Areas Related to Circles

Selected NCERT Questions

1. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Let r be the radius of circle, then circumference = 22 cm

$$\Rightarrow \quad 2\pi r = 22 \quad \Rightarrow \quad r = \frac{22}{2\times\frac{22}{7}} = \frac{22\times 7}{2\times 22} = \frac{7}{2}$$

$$\text{Now, area of a quadrant of a circle} = \frac{\pi r^2}{4} = \frac{\frac{22}{7}\times\left(\frac{7}{2}\right)^2}{4} = \frac{\frac{22}{7}\times\frac{49}{4}}{4}$$

$$= \frac{154}{16} = \frac{77}{8} = 9\frac{5}{8} = 9.625\ \text{cm}^2$$

2. Figure 11.11 depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

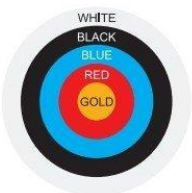


Fig. 11.11

Sol. The area of Gold region $= \pi(10.5)^2 = \frac{22}{7} \times 110.25$

$$= \frac{2425.5}{7}\ \text{cm}^2 = 346.5\ \text{cm}^2$$

$$= \frac{330}{360} \times 3.14 \times 16 \text{ cm}^2 = 46.05 \text{ cm}^2$$
$$= 46.1 \text{ cm}^2 \text{ (approx.)}$$

5. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Sol. Let R be the radius of required circle. Then, we have

$$\pi R^2 = \pi(8)^2 + \pi(6)^2$$
$$\Rightarrow \quad \pi R^2 = 64\pi + 36\pi \quad \Rightarrow \quad \pi R^2 = 100\,\pi$$
$$\therefore \quad R^2 = \frac{100\pi}{\pi} = 100 \quad \Rightarrow \quad R = 10 \text{ cm}$$

Hence, radius of required circle is 10 cm.

6. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Sol. Let R be the radius of required circle. Then, we have

$$2\pi R = 2\pi(19) + 2\pi\,(9)$$
$$\Rightarrow \quad 2\pi R = 2\pi(19 + 9) \quad \Rightarrow \quad R = \frac{2\pi \times 28}{2\pi} = 28$$

Hence, the radius of required circle is 28 cm.

7. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. Since the minute hand rotates through 6° in one minute, therefore, area swept by the minute hand in one minute is the area of a sector of angle 6° in a circle of radius 14 cm.

Hence, the area swept in 5 minutes $= \frac{\theta}{360^\circ} \times \pi r^2 \times 5$

$$= \frac{6^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 \times 5 = \frac{1}{60} \times 22 \times 28 \times 5$$
$$= \frac{154}{3} \text{ cm}^2 = 51\frac{1}{3} \text{ cm}^2$$

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (Fig. 11.13). Find

(*i*) the area of that part of the field in which the horse can graze;

(*ii*) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)

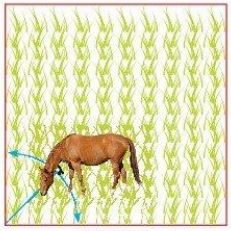


Fig. 11.13

Sol. Let the horse be tied at point O and the length of the rope is OH (Fig. 11.14). Thus,

(*i*) The area of the part of the field in which the horse can graze = area of the quadrant of a circle ($OAHB$)

$$= \frac{\pi r^2}{4} = \frac{1}{4} \times 3.14 \times 5 \times 5 = \frac{78.5}{4} = 19.625 \text{ m}^2$$

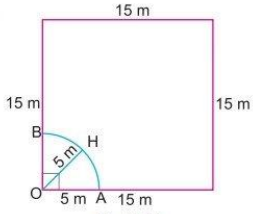


Fig. 11.14

(*ii*) Now $r = 10$ m and (Fig. 11.15)

$$\therefore \text{ Required area } = \frac{\pi r^2}{4}$$
$$= \frac{3.14 \times (10)^2}{4} = \frac{3.14 \times 100}{4}$$
$$= \frac{314}{4} = 78.5 \text{ m}^2$$

Increase in the grazing area

$= (78.5 - 19.625)\ m^2$

$= 58.875\ m^2$

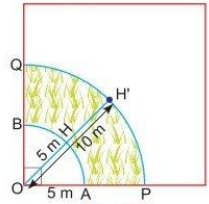


Fig. 11.15

9. An umbrella has 8 ribs which are equally spaced (Fig. 11.16). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Fig. 11.16

Sol. We have, $r = 45$ cm

$\therefore$ Area between two consecutive ribs $= \frac{1}{8} \times \pi r^2$

$= \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 = \frac{11 \times 45 \times 45}{4 \times 7}$

$= \frac{22275}{28} = 795.54\ cm^2$

10. Find the area of the shaded region in Fig. 11.17, where a circular arc of radius 6 cm has been drawn with vertex *O* of an equilateral triangle *OAB* of side 12 cm as centre. **[*CBSE (F)* 2016]**

Sol. We have, radius of circular region = 6 cm and each side of ΔOAB = 12 cm.

$\therefore$ Area of the circular portion

= area of circle – area of the sector

$= \pi r^2 - \frac{\theta}{360^\circ} \times \pi r^2$

$= \pi r^2 \left(1 - \frac{\theta}{360^\circ}\right) = \frac{22}{7} \times (6)^2 \left(1 - \frac{60^\circ}{360^\circ}\right)$

$= \frac{22}{7} \times 36 \times \frac{5}{6} = \frac{22 \times 30}{7} = \frac{660}{7}\ cm^2$

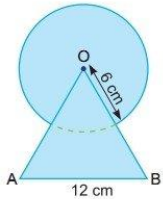


Fig. 11.17

Now, area of the equilateral triangle *OAB*

$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (12)^2 = \frac{\sqrt{3}}{4} \times 144 = 36\sqrt{3}\ cm^2$

$\therefore$ Area of shaded region = area of circular portion + area of equilateral triangle *OAB*

$= \left(\frac{660}{7} + 36\sqrt{3}\right) cm^2 = \frac{12}{7}(55 + 21\sqrt{3})\ cm^2$

11. From each corner of a square of side 4 cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 11.18. Find the area of the remaining portion of the square.

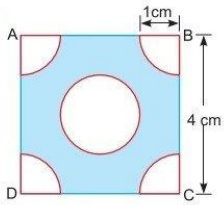


Fig. 11.18

Sol. We have, the side of the square $ABCD = 4$ cm

$\therefore$ Area of the square $ABCD = (4)^2 = 16$ cm^2

Since, each quadrant of a circle has radius 1 cm.

$\therefore$ The sum of the areas of four quadrants

$$= 4 \times \left(\frac{\pi r^2}{4}\right) = \pi r^2 = \frac{22}{7} \times (1)^2 = \frac{22}{7} \text{ cm}^2$$

Now, area of the circle of diameter 2 cm $= \pi \frac{d^2}{4} = \pi \times \frac{4}{4} = \pi = \frac{22}{7}$ cm^2

$\therefore$ Area of the remaining portion

= area of the square $ABCD$ – sum of the areas of four quadrants – area of the circle of diameter 2 cm

$$= 16 - \frac{22}{7} - \frac{22}{7} = \frac{112 - 22 - 22}{7} = \frac{68}{7} = 9.71 \text{ cm}^2$$

12. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 11.19(a). Find the area of the design.

[*Competency Based Question*]

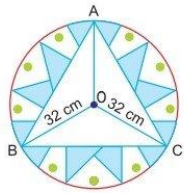


Fig. 11.19 (a)

Sol. Here, ΔABC is an equilateral triangle. Let O be the circumcentre of circumcircle.

Radius, $r = 32$ cm.

Now, area of circle $= \pi r^2$

$$= \frac{22}{7} \times 32 \times 32 = \frac{22528}{7} \text{ cm}^2$$

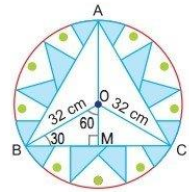


Fig. 11.19(b)

Draw $OM \perp BC$

$\therefore$ In ΔOMB

$$OM = OB \sin 30° = 32 \times \frac{1}{2} = 16 \text{ cm}$$

and, $$BM = OB \cos 30° = 32 \times \frac{\sqrt{3}}{2} = 16\sqrt{3}$$

$\therefore$ Area of $\Delta ABC = 3 \times$ area of ΔBOC

$$= 3 \times (2 \times \text{ar}(\Delta OMB))$$

$$= 6 \times \frac{1}{2} \times BM \times OM = 3 \times 16\sqrt{3} \times 16$$

$$= 768\sqrt{3} \text{ cm}^2$$

$\therefore$ Area of the design = area of the circle – area of ΔABC

$$= \left(\frac{22528}{7} - 768\sqrt{3}\right)$$

$$= (3218.28 - 1330.176) = 1888.1 \text{ cm}^2$$

13. Fig. 11.20, depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

(*i*) the distance around the track along its inner edge.

(*ii*) the area of the track.

