Previous Year Paper

30th May 2023 (Shift 1)

Q1. If A is an invertible matrix of order 2; then det(A^{-1}) is equal to:
(a) 1
(b) $\frac{1}{\det(A)}$
(c) $\det(A)$
(d) 0

Q2. The value of x, for which the function $f(x) = x(x-2)$ attains its minimum value, is:
(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) 0

Q3. In a linear programming problem, the constraints on the decision variables x and y are, $x - 3y \geq 0, y \geq 0, 0 \leq x \leq 3$. The feasible region:
(a) is not in the first quadrant
(b) is bounded in the first quadrant
(c) is unbounded in the first quadrant
(d) does not exist

Q4. The order of the differential equations representing the family of parabolas $y^2 = 4ax$ is:
(a) 1
(b) 2
(c) 3
(d) 4

Q5. If $y = 2x(x-1)(2-x)$; then $\frac{d^2y}{dx^2}$ is:
(a) $6 - 12x$
(b) $12(1+x)$
(c) $12(1-x)$
(d) $6 + 2x$

Q6. If A and B are two matrices of order $4 \times m$ and $4 \times n$, respectively and m = n then the order of matrix (5A — 2B) is:
(a) $4 \times m$
(b) 4×4
(c) 3×3
(d) $m \times 4$

Q7. The area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$ is:
(a) $\frac{16}{3}$ sq. units
(b) $\frac{32}{3}$ sq. units
(c) $\frac{8}{3}$ sq. units
(d) $\frac{11}{3}$ sq. units

Q8. The order of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 = \left(\frac{d^3y}{dx^3}\right)$ is:
(a) 3
(b) 2
(c) 1
(d) 4

Q9. Determinant is unaltered if:
(a) Elements of any two rows are interchanged
(b) Elements of any two columns are interchanged
(c) Every diagonal element is changed to zero
(d) All the rows and columns are interchanged

Q10. Amit, Sumit and Puneet were doing shooting. Amit hits the target 4 times in 5 shots, Sumit hits the target 3 times in 4 shots and Puneet hits the target 2 times in 3 shots.
Then the probability that Amit, Sumit and Puneet all may hit the target is:
(a) $\frac{1}{5}$
(b) $\frac{2}{5}$
(c) $\frac{3}{5}$
(d) $\frac{4}{5}$

Q11. $\int e^{2x^3 + 2\log x} dx =$
(a) $\frac{1}{3}e^{2x^3} + C$
(b) $\frac{1}{6}e^{2x^3} + C$
(c) $\frac{1}{2}e^{2x^3} + C$
(d) $\frac{1}{12}e^{2x^3} + C$

Q12. For an objective function $z = ax + by$ where a, b > 0. The points of feasible region determined by a set of constraints are (0, 20) (10, 10) (30, 30) and (0, 40). The condition on a and b such that max z occurs at both the points (30, 30) and (0, 40) is:
(a) $b - 3a = 0$
(b) $a = 3b$
(c) $a + 2b = 0$

(d) $2a - b = 0$

Q13. The mean number of heads in three tosses of a fair coin is:

(a) $\frac{1}{2}$
(b) 1
(c) $\frac{3}{2}$
(d) $\frac{1}{4}$

Q14. The points on the curve $\frac{x^2}{16} + \frac{y^2}{25} = 1$ at which tangents are parallel to x-axis are:

(a) $(\pm 5, 0)$
(b) $(\pm 4, 0)$
(c) $(0, \pm 5)$
(d) $(0, \pm 4)$

Q15. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ then:

(a) $A^{-1} = \frac{1}{11} A$
(b) $A^{-1} = \frac{1}{19} A$
(c) $A^{-1} = -\frac{1}{19} A$
(d) $A^{-1} = \frac{1}{7} A$

Q16. $(\vec{a}.\hat{\imath})^2 + (\vec{a}.\hat{\jmath})^2 + (\vec{a}.\vec{k})^2$ is equal to:

(a) 1
(b) $\vec{a}$
(c) $-\vec{a}$
(d) $|\vec{a}|^2$

Q17. Match **List-I** with **List-II**:

