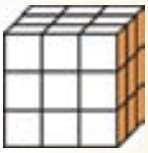
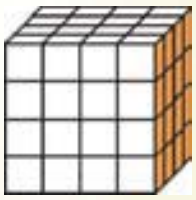
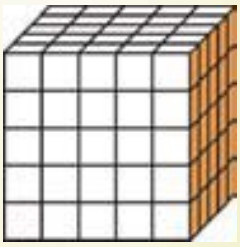
1 Cube and Cube Root

Let us remember :

You have learnt about cube and its volume in Standard VII. All sides of a cube are equal in measure. Aren't they ? Now fill in the blanks according to example :

Question	Figure	Volume (cm^3)
What will be the volume of cube having side of measure 1 cm ?		$1 \times 1 \times 1 = 1^3 = 1$
What will be the volume of cube having side of measure 2 cm ?		$2 \times 2 \times 2 = 2^3 = 8$
What will be the volume of cube having side of measure 3 cm ?		$3 \times 3 \times 3 = 3^3 = 27$
What will be the volume of cube having side of measure 4 cm ?		
What will be the volume of cube having side of measure 5 cm ?		

Let's learn new :

On the basis of Table the numbers 1, 8, 27, 64, 125 are obtained if same number is multiplied three times. Such obtained number is called cube of that number.

Cube of a number is obtained by multiplying that number with its square. Therefore, $x^2 = x \times x$.

Here, $1 \times 1 \times 1 = 1^3 = 1$. Therefore cube of 1 is '1' and cube root of '1' is '1'.

The symbol for cube root is '$\sqrt[3]{\ }$'.

Symbolically $\sqrt[3]{1} = 1$ (Read as : Cuberoot of 1 is equal to 1)

$2 \times 2 \times 2 = 2^3 = 8$. Therefore cube of 2 is '8' and cuberoot of 8 is '2'.

Symbolically $\sqrt[3]{8} = 2$ (Read as : Cuberoot of 8 is 2)

$3 \times 3 \times 3 = 3^3 = 27$. $\therefore$ Cube of 3 is '27' and $\sqrt[3]{27} = 3$

$4 \times 4 \times 4 = 4^3 = 64$ $\therefore$ Cube of 4 is '64' and $\sqrt[3]{64} = 4$

$5 \times 5 \times 5 = 5^3 = 125$ $\therefore$ Cube of 5 is '125' and $\sqrt[3]{125} = 5$

The numbers 1, 8, 27, 64, 125,... etc obtained above are called perfect cube numbers.

- **Now complete the following Table :**

Table 1

Number	Cube	Number	Cube
1	$1^3 = 1$	11	.....
2	$2^3 =$	12	$12^3 = 1728$
3	$3^3 =$	13	$13^3 = 2197$
4	$4^3 = 64$	14	.....
5	$5^3 =$	15	.....
6	$6^3 = 216$	16	$16^3 = 4096$
7	.....	17	$17^3 = 4913$
8	.....	18	.....
9	$9^3 =$	19	.....
10	.....	20	$20^3 = 8000$

- **Answer the following questions on the basis of Table 1 :**

(1) Is your roll number is a perfect cube number ? Why ?

.....

(2) Which are the perfect cube numbers from 1 to 100 ?

.....

(3) How many perfect cube numbers are there from 1 to 1000 ?

.....

- **Fill in the blank in the given Table 2 on the basis of Table 1 :**

Table 2

Digit at unit place of number	Digit at unit place of a number obtained by cubing number	To get more information of the numbers on the basis of Table 1 and Table 2, cube the following numbers : 21 to 30	Digit at unit place of number	Digit at unit place of a number obtained by cubing number
1	1		2	8
4	.....		8	.....
5	5		3	7
6	.....		7	.....
9	.....		0	.....

It is clear from the Table 2,

- If 1, 4, 5, 6, 9 and 0 are the digit at unit place of numbers then the digit at unit place of the cube of numbers will be 1, 4, 5, 6, 9 and 0 respectively.
- If digit at unit place of a number is 2 then 8 will be the digit at unit place of cube of that number and if 8 lies at unit place then cube of that number has 2 at its unit place. If 3 lies at unit place then its cube has 7 will be at its unit place and if 7 lies at unit place then its cube has 3 at its unit place.