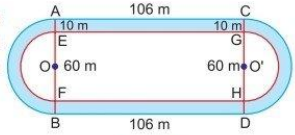


Fig. 11.20

Sol. Here, we have

$OE = O'G = 30$ m

$AE = CG = 10$ m

$OA = O'C = (30+10)$ m $= 40$ m

$AC = EG = FH = BD = 106$ m

(*i*) The distance around the track along its inner edge

$= EG + FH + 2 \times$ (circumference of the semicircle of radius $OE = 30$cm)

$$= 106 + 106 + 2\left(\frac{1}{2} \times 2\pi \times 30\right) = 212 + 60\pi$$

$$= 212 + 60 \times \frac{22}{7} = \left(212 + \frac{1320}{7}\right) = \left(\frac{1484 + 1320}{7}\right) = \frac{2804}{7} = 400\frac{4}{7} \text{m}$$

(*ii*) Area of the track = area of the shaded region

= area of rectangle $AEGC$ + area of rectangle $BFHD$ + 2 (area of the semicircle of radius 40 m – area of the semicircle with radius 30 m)

$$= [(10 \times 106) + (10 \times 106)] + 2\left\{\frac{1}{2} \times \frac{22}{7} \times (40)^2 - \frac{1}{2} \times \frac{22}{7} \times (30)^2\right\}$$

$$= 1060 + 1060 + \frac{22}{7}[(40)^2 - (30)^2]$$

$$= 2120 + \frac{22}{7} \times 700 = 2120 + 2200 = 4320 \text{ m}^2$$

14. In Fig. 11.21, *AB* and *CD* are two diameters of a circle (with centre *O*) perpendicular to each other and *OD* is the diameter of the smaller circle. If *OA* = 7 cm, find the area of the shaded region.

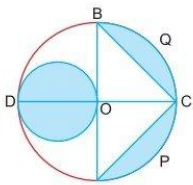


Fig. 11.21

Sol. Here, area of sector $OBQC = \frac{90°}{360°} \times \pi \times (7)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} \text{ cm}^2$$

and, area of $\Delta OBC = \frac{1}{2} \times OC \times OB = \frac{1}{2} \times 7 \times 7 = \frac{49}{2} \text{ cm}^2$

$\therefore$ Area of the segment BQC = area of sector $OBQC$ – area of ΔOBC

$$= \frac{77}{2} - \frac{49}{2} = \frac{28}{2} = 14 \text{ cm}^2$$

Similarly, area of the segment $APC = 14$ cm^2

Now, the area of the circle with OD as diameter $= \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{cm}^2$

Hence, the total area of the shaded region $= \left(14 + 14 + \frac{77}{2}\right)\text{cm}^2 = \left(28 + \frac{77}{2}\right)\text{cm}^2$

$= \left(\frac{56 + 77}{2}\right)\text{cm}^2 = \frac{133}{2}\text{cm}^2 = 66.5\text{ cm}^2$

15. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 11.22). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)

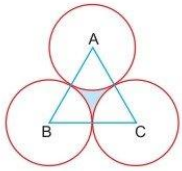


Fig. 11.22

Sol. Let each side of the equilateral triangle be x cm. Then,

Area of equilateral triangle ABC = 17320.5 cm^2 (Given)

$\Rightarrow \quad \frac{\sqrt{3}}{4}x^2 = 17320.5 \quad \Rightarrow \quad \frac{1.73205}{4}x^2 = 17320.5$

$\Rightarrow \quad x^2 = \frac{4 \times 17320.5}{1.73205} \quad \Rightarrow x^2 = 40000$

$\therefore \quad x = 200$ cm

Thus, radius of each circle $= \frac{200}{2}\text{ cm} = 100\text{ cm}$

Now, area of shaded region = area of $\Delta ABC - 3 \times$ area of a sector of angle 60° and radius 100 cm

$= 17320.5 - 3 \times \frac{60°}{360°} \times \pi \times (100)^2$

$= 17320.5 - \frac{1}{2} \times \pi \times 100 \times 100$

$= 17320.5 - 3.14 \times 5000$

$= 17320.5 - 15700 = 1620.5\text{ cm}^2$

16. On a square handkerchief, nine circular designs, each of radius 7 cm are made (see Fig. 11.23). Find the area of the remaining portion of the handkerchief.

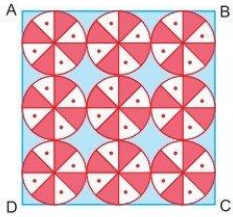


Fig. 11.23

Sol. Total area of circular design $= 9 \times$ area of one circular design

$= 9 \times \pi \times (7)^2$

$= 9 \times \frac{22}{7} \times 7 \times 7 = 1386\text{ cm}^2$

Now, each side of square $ABCD$ $= 3 \times$ diameter of circular design

$= 3 \times 14 = 42$ cm

$\therefore$ Area of square $ABCD$ $= (42)^2 = 1764\text{ cm}^2$

$\therefore$ Area of the remaining portion of handkerchief

= area of square $ABCD$ – total area of circular design

$= (1764 - 1386)\text{ cm}^2 = 378\text{ cm}^2$

17. In Fig. 11.24, $OACB$ is a quadrant of a circle with centre O and radius 3.5 cm. If $OD = 2$ cm, find the area of the (i) quadrant $OACB$, (ii) shaded region.

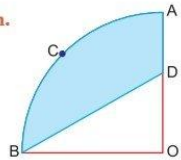


Fig. 11.24

Sol. (i) Area of quadrant $OACB = \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8}\text{ cm}^2 = 9\frac{5}{8}\text{ cm}^2$

(*ii*) Now, we find area of ΔOBD

We have area of $\Delta OBD = \frac{1}{2} \times OB \times OD$

$= \frac{1}{2} \times 3.5 \times 2 = 3.5 \text{ cm}^2 = \frac{7}{2} \text{ cm}^2$

Hence, area of shaded region = area of quadrant $OACB$ – area of ΔOBD

$= \left(\frac{77}{8} - \frac{7}{2}\right)\text{cm}^2 = \left(\frac{77-28}{8}\right)\text{cm}^2 = \frac{49}{8}\text{cm}^2 = 6\frac{1}{8}\text{cm}^2$

18. In Fig. 11.25 *ABPC* is a quadrant of a circle of radius 14 cm and a semicircle is drawn with *BC* as diameter. Find the area of the shaded region.

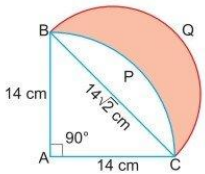


Fig. 11.25

Sol. In ΔABC, we have

$BC = \sqrt{(AC)^2 + (AB)^2}$ (By Pythagoras Theorem)

$= \sqrt{(14)^2 + (14)^2} = \sqrt{196 + 196} = \sqrt{392} = 14\sqrt{2}$ cm

Now, area of sector $ABPC = \frac{90°}{360°} \times \pi \times (14)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$

and, area of $\Delta ABC = \frac{1}{2} \times AC \times AB = \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$

$\therefore$ Area of segment BPC = area of sector $ABPC$ – area of ΔABC

$= (154 - 98) \text{ cm}^2 = 56 \text{ cm}^2$

Now, we have radius of semi-circle $BQC = \frac{14\sqrt{2}}{2}$ cm $= 7\sqrt{2}$ cm

$\therefore$ Area of semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2} = 154 \text{ cm}^2$

Hence, area of the shaded region = area of the semi-circle BQC – area of the segment BPC

$= (154 - 56) \text{ cm}^2 = 98 \text{ cm}^2$

19. Calculate the area of the designed region in Fig. 11.26, which is common between the two quadrants of circles of radius, 8 cm each. ***[Competency Based Question]***

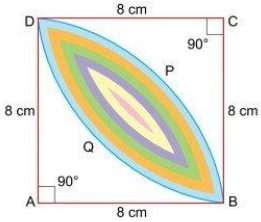


Fig. 11.26

Sol. Here, radius of each quadrant $ABPD$ and $BQDC = 8$ cm

Sum of areas of quadrants $= 2 \times \frac{1}{4}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times (8)^2 = \frac{11}{7} \times 64 = \frac{704}{7} \text{ cm}^2$

Now, area of the square $ABCD = 8 \times 8 = 64 \text{ cm}^2$

Hence, area of designed region = area of shaded region

= sum of areas of quadrants – area of the square $ABCD$

$= \frac{704}{7} - 64 = \frac{704 - 448}{7} = \frac{256}{7} = 36.57 \text{ cm}^2$

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. David draws a circle with diameter 6 units. He draws another circle by increasing the radius of the previously drawn circle by 4 units. What would be the quotient if he divides the circumference of the newly formed circle by its diameter?

