of the circuit. From equation (10.45)

$$\cos\phi = \frac{P_{av}}{P_{vir}} = \frac{\text{average power}}{\text{virtual power}}$$

The ratio of P_{av} and P_{rms} is equal to power factor $(\cos\phi)$. It is a dimensionless quantity and its value is between 0 and 1. If power factor of a circuit is zero, energy loss in the circuit is zero.

For L-C-R series Ac circuit

$$\tan\phi = \frac{X_L \sim X_C}{R}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L \sim X_C)^2}}$$

$$\cos\phi = \frac{R}{Z} \qquad \ldots (10.46)$$

The ratio of resistance and impedance in L-C-R circuit is called power factor.

Special Conditions

(i) For pure resistive circuit- In this circuit voltage and current are always in phase, i.e. $\phi = 0$ $\cos\phi$ = 1, power factor is 1. $P_{av} = V_{rms} I_{rms} = P_{vir}$ which means that in a pure resistive circuit power factor is unity, and the circuit consumes maximum power. And $P_{av} = P_{vir}$. The fig. (10.40) gives instantaneous values of V and I and it shows that power consumed by the circuit is maximum.

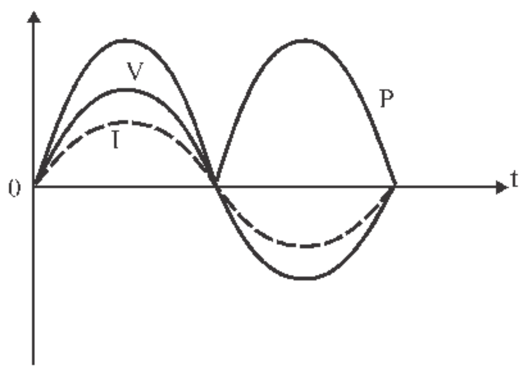


Fig. 10.40 Power in resistive circuit

(ii) For pure inductive circuit - In this circuit the applied voltage leads the current in the circuit by $\phi = \pi/2$; hence the power factor $\cos\phi = 0$ and P_{av} = 0. In pure inductive circuit the average power consumed over a complete cycle is zero. Fig (10.14) shows that the area of positive loop (energy absorbed) is exactly equal o the area of negative loop (energy returned).

(iii) For pure capacitive circuit - In this type of circuit, the current leads the applied potential by $\pi/2$ i.e. $\phi = -\pi/2$ hence the power factor $\cos\phi = 0$

From equ. (10.44) $\quad P_{av} = 0$

In a pure capacitive circuit $\cos\phi = R/Z$ also the average power over one cycle is zero. As evident from fig 10.18. The area of positive loop of power curve is equal to the area of the negative loop of the same curve.

(iv) For L-C-R series circuit - In this circuit, at resonance, V and I are in same phase, i.e $\phi = 0$ and $\cos\phi = 1$, the circuit has maximum power factor.

In electric fan and induction motor, the value of L is very large due to many turns of its winding, hence its ϕ is increased and power factor is decreased. To reduce ϕ, a condensor (capacitor) is used so that ϕ approaches zero and power factor approaches 1. The motor gets maximum power and fan moves faster. That is the reason of changing the condensor of a fan, when such problem arises.

10.9 Wattless Current

The AC circuit which has an inductor and a capacitor but no resistance, both voltage and current are present, but average power is zero.

$$P_{av} = V_{rms} I_{rms} \cos\left(\pm\frac{\pi}{2}\right) = 0$$

which means that although the currnet exists, its contribution to power (watt) is zero. Hence called wattless current.

Even when the current is having a phase $\phi = \pi/2$ 0, with voltage, there is a component of I, which has a phase difference of $\pi/2$ with voltage. this component does not contribute to the power, and called wattless current.