Examine the number of zero of the number obtained by cube of 10, 20, 30.

$10^3 = 10 \times 10 \times 10 = 1000$ $\quad$ $20^3 = 20 \times 20 \times 20 =$ .....

$30^3 = 30 \times 30 \times 30 =$ ..... $\quad$ $40^3 = 40 \times 40 \times 40 =$ .....

On the basis of this we can say that the number having zero at its unit's place, the number obtained by cube of them have zero at unit's ten's and hundred's place.

Example 1 : What will be the digit at unit place obtained by cubing the given numbers :

(1) 11 (2) 58 (3) 104 (4) 407 (5) 596 (6) 840

Number	Digit at unit place obtained by cube of number	Number	Digit at unit place obtained by cube of number
11	1	407	3
58	2	596	6
104	4	840	0

Example 2 : What will be the digit at unit place of cube-root of given perfect cube number ?

(1) 6859, (2) 3375, (3) 17576, (4) 39304, (5) 35397, (6) 64000

(1) 6859 : 9 is the digit at unit place of cube root of perfect cube number 6859.
(2) 3375 : 5 is the digit at unit place of cube root of perfect cube number 3375.
(3) 17576 : 6 is the digit at unit place of cube root of perfect cube number 17576.
(4) 39304 : 4 is the digit at unit place of cube root of perfect cube number 39304.
(5) 35397 : 7 is the digit at unit place of cube root of perfect cube number 35397.
(6) 64000 : 0 is the digit at unit place of cube root of perfect cube number 64000.

1. What will be the 'digit at unit place' obtained by cubing the following numbers :

(1) 401 (2) 258 (3) 344 (4) 47 (5) 66 (6) 25 (7) 79 (8) 10

2. What will be the 'digit at unit place' in the cube root of the following numbers :

(1) 729 (2) 4096 (3) 15625 (4) 13824
(5) 12167 (6) 8000 (7) 5832 (8) 1331

Examine that the given number is perfect cube or not :

Example 3 : Verify 64 is perfect cube or not.

2	64
2	32
2	16
2	8
2	4
2	2
	1

$64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3$

$= (2 \times 2)^3$

$= 4^3$

$\therefore$ 64 is a perfect cube number.

Example 4 : Verify 2312 is perfect cube or not.

2	2312
2	1156
2	578
17	289
17	17
	1

$2312 = \underline{2 \times 2 \times 2} \times \underline{17 \times 17}$

Here 2 is three times but 17 is two times only.

There 2312 is not a perfect cube number.

Practice 2

- **Which of the following numbers are perfect cube ? For what ?**

(1) 729 (2) 100 (3) 243 (4) 400 (5) 3375 (6) 127000 (7) 4913 (8) 4096

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Example 5 : By which smallest number 2312 must be multiplied to get a perfect cube.

According to Example 4, $2312 = \underline{2 \times 2 \times 2} \times \underline{17 \times 17}$

Here, 2 is three times but 17 is only two times. If here 17 will be three times, then the new number obtained will be perfect cube.

Therefore, $2312 \times 17 = \underline{2 \times 2 \times 2} \times \underline{17 \times 17 \times 17}$

Hence the smallest natural number by which 2312 should be multiplied to make it perfect cube is 17.

Example 6 : By which smallest number 1029 should be multiplied to get a perfect cube ?

Example 7 : By which smallest number 704 should be divided to get a perfect cube ?

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

$704 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$

Here 11 is only one time. If 11 is removed, therefore, if given number is divided by 11, then new number is a perfect cube.

Example 8 : By which smallest number 1600 should be divided to get a perfect cube number ?

Practice 3

1. **Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube ?**

(1) 256 (2) 100 (3) 576 (4) 81 (5) 1715

2. **Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube :**

(1) 88 (2) 875 (3) 1512 (4) 625 (5) 13500

To find cube root through factorisation method :

We have learnt to find square root through factorisation method. Now let's learn to find cube root through factorisation method.

Example 9 : Find cube root of 15625 by factorisation method.

