Manipal – 2023

Solved Paper

Physics

1. Two stones having different masses m_1 and m_2 are projected at angles θ and $(90^\circ - \theta)$ with same velocity from the same point. The ratio of their maximum heights is

A. 1:1

B. $1 : \tan\theta$

C. $\tan\theta : 1$

D. $\tan^2\theta : 1$

2. A force $F = (2 + x)$ acts on a particle in xx-direction, where F is in newton and x in metre. The work done by this force during'a displacement from $x = 1.0$ m to $x = 2.0$ m is

A. 2J

B. 3.5J

C. 4.5J

D. None of these

3. A disc of mass 5 kg and radius 50 cm rolls on the ground at the rate of 10 ms^{-1}. Find the kinetic energy of the disc in J .

Numerical

4. The acceleration due to gravity becomes g/2 (g = acceleration due to gravity on the surface of the earth) at a height equals to

A. R/4

B.R/2

C. R/3

D. R/5

5. Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, then the ratio of their diameters is found to be $\sqrt{k} : 1$. Find the value of k.

Numerical

6. The lower end of a capillary tube is dipped into water and it is seen that water rises through 7.5 cm in the capillary. Given, surface tension of water is 7.5×10^{-2} Nm^{-1} and angle of contact between water and glass capillary tube is zero.

What will be the diameter (in mm) of the capillary tube? (given, $g = 10\ ms^{-2}$)

Numerical

7. A function of time is represented as follows $\sin \omega t + \cos 2\omega t + \sin 4\omega t$. The motion represented by it is

A. non-periodic

B. periodic

C. both non-periodic and periodic

D. data insufficient

8. A charged capacitor when filled with a dielectric $K = 3$ has charge Q_0, voltage V_0 and field E_0. If the dielectric is replaced with another one having $K = 9$, the new values of charge voltage and field will be respectively

A. $3Q_0, 3V_0, 3E_0$

B. $Q_0, 3V_0, 3E_0$

C. $Q_0, \frac{V_0}{3}, 3E_0$

D. $Q_0, \frac{V_0}{3}, \frac{E_0}{3}$

9. The resistance of a wire at 20^oC is 20Ω and at 500^oC is 60Ω. At which temperature its resistance will be 25Ω ?

A. 50^oC

B. 60^oC

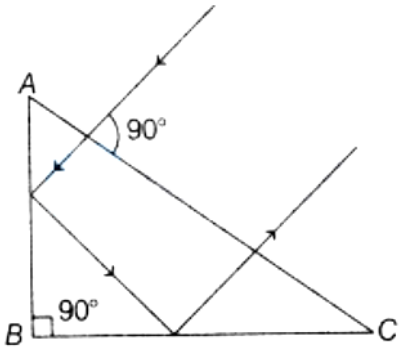
C. 70^oC

D. 80^oC

10. A vertical disc of diameter 10 cm makes 20 revolutions per second about a horizontal axis passing through its centre. A uniform magnetic field 10^{-1} T acts perpendicular to the plane of the disc. If the potential difference between its centre and rim is found to be $\pi/x \times 10^{-p}$ Volt, then find the ratio of x/p.

Numerical

11. A ray falls on a prism ABC($AB = BC$) and travels as shown in figure. The minimum refractive index of the prism material should be



A. $4/3$

B. $\sqrt{2}$

C. 1.5

D. $\sqrt{3}$

12. In Young's double slit experiment, if the source of light changes from orange to blue, then

A. the central bright fringe will become a dark fringe

B. the distance between consecutive fringes will decrease

C. the distance between consecutive fringes will increase.

D. the intensity of the minima will increase.

13. A proton accelerated through a potential difference of 100 V , has de-Broglie wavelength λ_0. The de-Broglie wavelength of an α-particle, accelerated through 800 V is

A. $\lambda_0/\sqrt{2}$

B. $\lambda_0/2$

C. $\lambda_0/4$

D. $\lambda_0/8$

14. Light of wavelength 2475Å is incident on barium. Photoelectrons emitted describe a circle of radius 100 cm by a magnetic field of flux density $1/\sqrt{17} \times$

10^{-5} tesla. The value of work function of the barium is ..... eV. . given, $e/m = 1.7 \times 10^{11}$)

Numerical

15. Ionisation potential of hydrogen atom is 13.6 V . Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV . The spectral lines emitted by hydrogen atoms according to Bohr's theory will be