From section 10.7 we have

instantaneous power $P = V_m I_m \sin\omega t \sin(\omega t - \phi)$

$$P = V_m I_m \sin\omega t(\sin\omega t \cos\phi - \cos\omega t \sin\phi)$$

$$P = V_m I_m \sin^2\omega t \cos\phi - V_m I_m \sin\phi \sin\omega t \cos\omega t$$

$$P = V_m I_m \cos\phi \sin^2\omega t - \frac{V_m I_m}{2} \sin\phi \sin 2\omega t$$

$$P = P_1 - P_2$$

The average value of component P_1 for one cycle is

$$\overline{P_1} = V_m (I_m \cos\phi)\overline{\sin^2\omega t} \qquad \ldots (10.47)$$

$$= \frac{V_m (I_m \cos\phi)}{2} \qquad (\because \overline{\sin^2\omega t} = \frac{1}{2})$$

Similarly average of P_2-

$$\overline{P_2} = \frac{V_m (I_m \sin\phi)}{2}\overline{\sin 2\omega t} \qquad \ldots (10.48)$$

The contribution of component

$P_2 = 0 \ (\because \overline{\sin 2\omega t} = 0)$ for a complete cycle average value of $(\because \overline{\sin 2\omega t} = 0)$ hence $P_2 = 0$. This component of current, is called the wattless current in this case.

From the above calculation it is clear that if a resistance is used with a reactive element X, the current can be resolved in two components, one component $I_{rms}\cos\phi\left(\frac{I_m}{\sqrt{2}}\cos\phi\right)$ is in phase with the applied voltage and called working current. Similarly the other component of current $I_{rms}\sin\phi$ has a phase difference of $\pi/2$ with voltage and contribution to power is zero, and is called wattless current.

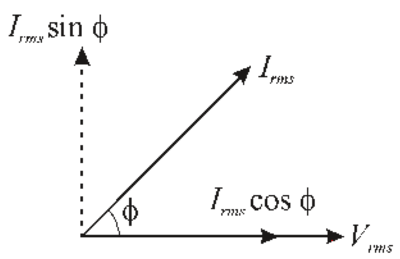


Fig 10.41 V-I graph for RC ac circuit

Example 10.29 : An ac circuit of source 200 V; 50 Hz has resistance of 10 Ω, and inpedance 14.14Ω. Find (i) Power factor (ii) Virtual power (iii) Average power (iv) the value of wattless current.

Solution : Given

$V_{rms} = 200\,\text{V},\ f = 50\ \text{Hz}, R = 10\ \Omega$

and $Z = 14.14\ \Omega$

(i) $\cos\phi = \frac{R}{Z} = \frac{10}{14.14} = \frac{1}{\sqrt{2}}$

(ii) Virtual Power $P_{vir} = V_{rms} I_{rms}$

$= V_{rms} \cdot \frac{V_{rms}}{Z} = \frac{200200}{14.14} = 2820\text{W}$

(iii) Average Power $P_{av} = V_{rms} I_{rms}\cos\phi$

$= 2820 \times \frac{1}{\sqrt{2}} = 2000\ \text{W}$

(iv) Wattless Current $= I_{rms}\sin\phi = \frac{V_{rms}}{Z}\times\sin\phi$

$\left(\cos\phi = \frac{1}{\sqrt{2}}\right)$

$\frac{200}{14.14}\times\sin\frac{\pi}{4} = \frac{200}{14.14}\times\frac{1}{\sqrt{2}} = 10\ \text{A}$

Example 10.30 : For an AC circuit, voltage and current are given by $V = 100\sin\omega t$ V

$I = \sin(\omega t + \frac{\pi}{3})$ A

Find : (i) Power factor (ii) Average Power (iii) Wattless current

Solution : Given $V_m = 100\,\text{V}, I_m = 1\,\text{A},\ \phi = \frac{\pi}{3}$

(i) Power factor $= \cos\phi \quad \cos\frac{\pi}{3} = \frac{1}{2}$

(ii) Average Power $= V_{rms} I_{rms}\cos\phi = \frac{V_m}{\sqrt{2}}\times\frac{I_m}{\sqrt{2}}\cos\phi$

$= \frac{100}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times\frac{1}{2} = 25\ \text{W}$

(iii) Wattless current $= I_{rms}\sin\phi = \frac{1}{\sqrt{2}}\times\frac{\sin\pi}{3} = \frac{\sqrt{3}}{2\sqrt{2}}$

$= 0.61$ A

10.10 Choke Coil

A coil with very high self inductance and very low reisitance is called a choke coil. It is made by winding thick, insulated copper wire over a laminated iron core.