   (*a*) 8π (*b*) 12π (*c*) π (*d*) 2π

2. If the area of a circle is 154 cm^2, then its perimeter is [*NCERT Exemplar*]

   (*a*) 11 cm (*b*) 22 cm (*c*) 44 cm (*d*) 55 cm

3. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is [*NCERT Exemplar*]

   (*a*) 31 cm (*b*) 25 cm (*c*) 62 cm (*d*) 50 cm

4. Savita has a lamp placed at the centre of her square yard, each side measuring 20 m. The light of lamp covers a circle of radius 10 m on yard. What area of the yard is not lit by the lamp? [*CBSE Question Bank*]

   (*a*) 400π sq. m (*b*) 100π sq. m (*c*) $(40 - 10\pi)$ sq. m (*d*) $(400 - 100\pi)$ sq. m

5. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is [*NCERT Exemplar*]

   (*a*) 56 cm (*b*) 42 cm (*c*) 28 cm (*d*) 16 cm

6. A fountain is enclosed by a circular fence of circumference 11 m and is surrounded by a circular path. The circumference of the outer boundary of the path is 16 m. A gardener increased the width of the pathway by decreasing the area enclosed by the fence such that the length of the fence is decreased by 3 m. The path is to be covered by the bricks which cost ₹ 125 per m^2. What will be the total cost, to the nearest whole number, required to cover the area by the bricks? $\left(\text{Use } \pi = \frac{22}{7}\right)$

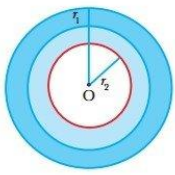


Fig. 11.27

   (*a*) ₹ 1,910 (*b*) ₹ 9,878 (*c*) ₹ 39,772 (*d*) ₹ 79,545

7. The area of the square that can be inscribed in a circle of radius 8 cm is [*NCERT Exemplar*]

   (*a*) 256 cm^2 (*b*) 128 cm^2 (*c*) $64\sqrt{2}$ cm^2 (*d*) 64 cm^2

8. Observe the figure below:

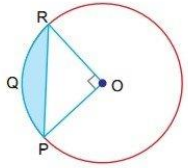


Fig. 11.28

   What is the area of the segment *PQR*, if the radius of the circle is 7 cm? $\left(\text{Use } \pi = \frac{22}{7}\right)$

   (*a*) 14 cm^2 (*b*) 17.3 cm^2 (*c*) 28 cm^2 (*d*) 91 cm^2

9. The area of the circle that can be inscribed in a square of side 6 cm is [*NCERT Exemplar*]

   (*a*) 36π cm^2 (*b*) 18π cm^2 (c) 12π cm^2 (*d*) 9π cm^2

10. Which of these is equivalent to π?

(*a*) $\frac{\text{Circumference}}{\text{Radius}}$ (*b*) $\frac{\text{Circumference}}{\text{Diameter}}$

(*c*) Circumference × Diameter (*d*) Circumference × Radius

11. Area of the largest triangle that can be inscribed in a semi-circle of radius *r* units is

[*NCERT Exemplar*]

(*a*) r^2sq. units (*b*) $\frac{1}{2}r^2$sq. units (*c*) $2r^2$sq. units (*d*) $\sqrt{2}\,r^2$sq. units

12. An arc of a circle of radius 14 cm, subtends an angle of 45° at the centre as shown:

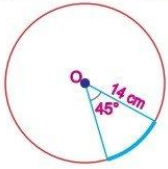


Fig. 11.29

Which of these options is correct?

(*a*) The arc shown is a minor arc and its length is 5.5 cm.

(*b*) The arc shown is a major arc and its length is 77 cm.

(*c*) The arc shown is a major arc and its length is 38.5 cm.

(*d*) The arc shown is a minor arc and its length is 11 cm.

13. The perimeter of a circle is equal to that of a square, then the ratio of their areas is

[*NCERT Exemplar*]

(*a*) 22 : 7 (*b*) 14 : 11 (*c*) 7 : 22 (*d*) 11 : 14

14. In the figure below, the square *JKLM* is inscribed within a circle and ΔJMN is a right-angled isosceles triangle. The point marked *O* is the centre of the circle.

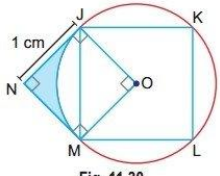


Fig. 11.30

What is the area of the shaded part of the figure? **[*Competency Based Question*]**

(*a*) $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ cm^2 (*b*) $\left(\pi-\frac{1}{2}\right)$ cm^2 (*c*) $\left(1-\frac{\pi}{4}\right)$ cm^2 (*d*) $(1-\pi)$ cm^2

15. The diameter of a wheel is 1 m. The number of revolutions it will make to travel a distance of 22 km will be

(*a*) 2,800 (*b*) 4,000 (*c*) 5,500 (*d*) 7,000

Answers

1. (*c*) **2.** (*c*) **3.** (*d*) **4.** (*d*) **5.** (*c*) **6.** (*a*) **7.** (*b*)
8. (*a*) **9.** (*d*) **10.** (*b*) **11.** (*a*) **12.** (*d*) **13.** (*b*) **14.** (*c*)
15. (*d*)

Very Short Answer Questions

Each of the following questions are of* 1 *mark.

1. A thin wire is in the shape of a circle of radius 77 cm, it is bent into a square. Find the side of the square. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Let side of square be x cm.

$\therefore$ Perimeter of the circle = Perimeter of the square

$2\pi r = 4x$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 77 = 4x \Rightarrow x = \frac{2 \times 22 \times 11}{4}$

$\Rightarrow x = 121$ cm

$\therefore$ Length of the side of the square = 121 cm.

2. If circumference and the area of a circle are numerically equal, find the diameter of the circle.

Sol. Given, $2\pi r = \pi r^2$

$\Rightarrow \quad 2r = r^2 \quad \Rightarrow \quad r^2 - 2r = 0$

$\Rightarrow \quad r(r-2) = 0 \quad$ or $\quad r = 2$

i.e., $\quad d = 4$ units

3. If the perimeter of a semi-circular protractor is 36 cm, find its diameter.

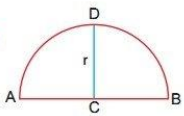


Fig. 11.31

Sol. Perimeter of a semicircular protractor = perimeter of a semicircle

$= (2r + \pi r)$ cm

Given, $\quad 2r + \pi r = 36$

$\Rightarrow \quad r\left(2 + \frac{22}{7}\right) = 36 \quad \Rightarrow \quad r\left(\frac{36}{7}\right) = 36 \Rightarrow \quad r = 7$ cm

Diameter $= 2r = 2 \times 7 = 14$ cm.

4. In Fig. 11.32, there is a sector of circle of radius 10.5 cm. Find the perimeter of the sector. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[*CBSE* 2020 (30/2/1)]**

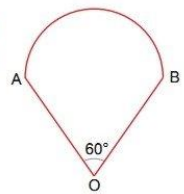


Fig. 11.32

Sol. Given, radius of the sector of the circle, $r = 10.5$ cm.

$\therefore$ Perimeter of the sector $= \frac{\theta}{360^\circ} \times 2\pi r + 2r$

$= \frac{60^\circ}{360^\circ} \times \frac{2 \times 22}{7} \times 10.5 + 2 \times 10.5$

$= \frac{1}{6} \times 44 \times 1.5 + 21$

$= 11 + 21 = 32$ cm

$\therefore$ Perimeter of the sector = 32 cm

5. Find the area of a sector of a circle whose radius is r and length of the arc is l.

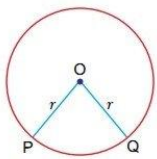


Fig. 11.33

Sol. Area of a sector of a circle with radius r

$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{\theta}{360^\circ} \times 2\pi r \frac{r}{2} = \frac{1}{2} lr$ sq. units $\quad \left(\because \ l = \frac{2\pi r\theta}{360^\circ}\right)$

6. Find the area of the circle inscribed in a square of side a cm.

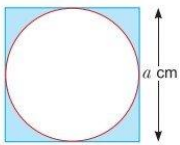


Fig. 11.34

Sol. Diameter of the circle = a cm

$\Rightarrow \quad$ Radius $= \frac{a}{2}$ cm

$\Rightarrow \quad$ Area $= \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$ cm^2

7. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?

Sol. Arc length of a circle of radius $r = \frac{\theta}{360^\circ} \times 2\pi r$

$\Rightarrow \quad 5\pi = \frac{\theta}{360^\circ} \times 2\pi \times 10 \quad \text{or} \quad \frac{\theta}{360^\circ} = \frac{5\pi}{20\pi} = \frac{1}{4}$

$\Rightarrow \quad \theta = \frac{360^\circ}{4} = 90^\circ$

8. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 km.

Sol. Number of revolutions $= \frac{11 \times 1000}{2 \times \frac{22}{7} \times 0.25} = \frac{11 \times 1000 \times 7}{11} = 7000.$

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left(\text{Use } \pi = \frac{22}{7}\right)$ **[*CBSE* 2020 (30/1/1)]**

Sol. Let r be the radius and θ be the angle subtended by the arc at the centre of the circle.

