Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 24

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. A quadratic polynomial whose zeros are $\frac{3}{5}$ and $\frac{-1}{2}$, is **[1]**

   a) $10x^2 - x + 3$ b) $10x^2 + x - 3$

   c) $10x^2 - x - 3$ d) $10x^2 + x + 3$

2. Given that 2x + 3y = 11, 2x - 4y = -24 and y = mx + 3, then the value of m is **[1]**

   a) 2 b) 0

   c) m= -1 d) 1

3. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and denominator. If 3 is added to both the numerator and denominator it becomes $\frac{5}{6}$, then the fraction is **[1]**

   a) $\frac{9}{7}$ b) $\frac{-9}{7}$

   c) $\frac{7}{9}$ d) $\frac{-7}{9}$

4. △ABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. If $\triangle DEF \sim \triangle ABC$ and EF = 4 cm, then perimeter of △DEF is **[1]**

   a) 30 cm b) 15 cm

   c) 22.5 cm d) 7.5 cm

5. An unbiased die is thrown once. The probability of getting a number between 2 and 6 is **[1]**

a) $\frac{1}{2}$
b) $\frac{2}{5}$
c) $\frac{1}{3}$
d) $\frac{2}{3}$

6. In triangle ABC, D, E and F are the midpoints of sides BC, CA and AB respectively and $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}, then$ **[1]**

a) $\Delta BCA \sim \Delta FDE$
b) $\Delta CBA \sim \Delta FDE$
c) $\Delta FDE \sim \Delta ABC$
d) $\Delta FDE \sim \Delta CAB$

7. The mean of the first 10 composite numbers is **[1]**

a) 11.2
b) 11.4
c) 112
d) 12.2

8. If $(\tan \theta + \cot \theta) = 5$ then $(\tan^2 \theta + \cot^2 \theta) = ?$ **[1]**

a) 23
b) 25
c) 24
d) 27

9. In $\triangle$ ABC, a line XY parallel to BC cuts AB at X and AC at Y. If BY bisects $\angle$XYC, then **[1]**

a) BC = CY
b) BC = BY
c) BC $\neq$ BY
d) BC $\neq$ CY

10. The product of two successive integral multiples of 5 is 1050. Then the numbers are **[1]**

a) 25 and 35
b) 25 and 30
c) 30 and 35
d) 35 and 40

11. If two positive integers ‘a’ and ‘b’ are written as $a = pq^2$ and $b = p^3q^2$, where ‘p’ and ‘q’ are prime numbers, then LCM(a, b) = **[1]**

a) pq
b) p^3q^2
c) p^2q^3
d) p^2q^2

12. If mode of a series exceeds its mean by 12, then mode exceeds the median by: **[1]**

a) 4
b) 8
c) 6
d) 10

13. The points A (-4, 0), B(4, 0) and C(0, 3) are the vertices of a **[1]**

a) isosceles triangle
b) scalene triangle

c) equilateral triangle

d) right triangle

14. If the altitude of the sun is 60°, the height of a tower which casts a shadow of length 90 m is **[1]**

a) 60 m

b) $90\sqrt{3}$ m

c) 90 m

d) $60\sqrt{3}$ m

15. If tan$\theta = \frac{m}{n}$, then $\frac{m \sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta}$= **[1]**

a) $\frac{m^2-n^2}{m^2+n^2}$

b) $\frac{m^2+n^2}{m^2-n^2}$

c) 1

d) $\frac{n^2-m^2}{n^2+m^2}$

16. The number of tangents that can be drawn from an external point to a circle is **[1]**

a) 1

b) 4

c) 2

d) 3

17. In a $\triangle$ ABC, $\angle A = 90^\circ$, AB = 5 cm and AC = 12 cm. Also $AD \perp BC$, Then AD = **[1]**

a) $\frac{2\sqrt{15}}{13}$cm

b) $\frac{60}{13}$cm

c) $\frac{13}{40}$cm

d) $\frac{13}{2}$cm

18. **Assertion (A):** Graph of a quadratic polynomial is always U shaped upward or downward.
Reason (R): Curve of any quadratic polynomial is always symmetric about the fixed-line. **[1]**

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. By a reduction of Re.1 per kg in the price of sugar, Radha can buy one kg sugar more for Rs.56. The original price of 1 kg of sugar is **[1]**

a) Rs.8

b) Rs.7

c) Rs.9

d) Rs.6

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2.
Reason (R): Total surface area of a cuboid is 2(lb + bh + hl) **[1]**

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

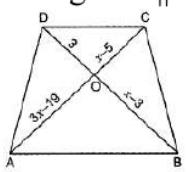
Section B

21. Find the ratio in which point P(-1, y) lying on the line segment joining points A(-3,10) and B(6, - 8) divides it. Also find the value of y. **[2]**

22. Find the roots of the quadratic equation : $2x^2 + x + 4 = 0$ by applying the quadratic formula: **[2]**

23. Find the largest four-digit number which when divided by 4, 7 and 13 leaves a remainder 3 in each case. **[2]**

24. In Fig. $AB \| DC$. Find the value of x. **[2]**



OR

If $\triangle ABC$ and $\triangle DEF$ are similar triangles such that AB = 3 cm, BC = 2 cm , CA = 2.5 cm and EF = 4 cm, calculate the perimeter of $\triangle DEF$