Due to high self inductance and very low resistance, $\phi \simeq 90^\circ$, and $\cos\phi \simeq 0$. There is a negligible power dissipation in the coil. Also due to high L, its inductive reactance X_L, is large, and it is highly effective in controlling AC current without significant losses. It is used in flourosent lamp and mercury/sodium lamp. In choke the most of the current is wattless current with high value of ϕ. If a resistance is used to controll the current, instead of choke, there is very high joule loss.

Metal Detector

It works on condition of resosance in AC circuit. When some metal/A person with metal comes in contact with coil of the circuit, its impedance changes which bring about a change in current, which is heared as a beep.

10.11 Transformer

A device by which we can change ac voltage. It is based on the principle of mutual inductance. It is called transformer.

10.11.1 Construction

As in fig. 10.42, it is made up of a rectangular or any other shaped laminated soft ion core. The soft iron laminea are placed one over the other and an insulating liquid (Lekar) is poured between them. Two coils of insulated wire of Cu/Al are wound over the iron core. The coil on which input AC is applied is called primary coil, while other, from which AC out put is drawn is called secondary coil.

10.11.2 Principle and Working

When AC is applied to the primary coil, the changing magnetic flux produced in iron core is associated with secondary coil. This changing magnetic flux produces *emf* in secondary coil according to Faraday's law. The magnetic flux is confined to iron core and there is very little leakage due to packed winding. The frequency of AC in secondary is same as that of AC in primary.

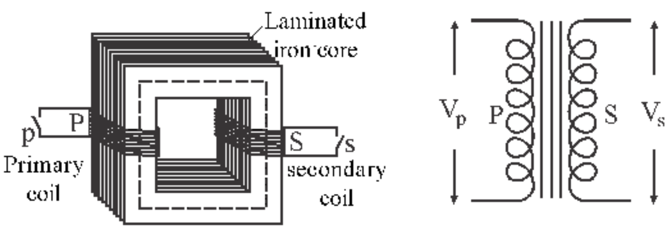


Fig 10.42 Transformer

Let the number of turns in primary and secondary be N_P and N_S. At any instant t, the voltage applied on primary coil V_P produces a flux ϕ, due to which emf induced between the ends of secondary coil of N_S turns

$$E_S = -N_S \frac{d\phi}{dt} \quad \ldots (10.49)$$

The changing flux ϕ also produces *emf* in primary which is called back *emf*.

$$E_P = -N_P \frac{d\phi}{dt} \quad \ldots (10.50)$$

When the secondary coil is in open circuit or very small current is drawn, then $E_P = V_P$ and $E_S \simeq V_S$.

$$E_P = V_P = -N_P \frac{d\phi}{dt} \quad \ldots (10.51)$$

$$E_S = V_S = -N_S \frac{d\phi}{dt} \quad \ldots (10.52)$$

deviding we get $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ ... (10.53)

If there is no energy loss in transformer, the input power is equal to out put power. (In ideal transformer)

$$I_P V_P = I_S V_S \quad \ldots (10.54)$$

From equ. 10.53 and 10.54 we get

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S} \quad \ldots (10.55)$$

The ratio of voltage in primary and secondary is equal to the ratio of number of turns in primary and secondary coils. The ratio of currents is primary and secondary coils is equal to the inverse of the ratio of number of turns in them.

10.11.3 Type of Transformers

(i) Step-up Transformer - When $N_S > N_P$, then $V_S > V_P$ and current $I_S < I_P$. Such type of transformer is called step up transformer.

(ii) Step-down Transformer - When the number of turns in secondary N_S is less than the number of turns in primary N_P, the out put voltage V_S is less than input voltage V_P and the current in secondary I_S is greater than current in primary I_P. Such transformer is called step-down transformer.

Hence when $N_S < N_P$ then $V_S < V_P$ and $I_S > I_P$. If there is no loss of energy in transfering power from primary coil to secondary coil, $P_P - P_S$. The efficiency of such transformer is 100%.

Practically no transformer has 100 % efficiency. There is always an energy loss in one way or the other.

The following are the energy losses in transformer.

(i) Copper losses : The primary and secondary coils have very small but non zero resistance, due to which heat is produced due to joule effect (power loss – I^2R). This loss is called copper loss. To reduce this loss, thick, copper wire is used for windings.