A. one

B. two

C. three

D. four

Chemistry

1. The number of Cl^- ions in 100 mL of 0.001 M NaCl solution is

A. 6.022×10^{23}

B. 6.022×10^{20}

C. 6.022×10^{19}

D. 6.022×10^{21}

2. Which of the following conclusion is correct regarding photoelectric effect?

A. Energy of a photon depends upon frequency of light absorbed.

B. Energy of a photon depends upon intensity of light.

C. Kinetic energy of electrons increases due to increase in intensity of light.

D. All the energy of photon is absorbed by metal súrface.

3. Which of the following given below species have same bond order?
I. H_2

II. B_2

III. O_2^{2-}

IV. Be_2

V. N_2

Correct option is

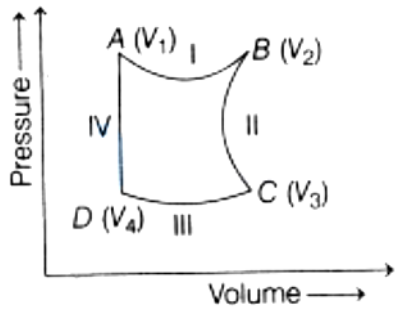
A. I and II

B. I, II and III

C. I, II and IV

D. I, IV and V

4. Consider the graph for cyclic process.



The isothermal compression process among them is

A. I

B. II

C. III

D. IV

5. The pK_a of acetic acid is 4.74. The concentration of CH_3COOH is 0.01 M . The pH of CH_3COOH is____

Numerical

6. The hydroxide of alkaline earth metal not soluble in water is

A. magnesium hydroxide

B. calcium hydroxide

C. beryllium hydroxide

D. barium hydroxide

7. Total number of $2c-2e^-$ and $3c-3e^-$ bonds are present in diborane is___

Numerical

8. Which of the following reagents are used to convert propene to propyne?

A. $SOCl_2$/Py, alc. KOH,H^+/H_2O

B. Br_2/CCl_4/ alc. KOH/Δ

C. alc. KCN,$SOCl_2$

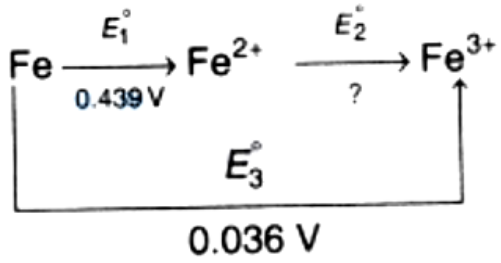
D. alc. KOH,B_2H_6/H_2O_2

9. A 5% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1gcm^{-3} molar mass of the substance will be___

Numerical

10.

Value of E_2^0 is in the given diagram (ignore -ve sign)



Numerical

11. Among Cu^{2+}, Fe^{3+}, Ti^{3+} and Zn^{2+} ions.

The number of ions that impart colour is___.

Numerical

12. Match the complex given in column-I with their hybridisation given in column-II.

	Column-I		Column-II
A.	$[Ni(CO)_4]$	1.	dsp^2
B.	$[Ni(CN)_4]^{2-}$	2.	sp^3
C.	$[Ni(NH_3)_6]^{2+}$	3.	d^2sp^3
D.	$[Fe(CN)_6]^{3-}$	4.	sp^3d^2

A.

A	B	C	D
1	2	3	4

B.

A	B	C	D
2	1	4	3

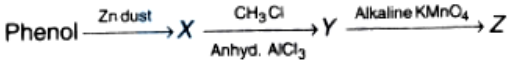
C.

A	B	C	D
1	3	2	4

D.

A	B	C	D
2	3	1	4

13. Find the product Z in the series of reaction.



A. benzaldehyde

B. benzene

C. benzoic acid

D. benzophenone

14. Which of the given reaction doesn't able to form benzaldehyde as product?

A. Rosenmund reaction

B. Etard reaction

C. Gattermann Koch reaction

D. Cannizzaro reaction

15. In which of the given form styrene exist at room temperature?

A. Liquid

B. Solid

C. Gas

D. Pseudo solid

Mathematics

1. If complex number z lies in the interior or on the boundary of circle of radius 3 units, then maximum and minimum values of |z + 1| are

A. (6,0)

B. (3,0)

C. (6,3)

D. (4,1)

2. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is___

Numerical

3. If a^2, b^2 and c^2 are in AP, then b + c, c + a and a + b are in

A. AP

B. GP

C. HP

D. None of these

4. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

A. $\left(14, \frac{272}{3}\right)$

B. $\left(16, \frac{272}{3}\right)$

C. $\left(16, \frac{251}{3}\right)$

D. $\left(14, \frac{251}{3}\right)$

5. Last two digits in $(9)^{50}$ be___

Numerical

6. The differential equation of all circles which pass through the origin and whose centres lies on YY-axis is

A. $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

B. $\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$

C. $\frac{dy}{dx} = \frac{2xy}{x^2-y^2}$

D. None of these

7. If F : R → R is a differentiable function and $f(3) = 6$, then $\lim_{x\to 3} \int_6^{f(x)} \frac{2tdt}{(x-2)}$ is equal to

A. $18f'(3)$

B. 0

C. $24f'(3)$

D. $3f'(3)$

8. The value of $f(x) = \lim_{x\to 0} \left(\frac{8^x+2^x+4^x}{3}\right)^{2/x}$, then $f(x)'$ equals to

Numerical

9. If $\sum_{i=1}^{9} (x_i - 5) = 9$ and $\sum_{i=1}^{9} (x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2 \ldots, x_9$ is

A. 9

B. 4

C. 3

D. 2

10. Let A = {1, 2, 3, 4, 5, 6, 7} and B = {3, 6, 7, 9}. Then, the number of elements in the set {C ⊆ AiC ∩ B ≠ ϕ} is

A. 112

B. 120

C. 108

D. 96

11. If $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$, then $B^T A^T$ is

A. null matrix

B. an identity matrix

C. scalar but not an identity matrix

D. such that $T_r(B^TA^T) = 4$

12. Let $f(x) = \frac{1}{\sqrt{1+x^2}}$, then

A. $f(x, y) = f(x) \cdot f(y)$

B. $f(x, y) \geq f(x) \cdot f(y)$

C. $f(x, y) \leq f(x) \cdot f(y)$

D. $f(x, y) = f(x) - f(y)$

13. Set of values of x lying in $[0,2\pi]$ satisfying the inequality $|\sin x| > 2\sin^2 x$ contains

A. $\left(0, \frac{\pi}{6}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

B. $\left(0, \frac{7\pi}{6}\right)$

C. $\pi/6$

D. None of these

14. $\int f(x)dx = g(x)$, then $\int \cos x\, f(\sin x)dx$ is equal to

A. $g(\cos x) + C$

B. $g(\sin x) + C$

C. $\int g(x) + \sin x + C$

D. $f(\sin x) + g(\cos x) + C$

15. The area bf the region bounded by the curves $y = |5 - x|$, $x = 1$, $x = 6$ and the X-axis

A. 15 sq units

B. 17/2 sq units

C. 13 sq units

D. 16 sq units

16. The area bounded by the curves $y = \tan x, -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}, y = \cot x, \frac{\pi}{6} \leq x \leq \frac{\pi}{2}$ and the X-axis is