$\therefore$ Length of arc = length of piece of wire

$\frac{\theta}{360} \times 2\pi r = 22$

$\Rightarrow \quad \frac{60}{360} \times 2\pi r = 22 \quad \Rightarrow \quad \frac{\pi r}{3} = 22$

$\Rightarrow \quad r = \frac{3 \times 22}{\pi} = \frac{3 \times 22}{\frac{22}{7}} = 21 \text{ cm}$

$\therefore$ Radius = 21 cm

Fig. 11.35

2. A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

Sol. Let the outer and inner radii of the ring be R m and r m respectively. Then, $2\pi R = 396$ and $2\pi r = 352$

$\Rightarrow \quad 2 \times \frac{22}{7} \times R = 396 \quad \text{and} \quad 2 \times \frac{22}{7} \times r = 352$

$\Rightarrow \quad R = 396 \times \frac{7}{22} \times \frac{1}{2} \quad \text{and} \quad r = 352 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow \quad R = 63 \text{ m} \quad \text{and} \quad r = 56 \text{ m}$

Hence, width of the track $= (R - r) = (63 - 56) = 7$ m

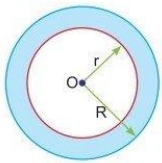


Fig. 11.36

3. In the Fig. 11.37, *ABCD* is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. **[*Competency Based Question*]**

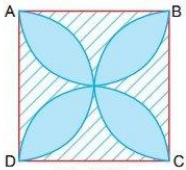


Fig. 11.37

Sol. Area of 1 segment = area of sector – area of triangle

$$= \left(\frac{90°}{360°}\right)\pi r^2 - \frac{1}{2}\times 7\times 7$$

$$= \frac{1}{4}\times\frac{22}{7}\times 7^2 - \frac{1}{2}\times 7\times 7$$

$$= 14 \text{ cm}^2$$

Area of 8 segments $= 8 \times 14 = 112 \text{ cm}^2$

Area of the shaded region $= 14 \times 14 - 112$

$= 196 - 112 = 84 \text{ cm}^2$

(*each petal is divided into 2 segments*) **[*CBSE Marking Scheme* 2021]**

4. In Fig. 11.38, a square *OABC* is inscribed in a quadrant *OPBQ*. If *OA* = 15 cm, find the area of the shaded region. (Use π = 3.14) **[*CBSE* 2019, (30/2/1)]**

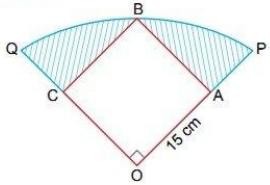


Fig. 11.38

Sol. Radius of quadrant = $OB = \sqrt{15^2 + 15^2} = 15\sqrt{2}$ cm.

Shaded area = Area of quadrant – Area of square

$$= \frac{1}{4}(3.14)(15\sqrt{2})^2 - (15)^2$$

$$= (15)^2\,(1.57 - 1) = 128.25 \text{ cm}^2$$ **[*CBSE Marking Scheme* 2019 (30/2/1)]**

Short Answer Questions–II

Each of the following questions are of 3 marks.

1. In Fig. 11.39, there are shown two arcs *PAQ* and *PBQ*. Arc *PAQ* is a part of circle with centre *O* and radius *OP* while arc *PBQ* is a semi-circle drawn on *PQ* as diameter with centre *M*. If *OP* = *PQ* = 10 cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)\text{cm}^2$. **[*CBSE (Delhi)* 2016]**

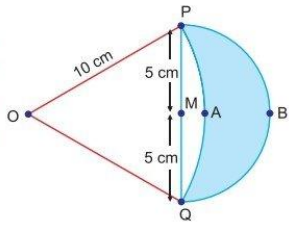


Fig. 11.39

Sol. Since $OP = PQ = QO$

$\Rightarrow$ ΔPOQ is an equilateral triangle.

$\therefore$ $\angle POQ = 60°$

Area of segment $PAQM$

$$= \frac{\theta}{360°}\pi r^2 - \frac{\sqrt{3}}{4}a^2 = \frac{60°}{360°}\pi\times 10^2 - \frac{\sqrt{3}}{4}\times 10^2$$

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)\text{cm}^2 = \left(\frac{50\pi}{3} - 25\sqrt{3}\right)\text{cm}^2$$

Area of semicircle with M as centre $= \frac{\pi}{2}(5)^2 = \frac{25\pi}{2}\text{ cm}^2$

Area of shaded region $= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right) = \frac{25}{2}\pi - \frac{50}{3}\pi + 25\sqrt{3}$

$= \frac{-25}{6}\pi + 25\sqrt{3} = 25\left(\sqrt{3} - \frac{\pi}{6}\right)\text{cm}^2$ Hence Proved.

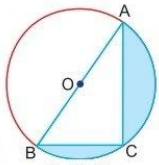


Fig. 11.40

2. In Fig. 11.40, *O* is the centre of a circle such that diameter *AB* = 13 cm and *AC* = 12 cm. *BC* is joined. Find the area of the shaded region. (Take π = 3.14)

[*CBSE* 2016 (30/2)]

Sol.

13) Q10 –

Radius of semicircle ACB = $\frac{13}{2}$ cm.

Area of semicircle = $\frac{\pi r^2 \times 1}{2}$

Its area = $\frac{1}{2} \times 3.14 \times \frac{13}{2} \times \frac{13}{2}\text{cm}^2$

$\frac{3.14 \times 169\text{cm}^2}{8} = \frac{530.66\text{cm}^2}{8}$

Semicircle subtend 90° at circle, ∠ ACB = 90°

In Δ ABC –

$AC^2 + BC^2 = AB^2 \Rightarrow 12^2 + BC^2 = 169\text{cm}^2$

$\Rightarrow BC^2 = (169 - 144)\text{cm}^2$ $BC^2 = 25\text{cm}$

$BC = 5\text{cm}$.

Area of Δ = $\frac{1}{2}$ × Base × Height

Area of Δ ABC = $\frac{1}{2} \times AC \times BC = \frac{1}{2} \times 12 \times 5 = 30\text{cm}^2$

Area of shaded region = $\frac{530.66\text{cm}^2}{8} - 30\text{cm}^2$

$\Rightarrow (66.3325 - 30)\text{cm}^2$

$\Rightarrow \boxed{36.3325\text{cm}^2}$

[*Topper's Answer* 2016]

3. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle 120°. Find the total area cleaned at each sweep of the blades. $\left(\text{Take } \pi = \frac{22}{7}\right)$

[*CBSE* 2019 (30/3/1)]

Sol. Total area cleaned = 2 × area of sector

$= 2 \times \frac{\pi r^2 \theta}{360°}$ **1**

$= 2 \times \frac{22}{7} \times 21 \times 21 \times \frac{120°}{360°}$ **1**

$= 924\text{ cm}^2$ **[*CBSE Marking Scheme* 2019 (30/3/1)] 1**

4. Find the area of the shaded region in Fig. 11.41, where arcs drawn with centres *A*, *B*, *C* and *D* intersect in pairs at mid-points *P*, *Q*, *R* and *S* of the sides *AB*, *BC*, *CD* and *DA* respectively of a square *ABCD* of side 12 cm. [Use $\pi = 3.14$] [*NCERT Exemplar, CBSE* 2018 (30/1/1)]

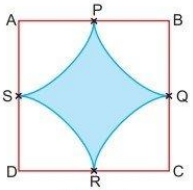
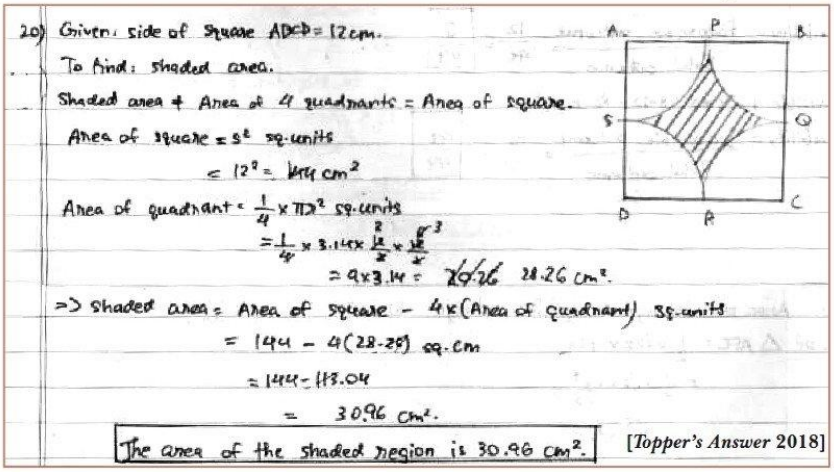


Fig. 11.41

Sol.



[*Topper's Answer* 2018]

5. In Fig. 11.42, three section of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre are shaded. Find the area of the shaded region. [*CBSE* 2019 (30/5/1)]

Sol. We have radius of circle = 7 cm

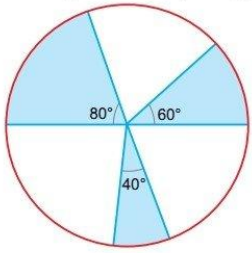


Fig. 11.42

Area of sector $= \frac{\theta}{360°} \times \pi r^2$

Area of sector containing 60°angle $= \frac{60°}{360°} \times \frac{22}{7} \times (7)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 49$

$= \frac{11 \times 7}{3} = \frac{77}{3} \text{cm}^2$

Also, area of sector containing 80° angle $= \frac{80°}{360°} \times \frac{22}{7} \times (7)^2$

$= \frac{2}{9} \times 22 \times 7 = \frac{308}{9} \text{cm}^2$

Again, area of sector containing 40° angle $= \frac{40°}{360°} \times \frac{22}{7} \times (7)^2$

$= \frac{1}{9} \times 22 \times 7 = \frac{154}{9} \text{cm}^2$

Area of total shaded region $= \frac{77}{3} + \frac{308}{9} + \frac{154}{9}$

$= \frac{231 + 308 + 154}{9}$

$= \frac{693}{9} = 77 \text{ cm}^2$

6. Three semicircles each of diameter 3 cm, a circle of diameter 4·5 cm and a semicircle of radius 4·5 cm are drawn in the given figure. Find the area of the shaded region.

[*CBSE* 2017 (30/3)]

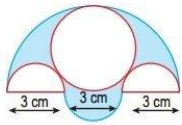


Fig. 11.43

Sol.

13. Area of shaded region = Area of semicircle with r = 4.5 cm
+ Area of semicircle with d = 3 cm
− 2× area of semicircle with d = 3 cm
− area of circle with d = 4.5 cm.