(ii) Losses due to leakage in magnetic flux - The whole flux produced by primary does not passes through secondary, due to faulty winding or air between laminea of iron core, there is a leakage of flux. To reduce this loss the two coils are wound over each other.

(iii) Eddy current losses - The change in magnetic flux through the iron core, induces small current loops in it, called eddy currents, which dissipates energy in the form of heat. To reduce this loss, the transformer core is laminated.

(iv) Hysteresis Losses - The magnetization of iron core is periodically reversed as per the AC frequency. Hysteresis loss ocures in each cycle, due to reversal of magnetization. The area of hysteresis loop, represents the energy loss per second per unit volume. The soft-iron core is used to reduce this loss, which has minimum area of its hysteresis loop.

Transformers are used in power transmission and impedance matching. Audio frequency transformer are used in telephony and radiotelephony and radio frequency transformers are used in radio communication. The transmission of electrical power to distant areas is done at very high voltage and low current, to reduce the transmission loss. For this step up transformers are used. It is clear from example 10.32.

Example 10.31 : The current in primary coil of a transformer is 1 A at input power of *4kW*. The voltage at secondary coil is 400 *V*. If number of turns in primary coil is 100, find number of turns in secondary coil.

Solution :

$$P_{in} = V_P I_P$$

given $I_P = 1$ A, $P_{in} = 4000$ W, $V_S = 400$ W

$N_P = 100$

$$V_P = \frac{P_{in}}{I_P} = \frac{4000}{1} = 4000\,\text{W}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$N_S = \frac{400}{4000} \times 100 = 10$$

Example 10.32 : A transmission line has a resistance of 20 Ω. The power to be transmitted is 6.6 kW. If the power is transmitted at (i) 22000 V (ii) 220 V. Find power loss and voltage drop in both the cases. What conclusion is drawn from this?

Solution : In first case, $P = VI$

$$I = \frac{P}{V}$$

given $P = 6.6\,\text{KW}, R = 20\,\Omega,$

$$V_1 = 22000\text{ V}, V_2 = 220\text{ V}$$

$$I = \frac{6600}{22000} = 0.3\text{ A}$$

Power loss due to heat is $I^2R = (0.3)^2 \times 20 = 1.8$ W

Voltage drop on line is IR = 0.3 x 20 = 6 V

For case (ii) $I = \frac{6600}{220} = 30\text{ A}$

Power loss due to heat is $I^2R = (30)^2 \times 20 = 1800$ W

Voltage drop on the line is IR = 30 x 20 = 600 V

It is concluded that all type of losses are less when power transmission is done at very high votage, i.e 220 kV or 400 kV.

Important Points

1. Sinosuidal Ac voltage or current is expressed by -

$$V = V_m \sin \omega t$$

or $$I = I_m \sin \omega t$$

2. Average value of AC voltage/current for whole cycle is zero for positive and negative half cycles the average value is

$$I_{av} = \pm \frac{2I_m}{\pi} = \pm 0.637\, I_m$$

3. Values given for AC voltage and currents are their I_{rms} values, it is also called effective values.

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

4. The obstacle produced by an inductor in AC circuit is called inductive reactance X_L and by capacitor its capacitive reactance X_C.

$$X_L = L\omega = L \times 2\pi f$$

$$X_C = \frac{1}{c\omega} = \frac{1}{c \times 2\pi f}$$

5. Obstacle produced in R-L, R-C and L-C-R circuit is called impedance Z.

for R-L circuit $Z = \sqrt{R^2 + X_L^2}$

R-C circuit $Z = \sqrt{R^2 + X_C^2}$

L-C-R circuit $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$

6. Phase angle for different AC circuits the phase difference between voltage and current is as given -

for pure resitaive circuit $\tan \phi = 0,\ \phi = 0$

pure inductive circuit $\tan \phi = \infty,\ \phi = \pi / 2$

Pure capacitor circuit $\tan \phi = -\infty \quad \phi = -\pi / 2$

R-L circuit $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

R-C circuit $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

L-C-R circuit $\phi = \tan^{-1}\left(\frac{X_L \sim X_C}{R}\right)$

7. For resonant L-C-R circuit

$X_L = X_C \qquad \omega_r = \frac{1}{\sqrt{LC}}$

$Z_{\min} = R \qquad f_r = \frac{1}{2\pi\sqrt{LC}}$

$\phi = 0$

$(\cos\phi)_{\max} = 1$

8. The frequencies lower and higher than resosnant frequency, where the current in the circuit becomes $\frac{1}{\sqrt{2}}$ I_{max}, are called half power frequencies.