A. $\ln\sqrt{3}$

B. $\ln\sqrt{2}$

C. $\ln 2$

D. $\ln(3/2)$

17. If the vectors $p = (a+1)\hat{\mathbf{i}} + a\hat{\mathbf{j}} + a\hat{\mathbf{k}}, q = a\hat{\mathbf{i}} + (a+1)\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ and $r = a\hat{\mathbf{i}} + a\hat{\mathbf{j}} + (a+1)\hat{\mathbf{k}}$ (a ∈ R) are coplanar, then the value of aa is____

Numerical

18. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to

A. -1

B. $2/9$

C. $9/2$

D. 0

19. If the foot of the perpendicular drawn from the point (2, 0, 1) on a line passing through (α, 5, 1) is (7/3, 5/3, 15/4), then αα is equal to____

Numerical

20. An unbaised coin is tossed n times. If the probability of getting 5 heads is equal to the probability of getting 6 heads, then the probability of getting 3 heads is

A. $^{11}C_5\left(\frac{1}{2}\right)^5$

B. $^{11}C_6\left(\frac{1}{2}\right)^6$

C. $^{11}C_3\left(\frac{1}{2}\right)^{11}$

D. $\frac{11}{1024}$

Solution

Physics

Ans 1

Correct Option. D

Solution:

Maximum height, $H_1 = \frac{u^2 \sin^2\theta}{2g} \ldots$ (i)

and $H_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} \ldots$ (ii)

From Eqs. (i) and (ii), we get

$\frac{H_1}{H_2} = \frac{\sin^2\theta}{\sin^2(90^\circ - \theta)}, \frac{H_1}{H_2} = \frac{\tan^2\theta}{1}$

$\therefore \quad H_1 : H_2 = \tan^2\theta : 1$

Ans 2

Correct Option. B

Solution:

The work done in small displacement from x to x + dx is

$dW = Fdx = (2 + x)dx$

Hence,

$W = \int_1^2 dW = \int_1^2 (2 + x)dx = \int_1^2 2dx + \int_1^2 xdx$

$= \left[2x + \frac{x^2}{2}\right]_1^2 = 3.5 \text{ J}$

Ans 3

Correct Answer. 375

Solution:

Here, mass of the disc, M = 5 kg

Radius of the disc, R = 50 cm = 1/2 m Linear velocity of the disc, v = 10 ms^{-1}

As, v = Rω

$\therefore 10 = 1/2\omega$ or $\omega = 10 \times 2 = 20 \text{rads}^{-1}$

Also, moment of inertia of disc about an axis through its centre,

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 5 \times \left(\frac{1}{2}\right)^2 = \frac{5}{8}\ \text{kg}^2 - \text{m}$$

$$\therefore \text{KE of translation } = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times (10)^2 = 250\ \text{J}$$

$$\text{Rotational kinetic energy } = \frac{1}{2}/\omega^2 = \frac{1}{2}\left(\frac{5}{8}\right)(20)^2 = 125\ \text{J}$$

$$\text{Total kinetic energy } = \text{ Translational } + \text{ Rotational}$$
$$= (250 + 125) = 375\ \text{J}$$

Ans 4

Correct Option. A

Solution:

The acceleration due to gravity,

$g = \frac{GM}{R^2} \ldots$ (i)

At a height h above the surface of the earth, the acceleration due to gravity is

$g' = \frac{GM}{(R+h)^2} \ldots$ (ii)

From Eqs. (i) and (ii), we get

$$\frac{g}{g'} = \left(\frac{R+h}{R}\right)^2 = \left(1 + \frac{h}{R}\right)^2$$

$$\Rightarrow \quad \frac{g'}{g'} = \left(1 + \frac{h}{R}\right)^{-2} = \left(1 - \frac{2h}{R}\right)^2$$

Here,

$$g' = \frac{g}{2}$$

$$\therefore \quad \frac{g/2}{g} = \left(1 - \frac{2h}{R}\right) \Rightarrow \frac{2h}{R} = \frac{1}{2} \text{ or } h = \frac{R}{4}$$

Ans 5

Correct Answer. 2

Solution:

Elastic potential energy stored in a loaded'wire,

$U = \frac{1}{2} \times$ (Stress $\times$ Strain $\times$ Volume)

$\therefore$ Energy stored per unit volume,

$U = \frac{U}{\text{Volume}} = \frac{1}{2} \times$ Stress $\times$ Strain $= \frac{1}{2}\left(\frac{F}{A}\right)^2 \times \frac{1}{Y}$

$\frac{U_A}{U_B} = \frac{\frac{1}{2Y} \cdot \frac{F^2}{A_A^2}}{\frac{1}{2Y} \cdot \frac{F^2}{A_B^2}} = \frac{A_B^2}{A_A^2}, \frac{U_A}{U_B} = \frac{d_B^4}{d_A^4}$