List-I		List-II	
(A)	Derivative of $\lvert x-1 \rvert + \lvert x-3 \rvert at\ x = 2$	(I)	-1
(B)	$y = \log_e \sqrt{\tan x}\ then \frac{dy}{dx} at\ x = \frac{\pi}{4}$	(II)	1
(C)	$\sin(x+y) = \log(x+y)$ then $\frac{dy}{dx}$ is	(III)	2
(D)	Value of C in Lagrange's Mean Value Theorem for $f(x) = x^2 + x + 1, x \in [0, 4]$	(IV)	0

Choose the **correct** answer from the options given below:

(a) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(b) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
(c) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(d) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Q18. The area bounded by the curve $y = -x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by:

(a) $\frac{4}{3}$
(b) $\frac{2}{3}$
(c) $\frac{1}{3}$
(d) 0

Q19. x speaks truth in 60% and y in 50% of the cases. The probability that they contradict each other while narrating the same fact is:

(a) $\frac{1}{4}$
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$
(d) $\frac{2}{3}$

Q20. The solution of the differential equation $(coty)dx = x\,dy$ is:

(a) $x = C\ cosy$
(b) $x = C\ secy$
(c) $y = C\ cosx$
(d) $y = C\ secx$

Q21. A value of p for which the points (1, 1, p) and (-3, 0, 1) are equidistant from the plane $\vec{r}.(3\hat{\imath} + 4\hat{\jmath} - 12\hat{k}) + 13 = 0$ is:

(a) $p = 1$
(b) $p = \frac{2}{3}$
(c) $p = 2$
(d) $p = \frac{1}{3}$

Q22. $\tan^{-1} 3 + \tan^{-1} \lambda = \tan^{-1} \frac{3+\lambda}{1-3\lambda}$ is valid for which values of λ?

(a) $\lambda \in \left(-\frac{1}{3}, \frac{1}{3}\right)$
(b) $\lambda > \frac{1}{3}$
(c) $\lambda < \frac{1}{3}$
(d) All real values of λ

Q23. The value of $\tan^2(\sec^{-1} 2)$ is:

(a) 5
(b) 3
(c) 4
(d) 6

Q24. The function $f: R \rightarrow R$ given by $f(x) = -|x-1|$ is

(a) continuous as well as differentiable at $x = 1$
(b) not continuous but differentiable at $x = 1$
(c) continuous but not differentiable at $x = 1$
(d) neither continuous nor differentiable at $x = 1$

Q25. The maximum value of $4 \sin^2 x + 3 \cos^2 x$ is:

(a) 3
(b) 4
(c) 5
(d) 7

Q26. Given that the value of a determinant of order three is zero; which one of the following is not correct?
(a) The elements of any two rows are proportional
(b) The elements of any two column are proportional
(c) The elements of the diagonal are all non-zero and all other elements are zero
(d) The elements of second row are average of elements of first and second row

Q27. If $\int_0^1 \frac{e^x}{1+x}dx = K$, then $\int_0^1 \frac{e^x}{(1+x)^2}dx$ is equal to:
(a) $K-1+\frac{e}{2}$
(b) $K+1-\frac{e}{2}$
(c) $K-1-\frac{e}{2}$
(d) $K+1+\frac{e}{2}$

Q28. A linear programming problem is as follows:
Maximize/minimize objective function $z = 2x - y + 5$ subject to constraints.
$3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$.
If the corner points of feasible region are A(0, 10) B(12, 6) C(20, 0), 0(0, 0), then which of following is true.
(a) Maximum value of z is 40
(b) Minimum value of z is -5
(c) Difference of maximum and minimum values of z is 35
(d) At two corner points value of z are equal.

Q29. The degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 = 0$ is:
(a) 2
(b) 1
(c) 3
(d) 4

Q30. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then A^2=
(a) $\begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$
(b) $\begin{bmatrix} -\sin 2\theta & \sin 2\theta \\ cos2\theta & \cos 2\theta \end{bmatrix}$
(c) $\begin{bmatrix} cos2\theta & sin2\theta \\ -sin2\theta & cos2\theta \end{bmatrix}$
(d) $\begin{bmatrix} -cos2\theta & sin2\theta \\ -sin2\theta & cos2\theta \end{bmatrix}$

Q31. (A) $\frac{d}{dx}a^x = \frac{a^x}{\log_e a}$
(B) $\frac{d}{dx}e^{ax} = ae^{ax}$
(C) $\frac{d}{dx}x^x = x.x^{x-1}$
(D) $\frac{d}{dx}x^a = ax^{a-1}$
(E) $\frac{d}{dx}a^a = 0$

