☆ Boolean Algebra :

⇒ 'AND' Law | 'OR' Law

Identity element ⇒ '1' | '0'

$A \cdot 0 = 0$

$A \cdot \boxed{1} = A$

$A + \boxed{0} = A$

$A + 1 = 1.$

② Commutative Law:

Dual

$A + B = B + A.$

$A \cdot B = B \cdot A.$

NAND : $\uparrow$

$A \uparrow B = B \uparrow A$

Dual

$\therefore \overline{A \cdot B} = \overline{B \cdot A}$

$A \downarrow B = B \downarrow A$

⇒ Inhibition : $(/)$

$\Rightarrow \quad x \mid y = x \cdot \bar{y}$

$$\boxed{\begin{array}{c} A \mid B \not\Rightarrow B / A. \\ \therefore A \cdot \bar{B} \neq B \cdot \bar{A} \end{array}}$$

→ 'Inhibition is not Commutative.'

ⓑ Associative Law :

Dual

$(A+B) + C = A + (B+C)$.

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

$$(\overset{1}{A} \uparrow \overset{1}{B}) \uparrow \overset{0}{C} \neq \overset{1}{A} \uparrow (\overset{1}{B} \uparrow \overset{0}{C}).$$

$1 \uparrow 0$

$0 \uparrow 0 \qquad\qquad 1 \uparrow 1$

$= 1. \qquad\qquad = 0$

→ The NAND and Nor operation are Commutative but not Associative.

ⓒ Consensus Law :

$A \cdot B + \bar{A} \cdot C + B \cdot C = A \cdot B + \bar{A} \cdot C$.

$\therefore A \cdot B + \bar{A} C + B \cdot C (A + \bar{A})$

$= A \cdot B + \bar{A} C + ABC + \bar{A}BC$

$= \cancel{A \cdot B + \bar{A} C} +$

$= ABC(1+C) + \bar{A}C(1+C).$

$= AB + \bar{A}C$

$$\boxed{x \cdot y + \bar{y} \cdot z + WVxz = x \cdot y + \bar{y} \cdot z}$$

Dual $(A+B) \cdot (\bar{A}+C) \cdot (B+C) = (A+B) \cdot (\bar{A}+C)$.

(4) Distribution Law:

$\rightarrow$ $A \cdot (B+C) = AB + AC$.

Dual

$A + (B \cdot C) = (A+B) \cdot (A+C)$

i) $\bar{X} + X\bar{Y} = (\bar{X}+X)(\bar{X}+\bar{Y})$

$\therefore \boxed{\bar{X} + X\bar{Y} = \bar{X} + \bar{Y}}$

(ii) $AB + \overline{AB}C = (AB + \overline{AB})(AB + C)$.

$= AB + C$.

(5) Transposition Law:

$\rightarrow$ $AB + \bar{A}C = (\bar{A}+B)(A+C)$.

$= \bar{A} \cdot A + \bar{A}C + AB + BC$.

$= \bar{A}C + AB + BC$.

$= \bar{A}C + AB$ ($\because$ Consensus Law).

$=$ L.H.S

$\rightarrow$ $\boxed{x \cdot y + \bar{y} \cdot z = (x+\bar{y})(y+z)}$.

(6) DeMorgan's Law:

→ (a) NOR gate = Bubbled AND gate

$$\boxed{\overline{A+B+C+\cdots} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdots}$$

→ (b) NAND gate = Bubbled OR gate.

$$\boxed{\overline{A\cdot B\cdot C\cdot D\cdots} = \bar{A}+\bar{B}+\bar{C}+\bar{D}+\cdots}$$

(7) Shannon's Law:-

→ To find Complement of a fun 'F'.

(i) Find the Dual of F i.e. 'F_D'.
(ii) Complement all variables.

Ex-1 $F = AB + BC + CA$ then $\bar{F} = \bar{A}\cdot\bar{B} + \bar{B}\cdot\bar{C} + \bar{C}\cdot\bar{A}$

[T/F].