Here, $\frac{U_A}{U_B} = \frac{1}{4}, \frac{d_B^4}{d_A^4} = \frac{1}{4}$ or $\frac{d_B}{d_A} = \frac{1}{\sqrt{2}}$

$\frac{d_A}{d_B} = \sqrt{2} : 1 = \sqrt{k} : 1$ (given)

$\therefore \quad k = 2$

Ans 6

Correct Answer. 0.4

Solution:

Given, $h = 7.5 \text{ cm} = 7.5 \times 10^{-2} \text{ m}$

$S = 7.5 \times 10^{-2} \text{Nm}^{-1}$
$\theta = \sigma^\circ, r = ?$

As, $h = \frac{2S\cos\theta}{r\rho g} \Rightarrow 2r = \frac{4S\cos\theta}{h\rho g}$

$= \frac{4 \times 7.5 \times 10^{-2} \times \cos 0^\circ}{7.5 \times 10^{-2} \times 10^3 \times 10} = 4 \times 10^{-4} = 0.4 \text{ mm}$

Ans 7

Correct Option. B

Solution:

This is an example of a periodic motion. It can be noted that each term represents a periodic function with different angular frequency. Since, period is the least interval of time after which a function repeats its value, sin ωt has a period

$T_0 = \frac{2\pi}{\omega}$

$\cos 2\omega t$ has a period $\frac{\pi}{\omega} = \frac{T_0}{2}$ and $\sin 4\omega t$ has a period $\frac{2\pi}{4\omega} = \frac{T_0}{4}$

The period of the first term is a multiple of the periods of the last two terms. Therefore, the smallest interval of time after which the sum of the three terms repeats is T_0 and thus, the sum is a periodic function with a period $2\pi/\omega$.

Ans 8

Correct Option. D

Solution:

When charging battery is disconnected then charge remains constant, V becomes 1/K times and E becomes 1/K times.

So, new value of charge is Q_0.

New value of $V = \frac{V_0 \times 3}{9} = \frac{V_0}{3}$

New value of $E = \frac{E_0 \times 3}{9} = \frac{E_0}{3}$

Ans 9

Correct Option. D

Solution:

Resistance, $R_t = R_0(1 + \alpha t)$

$20 = R_0(1 + 20\alpha)$ and $60 = R_0(1 + 500\alpha)$

From this equation, we can find

$R_0 = 18.33\Omega$
$\alpha = 4.54 \times 10^{-3}$
$R_t = 25\Omega$

Again, $R_t = R_0(1 + \alpha t)$
$25 = 18.33\left(1 + 4.54 \times 10^{-3} t\right)$

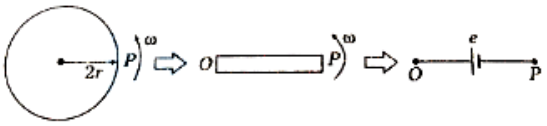
We find $t = 80^{\circ}\mathrm{C}$

Ans 10

Correct Answer. 1

Solution:

As we know, disc is equivalent to a rod of length r. Divide disc into rods, each rod can be replaced by a battery and these batteries are would be paralial in arrangement.



According to the question, radius of disc, $r = \frac{10}{2} = 5\ \text{cm} = 0.05\ \text{m}$

Angular velocity, $\omega = 20 \times 2\pi = 40\pi \text{rad/s}$

Magnetic field, $B = 10^{-1}\ \text{T}$

The emf of a disc is given as

$e = \frac{1}{2}B\omega r^2 = \frac{1}{2}(10^{-1})\,(40\pi)(0.05)^2$

$\Rightarrow \quad V_0 - V_A = 1.57 \times 10^{-2}\ \text{V}$

$= \frac{\pi}{2} \times 10^{-2}\ \text{V} = \frac{\pi}{x} \times 10^{-p}\ \text{V}$ (given)

$\therefore \quad \frac{x}{p} = \frac{2}{2} = 1$

Ans 11

Correct Option. B

Solution:

Angle i at both focus will be 45^0. For TIR to take place,

$i > \theta_c$ or $\sin i > \sin \theta_c$

$\therefore \quad \frac{1}{\sqrt{2}} > \frac{1}{\mu}$ or $\mu > \sqrt{2}$

Ans 12

Correct Option. B

Solution:

We know that in Young's double slit experiment, fringe width $\beta = \frac{D\lambda}{d}, \beta \propto \lambda$

Since, $\lambda_{\text{blue}} < \lambda_{\text{orange}}$

Thus, $\beta_{\text{blue}} < \beta_{\text{orange}}$

∴ In Young's double slit experiment, if the source of light changes from orange to blue, then distance between the consecutive fringes will decrease.

Ans 13

Correct Option. D

Solution:

As, $\lambda = \frac{h}{\sqrt{2qVm}} \propto \frac{1}{\sqrt{qVm}}$

$\therefore \quad \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{q_p V_p m_p}{q_\alpha V_\alpha m_\alpha}} = \sqrt{\frac{1\times100\times1}{2\times800\times4}} = \frac{1}{8}$

$\therefore \quad \lambda_\alpha = \frac{\lambda_p}{8} = \frac{\lambda_0}{8}$

Ans 14

Correct Answer. 4.5

Solution:

Radius of circular path described by a charged particle in a magnetic field B is given by

$r = \sqrt{\frac{2mK}{qB}} \Rightarrow K = \frac{q^2B^2r^2}{2m} = \left(\frac{e}{m}\right)\frac{eB^2r^2}{2}$

where, K = kinetic energy of a particle

q = charge on a particle

r = radius of circular path

$= \frac{1}{2} \times 1.7 \times 10^{11} \times 1.6 \times 10^{-19} \times \left(\frac{1}{\sqrt{17}} \times 10^{-5}\right)^2 \times (1)^2$