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$$15625 = \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$$
$$= 5^3 \times 5^3$$
$$= (5 \times 5)^3$$
$$= (25)^3$$
$$\therefore \sqrt[3]{15625} = 25$$

Example 10 : Find the cube root of 10648 by factorisation method.

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$$10648 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$$
$$= 2^3 \times 11^3$$
$$= (2 \times 11)^3$$
$$= (22)^3$$
$$\therefore \sqrt[3]{10648} = 22$$

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To find cube-root of fractional and decimal-fractional numbers :

As we have learnt to find out the cube of perfect cube integers, similarly let us learn to find the cube-root of perfect cube fractions.

Example 11 : Find cube root of $\sqrt[3]{\frac{27000}{8000}}$ or $\frac{27000}{8000}$ by factorisation method

2	27000
2	13500
2	6750
5	3375
5	675
5	135
3	27
3	9
3	3
	1

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{27000}{8000} = \frac{2\times2\times2\times5\times5\times5\times3\times3\times3}{2\times2\times2\times2\times2\times2\times5\times5\times5}$$
$$= \frac{2^3\times5^3\times3^3}{2^3\times2^3\times5^3}$$
$$= \frac{(2\times5\times3)^3}{(2\times2\times5)^3}$$
$$= \frac{(30)^3}{(20)^3}$$
$$\sqrt[3]{\frac{27000}{8000}} = \sqrt{\frac{(30)^3}{(20)^3}} = \frac{30}{20}$$

Example 12 : Find cube root of $\sqrt[3]{\frac{19683}{15625}}$ or $\frac{19683}{15625}$ by factorisation method.

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$$\frac{19683}{15625} = \frac{3\times3\times3\times3\times3\times3\times3\times3\times3}{5\times5\times5\times5\times5\times5}$$

$$= \frac{3^3\times3^3\times3^3}{5^3\times5^3}$$

$$= \frac{(3\times3\times3)^3}{(5\times5)^3}$$

$$= \frac{(27)^3}{(25)^3}$$

$$\sqrt[3]{\frac{19683}{15625}} = \sqrt{\frac{(27)^3}{(25)^3}} = \frac{27}{25}$$

Example 13 : Find cube root of $\sqrt[3]{1.331}$ or 1.331 by factorisation method.

$1.331 = \frac{1331}{1000}$

11	1331
11	121
11	11
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{1331}{1000} = \frac{11\times11\times11}{2\times2\times2\times5\times5\times5}$$

$$= \frac{11^3}{(2^3\times5^3)}$$

$$= \frac{11^3}{(2\times5)^3}$$

$$= \frac{11^3}{10^3}$$

$$\sqrt[3]{1.331} = \sqrt{\frac{11^3}{10^3}} = \frac{11}{10} = 1.1$$

Example 14 : Find cube root of $\sqrt[3]{12.167}$ or 12.167 by factorisation method.

$12.167 = \frac{12167}{1000}$

23	12167
23	529
23	23
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{12167}{1000} = \frac{23\times23\times23}{2\times2\times2\times5\times5\times5}$$

$$= \frac{23^3}{2^3\times5^3}$$

$$= \frac{23^3}{(2\times5)^3}$$

$$= \frac{23^3}{10^3}$$

$$\sqrt[3]{12.167} = \sqrt{\frac{23^3}{10^3}} = \frac{23}{10} = 2.3$$

- **To find cube root through estimation method :**

If given number is perfect cube then the cube root of same number can be found by estimation method.

Example 15 : Find the cube root of 4913 through estimate.

$\sqrt[3]{4913}$

4913

1 7

Explanation :

Step 1 : Divide the given number into two parts. One part of number having hundred's, ten's and unit's digit and other part of number. Here, in 4913, 913 is first part and 4 is the second part.

Step 2 : Here, first part is 913 whose unit place digit is 3. Here 3 comes at the unit's place of a number between 1 and 9 when it's cube root ends in 3 ? This number is 7.

Step 3 : Here, the number of second part is the smallest number out of the numbers between the cube of two numbers. Here, the second part is 4, which lies between $1^3 = 1$ and $2^3 = 8$. 1 is smaller between 1 and 2. Therefore, 1 will be taken at ten's place of cube root of a perfect cube number.

$\therefore \sqrt[3]{4913} = 17$

Example 16 : Find the cube root of 15625 through estimation.