Choose the **correct** answer from the options given below:
(a) (A), (B) Only
(b) (B), (D), (E) Only
(c) (D), (E) Only
(d) (A), (E), Only

Q32. The maximum value of $z = 4x + 3y$ subject to constraint $x + y \leq 6, x, y \geq 0$ is:
(a) 18
(b) 24
(c) 40
(d) 34

Q33. The value of λ for which the following system of equations has unique solution.
$\lambda x + 3y - z = 1$
$x + 2y + z = 2$
$-\lambda x + y + 2z = -1$ are
(a) $\lambda \neq \frac{5}{2}$
(b) $\lambda \neq \frac{3}{2}$
(c) $\lambda \neq \frac{7}{2}$
(d) $\lambda \neq \frac{-7}{2}$

Q34. If $f: [3, \infty] \to A$ defined by $f(x) = x^2$ is an onto function, then A is:
(a) R
(b) $[5, \infty)$
(c) $[6, \infty)$
(d) $[9, \infty)$

Q35. The function $f(x) = x + \cot^{-1} x$ is increasing in the interval:
(a) $(-\infty, \infty)$
(b) $(-1, \infty)$
(c) $(0, \infty)$
(d) $(1, \infty)$

Q36. If $|\vec{a}| = 7$ then $|3\vec{a}|$ is:
(a) 63
(b) 21
(c) 7
(d) $\sqrt{21}$

Q37. If a function $f: R \to R$ is defined by $f(x) = x^2 + 1$, then the pre images of 17 and -3 respectively are:
(a) Φ, {4, -4}
(b) {3, -3}, Φ
(c) {4, -4}, Φ

(d) {4, -4}, {2, -2}

Q38. Given $p \neq 1$, then $\int \frac{dx}{x(\log_e x)^p}$ is equal to:

(a) $\frac{(\log_e x)^{1+p}}{1+p}$

(b) $\frac{1+p}{(\log_e x)^{1+p}}$

(c) $\frac{(\log_e x)^{1-p}}{1-p}$

(d) $\frac{1-p}{(\log_e x)^{1-p}}$

Q39. If $f(x) = e^x g(x), g(0) = 2, g'(0) = 1$ then $f'(0)$ is equal to:

(a) 0

(b) 1

(c) 2

(d) 3

Q40. The value of the determinant $\begin{vmatrix} a\,cos\theta & a\,sin\theta \\ -a\,sin\theta & a\,cos\theta \end{vmatrix}$ is:

(a) a

(b) a^2

(c) 1

(d) 2a

Q41. If $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \lambda a^2 b^2 c^2$, then the value of λ is:

(a) 4

(b) 3

(c) 2

(d) 1

Q42. A card is picked up at random from 52 playing cards. Given that the picked card is a queen, then the probability of this card be a card of spade is:

(a) $\frac{1}{3}$

(b) $\frac{4}{13}$

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

Q43. The area bounded by $y^2 = x, y = 5$ and y axis is:

(a) 25 sq. units

(b) $\frac{125}{2}$ sq. units

(c) $\frac{125}{3}$ sq. units

(d) 75 sq. units

Q44. If A and B are symmetric matrices then (AB + BA) is:

(a) Symmetric matrix

(b) Skew-symmetric matrix

(c) Diagonal matrix

(d) Scalar matrix

Q45. Direction ratios of the line perpendicular to the lines $\frac{x-3}{2} = \frac{y+7}{-3} = \frac{z-2}{1}$ and $\frac{x+2}{1} = \frac{y+3}{2} = \frac{z-5}{-2}$ are:

(a) <4, -5, 7>

(b) <-4, 5, 7>

(c) <4, 5, -7>

(d) <4, 5, 7>

Q46. The relation R on the set A = {1, 2, 3, 4, 5}, given by R = {(a, b) : |a-b| is even}, is :

(a) Reflexive only

(b) Reflexive and symmetric only

(c) Symmetric and Transitive only

(d) Equivalence

Q47. The projection of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ on the vector $\hat{\jmath}$ is:

(a) 0

(b) 1

(c) 2

(d) -1

Q48. The value of m so that the lines $\frac{1-x}{3} = \frac{7y-14}{2m} = \frac{z-3}{2}$ and $\frac{7-7x}{3m} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angle is:

(a) $\frac{70}{11}$

(b) $\frac{50}{11}$

(c) $\frac{13}{11}$

(d) $\frac{17}{11}$

Q49. The equation of the normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $x = 1$ is:

(a) $2x - y - 1 = 0$

(b) $x + 2y + 7 = 0$

(c) $2x - y + 1 = 0$

(d) $x + 2y - 7 = 0$

Q50. If the given function $f(x)$, defined as

$$f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21 & \text{if } x \geq 10 \end{cases}$$

is continuous, then value of 2a+b is:

(a) 2

(b) 7

(c) 5

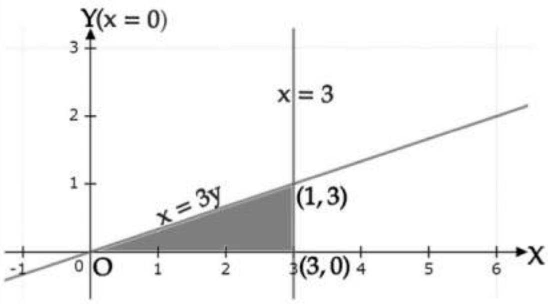
(d) 8

SOLUTIONS

S1. **Ans. (b)**
Sol. If A is square matrix of order 3, then $\det(A^{-1}) = \frac{1}{detA}$.

S2. **Ans. (b)**
Sol. Given function
$f(x) = x(x-2) = x^2 - 2x$
$f'(x) = 2x - 2$
$f'(x) = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1$
$f''(x) = 2 > 0$ so, $f(x)$ has minima at $x = 1$

S3. **Ans. (b)**
Sol. Given
$x - 3y \geq 0$
$x - 3y = 0$
If $x = 0, then\ y = 0 \Rightarrow point(0,0)$
$y = 1 \Rightarrow x = 3 \Rightarrow point\ (3,1)$
also given $0 \leq x \leq 3$
feasible region is bounded in first quadrant.



S4. **Ans. (a)**
Sol. Given parabola is
$y^2 = 4ax$ .... (i)
D. w. r. to x,
$2y\frac{dy}{dx} = 4a$
From eq. (i),
$y^2 = 2xy\frac{dy}{dx}$
$\frac{dy}{dx} = \frac{y}{2x} \Rightarrow \frac{dy}{dx} - \frac{y}{2x} = 0$
So, order of differential equation is 1.

S5. **Ans. (c)**
Sol. Given
$y = 2x(x-1)(2-x) = (2x^2 - 2x)(2-x)$
$= 4x^2 - 2x^3 - 4x + 2x^2 = 6x^2 - 2x^3 - 4x$
D. w. r. to x,
$\frac{dy}{dx} = 12x - 6x^2 - 4$
Again d. w. r. to x
$\frac{d^2y}{dx^2} = 12 - 12x = 12(1-x)$

S6. **Ans. (a)**
Sol. Given order of A and B are respectively $4 \times m$ and $4 \times n$.
If $m = n$, then order of $5A - 2B$ is $4 \times m$.

S7. **Ans. (b)**
Sol. Given curve $y^2 = 8x$
Area of bounded region $= 2\int_0^2 y\,dx =$
$2\int_0^2 2\sqrt{2x}\,dx = 4\sqrt{2} \times \left\{\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right\}_0^2$
$= \frac{8\sqrt{2}}{3} \times (2)^{\frac{3}{2}} = \frac{8\sqrt{2}}{3} \times 2\sqrt{2} = \frac{32}{3}$ sq. unit

S8. **Ans. (a)**
Sol. Given differential eq. is
$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 = \left(\frac{d^3y}{dx^3}\right)$
Here order of differential eq. is 3.

S9. **Ans. (d)**
Sol. Determinant is unaltered if all rows and columns are interchanged.