Ans:

$F = AB + BC + CA.$

$\therefore \cancel{\bar{F} =} \cancel{(\bar{A}+\bar{B})}\cdot\cancel{(\bar{B}+\bar{C})}\cdot\cancel{(\bar{C}+\bar{A})}.$

$\cancel{= (\bar{A}\bar{B} + \bar{A}\cdot\bar{C})\,(\bar{C}+\bar{A})}.$

$\cancel{= \bar{A}\bar{B} + \bar{A}\cdot\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\cdot\bar{C}}.$

$= \cancel{\bar{A}\bar{B} + \bar{A}\cdot\bar{C} +}$

i) $F_D = (A+B)\cdot(B+C)\cdot(C+A).$

(ii) $\bar{F} = (\bar{A}+\bar{B})(\bar{B}+\bar{C})(\bar{C}+\bar{A})$

$= (\bar{B} + \bar{A}\bar{C}) \cdot (\bar{C}+\bar{A})$.

$= \bar{B}\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{A}\bar{C}$.

$\therefore \bar{F} = \bar{A}\bar{B} + \bar{B}\cdot\bar{C} + \bar{C}\cdot\bar{A}$.

Ex-2 Simplify the following boolian expression to four literals.

① $F = \bar{A}C + \bar{C}D + \bar{B}C + AB$. → 4 literals.

Note: Literal: variable (or) Complement of variables.

Eg: $F(A, B, C) \Rightarrow A, \bar{A}, B, \bar{B}, C, \bar{C}$.

$F = (\bar{A}+\bar{B})C + \bar{C}D + AB$.

$F = \overline{AB} \cdot C + \bar{C}D + AB$.

$\therefore F = AB + C + \bar{C}D$.

$\boxed{F = AB + C + D}$

Ex-3 Determine the number of two input NAND gates required to implement the following:

Ans: (a) $F = (\bar{X}+\bar{Y})(W+Z)$.

$\therefore$ ~~$F = \overline{(\bar{X}+\bar{Y}) \cdot (W+Z)}$~~.

$F = \overline{X \cdot Y}\,(W+Z)$.

$\therefore F = \overline{X \cdot Y}\,W + \overline{X \cdot Y} \cdot Z$

$= PW + PZ$

$P = \overline{X \cdot Y}$

$F = \overline{\overline{PW} \cdot \overline{PZ}}$

4 NAND gate req.

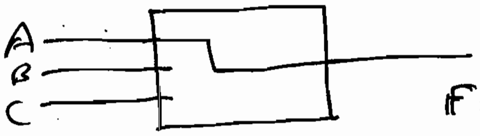
(b) $F = A + AB + ABC$

$= A + AB(1+C)$

$= A + AB$

$= A(1+B)$

$\therefore \boxed{F = A}$



$F = A$.

'0' NAND gates.

$\Rightarrow$ 0 NAND gate is required.

(c) n - input AND gate = ?

NAND gates

$\rightarrow$ 2 i/p AND $\Rightarrow F = \overline{\overline{A\cdot B}}$ = 2

3 i/p AND $\Rightarrow F = ABC$

$= \overline{\overline{\overline{\overline{AB}}\,C}}$ = 4

4 i/p AND $\Rightarrow F = \overline{\overline{\overline{\overline{AB}}\,\overline{\overline{CD}}}}$ = 6.

=

⋮

n i/p AND $\Rightarrow$ ~~for~~ $(2n-2)$ no. of 2 i/p NAND gate req.

Ex-4 Implement EX-OR gate using minimum no. of (a) NAND gate (b) NOR gate.

Ans:

$\rightarrow A \oplus B = \bar{A}.B + A.\bar{B}$

$= \bar{A}\cdot B + A\cdot\bar{B} + \boxed{A\cdot\bar{A} + B\cdot\bar{B}}$

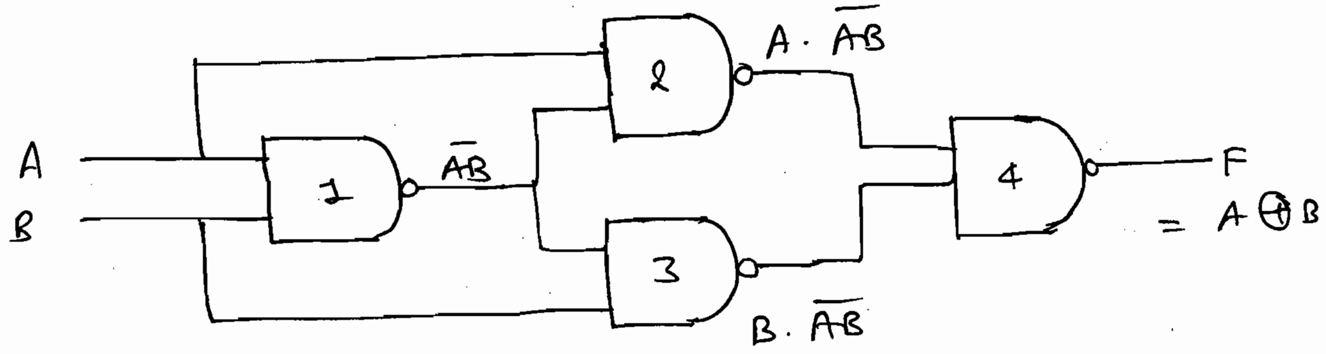
$= B(\bar{A}+\bar{B}) + A(\bar{A}+\bar{B})$.