$= \frac{1}{2} \times \pi \times (4.5)^2 + \left(\frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2\right) - 2\times\left(\frac{1}{2} \times \pi \left(\frac{3}{2}\right)^2\right) - \pi\left(\frac{4.5}{2}\right)^2$

$= \frac{1}{2} \times \pi \times \frac{20.25}{4} + \frac{\pi}{2} \times \frac{9}{4} - \pi \times \frac{20.25}{4}$

$= \frac{\pi}{4}\left[2 \times 20.25 - \frac{9}{2} - 20.25\right]$

$= \frac{\pi}{4}\left[40.5 - 4.5 - 20.25\right]$

$= \frac{\pi}{4}\left[20.25 - 4.5\right]$

$= \frac{\pi}{4}(15.75)$

$= \frac{22}{7} \times \frac{15.75}{4}$

$= \frac{11 \times 2.25}{2}$

$= \frac{24.75}{2}$

$= 12.375 \text{ cm}^2$

area of shaded region is 12.375 cm^2 **[*Topper's Answer* 2017]**

7. In Fig. 11.44, *ABCD* is a trapezium of area 24.5 sq. cm. In it, *AD* || *BC*, ∠*DAB* = 90°, *AD* = 10 cm and *BC* = 4 cm. If *ABE* is a quadrant of a circle, find the area of the shaded region. (Take $\pi = \frac{22}{7}$)

[*CBSE* (AI) 2014]

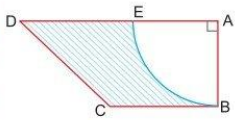


Fig. 11.44

Sol. Area of trapezium = 24.5 cm^2

$\frac{1}{2}[AD + BC] \times AB = 24.5$

$$\frac{1}{2}[10+4] \times AB = 24.5$$

$AB = 3.5$ cm $\Rightarrow$ $r = 3.5$ cm

$$\text{Area of quadrant} = \frac{1}{4}\pi r^2$$

$$= 0.25 \times \frac{22}{7} \times 3.5 \times 3.5 = 9.625 \text{ cm}^2$$

The area of shaded region = $24.5 - 9.625 = 14.875$ cm^2

8. Two circles touch internally. The sum of their areas is 116π cm^2 and distance between their centres is 6 cm. Find the radii of the circles. **[*CBSE (F)* 2017, *HOTS*]**

Sol. Let R and r be the radii of the circles [Fig. 11.45].

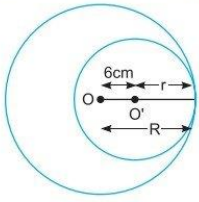


Fig. 11.45

Then, according to question,

$\Rightarrow$ $\pi R^2 + \pi r^2 = 116\pi$

$\Rightarrow$ $R^2 + r^2 = 116$ ...(*i*)

Distance between the centres = 6 cm

$\Rightarrow$ $OO' = 6$ cm

$\Rightarrow$ $R - r = 6$ ...(*ii*)

Now, $(R+r)^2 + (R-r)^2 = 2(R^2 + r^2)$

Using the equation (*i*) and (*ii*), we get

$(R+r)^2 + 36 = 2 \times 116$

$\Rightarrow$ $(R+r)^2 = (2 \times 116 - 36) = 196$

$\Rightarrow$ $R + r = 14$ ...(*iii*)

Solving (*ii*) and (*iii*), we get $R = 10$ and $r = 4$

Hence, radii of the given circles are 10 cm and 4 cm respectively.

9. In the given Fig. 11.46, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where *O* and *O′* are centres of the circles. Find the area of shaded region. **[*CBSE Delhi* 2017, *HOTS*]**

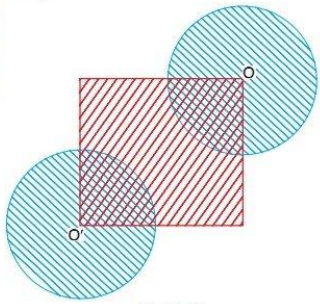


Fig. 11.46

Sol. Area of shaded region

= area of square + area of 2 major sectors having angle 270° at centre

$$= \text{side} \times \text{side} + 2 \times \frac{\pi r^2 \theta}{360^\circ}$$

$$= \left[28 \times 28 + 2 \times \frac{22}{7} \times 14 \times 14 \times \frac{270^\circ}{360^\circ}\right]$$

$$= 28 \times 28 \left(1 + \frac{11}{7} \times \frac{3}{4}\right)$$

$$= 28 \times 28 \left(1 + \frac{33}{28}\right) = 1708 \text{ cm}^2$$

10. The area of a circular play ground is 22176 cm^2. Find the cost of fencing this ground at the rate of ₹ 50 per metre. **[*CBSE* 2020(30/2/1)]**

Sol. Given, Area of circular playground = 22176 cm^2

$\Rightarrow$ $\pi r^2 = 22176$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 22176$

$\Rightarrow \quad r^2 = \frac{22176 \times 7}{22} = 7056$

$\Rightarrow \quad r^2 = 7056 \quad \Rightarrow r = \sqrt{7056} = 84$

$\therefore \quad r = 84$ cm

Now, circumference of the circular ground $= 2\pi r$

$= 2 \times \frac{22}{7} \times 84 = 2 \times 22 \times 12 = 528$ cm

$\therefore$ Total cost of fencing the ground $= \frac{528 \times 50}{100} = ₹264$

11. In Fig. 11.47, a square *OPQR* is inscribed in a quadrant *OAQB* of a circle. If the radius of circle is $6\sqrt{2}$ cm, find the area of the shaded region. **[*CBSE* 2020(30/4/1)]**

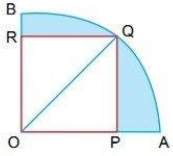


Fig. 11.47

Sol. Given, *OPQR* is a square and *OAQB* is the quadrant of a circle of radius $6\sqrt{2}$ cm.

$\therefore \quad OQ = 6\sqrt{2}$ cm

We know that diagonal of square $= \sqrt{2}a$, where a is the side of square.

$\Rightarrow \quad 6\sqrt{2} = \sqrt{2}a \quad \Rightarrow \quad a = 6$ cm

$\therefore$ Area of square $OPQR = (6)^2 = 36$ cm^2

Also, area of quadrant $OAQB = \frac{1}{4} \times \pi \times (6\sqrt{2})^2 = \frac{1}{4} \times \pi \times 72 = 18\pi$ cm^2

$\therefore$ Area of shaded region = area of quadrant of circle *OAQB* – area of square *OPQR*

$= (18\pi - 36)$ cm^2 $= 18(\pi - 2)$ cm^2

Long Answer Questions

Each of the following questions are of* 5 *marks.

1. An elastic belt is placed around the rim of a pulley of radius 5 cm. (Fig. 11.48). From one point *C* on the belt, the elastic belt is pulled directly away from the centre *O* of the pulley until it is at *P*, 10 cm from the point *O*. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$) **[*CBSE Delhi* 2016]**

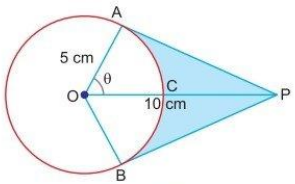


Fig. 11.48

Sol. In ΔAOP, $\cos\theta = \frac{5}{10}$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \quad \theta = 60°$

$\Rightarrow$ Reflex $\angle AOB = 360° - 2 \times 60° = 240°$

$\therefore$ Length of belt in contact with pully $= \frac{\theta}{360^\circ} \times 2\pi r = \frac{2\times 3.14\times 5\times 240}{360} = 20.93$ cm

Now, $\frac{AP}{OA} = \tan 60^\circ$

$PA = 5\sqrt{3}$ cm $= BP$ (Tangents from an external point are equal)

Area $(\Delta OAP + \Delta OBP) = 2\left(\frac{1}{2}\times 5\times 5\sqrt{3}\right) = 25\sqrt{3} = 43.25$ cm^2

Area of sector $OACB = \frac{\theta}{360^\circ}\pi r^2 = \frac{25\times 3.14\times 120}{360} = 26.17$ cm^2

Shaded area $= 43.25 - 26.17 = 17.08$ cm^2

2. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90^\circ$. Find the area of the shaded region. **[*CBSE* 2017(30/3/1)]**

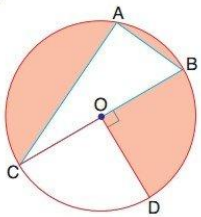
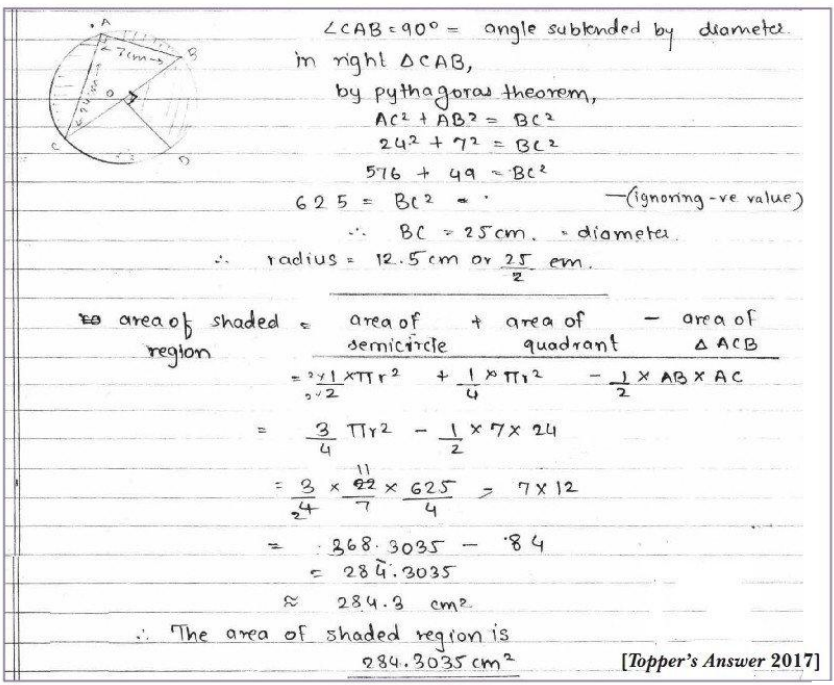


Fig. 11.49

Sol.