S10. **Ans. (b)**
Sol. Probability that Amit hit the target $= \frac{4}{5}$
Probability that Sumit hit the target $= \frac{3}{4}$
Probability that Puneet hit the target $= \frac{2}{3}$
Probability they all hit the target $= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$

S11. **Ans. (b)**
Sol. Given integral can be written as
$\int e^{2x^3 + 2\log x}\,dx = \int e^{2x^3} \times e^{\log x^2}\,dx$
$= \int (e^{2x^3} \times x^2)\ dx$
Put $2x^3 = t \Rightarrow 6x^2dx = dt \Rightarrow x^2dx = \frac{1}{6}dt$
$= \frac{1}{6}\int e^t dt = \frac{1}{6}e^t = \frac{1}{6}e^{2x^3} + c$

S12. **Ans. (a)**
Sol. Given
$Z = ax + by$
ATQ.
$30a + 30b = a(0) + b(40)$
$30a = 10b$
$3a = b$

S13. **Ans. (c)**
Sol. If three coins are tossed, then
$S = \{HHH, HHT, HTH, HTT, THT, TTH, THH, TTT\}$
Let X denoted number of heads, then
$P(X = 0) = \frac{1}{8}$
$P(X = 1) = \frac{3}{8}$
$P(X = 2) = \frac{3}{8}$
$P(X = 3) = \frac{1}{8}$
Mean $= \sum X.P(X) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$
$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$

S14. **Ans. (c)**
Sol. Given Curve is
$\frac{x^2}{16} + \frac{y^2}{25} = 1$
D. w. r. to x,
$\frac{2x}{16} + \frac{2y}{25} \times \frac{dy}{dx} = 0$

$\frac{2y}{25} \times \frac{dy}{dx} = -\frac{x}{8}$
$\frac{dy}{dx} = -\frac{25x}{16y}$
If tangent is parallel to x −axis, then
$\frac{dy}{dx} = 0$
$-\frac{25x}{16y} = 0 \Rightarrow x = 0$
$\Rightarrow y = \pm 5$
Points are $(0, \pm 5)$

S15. Ans. (b)
Sol. Given
$A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$
$|A| = -4 - 15 = -19$ and $adj(A) = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$
$A^{-1} = \frac{Adj(A)}{|A|} = -\frac{1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$

S16. Ans. (d)
Sol. Let $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
So, $\vec{a}.\hat{\imath} = x, \vec{a}.\hat{\jmath} = y$ & $\vec{a}.\hat{k} = z$
$(\vec{a}.\hat{\imath})^2 + (\vec{a}.\hat{\jmath})^2 + (\vec{a}.\hat{k})^2 = x^2 + y^2 + z^2 = |\vec{a}|^2$

S17. Ans. (a)
Sol. (A) $|x-1| + |x-3|$ at $x = 2$
L.H.D.
$\lim_{h\to 0}\frac{f(2-h)-f(2)}{-h} = \lim_{h\to 0}\frac{|2-h-1|+|2-h-3|-(1+1)}{-h}$
$= \lim_{h\to 0}\frac{|1-h|+|-1-h|-2}{-h} = 0$
(B) $y = \log\sqrt{\tan x}$, then $\frac{dy}{dx} = \frac{1}{\sqrt{\tan x}} \times \frac{1}{2\sqrt{\tan x}} \times \sec^2 x = \frac{\sec^2 x}{2\tan x}$
At $x = \frac{\pi}{4}, \frac{dy}{dx} = 1$
(C) $\sin(x+y) = \log(x+y)$
$\cos(x+y) \times \left(1 + \frac{dy}{dx}\right) = \frac{1}{x+y} \times \left(1 + \frac{dy}{dx}\right)$
$\{\cos(x+y) - \frac{1}{x+y}\}(1 + \frac{dy}{dx}) = 0$
$1 + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -1$
(D) $f(x) = x^2 + x + 1, x \in [0, 4]$
$f'(x) = 2x + 1$
$f'(c) = 2c + 1$
$f'(c) = \frac{f(b)-f(a)}{b-a}$
$2c + 1 = \frac{(4)^2+4+1-[(0)^2+0+1]}{4-0} = \frac{16+4}{4} = 5$
$2c + 1 = 5 \Rightarrow c = 2$

S18. Ans. (b)
Sol. Given $y = -x|x|, x = -1$ *to* $x = 1$
$y = \begin{cases} x^2, -1 \le x \le 0 \\ -x^2, 0 \le x \le 1 \end{cases}$
Area of bounded region $= \int_{-1}^{0} x^2 dx + \int_0^1 -x^2 dx$
$\left|\left\{\frac{x^3}{3}\right\}_{-1}^{0}\right| + \left|\left\{-\frac{x^3}{3}\right\}_0^1\right| = \left|0 + \frac{1}{3}\right| + \left|-\frac{1}{3} - 0\right| = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