$= A\cdot\overline{AB} + B\cdot\overline{AB}$.

$$\overline{\overline{F}} = \overline{\overline{A\cdot\overline{AB} + B\cdot\overline{AB}}}$$

$$\therefore\ F = \overline{\overline{A\cdot\overline{AB}} \cdot \overline{B\cdot\overline{AB}}} .$$

→



(b) Using NOR gate:

$$F = A \oplus B$$

$$F = A(\bar{A}+\bar{B}) + B(\bar{A}+\bar{B}).$$

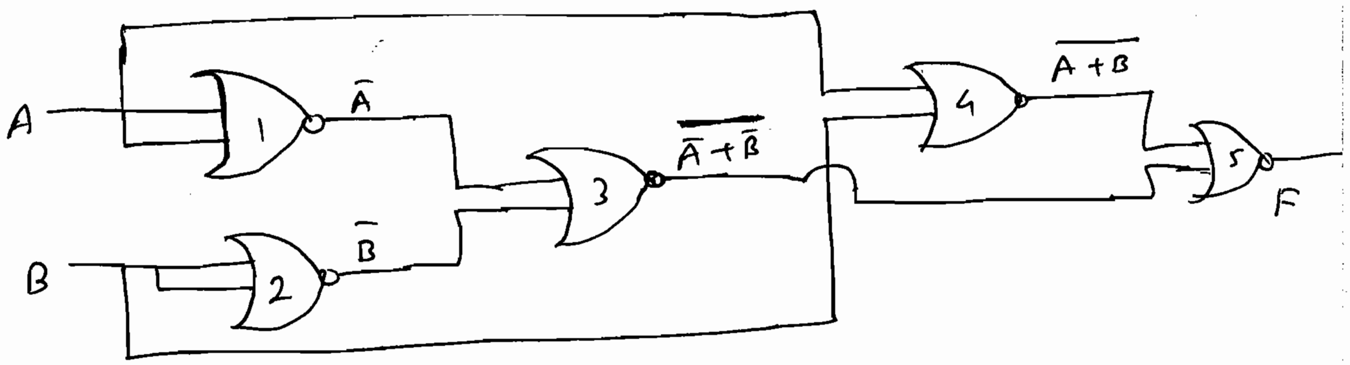
$$\therefore\ \cancel{F/F\ \overline{\overline{A(\bar{A}+\bar{B})+B(\bar{A}+\bar{B})}}}$$

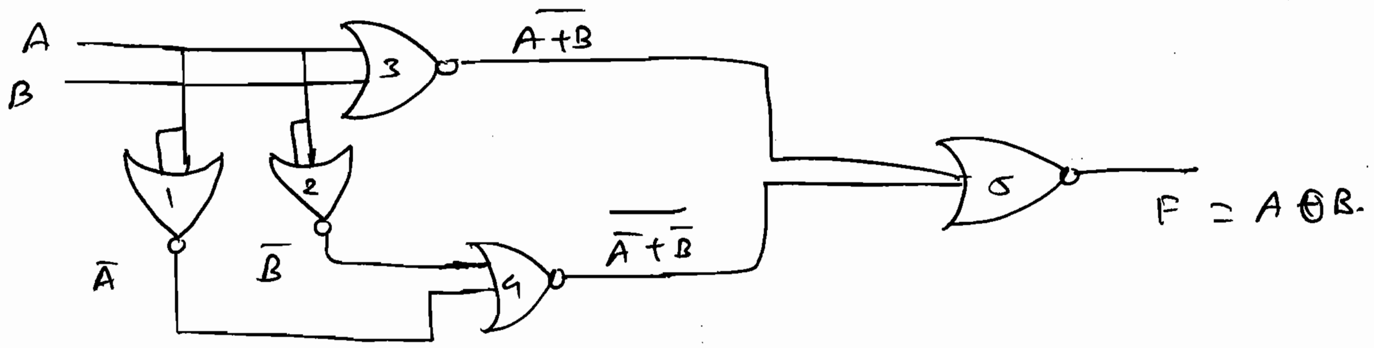
$$\bar{F} = \boxed{(\bar{A}+\bar{B})(A+B).}$$