$= 8 \times 10^{-20}\ \text{J} = 0.5\text{eV}$

Using the relation,

$E = \phi + K_{\max} \Rightarrow \phi = E - K_{\max}$

where, $E = h\nu = \frac{hc}{\lambda} = \frac{12375}{\lambda(\text{in Å})}$

$= \left(\frac{12375}{2475}\right)\text{eV} - 0.5\text{eV} = 4.5\text{eV}$

Ans 15

Correct Option. C

Solution:

Final energy of electron

$= -13.6 + 12.1 = -1.51\text{eV}$

This energy corresponds to third level i.e., n = 3

Hence, number of spectral lines emitted

$= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$

Chemistry

Ans 1

Correct Option. C

Solution:

100 mL of $0.001 \mathrm{MNaCl}^{-2}$ contain Cl^{-1} ions

$$= \frac{0.001}{1000} \times 100 = 10^{-4} \text{ mol}$$
$$= 10^{-4} \times \left(6.022 \times 10^{23}\right) = 6.022 \times 10^{19} \mathrm{Cl}^{-} \text{ ions}$$

Ans 2

Correct Option. A

Solution:

Energy of photon depends upon frequency and not on intensity. As on increasing the intensity of light of a particular frequency v, the number of electrons ejected increases but not their kinetic energy.

Ans 3

Correct Option. B

Solution:

$\mathrm{H}_2,\ \mathrm{B}_2, \mathrm{O}_2^{2-}$ have bond order equal which is to 1 and Be_2 and N_2 have 0 and 3 bond order respectively.

Ans 4

Correct Option. C

Solution:

I. → Isothermal expansion

II. → Adiabatic expansion

III. → Isothermal compression

IV. → Adiabatic compression

Ans 5

Correct Answer. 3.37

Solution:

For a weak acid,

$$\begin{aligned}\mathrm{pH} &= \tfrac{1}{2}[\mathrm{p}K_a - \log C] \\ &= \frac{1}{3}\left[4.74 - \log 10^{-2}\right] \\ &= \frac{1}{2} \times 6.74 = 3.37\end{aligned}$$

Ans 6

Correct Option. C

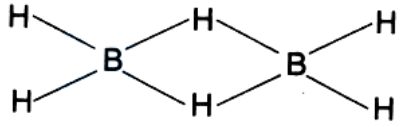
Solution:

As we move down the group, solubility increases. So, $Be(OH)_2$ is insoluble and $Ba(OH)_2$ is highly soluble in water.

Ans 7

Correct Answer. 6

Solution:



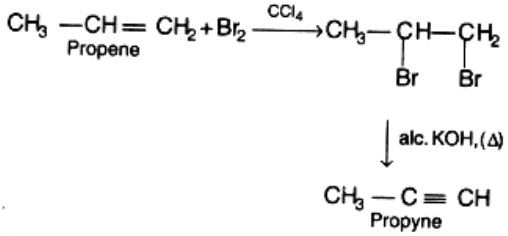
Number of $3c - 2e^{-1}$ and $2c - 2e^{-}$ bond present in diborane are 2 and 4, respectively.

Thus, total number of $3c - 2e^{-1}$ and $2c - 2e^{-}$ bonds are 6.

Ans 8

Correct Option. B

Solution:



Ans 9

Correct Answer. 200

Solution:

Solution with the same osmotic pressure are isotonic.

Let the molar mass of the substance be M.

$\therefore \quad \pi_1 = \pi_2$

$C_1RT = C_2RT$

$C_1 = C_2$

As density are also same

So, $\frac{5}{M} = \frac{1.5}{60}$

$\Rightarrow \quad M = \frac{5 \times 60}{1.5} = 200 \text{ g mol}^{-1}$

Ans 10

Correct Answer. 0.77

Solution:

$\Delta G_3 = \Delta G_1 + \Delta G_1$

$n_3FE_3^\circ = n_1FE_1^\circ + n_2FE_2^\circ$

$3 \times 0.036 = 2 \times 0.439 + 1 \times E_2^\circ$

$\Rightarrow \quad E_2^\circ = 0.108 - 0.878$

$= -0.77 \text{ V}$

Ans 11

Correct Answer. 3

Solution:

Cu^{2+}, Fe^{3+}, Ti^{3+} have unpaired electron which impart colour to the ions whereas Zn^{2+} has no unpaired, thus it does not impart any colour.

Ans 12

Correct Option. B

Solution:

$[Ni(CO)_4]$ – sp^3-hybridisation

$[Ni(CN)_4]^{2-}$ – dsp^2-hybridisation

$[Ni(NH_3)_6]^{2+}$ – sp^3d^2-hybridisation

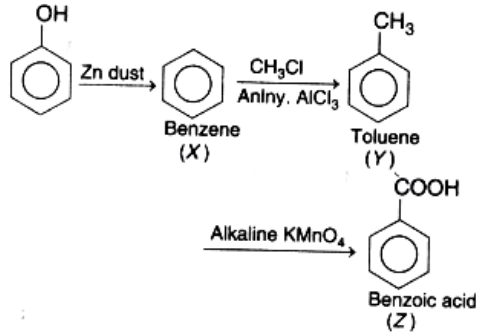
$[Fe(CN)_6]^{3-}$ – d^2sp^3-hybridisation

The correct answer is A-2, B-1, C-4, D-3.