S19. Ans. (c)
Sol. Probability x speaks truth= $P(x) = \frac{60}{100} = \frac{3}{10}$
Probability y speaks truth $= P(y) = \frac{50}{100} = \frac{1}{2}$
Probability that they contradict each other while narrating the same fact
$= P(x).P(y') + P(y).P(x')$
$= \frac{3}{10} \times \left(1 - \frac{1}{2}\right) + \frac{1}{2} \times \left(1 - \frac{3}{10}\right)$
$= \frac{3}{10} \times \frac{1}{2} + \frac{1}{2} \times \frac{7}{10}$
$= \frac{3}{20} + \frac{7}{20} = \frac{10}{20} = \frac{1}{2}$

S20. Ans. (b)
Sol. $\cot y\, dx = x\, dy$
$\int \frac{dy}{\cot y} = \int \frac{dx}{x}$
$\int \tan y\, dy = \int \frac{dx}{x}$
$\log x = \log|\sec y| + \log C$
$\log x = \log C\, secy$
$x = C\, secy$

S21. Ans. (a)
Sol. Eq. of plane $3x + 4y - 12z + 13 = 0$
ATQ.
$\left|\frac{3(1)+4(1)-12(p)+13}{\sqrt{(3)^2+(4)^2+(-12)^2}}\right| = \left|\frac{3(-3)+4(0)-12(1)+13}{\sqrt{(3)^2+(4)^2+(-12)^2}}\right|$
$\left|\frac{20-12p}{13}\right| = \left|\frac{-8}{13}\right|$
$20 - 12p = 8$
$12 - 12p = 0$
$p = 1$

S22. Ans. (c)
Sol. $\tan^{-1} 3 + \tan^{-1}\lambda = \tan^{-1}\frac{3+\lambda}{1-3\lambda}$
Is valid for $3\lambda < 1 \Rightarrow \lambda < \frac{1}{3}$

S23. Ans. (b)
Sol. We have
$\tan^2(\sec^{-1} 2) = \sec^2(\sec^{-1} 2) - 1 = \{\sec(\sec^{-1} 2)\}^2 - 1$
$= (2)^2 - 1 = 4 - 1 = 3$

S24. Ans. (c)
Sol. Given $f(x) = -|x-1|$ is continuous at $x = 1$ but not differentiable at $x = 1$.

S25. Ans. (b)
Sol. Given
$4\sin^2 x + 3\cos^2 x = 4\sin^2 x + 3(1 - \sin^2 x) = 3 + \sin^2 x$
$-1 \le \sin x \le 1 \Rightarrow 0 \le \sin^2 x \le 1$
Maximum value $= 3 + 1 = 4$

S26. Ans. (c)
Sol. The elements of diagonal are all non − zero and all other elements are zero is not correct statement.

S27. Ans. (b)
Sol. We have
$\int_0^1 \frac{e^x}{(1+x)^2}dx = \int_0^1 \frac{(1+x-x)}{(1+x)^2}e^x dx = \int_0^1 \left(\frac{1}{1+x} - \frac{x}{(1+x)^2}\right)e^x dx$
$= \int_0^1 \frac{e^x}{1+x}dx - \int_0^1 \frac{x}{(1+x)^2}e^x\, dx$

$= K - \left\{\int_0^1 \left(\frac{1+x-1}{(1+x)^2}\right) e^x\, dx\right\}$ { Since $\int_0^1 \frac{e^x}{1+x} dx = K$ }

$= K - \left\{\int_0^1 \left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right) e^x\, dx\right\}$

$= K - \left\{\frac{e^x}{1+x}\right\}_0^1$ { Since $\int e^x(f(x) + f'(x))dx = e^x f(x) + C$ }

$= K - \left\{\frac{e^1}{1+1} - \frac{e^0}{1+0}\right\} = K - \left\{\frac{e}{2} - 1\right\} = K + 1 - \frac{e}{2}$

S28. Ans. (b)

Sol. Given

$Z = 2x - y + 5$

Corner points of feasible region are (0, 10), (12, 6), (20, 0) & (0, 0).