$$= \overline{\overline{(\bar{A}+\bar{B})(A+B)}}$$

$$F = \overline{\overline{\bar{A}+\bar{B}} + \overline{A+B}}$$

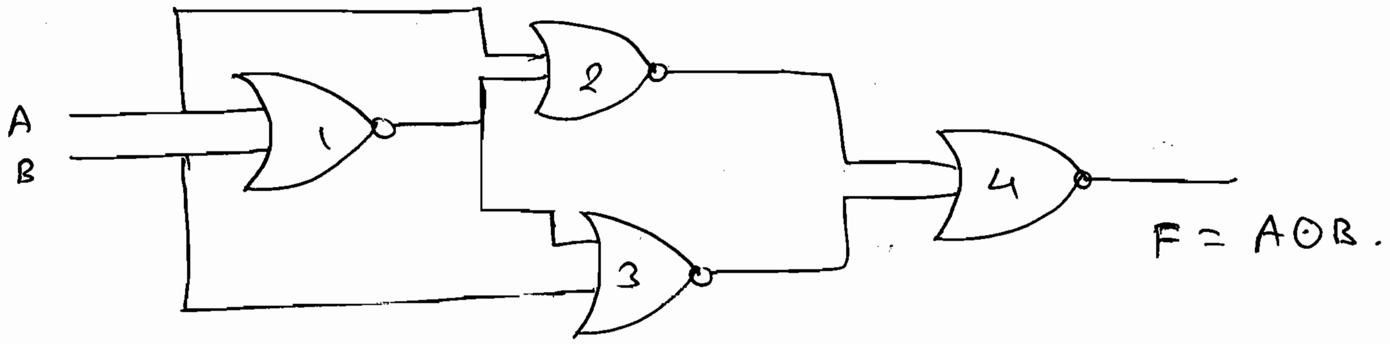
∴ 5 NOR gate.



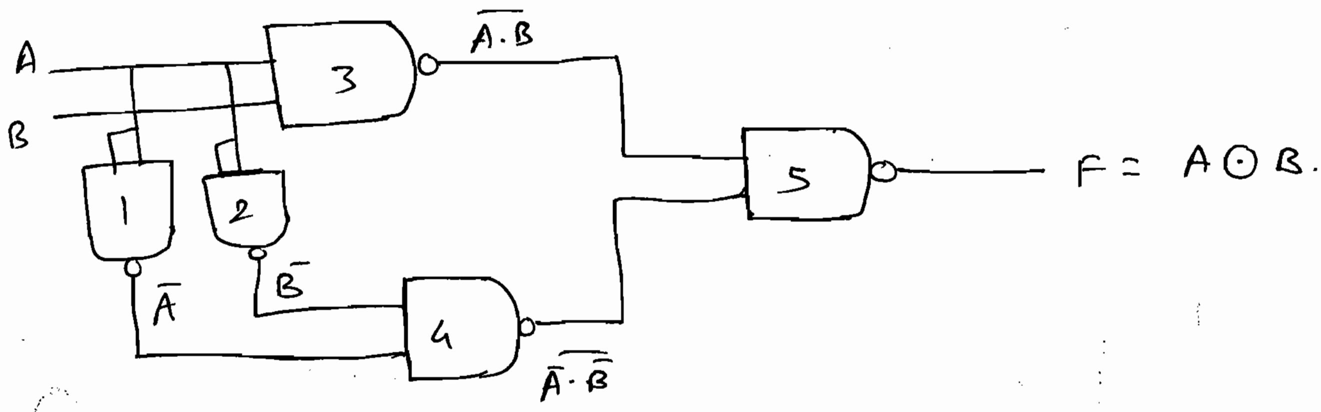


→ If complement of i/p variables are given i.e. $\bar{A}$, $\bar{B}$ are available then only 3 NOR gates are required to implement $A \oplus B$.