Ans 13

Correct Option. C

Solution:

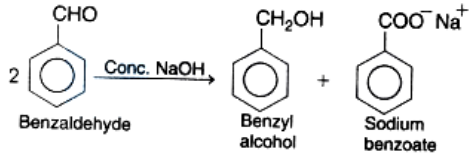


Ans 14

Correct Option. D

Solution:

In Cannizzaro reaction benzaldehyde acts as reactant not the product.



Ans 15

Correct Option. A

Solution:

Styrene exist as liquid at room temperature.

Mathematics

Ans 1

Correct Option. D

Solution:

Here, we have

Radius of circle = 3 units we find the maximum and minimum value of |z + 1|

for z lying in the interier or boundary of the circle

|z| = 3

For $\mathbf{Max}\,|z+1|$, z must lies on the bondary of the circle, then $|z| = 3$ and $z = 3e^{\theta}$, where θ is the argument of z.

Now, $|z+1| = \left|3e^{i\theta}+1\right|$

$$= |(3\cos\theta+1)+(3i\sin\theta)|$$
$$= \sqrt{(3\cos\theta+1)^2+(3\sin\theta)^2}$$

$$= \sqrt{9+1+6\cos\theta}$$
$$= \sqrt{10+6\cos\theta}$$

$\sin\theta, \cos\theta$ ranges from -1 to 1 , the maximum value of $|z+1|$ occur, when $\cos\theta = 1$. Therefore,
$\mathbf{Max}\,|z+1| = \sqrt{10+6} = \sqrt{16} = 4$

For minimum value The minimum value of $z+1$ occures, when z is at the centre of the circle $|z| = 3$. In this case $z = 0$.

On putting $z = 0$ into the expression, we have $\mathbf{Min}(|z+1|) = |0+1| = 1$

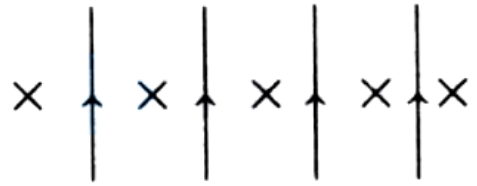
Hence, maximum value of $|z+1| = 4$ minimum value of $|z+1| = 1$

Ans 2

Correct Answer. 120

Solution:

Given, word is LETTER, having vowels E, E and consonants $\mathbf{L, T, T}$ and R . Now, the number of ways to arrange the consonants are $\frac{4!}{2!} = 12$ Now, we have five place to put vowels E, E.



So, number of ways to arrange vowels is ${}^5C_2 = 10$ So, number of required words $= 12 \times 10 = 120$

Ans 3

Correct Option. C

Solution:

We have, a^2, b^2 and c^2 are in AP adding $ab + bc + ca$ to each of these terms, we get $a^2 + ab + bc + ca, b^2 + ab + bc + ca$ and $c^2 + ab + bc + ca$ are in AP.

$\Rightarrow (a + b)(a + c), (b + c)(b + a)$ and $(c + a)$

$(c + b)$ are in AP.

Dividing each term by $(b + c)(c + a)(a + b)$, we get

$\frac{1}{b+c}, \frac{1}{c+a}$ and $\frac{1}{a+b}$ are in AP

$\Rightarrow b + c, c + a$ and $a + b$ are in HP.

Ans 4

Correct Option. B

Solution:

In the expansion of $\left(1 + ax + bx^2\right)(1 - 2x)^{18}$ coefficient of x^3 in $\left(1 + ax + bx^2\right)(1 - 2x)^{18}$

= Coefficient of x^3 in $(1 - 2x)^{18}$+ coefficient of x^2 in $a(1 - 2x)^{18}$+ coefficient of x in $b(1 - 2x)^{18}$.

$= -{}^{18}C_3 \cdot 2^3 + a{}^{18}C_2 \cdot 2^2 - b{}^{18}C_4 \cdot 2$

$\because \quad -{}^{18}C_3 \cdot 2^3 + a{}^{18}C_2 \cdot 2^2 - b{}^{18}C_4 \cdot 2 = 0$

$\Rightarrow \frac{18\times17\times16}{3\times2} \cdot 8 + a \cdot \frac{18\times17}{2} 2^2 - b \cdot 18 \cdot 2 = 0$

$\Rightarrow 17a - b = \frac{34\times16}{3} \ldots$ (i)

Similarly, coefficient of x^4

${}^{18}C_4 \cdot 2^4 - a \cdot {}^{18}C_3 2^3 + b \cdot {}^{18}C_2 \cdot 2^2 = 0$

$\therefore \quad 32a - 32b = 240 \ldots$ (ii)

From Eqs. (i) and (ii), we get

$a = 16$ and $b = \frac{272}{3}$

Ans 5

Correct Answer. 01

Solution:

$\because (9)^{50} = (10-1)^{50}$

$= [{}^{50}C_0(10)^{50}(-1)^0 - {}^{50}C_1(10)^{49}(1) + \ldots.] - {}^{50}C_{49}(10)^1 + {}^{50}C_{50}(10)^0$

$\because$ All the terms in the bracket are multiples of 10^2.

So, we can write as 100λ

$= 100\lambda - 50 \times 10 + 1 = 100\lambda - 500 + 1$

so, last 2 digits will be $= 01$.

Ans 6

Correct Option. C

Solution:

The equation of family of circles which passess through the origin and whose centrer lies on Y-axis is

$(x-0)^2 + (y-k)^2 = k^2$

$\Rightarrow x^2 + y^2 + k^2 - 2yk = k^2$
$\Rightarrow x^2 + y^2 - 2yk = 0$

where, k is parameter.