9. The difference between half power frequencies is called band width.

$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$ or $\Delta f = f_2 - f_1 = \frac{R}{2\pi L}$

$f_2 - f_1 = \frac{R}{2\pi L}$

10. In resosnant circuit, the ratio of resosnant frequency f_r and band width is called quality factor Q.

$Q = \frac{fr}{f_2 - f_1} = \frac{L\omega_r}{R}$

11. In ac circuit the average power $P_{av} = V_{rms} I_{rms} \cos\phi$ for resistive circuit $P_{av} = V_{rms} I_{rms}$ for inductive or capacitive circuit $P_{av} = 0$

12. Power factor $\cos\phi = \frac{P_{av}}{P_{\text{आभासी}}} = \frac{R}{Z}$ for pure resistance $\cos\phi = 1$; for inductive or capacitive circuit $\cos\phi = 0$

13. The component of the current $I_m \sin\phi$, which is out of phase by $\pi/2$ with voltage is called wattless current since it don't contribute to power.

14. The transformer works on the principle of mutual induction, with its help the AC voltage can be steped up or steped down.

Transformer formula $\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$

15. Transformers are of two types, step-up and step-dn transformer for step up $N_S > N_P$ for step *dn* $N_S < N_P$

16. The energy loss in a tansformer is due to (i) joule effect (ii) eddy current (iii) hysterissis (iv) flux leakage due to faulty winding.

Questions for Practice

Multiple Choice Question -

1. RMS value of alternating current is -

(a) Double the peak value

(b) Half of the peak value

(c) Equal to peak value

(d) $\frac{1}{\sqrt{2}}$ of the peak value

2. Due to which component of the current in AC circuit leads the voltage in phase -

(a) Pure resistor (b) Pure Inductor

(c) Pure capacitor (d) None of these

3. The AC current lags the voltage by $\frac{\pi}{2}$ in phase, when the circuit has -

(a) Only resistor (b) Only inductor

(c) Only capacitor (d) Capacitor and resistor

4. The unit of ωC is -

(a) Ohm (b) mho

(c) Volt (d) Amp

5. Role of capacitor a circuit -

(a) Allows ac current to pass through

(b) Stops ac current

(c) Allows dc current

(d) Stops ac current and allows dc current

6. Which of these does not have same units -

(a) $\frac{1}{\sqrt{LC}}$ (b) $\sqrt{LC}$

(c) RC (d) $\frac{L}{R}$

7. An ac circuit is resonant at 10 k Hz. If frequency is raised to 12 k Hz. The impedance of circuit will -

(a) Remain uneffected

(b) Increased by 1.2 times

(c) Will increase and becomes capacitive

(d) increases and becomes inductive

8. In a circuit current lags the voltage by $\pi/3$, the elements in the circuit are -

(a) R and C (b) R and L

(c) L and C (d) Only L

9. Power factor of a pure inductor or pure capacitor is -

(a) One (b) Zero

(c) π (d) Greater than zero

10. The current can be reduced in AC circuit witout power loss by -

(a)Using a pure inductor

(b) Using pure resistor

(c) Using a resistor and inductor

(d) Using resistor and capacitor

11. In an ac circuit the voltage and current are $V = V_m \sin \omega t$ and $I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$; the power disspated in the circuit will be -