3. In Fig. 11.50, a sector OAP of a circle with centre O, containing angle θ. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right]$. [*CBSE (AI)* 2016]

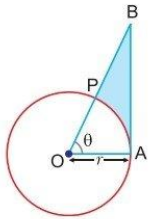


Fig. 11.50

Sol.

2s) Given – OAP is sector of circle with centre O, $\angle POA = \theta$ and $OA \perp AB$

To prove –

Perimeter of shaded region = $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$

Proof –

Perimeter of shaded region = BP + AB + arc AP — (IV)

Now – $= (\theta)$

$\tan\theta = \frac{AB}{r} \Rightarrow r\tan\theta = AB$ — (1)

$\sec\theta = \frac{OB}{r} \Rightarrow r\sec\theta = OB$

$OB - OP = BP \Rightarrow r\sec\theta - r = BP$ — (2)

Length of arc AP – $\frac{\theta}{360} \times 2\pi r = \frac{\theta}{360} \times 2\pi r = \frac{\theta\pi r}{180}$ — (3)

Putting value from eq (1), (2), (3) in eq IV

$\Rightarrow$ Perimeter of shaded region

$= r\tan\theta + r\sec\theta - r + \frac{\theta\pi r}{180}$

$= \left[r\left\{\tan\theta + \sec\theta + \frac{\theta\pi}{180} - 1\right\}\right]$

Hence proved. [~~Topper's Answer~~ 2016]

4. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle. [*CBSE Delhi* 2017]

Sol. Area of minor segment

= area of minor sector having angle 60° at centre – area of equilateral ΔOPQ

$$= \frac{22}{7} \times 10 \times 10 \times \frac{60°}{360°} - \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= 10 \times 10 \left[\frac{22}{7} \times \frac{1}{6} - \frac{\sqrt{3}}{4}\right]$$

$$= \frac{100}{84}(44 - 21\sqrt{3})\text{cm}^2 \quad \text{or} \quad \frac{25}{21}(44 - 21\sqrt{3})\text{cm}^2$$

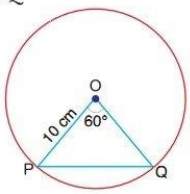


Fig. 11.51

Area of major segment = area of circle – area of minor segment

$$= \left[\frac{22}{7} \times 10 \times 10 - \frac{25}{21}(44 - 21\sqrt{3})\right]$$

$$= \frac{2200}{7} - \frac{25}{21}(44 - 21\sqrt{3}) = \frac{6600 - 1100 + 25 \times 21\sqrt{3}}{21}$$

$$= \frac{5500 + 25 \times 21\sqrt{3}}{21} = \frac{25}{21}(220 + 21\sqrt{3})\text{cm}^2$$

5. In Fig 11.52, a circle is inscribed in an equilateral triangle *ABC* of side 12 cm. Find the radius of inscribed circle and the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

[*CBSE Delhi* 2014]

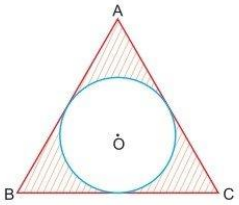


Fig. 11.52

Sol. **Construction:** Join *OA*, *OB* and *OC*

Draw $OZ \perp BC$, $OX \perp AB$ and $OY \perp AC$. (Fig 11.53)

Let the radius of the circle be *r* cm.

Area of ΔABC = area of ΔAOB + area of ΔBOC + area of ΔAOC

$$\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{1}{2} \times AB \times OX + \frac{1}{2} \times BC \times OZ + \frac{1}{2} \times AC \times OY \quad \text{[From Fig. 11.53]}$$

$$\frac{\sqrt{3}}{4}(12)^2 = \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 12 \times r + \frac{1}{2} \times 12 \times r$$

$$\frac{\sqrt{3}}{4} \times 12 \times 12 = 3 \times \frac{1}{2} \times 12 \times r$$

$$r = 2\sqrt{3} \text{ cm}$$

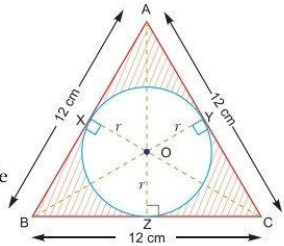


Fig. 11.53

Area of shaded region = area of ΔABC – area of inscribed circle

$$= \left[\frac{\sqrt{3}}{4}(12)^2 - \pi(2\sqrt{3})^2\right]\text{cm}^2$$

$$= \frac{\sqrt{3}}{4} \times 12 \times 12 - 3.14 \times 4 \times 3$$

$$= 1.73 \times 3 \times 12 - 3.14 \times 4 \times 3$$

$$= 62.28 - 37.68 = 24.6 \text{ cm}^2$$

6. In Fig. 11.54, from a rectangular region $ABCD$ with $AB = 20$ cm, a right triangle AED with $AE = 9$ cm and $DE = 12$ cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region. [Use $\pi = 3.14$] ***[CBSE (F) 2014, HOTS]***

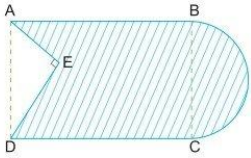


Fig. 11.54

Sol. Area of shaded region

= area of rectangle – area of triangle + area of semicircle.

In right ΔADE

$$AD^2 = AE^2 + DE^2$$

$$AD = \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15 \text{ cm}$$

Area of $\Delta AED = \frac{1}{2} \times DE \times AE = \frac{1}{2} \times 12 \times 9 = 54 \text{ cm}^2$

In semicircle at BC, diameter $= BC = 15$ cm.

Radius of semicircle $= \frac{1}{2} \times 15 = 7.5$ cm

Area of semicircle $= \frac{\pi r^2}{2} = \frac{3.14 \times 7.5 \times 7.5}{2} = 88.31 \text{ cm}^2$

Area of rectangle $= AB \times BC = 20 \times 15 = 300 \text{ cm}^2$

Area of shaded region $= 300 + 88.31 - 54 = 334.31 \text{ cm}^2$

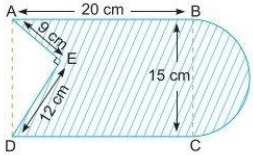


Fig. 11.55

Case Study-based Questions

Each of the following questions are of 4 marks.

1. Read the following and answer any four questions from (*i*) to (*v*).

A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat.

Designs of some brooches are shown below. Observe them carefully.

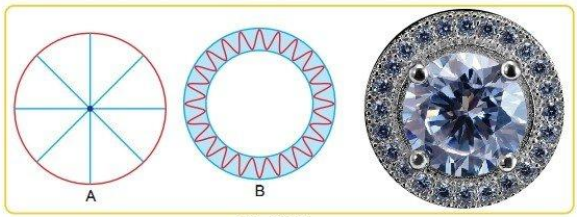


Fig. 11.56

Design *A*: Brooch A is made with silver wire in the form of a circle with diameter 28 mm. A wire used for making 4 diameters which divide the circle into 8 equal parts.

Design *B*: Brooch B is made of two colours gold and silver. Outer part is made with gold. The circumference of silver part is 44 mm and the gold part is 3 mm wide everywhere.

[CBSE Question Bank]

Refer to design *A*

(*i*) The total length of silver wire required is

(*a*) 180 mm (*b*) 200 mm (*c*) 250 mm (*d*) 280 mm

(*ii*) The area of each sector of the brooch is

(*a*) 44 mm^2 (*b*) 52 mm^2 (*c*) 77 mm^2 (*d*) 68 mm^2

Refer to design *B*

(*iii*) The circumference of outer part (golden) is

(*a*) 48.49 mm (*b*) 82.2 mm (*c*) 72.50 mm (*d*) 62.86 mm

(*iv*) The difference of areas of golden and silver parts is

(*a*) 18π (*b*) 44π (*c*) 51π (*d*) 64π

(*v*) A boy is playing with brooch *B*. He makes revolution with it along its edge. How many complete revolutions must it take to cover 80π mm ?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

Sol. (*i*) We have diameter of the wire = 28 mm

$\Rightarrow$ $d = 28$ mm

$\therefore$ Radius (r) = 14 mm

Total length of silver wire required = circumference of circle + 4 × diameter of the circle

$$= 2\pi r + 4 \times d$$

$$= 2 \times \frac{22}{7} \times 14 + 4 \times 28$$

$$= 88 + 112 = 200 \text{ mm}$$

$\therefore$ Option (*b*) is correct.