$\sqrt[3]{15625}$

15 625

2 5

Explanation :

Step 1 : Divide the given number into two parts. One part of number having handred's, ten's and unit's digit and other part of number having remaining digits. Here, dividing 15625 into two parts, 625 is first part and 15 is second part.

Step 2 : Here, first part is 625 whose digit at unit's place is 5. Therefore, 5 will be at unit place of the cube root of this perfect cube number.

Step 3 : Here, the number second part is the smallest number out of the numbers between cube of two numbers. Here, the second part is 15, which lies between $2^3 = 8$ and $3^3 = 27$. 2 is smaller between 2 and 3. Therefore, 2 will be taken at ten's place of cube root of a perfect cube number.

$\therefore \sqrt[3]{15625} = 25$

Note : If the second part is perfect cube then write the digit of its cube root.

Practice 4

1. Find the cube root of the following numbers by factorisation method :

(1) $\frac{8}{27}$ (2) $\frac{27}{125}$ (3) $\frac{125}{729}$ (4) $\frac{2744}{2197}$ (5) $\frac{3375}{4096}$

(6) 0.8 (7) 0.125 (8) 0.216 (9) 4.913 (10) 5.832

2. Find the cube root of the following numbers by estimation method :

(1) 8000 (2) 9261 (3) 13824 (4) 15625 (5) 19683

Exercise

1. **What will be the 'digit at unit place' in the perfect cube of the following numbers on the basis of digit at unit's place of the given numbers :**
   (1) 711 (2) 408 (3) 544 (4) 57 (5) 26 (6) 70
2. **What will be the 'digit at unit place' in the cube root of the following numbers on the basis of digit at unit's place of the given numbers :**
   (1) 216 (2) 2197 (3) 2744 (4) 6859 (5) 42875 (6) 125000
3. **Which numbers of the following numbers are perfect cube ? For which ?**
   (1) 400 (2) 9000 (3) 343 (4) 17576
4. **Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube :**
   (1) 675 (2) 392 (3) 968 (4) 875
5. **Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube :**
   (1) 1536 (2) 8019 (3) 7000 (4) 5400
6. **Find the cube root of the following numbers through factorisation method :**
   (1) 512 (2) 3375 (3) 17576 (4) 35937 (5) 32768 (6) 29791
7. **Find the cube root of the following numbers through estimation :**
   (1) 4096 (2) 42875 (3) 85184 (4) 54872 (5) 74088 (6) 140608

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Answers

Practice 1

1. (1) 1 (2) 2 (3) 4 (4) 3 (5) 6 (6) 5 (7) 9 (8) 0
2. (1) 9 (2) 6 (3) 5 (4) 4 (5) 3 (6) 0 (7) 8 (8) 1

Practice 2

1. (1) a perfect cube number (2) not a perfect cube number (3) not a perfect cube number (4) not a perfect cube number (5) a perfect cube number (6) a perfect cube number (7) a perfect cube number (8) a perfect cube number

Practice 3

1. (1) 2 (2) 10 (3) 3 (4) 9 (5) 25 **2.** (1) 11 (2) 7 (3) 7 (4) 5 (5) 4

Practice 4

1. (1) $\frac{2}{3}$ (2) $\frac{3}{5}$ (3) $\frac{5}{9}$ (4) $\frac{14}{13}$ (5) $\frac{15}{16}$ (6) 0.2 (7) 0.5 (8) 0.6 (9) 1.7 (10) 1.8
2. (1) 20 (2) 21 (3) 24 (4) 25 (5) 27

Exercise

1. (1) 1 (2) 2 (3) 4 (4) 3 (5) 6 (6) 0 **2.** (1) 6 (2) 3 (3) 4 (4) 9 (5) 5 (6) 0
3. (1) not a perfect cube number (2) not a perfect cube number
   (3) a perfect cube number (4) a perfect cube number
4. (1) 5 (2) 7 (3) 11 (4) 49 **5.** (1) 3 (2) 11 (3) 7 (4) 25
6. (1) 8 (2) 15 (3) 26 (4) 33 (5) 32 (6) 31
7. (1) 16 (2) 35 (3) 44 (4) 38 (5) 42 (6) 52