* X-NOR using min. no. of NOR gate:



* X-NOR using min. no. NAND gates.



* Minterms, Maxterms & Properties.

→ Minterms → Standard Product term.

→ maxterms → Standard Sum term.

⇒ Minterms → 8

$m_0 \quad \overset{0}{\bar{A}} \cdot \overset{0}{\bar{B}} \cdot \overset{0}{\bar{C}}$

$m_1 \quad \overset{0}{\bar{A}} \cdot \overset{0}{\bar{B}} \cdot \overset{1}{C}$

$m_2 \quad \overset{0}{\bar{A}} \cdot \overset{1}{B} \cdot \overset{0}{\bar{C}}$

$m_3 \quad \bar{A} \cdot B \cdot C$

$m_4 \quad A \cdot \bar{B} \cdot \bar{C}$

$m_5 \quad A \cdot \bar{B} \cdot C$

$m_6 \quad \overset{1}{A} \cdot \overset{1}{B} \cdot \overset{0}{\bar{C}}$

$m_7 \quad \overset{1}{A} \cdot \overset{1}{B} \cdot \overset{1}{C}$

$$\boxed{\begin{array}{l} Var = 1 \\ \overline{Var} = 0 \end{array}}$$

Maxterms → 8

$M_7 \quad \overset{\phi}{\bar{A}} + \overset{\phi}{\bar{B}} + \overset{\phi}{\bar{C}}$

$M_6 \quad \overset{1}{\bar{A}} + \overset{1}{\bar{B}} + \overset{0}{C}$

$M_5 \quad \overset{1}{\bar{A}} + \overset{0}{B} + \overset{1}{\bar{C}}$

$M_4 \quad \overset{1}{\bar{A}} + \overset{0}{B} + \overset{0}{C}$

$M_3 \quad \overset{0}{A} + \overset{1}{\bar{B}} + \overset{1}{\bar{C}}$

$M_2 \quad A + \bar{B} + C.$

$M_1 \quad A + B + \bar{C}.$

$M_0 \quad \overset{0}{A} + \overset{0}{B} + \overset{0}{C}$

$$\boxed{\begin{array}{l} Var = 0 \\ \overline{Var} = 1 \end{array}}$$

Ex-1 $F(A, B, C, D, E)$ and find $m_{23} = ?$, $M_{19} = ?$

Ans:

$$m_{23} = \overset{1}{A} \cdot \overset{0}{\bar{B}} \cdot \overset{1}{C}\, \overset{1}{D}\, \overset{1}{E}$$

$$M_{19} = \overset{1}{\bar{A}} + \overset{0}{B} + \overset{0}{C} + \overset{1}{\bar{D}} + \overset{1}{\bar{E}}.$$

* **Properties:**

① n-var Function ⇒ $\boxed{\begin{array}{l} 2^n \text{ minterms} \\ 2^n \text{ maxterms.} \end{array}}$

② $\boxed{M_i = \overline{m_i}}$ and vice versa.

③ $\boxed{m_i^D = M_{(2^n - 1 - i)}.}$ D = dual

e.g. $m_3 = \overset{0}{\bar{A}}\overset{1}{B}\overset{1}{C}$

$m_3^D = \overset{1}{\bar{A}} + \overset{0}{B} + \overset{0}{C}$

$= M_4$

$M_3^D = M_{2^3-1-3}.$

④ ⓐ Sum of all minterms = 1. $\boxed{\text{i.e. } \sum_{i=0}^{2^n-1} m_i = 1.}$

ⓑ Product of all maxterms = 0 $\boxed{\text{i.e. } \prod_{j=0}^{2^n-1} M_j = 0.}$

BEL ✡✡

Ex-1 How many minterms are present at the o/p of 6 i/p Ex-OR gate: $2^{6-1} = 32$ no. of minterms.

Ans:

$A \oplus B = A \cdot \bar{B} + \bar{A} \cdot B.$

~~A⊕B⊕C~~ ~~=~~ ~~AB~~

$A \oplus B = m_1 + m_2$ [2 out of 4 minterms].