(a) $\frac{V_m I_m}{R}$ (b) $\frac{V_m I_m}{\sqrt{2}}$

(c) $\frac{VI}{2}$ (d) Zero

12. In LCR series circuit the value of C = 1F, L = 1 H at resosnace, the freqency will be -

(a) 10^6 (b) ½ $2\pi \times 10^6$

(c) $\frac{10^6}{2\pi}$ (d) $2\pi \times 10^{-6}$

13. The transformer core is laminated, so that -

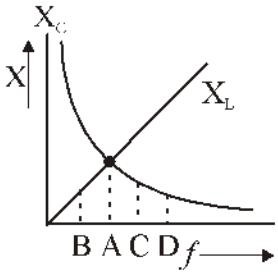
(a) Magnetic field is increased

(b) Residual magnetization is reduced in the core

(c) Magnetic saturation of core is increased

(d) Energy loss due to eddy currents is reduced

14. In the diagram, the point which indicated, resosnance is -



(a) A

(b) B

(c) C

(d) D

15. The ratio of currents in primary and secondary coils of a transformer with 100% efficiency is 1 : 4. the ratio of the voltage across the coils will be -

(a) 1 : 4 (b) 4 : 1

(c) 1 : 2 (d) 2 :1

Very Short Answer type questions -

1. The AC voltage is given by equation $V = 200\sqrt{2}\sin 100\pi t$ write its RMS value and frequency.
2. Write down the realtion between Peak and RMS value of AC current.
3. The current in an inductive circuit is given by $I = I_m \sin \omega t$. Give the equation for voltage.
4. The voltage in an AC circuit is given by $V = 200\sin 314t$. Give the frequency of AC voltage.
5. How the inductive and capacitive reactances are effected by increasing AC frequency?
6. A coil has inductance of 0.1 H. Find its reactance at 50 Hz.
7. What will be the phase difference between voltage and current in series L-C-R circuit?
8. What will be phase difference between voltage across inductor anf voltage across capacitor in series L-C-R resosnant AC circuit?
9. What will be impedance in series L-C-R resonant circuit at resonance?
10. What will be value of power factor in Ac circuit for inductor, capacitor and resistor?
11. What is the unit of $\sqrt{LC}$?
12. In series L-C-R resosnant circuit, capacity is changed to 4 times. What will be new inductance for the same resonanace frequency?
13. What will be RMS value of wattless current?
14. The ratio of number of turns in primary and secondary of a transformer is 1:4. What type of transformer it is?
15. Write down the expression for wattless current in ac circuit.

Short Answer Type Question -

1. Why ac is prefered to dc? Please explain.
2. 220 V ac is more danserous then 220 V dc. Why?
3. Draw graph between frequency and X_L and X_C.
4. A capacitor stops DC current, while it allows ac current. Why?
5. A coil has ohmic resistance of 6Ω, if its impedance is 10Ω, find inductive reactance X_L.
6. If f_r is the resonant frequency of ac circuit. Give phase realtionship between voltage and current for (i) $f = f_r$ (ii) $f < f_r$ (iii) $f > f_r$.
7. What is band width? Write its expression for L-C-R series circuit.
8. What are half-power frequencies? What will be the current at these frequencies?
9. If the value of resistance and reactance are same for a coil, what will be its power factor?
10. In the transmission of electrical energy, less power factor of circuit means, more power loss. Please explain.
11. Write down the expression for impedance, frequency and power factor for a series L-C-R ac circuit?
12. Write down the principle of a transformer and its uses.
13. Find average value of ac current in its first positive half cycle.
14. On what factors, the power losses in a transformer depends? How its can be reduced?
15. Find the expression for impedance and phase difference between voltage and current in series R-L ac circuit.

Essay Types Question -

1. Find current, phase difference, reatance and average power used in a pure inductive AC circuit. Also draw phasor diagram.
2. Derive expression for impedance and current in a R-L series AC circuit. Draw phasor diagram.
3. What do you mean by resosnant circuit. Give the required condition for series L-C-R resonant circuit derive the expression for its resonance frequency. Where this circuit is used?
4. Draw a graph between frequency and current for a series L-C-R ac circuit. Derive expression for band width, showing half power frequencies on the graph.
5. Derive expression for power in AC circuit. How this formula will change if the circuit does not have reactance and resistance? Also define power factor.

Answer (Mutliple Choice Questions)

1. (D) 2. (C) 3. (B) 4. (B) 5. (A) 6. (A) 7. (D) 8. (B) 9. (B) 10. (A) 11. (D) 12. (C) 13. (D) 14. (A) 15. (B)

Very Short Answer Type Question -

(1) 200 V, 50 Hz (2) $I_{rms} = \frac{I_m}{\sqrt{2}}$

(3) $V = V_m \sin\left(\omega t + \frac{\pi}{2}\right)$

(4) $2\pi f = 314$ so $f = 50Hz$

(5) Inductive reactance increases and capacitive reactance decreases.