(*ii*) Angle of the each sector $= \frac{360^\circ}{8} = 45^\circ$

$\therefore$ Area of each sector of the brooch $= \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{8} \times 22 \times 2 \times 14 = 77 \text{ mm}^2$$

$\therefore$ Option (*c*) is correct.

(*iii*) Now, refer to design *B*, we have

Circumference of silver part = 44 mm

$2\pi r = 44$ (where r is the radius of inner circle)

$\Rightarrow$ $r = \frac{44}{2\pi}$

$\Rightarrow$ $r = \frac{44}{2 \times \frac{22}{7}} = 7$ mm

$\therefore$ Radius of outer part (R) = (7 + 3)

= 10 mm

Circumference of outer part $= 2\pi R$

$= 2\pi \times 10 = 20\,\pi$

$= 20 \times \frac{22}{7}$

$= \frac{440}{7} = 62.86$ mm

$\therefore$ Option (*d*) is correct.

(*iv*) Difference of areas of golden and silver part $= \pi R^2 - \pi r^2$

$= \pi\,[(10)^2 - (7)^2]$

$= \pi(100 - 49)$

$= 51\pi$ mm^2

$\therefore$ Option (*c*) is correct.

(*v*) Circumference of outer part(circular) $= 2\pi R$

$= 2\pi \times 10 = 20\pi$

$\therefore$ Number of revolution $= \frac{80\pi}{20\pi} = 4$.

$\therefore$ Option (*c*) is correct.

2. Gauri got her wall painted in a different manner. The whole wall was painted pink, leaving a circular portion of diameter 4.2 m. In this circle, she asked the painter to paint a beautiful scenery in one half of it by drawing a full size triangle possible (as shown in the figure). In the other half of the circle, she drew the largest circle possible and pasted some of her pictures. The remaining part of the big circle was filled with dotted design.

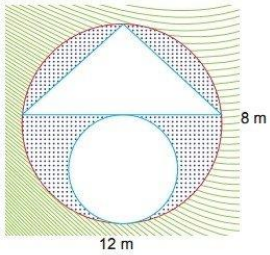


Fig. 11.57

Based on above information answer the following questions.

(*i*) (*a*) What is the radius of the circle allotted for her pictures?

(*b*) What is the area of the wall that is not painted pink?

(*ii*) What is the area of the dotted design?

Sol. (*i*) (*a*) Radius of the circle alloted for pictures $= \frac{2.1}{2} = 1.05$ m

(*b*) Area of the wall not painted pink $= 12 \times 8 - \pi(2.1)^2 = (96 - 4.41\pi)$ m^2

(*ii*) Area of dotted design $= \pi(2.1)^2$ – Area of triangle – Area of small circle

$= \pi(2.1)^2 - 4.41 - \pi\left(\frac{2.1}{2}\right)^2 = \pi\left[4.41 - \frac{4.41}{4}\right] - 4.41$

$= 4.41\left[\pi\left(1 - \frac{1}{4}\right) - 1\right] = 4.41\left(\frac{3}{4} \times \frac{22}{7} - 1\right)$

$= 4.41 \times \frac{19}{14} = 6$ m^2 (approx.)

PROFICIENCY EXERCISE

■ Objective Type Questions: [1 mark each]

1. Choose and write the correct option in each of the following questions.

(*i*) If the perimeter of a semicircular protractor is 36 cm then its diameter is

(*a*) 14 cm (*b*) 16 cm (*c*) 18 cm (*d*) 12 cm

(*ii*) In the figure given below, O is the centre of the circle. PR and RQ are chords of the circle. The radius of the circle is 5 cm. $PR = 8$ cm, $QR = 6$ cm and $\angle PRQ = 90°$.

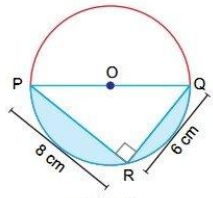


Fig. 11.58

What is the approximate area of the shaded region? **[*CBSE Question Bank*]**

(*a*) $\left(\frac{25}{4}\pi - 24\right)\text{cm}^2$ (*b*) $\left(\frac{25}{2}\pi - 24\right)\text{cm}^2$ (*c*) $\left(\frac{25}{4}\pi\right)\text{cm}^2$ (*d*) $\left(\frac{25}{2}\pi\right)\text{cm}^2$

(*iii*) In the figure below $RT = 1$ cm and $OQ = 3$ cm. **[*Competency Based Question*]**

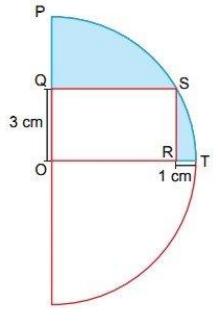


Fig. 11.59

What is the area of the shaded region? **[*CBSE Question Bank*]**

(*a*) $(12.5\pi - 12)\text{cm}^2$ (*b*) $(6.25\pi - 12)\text{cm}^2$ (*c*) $(12.5\pi - 15)\text{ cm}^2$ (*d*) $(6.25\pi - 15)\text{ cm}^2$

(*iv*) The area of a sector of a circle with radius 14 cm and central angle 45° is

(*a*) 76 cm^2 (*b*) 77 cm^2 (*c*) 66 cm^2 (*d*) 55 cm^2

(*v*) Which of these is equivalent to the sum of the lengths of arc corresponding to the minor and major segment of a circle of radius 12 cm?

(*a*) 24π cm (*b*) 48π cm (*c*) 12π cm (*d*) 144π cm

■ Very Short Answer Questions: [1 mark each]

2. What is the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm? **[*NCERT Exemplar*]**

3. The area of a circle is 220 cm^2. What will be the area of a square inscribed in it?
4. The circumference of a circle is 50 cm. What will be the side of a square that can be inscribed in the circle?
5. If the area of a circle increases from 9π to 16π, then what will be the ratio of the circumference of the first circle to the second circle?
6. A wire can be bent in the form of a circle of radius 35 cm. If it is bent in the form of a square, then what will be its area?

■ Short Answer Questions-I: [2 marks each]

7. What is the ratio of areas of two circles whose circumferences are in the ratio 3 : 4?
8. If the area of a sector of a circle is $\frac{5}{18}$ th of the area of that circle, then find the central angle of the sector.
9. If a circle is inscribed in a square, what is the ratio of the area of the circle and the square?
10. What is the length of an arc in terms of π that subtends an angle of 72° at the centre of a circle of radius 10 cm?
11. In a circle of radius 8 cm, an arc subtends an angle of 108° at the centre. What is the area of the sector in terms of π?
12. Find the perimeter of a square circumscribing a circle of radius a cm.
13. What is the angle subtended at the centre of a circle of radius 5 cm by an arc length 4π cm?
14. Find the area of a quadrant of a circle whose circumference is 616 cm.
15. Find the radius of a semicircular protractor if its perimeter is 36 cm.

■ Short Answer Questions-II: [3 marks each]

16. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 48 hours. **[*CBSE* 2018, (*C*) (30/1)]**
17. The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square. **[*CBSE* 2018, (*C*) (30/1)]**
18. Find the area of the shaded region in Fig. 11.60, if *ABCD* is a rectangle with sides 8 cm and 6 cm and *O* is the centre of circle. (Take $\pi = 3.14$) **[*CBSE* 2019, (30/1/1)]**

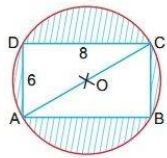


Fig. 11.60

19. Find the area of the segment shown in Fig. 11.61, if radius of the circle is 21 cm and $\angle AOB = 120°$ $\left(\text{Use } \pi = \frac{22}{7}\right)$ **[*CBSE* 2019, (30/1/2)]**

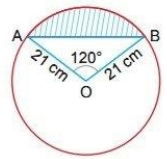


Fig. 11.61

20. In Fig. 11.62, *ABCD* is a square with side $2\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region. (Use $\pi = 3.14$) **[*CBSE* 2019, (30/2/1)]**

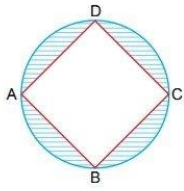


Fig. 11.62

21. A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$) **[*CBSE* 2019, (30/3/3)]**

22. In Fig. 11.63, two concentric circles with centre *O*, have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. **[*CBSE* 2019, (30/4/2)]**

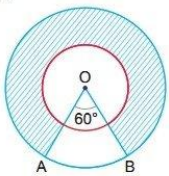


Fig. 11.63

23. In Fig. 11.64, find the area of the shaded region, where *ABCD* is a square of side 14 cm in which four semi-circles of same radii are drawn as shown. (Take $\pi = 3.14$) **[*CBSE* 2019, (*C*) (30/1/2)]**

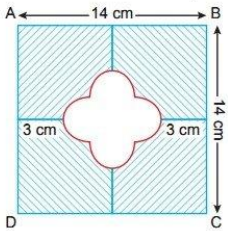


Fig. 11.64

24. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each vertex as centre, circles are described with radius equal to half the length of the side of the triangle. Find the area of the part of the triangle not included in the circles. (Take $\sqrt{3} = 1.73$, $\pi = \frac{22}{7}$)

25. In Fig. 11.65, the boundary of shaded region consists of four semicircular arcs, two smallest being equal. If diameter of the largest is 14 cm and that of the smallest is 3.5 cm, calculate the area of the shaded region. (Use $\pi = \frac{22}{7}$)

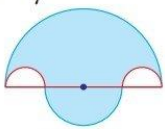


Fig. 11.65

26. The inner circumference of a circular track is 132 m. The track is 2.5 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹3.50 per metre.