$\therefore A \oplus B \oplus C = m_1 + m_2 + m_4 + m_7$ [4 out of 8 minterms]

∴ ~~A⊕B⊕C~~ n-input Ex-OR gate output contains $= \frac{2^n}{2}$

$= 2^{(n-1)}$ no of ~~output~~ minterms.

NOTE: Same for X-NOR.

Ex-2 How many boolian F^ns can be formed as using n- boolian variables?

Ans: 'n' Boolian variable → Boolian functions $= 2^{2^n}$.

⇓

$x = 2^n$ minterms ↗ $2^x = 2^{2^n}$.

F(A, B)

	AB	F_0	F_1	F_2	F_3	....	F_{15}
m_0 ←	00	0	0	0	0		1
m_1 ←	01	0	0	0	0		1
m_2 ←	10	0	0	1	1		1
m_3 ←	11	0	1	0			1

F_0 ↓ ∅ (Null); F_1 ↓ AND; F_2 ↓ Inhibition (A|B); F_3 → Transfer; F_{15} ↓ Identity.

* Forms of Boolean Functions:

1) a) Sum of Products (SOP) form → DNF

b) Product of Sum (POS) form. → CNF

2) a) Canonical (or) Standard SOP form (sum of minterms) → DCF

b) Canonical (or) Standard POS form. (Product of max terms) → CCF

- DNF = Disjunctive Normal form
- CNF = Conjunctive normal form.
- DCF = Disjunctive canonical form
- CCF = Conjuctive Canonical form.

Ex-1 $F(A,B,C,D) = \overset{0}{\bar{A}} + \overset{0\,1\,0}{\bar{A} C \bar{D}} + \overset{0\,1}{\bar{B} C}$ to canonical SOP form. (sum of minterms form)

Ans: $\cancel{F(A,B,C,D) = \bar{A}B + \bar{A}\bar{B} + \bar{A}BC\bar{D} + \bar{A}\bar{B}C\bar{D} +}$

$$F(A,B,C,D) = \underbrace{\bar{A}}_{\downarrow} + \underbrace{\bar{A}C\bar{D}}_{\downarrow} + \underbrace{\bar{B}C}_{\downarrow}$$

$\bar{A}$		$\bar{A}$ _ $C\bar{D}$	
$m_0 \leftarrow$	0 0 0 0	~~0 0 1 0~~	~~0 0 1 0~~
⋮	0 0 0 1	~~0 1 1 0~~	~~0 0 1 1~~
	0 0 1 0		1 0 1 0 $\rightarrow m_{10}$
	0 0 1 1		1 0 1 1 $\rightarrow m_{11}$
	0 1 0 0		
	0 1 0 1		
	0 1 1 0		
m_7	0 1 1 1		

$\therefore F(A,B,C,D) = \sum m(0,1,\ldots 7,10,11) \leftarrow$ Can SOP form.

$\therefore F(A,B,C,D) = \prod M(8,9,12,13,14,15) \leftarrow$ Can POS form.

Ex-2 Convert $F(A,B,C) = A \cdot B + \bar{A} \cdot C$ into canonical POS form (product of maxterms)

Ans: $\cancel{F(A,B,C) =}$ $A \cdot B + \bar{A} C$

~~M_0 0 0 0~~   ~~1 0 0 M_4~~

~~M_1 0 0 1~~   1 0 1 M_5.

$\therefore$ SOP

Can SOP $\rightarrow$ Can POS

POS $\rightarrow$ Can POS.

$F = A \cdot B + \bar{A} \cdot C.$

$F = (A + C) \cdot (\bar{A} + B)$

$\boxed{F = \Pi M(0, 2, 4, 5).}$

	A	B	C
M_0	0	0	0
M_2	0	1	0

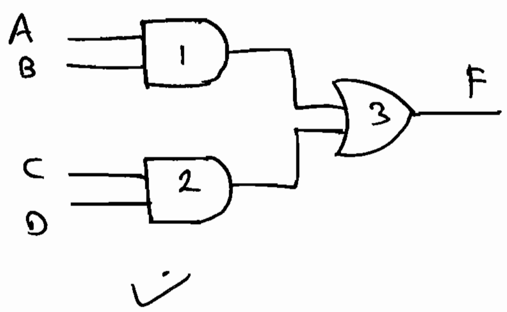
A	B	C	
1	0	0	M_4
1	0	1	M_5

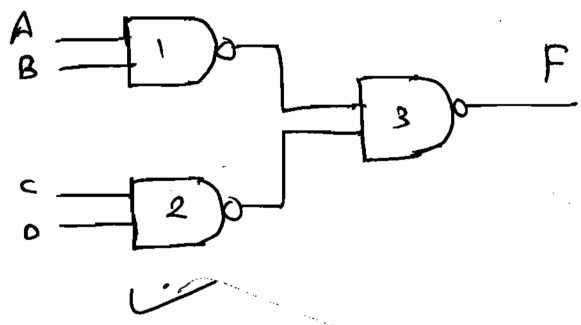
* Two Level Logic:

→ (a) SOP form:

$F = AB + CD$

AND - OR Logic ≡ NAND - NAND Logic

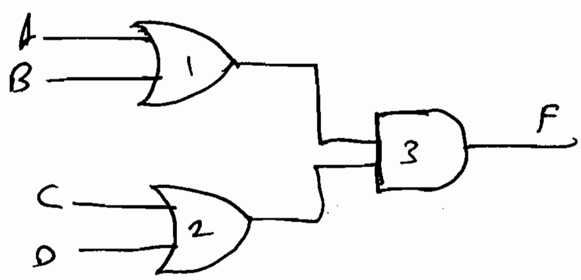


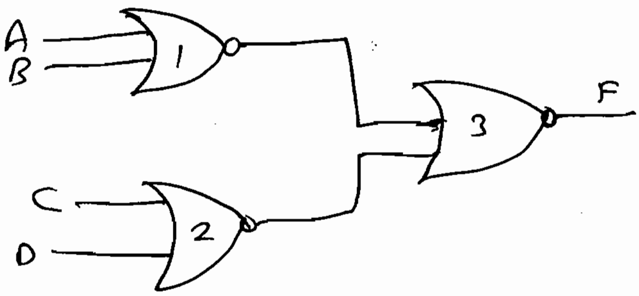


(2) POS - form:

$F = (A + B) \cdot (C + D).$

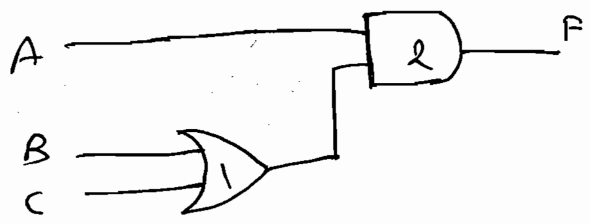
OR - AND Logic : ≡ NOR - NOR Logic





* Hybrid Logic.

$F = AB + AC = A \cdot (B + C)$



→ The advantages of two level logic is the propogation for all the input variables is same.

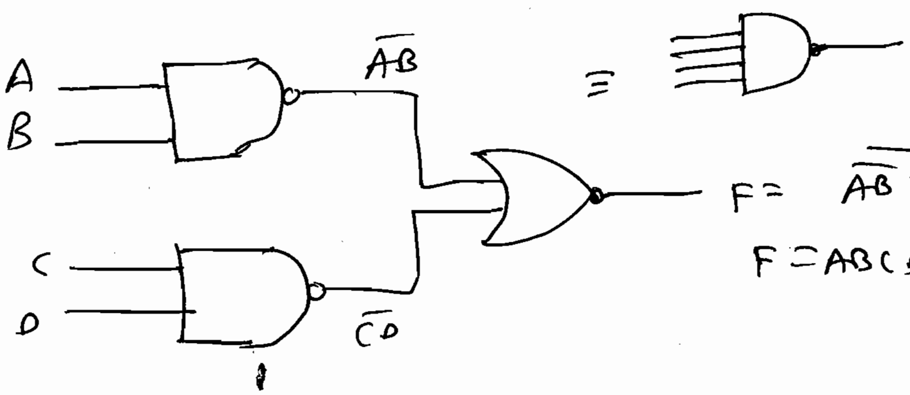
☆ Types of Two Level logic:

→ ~~Eg.~~ (1) Degenerative
(2) Non-Degenerative.

① Degenerative.

⇒ There are only one logical operation in o/p, then it is called Degenerative type.

e.g. NAND - NOR Logic



$F = \overline{\overline{AB} + \overline{CD}}$

$F = ABCD$

→ The advantages of Degenerative form is the ~~fanning~~ fan-in of the gate is increase.