On differentiating w.r.t . x, we get

$2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx}k = 0$

$\Rightarrow \quad x + (y-k)\frac{dy}{dx} = 0$
$\Rightarrow \quad x + \left\{y - \frac{x^2+y^2}{2y}\right\}\frac{dy}{dx} = 0$

$\Rightarrow \quad x + \frac{1}{2y}\left(y^2 - x^2\right)\frac{dy}{dx} = 0$
$\Rightarrow \quad 2xy = \left(x^2 - y^2\right)\frac{dy}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2xy}{x^2-y^2},$

which is required equation.

Ans 7

Correct Option. A

Solution:

Let $I = \lim_{x\to 3} \int_6^{f(x)} \frac{2t}{x-2} dt \left[\frac{0}{0} \text{ form, as } f(3) = 6\right]$

On applying the L' Hospital rule, we get

$I = \lim_{x\to 3} \int_6^{f(x)} \frac{2f(x)f'(x)}{1} (\because f(3) = 6)$

So, $I = 3f(3) \cdot f'(3) = 18f'(3)$

$\therefore \quad \lim_{x\to 3} \int_6^{f(x)} \frac{2tdt}{x-2} = 18f'(3)$

Ans 8

Correct Answer. 16

Solution:

$y = \lim_{x\to 0} \left(\frac{8^x+2^x+4^x}{3}\right)^{2/x}$

$$\Rightarrow \log y = \lim_{x\to 0} \frac{2}{x} \log\left(\frac{8^x + 2^x + 4^x}{3}\right)$$
$$= 2 \lim_{x\to 0} \frac{1}{x} [\log(8^x + 2^x + 4^x) - \log 3]$$

$= 2 \lim_{x\to 0} \frac{[\log(8^x+2^x+4^x)-\log 3]}{x} \quad \left[\frac{0}{0} \text{ form }\right]$

On applying L' Hospital rule, we get

$$= 2 \lim_{x\to 0} \frac{[8^x \log 8 + 2^x \log 2 + 4^x \log 4]}{8^x + 2^x + 4^x}$$
$$= 2\left[\frac{\log 8 + \log 2 + \log 4}{3}\right]$$

$= \frac{2}{3}[3\log 2 + \log 2 + 2\log 2] \quad [\log(m)^n = n \log m]$

$$= \frac{2}{3} \times 6 \log 2$$
$$\log y = 4 \log 2 = \log 16 \Rightarrow y = 16$$

Ans 9

Correct Option. D

Solution:

Given, $\sum_{i=1}^{y}(x_i-5)=9$

$\sum x_i-45=9 \Rightarrow \sum x_i=54$

Also, $\sum_{i=1}^{9}(x_i-5)^2=45$

$\Rightarrow \quad \sum x_i^2-10\times 54+25\times 9=45$

$\sum x_i^2=360$

$\therefore$ Standard deviation $(\sigma)=\sqrt{\frac{360}{9}-\left(\frac{54}{9}\right)^2}=2.$

Ans 10

Correct Option. A

Solution:

$A=\{1,2,3,4,5,6,7\}$ and $B=\{3,6,7,9\}$.

Total subset of $A=2^7=128$ $C\cap B=\phi$, when set C contains the elements 1,2 , 4,5 .

$S=\{C\subseteq A; C\cap B\neq\phi\}$

Total $-(C\cap B=\phi)=128-2^4=112$

Ans 11

Correct Option. B

Solution:

$B^TA^T=(AB)^T$

$=\left(\begin{bmatrix}2 & -1\\ -7 & 4\end{bmatrix}\begin{bmatrix}4 & 1\\ 7 & 2\end{bmatrix}\right)^T=\begin{bmatrix}8-7 & 2-2\\ -28+28 & -7+8\end{bmatrix}^T$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}^T=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

Ans 12

Correct Option. B

Solution:

Given that

$f(x) = \frac{1}{\sqrt{1+x^2}}$

$\therefore \quad f(y) = \frac{1}{\sqrt{1+y^2}}$

and $f(x,y) = \frac{1}{\sqrt{1+x^2y^2}}$

Now, $f(x,y) = f(x) \cdot f(y)$

$\frac{1}{\sqrt{1+x^2y^2}} = \frac{1}{\sqrt{1+x^2}} \times \frac{1}{\sqrt{1+y^2}}$

$\frac{1}{\sqrt{1+x^2y^2}} = \frac{1}{\sqrt{(1+x^2)(1+y^2)}}$

On squaring both sides, we get

$\frac{1}{1+x^2y^2} = \frac{1}{(1+x^2)(1+y^2)}$

$\frac{1}{1+x^2y^2} \geq \frac{1}{(1+x^2)(1+y^2)}$

$\left[\because 1 + x^2y^2 \leq 1 + x^2 + y^2 + x^2y^2\right]$

Hence, $f(x,y) \geq f(x) \cdot f(y)$

Ans 13

Correct Option. A

Solution:

$|\sin x| > 2\sin^2 x$

$\Rightarrow |\sin x|(2|\sin x| - 1) < 0 \Rightarrow 0 < |\sin x| < \frac{1}{2}$

$\Rightarrow x \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right) \cup \left(\pi, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

Ans 14

Correct Option. B

Solution:

Here, $\int f(x)dx = g(x)$

Now, $\int \cos x f(\sin x)dx$

Let, $\sin x = k \Rightarrow \cos x dx = dk$

$\therefore \int f(k)dk = g(k) + C = g(\sin x) + C$

Ans 15

Correct Option. B

Solution:

Required area $= \left|\int_1^6 |5 - x|dx\right|$

$= \left|\int_1^5 -(5-x)dx + \int_5^6 (5-x)dx\right|$

$= \left|\left[-5x + \frac{x^2}{2}\right]_1^5 + \left[5x - \frac{x^2}{2}\right]_5^6\right|$

$= \left|\left[-25 + \frac{25}{2} + 5 - \frac{1}{2}\right] + \left[30 - 18 - 25 + \frac{25}{2}\right]\right|$

$= \left[\frac{-50+25+10-1}{2}\right] + \left[\frac{-26+25}{2}\right]$

$= \left|-8 - \frac{1}{2}\right| = \frac{17}{2}$ sq units

Ans 16

Correct Option. C

Solution:

The given curves, $y = \tan x$ and $y = \cot x$ meet at $x = \frac{\pi}{4}$.

$\therefore$ Required area $= \int_0^{\pi/4} \tan x dx + \int_{\pi/4}^{\pi/2} \cot x dx$

$= (\ln \sec x)_0^{\pi/4} + (\ln \sin x)_{\pi/4}^{\pi/2}$

$= \ln \sqrt{2} - \ln \frac{1}{\sqrt{2}} = 2 \ln \sqrt{2} = \ln 2$

Ans 17

Correct Answer. -0.33

Solution:

Given, vectors are

$$\mathbf{p} = (a+1)\hat{\mathbf{i}} + a\hat{\mathbf{j}} + a\hat{\mathbf{k}}$$
$$\mathbf{q} = a\hat{\mathbf{i}} + (a+1)\hat{\mathbf{j}} + a\hat{\mathbf{k}}$$

and $\mathbf{r} = a\hat{\mathbf{i}} + a\hat{\mathbf{j}} + (a+1)\hat{\mathbf{k}}$

$[pqr] = 0$

$$\Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$
$$\Rightarrow (a+1)\left[(a+1)^2 - a^2\right] - a\left[a(a+1) - a^2\right] + a\left[a^2 - a(a+1)\right] = 0$$

$$\Rightarrow (a+1)[2a+1] - 2a[a] = 0$$
$$\Rightarrow 2a^2 + 3a + 1 - 2a^2 = 0$$
$$\Rightarrow a = -\frac{1}{3} = -0.33$$

Ans 18

Correct Option. C

Solution:

Given, two lines

$L_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

and $L_2 : \frac{x-3}{1} = \frac{y-k}{2} = \frac{z-0}{1}$

Let $L_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = p$

and $L_2 : \frac{x-3}{1} = \frac{y-k}{2} = \frac{z-0}{1} = q$

Any point P on line L_1 is of type

$P(2p+1, 3p-1 \text{ and } 4p+1)$ and any point Q on the line L_2 is of type $Q(q+3, 2q+k \text{ and } q)$

Since, L_1 and L_2 are intersecting each other, hence, both point P and Q should coincide at the point of intersection. Corresponding co-ordinates of P and Q should be same.

$2p+1=q+3, 3p-1=2q+k$ and $4p+1=q$

On solving, $2p+1=q+3$ and $4p+1=q$, we get

$p=-\frac{3}{2}$ and $q=-5$

On substituting the values of p and q in the third equation $3p-1=2q+k$, we get

$3\left(-\frac{3}{2}\right)-1=2(-5)+k$

$\therefore \quad k=\frac{9}{2}$

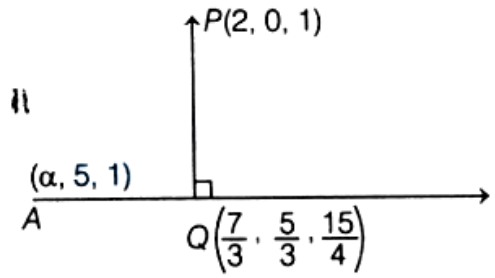
Ans 19

Correct Answer. 4.372

Solution:

It is given that $Q\left(\frac{7}{3}, \frac{5}{3}, \frac{15}{4}\right)$ is foot of

perpendicular of point $P(2,0,1)$ to a line passes through point $A(\alpha,5,1)$. So $PQ \perp AQ$



$\because$ Direction ratios of line segments PQ is $\left(\frac{7}{3}, \frac{5}{3}, \frac{15}{4}\right)$ and direction ratios of line AQ is

$\left(\alpha-\frac{7}{3}, \frac{10}{3}, -\frac{11}{4}\right)$

Since, PQ and AQ is perpendicular to each other,

so $PQ \cdot AQ = 0$

$\frac{7}{3}\left(\alpha-\frac{7}{3}\right)+\frac{5}{3}\times\frac{10}{3}+\frac{15}{4}\times\left(-\frac{11}{4}\right)=0$

$\Rightarrow \quad \frac{7(3\alpha-7)}{9}+\frac{50}{9}-\frac{165}{16}=0$

$\Rightarrow \quad \frac{336\alpha-784+800-1485}{144}=0$

$\Rightarrow \quad 336\alpha=1469 \Rightarrow \alpha=\frac{1469}{336}=4.372$

Ans 20

Correct Option. C

Solution:

The coin is tossed n times.

Let x denotes the number of heads in n trials and it is given that p(x = 5) = p(x = 6)

$$\Rightarrow {}^{n}C_5\left(\frac{1}{2}\right)^n = {}^{n}C_6\left(\frac{1}{2}\right)^n \Rightarrow n = 11$$

$$\therefore p(x = 3) = {}^{11}C_3\left(\frac{1}{2}\right)^{11}$$