27. A race track is in the form of a ring whose inner and outer circumferences are 44 cm and 66 cm respectively. Find the width of the track.

28. A circular park is surrounded by a road 28 m wide. Find the area of the road if the circumference of the park is 880 m.

29. Find the area of the flower bed (with semicircular ends.) in Fig. 11.66.

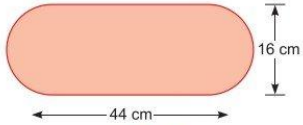


Fig. 11.66

30. Find the area of the shaded field shown in Fig. 11.67. ***[NCERT Exemplar]***

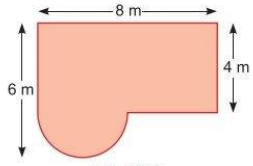


Fig. 11.67

31. In Fig. 11.68, arcs are drawn by taking vertices A, B, and C of an equilateral triangle of side 10 cm to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. (Use $\pi = 3.14$) ***[NCERT Exemplar]***

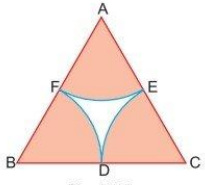


Fig. 11.68

32. Prove that the area of a circular path of uniform width h surrounding a circular region of radius is $\pi h\,(2r + h)$.

33. In Fig. 11.69, $ABCD$ is a rectangle with $AB = 14$ cm and $BC = 7$ cm. Taking DC, BC and AD as diameter, three semicircles are drawn. Find the area of the shaded region.

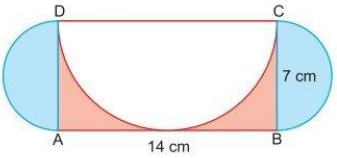


Fig. 11.69

34. In Fig. 11.70, three semicircles A, B and C are drawn having diameters 3 cm each and a circle D is drawn with diameter 4.5 cm. Calculate (*i*) the area of the shaded region. (*ii*) the cost of painting the shaded region at the rate of 25 paise per cm^2.

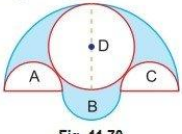


Fig. 11.70

■ Long Answer Questions: [5 marks each]

35. Three circles each of radius 7 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between the circles.

36. The area of a circular playground is 88704 m^2. Find the cost of fencing this ground at the rate of ₹65 per metre.

37. The diameter of front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions. ***[NCERT Exemplar]***

38. Find the area of the segment of a circle of radius 12 cm whose corresponding sector has a central angle of 60°. (Use $\pi = 3.14$)

39. Find the difference of the area of a sector of angle 90° and its corresponding major sector of a circle of radius 9.8 cm.

40. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre.

41. On a square cardborad sheet of area 784 cm^2, four congruent circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to two circular plates. Find the area of the square sheet not covered by the circular plates.

42. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 2464 cm^2. (Hint: radius of circle = $\frac{1}{2}$ diagonal)

43. Find the number of revolutions made by a circular wheel of area 6.16 m^2 in rolling a distance of 572 m.

44. With the vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 6 cm each in Fig. 11.71. If $AB = 20$ cm, $BC = 48$ cm and $CA = 52$ cm, then find the area of the shaded region. (Use $\pi = 3.14$)

Fig. 11.71

45. In Fig. 11.72, $PQRS$ is a square lawn with side $PQ = 42$ metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts). ***[CBSE (AI) 2015]***

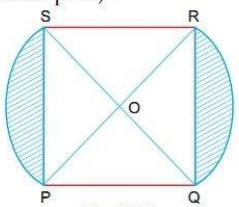


Fig. 11.72

Answers

1. (*i*) (*a*) (*ii*) (*b*) (*iii*) (*b*) (*iv*) (*b*) (*v*) (*a*)

2. 50 cm **3.** 140 cm^2 **4.** $\frac{25\sqrt{2}}{\pi}$ cm **5.** 3 : 4 **6.** 3025 cm^2 **7.** 9 : 16

8. 100° **9.** $\pi : 2$ **10.** 4π cm **11.** 19.2π cm^2 **12.** $8a$ cm **13.** 144°

14. 7546 cm^2 **15.** 7 cm **16.** 608π cm **17.** 25π cm^2 **18.** 30.5 cm^2

19. 271.3 cm^2 (Approx) **20.** 4.56 cm^2 **21.** 17.90 cm^2 (Approx) **22.** 3465 cm^2

23. 154.88 cm^2 **24.** 7.77 cm^2 **25.** 86.625 cm^2 **26.** ₹129.15

27. 3.5 cm **28.** 27104 m^2 **29.** $(704 + 64\pi)$ cm^2 or 905.14 cm^2 **30.** $(32 + 2\pi)m^2$

31. 39.25 cm^2 **33.** 59.5 cm^2 **34.** (*i*) 12.375 cm^2 (*ii*) ₹ 3.10

35. 7.86 cm^2 **36.** ₹ 68640 **37.** 560 **38.** 13.08 cm^2 **39.** 150.92 cm^2

40. $\left(\frac{25\pi}{4} + \frac{25}{2}\right)$ cm^2 **41.** 168 cm^2 **42.** 1568 cm^2 **43.** 65 **44.** 423.48 cm^2

45. 504 m^2

Self-Assessment

Time allowed: 1 hour **Max. marks: 40**

SECTION A

1. Choose and write the correct option in the following questions. **(3 × 1 = 3)**

(*i*) If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then **[*NCERT Exemplar*]**

(*a*) $R_1 + R_2 = R$ (*b*) $R_1 + R_2 > R$ (*c*) $R_1 + R_2 < R$ (*d*) None of these

(*ii*) The ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal is

(*a*) $\pi : \sqrt{2}$ (*b*) $\pi : \sqrt{3}$ (*c*) $\sqrt{3} : \pi$ (*d*) $\sqrt{2} : \pi$

(*iii*) The area of a quadrant of a circle whose circumference is 616 cm will be

(*a*) 7546 cm^2 (*b*) 7500 cm^2 (*c*) 7456 cm^2 (*d*) 7564 cm^2

2. Solve the following questions. **(2 × 1 = 2)**

(*i*) Find the area of a square inscribed in a circle of diameter p cm.

(*ii*) If the diameter of a semicircular protractor is 14 cm, then find its perimeter.

SECTION B

■ **Solve the following questions.** **(4 × 2 = 8)**

3. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

4. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹50 per m.

5. Difference between the circumference and radius of a circle is 37 cm. Find the area of circle.

6. Find the area of the shaded region in Fig. 11.73, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40°$.

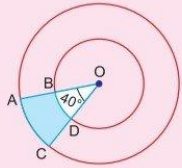


Fig. 11.73

■ **Solve the following questions.** **(4 × 3 = 12)**

7. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour which the boy is cycling.

8. In Fig. 11.74, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 20$ cm, find the area of the shaded region. (Use $\pi = 3.14$). **[*CBSE Delhi* 2014]**

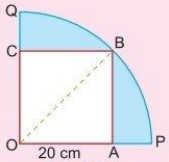


Fig. 11.74

9. Find the area of the shaded region in Fig. 11.75, where $ABCD$ is a square of side 14 cm each.

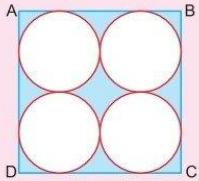


Fig. 11.75

10. A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14 m as shown in Fig. 11.76. Find the area of the remaining part of the park.

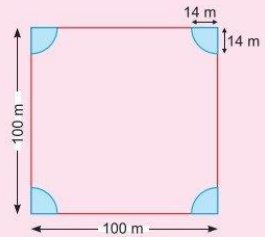


Fig. 11.76

■ **Solve the following questions.** **(3 × 5 = 15)**

11. In Fig. 11.77, a chord AB of a circle, with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $ALBQA$. (Use $\pi = 3.14$)

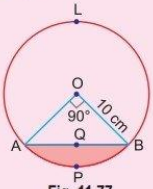


Fig. 11.77

12. A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0·7 m. Find the expenditure of cementing the path, if its cost is ₹110 per sq. m.

13. In Fig. 11.78, $ABCD$ is a trapezium with $AB \parallel DC$, $AB = 18$ cm, $DC = 32$ cm and distance between AB and DC is 14 cm. If arcs of equal radii 7 cm with centres A, B, C and D have been drawn, then find the area of the shaded region of the figure. ***[NCERT Exemplar]***

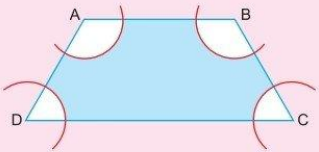


Fig. 11.78

Answers

1. (*i*) (*a*) (*ii*) (*b*) (*iii*) (*a*)

2. (*i*) $\frac{p^2}{2}$ cm^2 (*ii*) 36 cm

3. 3.92 cm **4.** ₹26400 **5.** 154 cm^2 **6.** 51.33 cm^2 **7.** 15.84 km/h

8. 228 cm^2 **9.** 42 cm^2 **10.** 9384 cm^2 **11.** 28.5 cm^2, 285.5 cm^2

12. ₹1863.40 **13.** 196 cm^2