(6) 31.4 Ω

(7) Between 0 and $\pm\frac{\pi}{2}$

(8) 180^0

(9) Equal to resistance

(10) Zero, zero and one

(11) Second

(12) $\frac{L}{4}$ (13) $\frac{I_m}{\sqrt{2}} \sin\theta$ (14) Step-up

(15) $I_{rms} \sin\phi$

Numerical Questions -

1. If $V = 50\sin(157t + \phi)$ V for ac circuit, find (A) RMS value of ac voltage (B) Frequency of ac voltage.

(35.35 V, 25 Hz)

2. At what instant the value of ac current will be equal (i) half its peak value (ii) $\frac{\sqrt{3}}{2}$ of its peak value for a sinosuidal AC current.

(T/12 s; T/6 s)

3. $V = 100\cos\omega t$ is applied to a circuit containing 10Ω resistor and 100 mH inductor in series. find the current in the circuit and phase difference between voltage and current. (ω =100)

$(\pi/4)$

4. Find inductive reactance for 100 mH inductor at 1 kHz frequency. Find current in the inductor if voltage applied is 6.28 V.

(X_L = 628 Ω; I = 0.1 A)

5. An inductor has inductance 1 H (i) At what frequency its reactance will be 3140 Ω ? What will be capacity of a capacitor to have same reactance at same frequency?

(500 Hz; 0.11 F)

6. A capacitor of capacity 120 µF is joined to an AC source of freqency 50 Hz. Find its capacitive reactance. If the frequency is changed to 5 MHz, what will be the change in its reactance?

(26.54Ω, reactance will reduced to 2.654×10^{-4}Ω)

7. An inductor of $R = 10\Omega$ and 0.4 H is connected to 6.5 V, $\frac{30}{\pi}Hz$ Ac source. Find average power dissipated in the circuit. (5/8 W)

8. A 60 V, 10 Ω bulb is connected to 10 V, 60 Hz, AC source. A coil is connected in series. Find the value of inductance of the coil for full illumination of the bulb. (1.28 H)

9. A series L-C-R circuit containing $R = 20\Omega$, L = 200 m H and C = 40 F is joined to a 120 V, 60 Hz AC source.

Find (i) Total reactance (ii) impedance (iii) power factor (iv) average power.

($X = 9\Omega, Z = 21.94\Omega,\ \cos\theta = 0.912$ P = 598.58W)

10. Find the resosance frequency of series L-C-R series circuit containing $L = 0.1\ H, C = 20\ \mu F;$ $R = 10\Omega$. (112.6 Hz)

11. A source of $V = 15\cos\omega t$ V is connected to a series L-C-R circuit having $L = 10\ mH, R = 3\ \Omega$ and C = 1μ F. Find the peak value of current at frequency 10% less than the resonant frequency.

(0.704 A)

12. A series L-C-R circuit of $L = 200\ mH, C = 500\ \mu F;$ $R = 100\Omega$ is connected to 100 V ac source. Find -

(i) Frequency at which power factor of circuit is 1.

(ii) Peak value of current at this frequency.

(iii) Quality factor

(15.9 Hz, 1.414 A, 0.2 A)

13. A coil has a power factor 0.707 at 60 Hz. What will be power factor at 120 Hz?

(0.44)

14. A series L-C-R circuit of $L = 5\ H, C = 80\ \mu F,$ $R = 40\Omega$ is joined to a source of 230 V. Find (i) Resonant frequency (ii) impedance of circuit and peak alue of current at resonance (iii) RMS value of voltage at all the three elements.

(50 Hz, 40 Ω, 8.1 A, 230 V, 1437.5 V, 1437.5 V)

15. A transformer having 5000 turns in its primary steps down 2200 V to 220 V. If the efficiency of the transfer is 80% and out put power is 8 kW, find (i) N_S (ii) I_P (iii) I_S (iv) Input power where the symbols have their usual meaning.

(500, 4.54 A, 36.36 A, 10KW)