At (0, 10) we have $z = 2(0) - 10 + 5 = -5$ (Minimum)

At (12, 6) we have $z = 2(12) - 6 + 5 = 24 - 1 = 23$

At (20, 0) we have $z = 2(20) - 0 + 5 = 45$ (Maximum)

At (0, 0) we have $z = 2(0) - 0 + 5 = 5$

S29. Ans. (a)

Sol. Given differential equation is

$\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 = 0$

Degree of differential equation is 2.

S30. Ans. (c)

Sol. We have

$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta - \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

S31. Ans. (b)

Sol. We have

(A) $\frac{d}{dx}(a^x) = a^x \log a$

(B) $\frac{d}{dx}(e^{ax}) = ae^{ax}$

(C) Let $y = x^x$

$\log y = x \log x$

$\frac{1}{y} \times \frac{dy}{dx} = x \times \frac{1}{x} + \log x \times 1$

$\frac{dy}{dx} = x^x(1 + \log x)$

(D) $\frac{d}{dx}(x^a) = ax^{a-1}$

(E) $\frac{d}{dx}(a^a) = 0$

S32. Ans. (b)

Sol. We have

$z = 4x + 3y$

$x + y = 6$

Points are (0, 6) & (6, 0).

$z = 4(0) + 3(6) = 18$ (minimum)

$z = 4(6) + 3(0) = 24$ (maximum)

S33. Ans. (d)

Sol. Given system of equations are

$\lambda x + 3y - z = 1$

$x + 2y + z = 2$

$-\lambda x + y + 2z = -1$

Which is of the form $AX = B$

Here $A = \begin{bmatrix} \lambda & 3 & -1 \\ 1 & 2 & 1 \\ -\lambda & 1 & 2 \end{bmatrix}$

We have

$|A| = \lambda(4 - 1) - 3(2 + \lambda) - 1(1 + 2\lambda) \neq 0$

$3\lambda - 6 - 3\lambda - 1 - 2\lambda \neq 0$

$-2\lambda - 7 \neq 0$

$\lambda \neq -\frac{7}{2}$

S34. Ans. (d)

Sol. The function $f: [3, \infty) \to A$ defined by $f(x) = x^2$ is an onto function, then A is $[9, \infty)$.

Since for all $y \in [9, \infty)$ there exist $x \in [3, \infty)$.

S35. Ans. (a)

Sol. Given $f(x) = x + \cot^{-1} x$

$f'(x) = 1 - \frac{1}{1+x^2} = \frac{x^2}{1+x^2} > 0$ for all real numbers.

S36. Ans. (b)

Sol. We have

$|\vec{a}| = 7 \Rightarrow |\vec{a}|^2 = 49$

$|3\vec{a}|^2 = 9 \times |\vec{a}|^2 = 9 \times 49 = 441$

$\Rightarrow |3\vec{a}| = 21$

S37. Ans. (c)

Sol. Given

$y = f(x) = x^2 + 1$

$x = \sqrt{y - 1}$

For $y = 17$

$x = \sqrt{17 - 1} = \sqrt{16} = \pm 4$

For $y = -3$

$x = \emptyset$

S38. Ans. (c)

Sol. $\int \frac{dx}{x(\log x)^p}$

Put $\log x = t$

$\frac{dx}{x} = dt$

$\int \frac{dt}{t^p} = \int t^{-p} dt = \frac{t^{-p+1}}{-p+1} = \frac{(\log x)^{1-p}}{1-p}$

S39. Ans. (d)

Sol. $f(x) = e^x g(x)$

$f'(x) = e^x g'(x) + e^x g(x)$

$f'(0) = e^0 g'(0) + e^0 g(0) = 1 \times 1 + 1 \times 2 = 3$

S40. Ans. (b)

Sol. We have

$\begin{vmatrix} a\cos\theta & a\sin\theta \\ -a\sin\theta & a\cos\theta \end{vmatrix} = a^2\cos^2\theta + a^2\sin^2\theta = a^2$

S41. Ans. (a)

Sol. We have

$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} =$

$a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= a^2b^2c^2\{-1(1-1) - 1(-1-1) + 1(1+1)$
$\lambda a^2b^2c^2 = a^2b^2c^2(4) = 4a^2b^2c^2$
$\lambda = 4$

S42. Ans. (c)
Sol. Total cards = 52
Cards of spade = 13
Queen of spade = 1
n(Queen of Spade) $= 13_{C_1} = 13$
n(Total no. of cases) $= 52_{C_1} = 52$
Required probability $= \frac{13}{52} = \frac{1}{4}$

S43. Ans. (c)
Sol. $y^2 = x$
Area of bounded region $= \int_0^5 x dy = \int_0^5 y^2 dy =$
$\left[\frac{y^3}{3}\right]_0^5 = \frac{125}{3}$ sq. unit

S44. Ans. (a)
Sol. Given A and B are symmetric matrices. So,
$A' = A$ & $B' = B$
$(AB + BA)' = (AB)' + (BA)' = B'A' + A'B' =$
$BA + AB = AB + BA$
So, $AB + BA$ is symmetric matrix.

S45. Ans. (d)
Sol. Given lines are
$\frac{x-3}{2} = \frac{y+7}{-3} = \frac{z-2}{1}$ & $\frac{x+2}{1} = \frac{y+3}{2} = \frac{z-5}{-2}$
We have
The line whose direction ratio are $< 4, 5, 7 >$ is perpendicular to given lines.
Since $4 \times 2 + 5 \times -3 + 2 \times 1 = 8 - 15 + 7 = 15 - 15 - 0$
& $4 \times 1 + 5 \times 2 + 7 \times -2 = 4 + 10 - 14 = 0$

S46. Ans. (d)
Sol. Given set $A = \{1, 2, 3, 4, 5\}$ and $R = \{(a, b): |a - b|\ is\ even\}$
$(a, a) \in R$ since $a - a = 0$ is even. So, R is reflexive.
If $(a, b) \in R$, then $a - b$ is even.
$\Rightarrow b - a$ is also even.
$\Rightarrow (b, a) \in R$
So, R is symmetric.
$(a, b) \in R, (b, c) \in R$, then $a - b$ & $b - c$ both are even.
$a - c = a - b + b - c$ is also even.
So, R is transitive.
So, R is equivalence relation.

S47. Ans. (b)
Sol. Projection of $\hat{\imath} + \hat{\jmath} + \hat{k}$ on $\hat{\jmath} = \frac{(\hat{\imath}+\hat{\jmath}+\hat{k}).\hat{\jmath}}{|\hat{\jmath}|} = \frac{1}{1} = 1$

S48. Ans. (a)
Sol. Given equations of lines can be written as
$\frac{x-1}{-3} = \frac{y-2}{\frac{2m}{7}} = \frac{z-3}{2}$ and $\frac{x-1}{-\frac{3m}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$
Lines are perpendicular if $-3 \times \left(-\frac{3m}{7}\right) +$
$\left(\frac{2m}{7}\right) \times 1 + 2 \times -5 = 0$
$\frac{9m}{7} + \frac{2m}{7} - 10 =$
$\frac{11m}{7} - 10 = 0$
$\frac{11m}{7} = 10$
$m = \frac{70}{11}$

S49. Ans. (d)
Sol. Given curve
$y = x^4 - 6x^3 + 13x^2 - 10x + 5$
$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$
At $x = 1$
$\frac{dy}{dx} = 4 - 18 + 26 - 10 = 2$
Slope of normal $= -\frac{1}{2}$
$x = 1 \Rightarrow y = 1 - 6 + 13 - 10 + 5 = 3$
Equation of normal is
$y - 3 = -\frac{1}{2}(x - 1)$
$2y - 6 = -x + 1$
$x + 2y - 7 = 0$

S50. Ans. (c)
Sol. Given function is
$$f(x) = \begin{cases} 5, x \leq 2 \\ ax + b, 2 < x < 10 \\ 21, x \geq 10 \end{cases}$$
Continuity at $x = 2$
L.H.L. = 5 & $f(2) = 5$
R.H.L.
$\lim_{x \to 2^+} ax + b = \lim_{h \to 0} a(2 + h) + b = 2a + b$
Continuity at $x = 10$
L.H.L.
$\lim_{x \to 10^-} ax + b = \lim_{h \to 0} a(10 - h) + b = 10a + b$
R.H.L. = 21 & $f(10) = 21$
We have
$2a + b = 5$
$10a + b = 21$