25. Prove that: $\left(cosec^2 \theta - 1\right) \tan^2 \theta = 1$ **[2]**

OR

Using the formula, cos A = $\sqrt{\frac{1+\cos 2A}{2}}$ find the value of cos 30°, it being given that $\cos 60° = \frac{1}{2}$

Section C

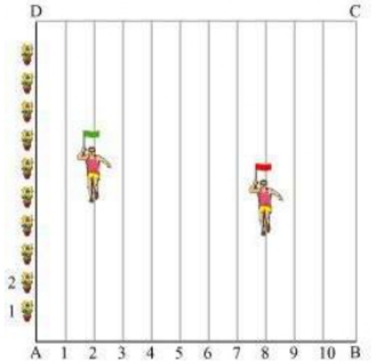
26. In a ΔABC, D and E are points on the sides AB and AC respectively such that $DE \| BC$. If AD = 8x - 7, DB = 5x - 3, AE = 4x - 3 and EC = (3x -1), find the value of x. **[3]**

27. Solve $(x - 3)(x - 4) = \frac{34}{(33)^2}$ **[3]**

28. Find the HCF of the following polynomials: **[3]**
$2\left(x^4 - y^4\right), 3\left(x^3 + 2x^2y - xy^2 - 2y^3\right)$

29. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. Niharika runs $\frac{1}{4}$th of the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th of the distance AD on the 8th line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she **[3]**

post her flag?



OR

Show that the points A(3, 5), B(6, 0), C(1, -3) and D(-2, 2) are the vertices of a square ABCD.

30. If the mean of the following frequency distribution is 62·8, then find the missing frequency x : **[3]**

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	8	x	12	7	8

31. The angle of elevation of the top of a tower at a point on the level ground is 30^{o}. After walking a distance of 100 m towards the foot of the tower along the horizontal line through the foot of the tower on the same level ground the angle of elevation to the top of the tower is 60^{o}, find the height of the tower. **[3]**

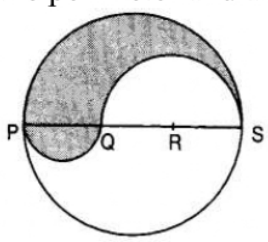
OR

From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45^{o} and 60^{o}. Find the distance between the objects.

Section D

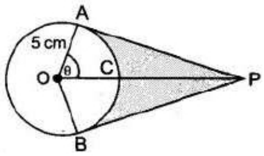
32. QR is the tangent to the circle whose centre is P. If QA || RP and AB is the diameter, prove that RB is a tangent to the circle. **[5]**

33. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region **[5]**



OR

An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.



34. Solve system of equations: **[5]**

$\frac{x}{7}+\frac{y}{3}=5$

$\frac{x}{2}-\frac{y}{9}=6$

OR

Form the pair of linear equations in the problem, and find its solution (if it exists) by the elimination method:
A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Mona paid Rs.27 for a book kept for seven days, while Tanvy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

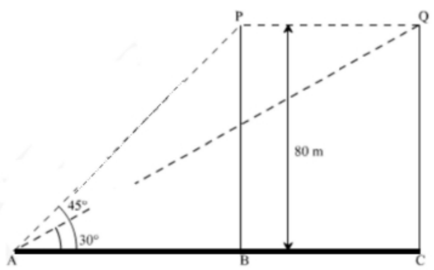
35. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is: **[5]**

i. of red colour
ii. a queen
iii. an ace
iv. a face card.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.



(i) Find the distance between observer and the bottom of the tree?

(ii) Find the speed of the bird?

(iii) Find the distance between second position of bird and observer?

OR

Find the distance between initial position of bird and observer?

37. **Read the text carefully and answer the questions:** **[4]**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) How many rows are there of rose plants?

(ii) Also, find the total number of rose plants in the garden.

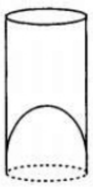
OR

If total number of plants are 80 in the garden, then find number of rows?

(iii) How many plants are there in 6th row.

38. **Read the text carefully and answer the questions:** **[4]**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.



(i) If the height of a glass was 10 cm, find the apparent capacity of the glass.

(ii) Also, find its actual capacity. (Use π = 3.14)

OR

How many glasses he serves if the container is full?

(iii) Find the capacity of the container in liter?

SOLUTION

Section A

1. **(c)** 10x^2 - x -3

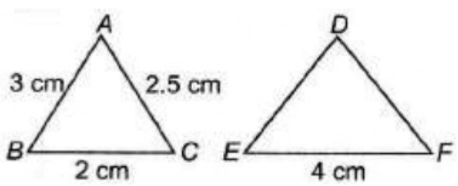
   Explanation: $\alpha+\beta=\left(\frac{3}{5}-\frac{1}{2}\right)=\frac{1}{10}, \alpha\beta=\frac{3}{5}\times\left(\frac{-1}{2}\right)=\frac{-3}{10}$

   Required polynomial is $x^2-\frac{1}{10}x-\frac{3}{10}$, i.e., 10x^2 - x - 3
2. **(c)** m= -1

   Explanation: Given: 2x + 3y = 11 ... (i)
   2x - 4y = -24 ... (ii)
   Subtracting eq. (ii) from eq. (i), we get
   7y = 35
   ⇒ y = 5
   Putting the value of y in eq. (i), we get
   2x + 3 × 5 = 11
   ⇒ 2x = -4
   ⇒ x = -2
   Now, y = mx + 3
   ⇒ 5 = m × (-2) + 3
   ⇒ 2m = 3 - 5
   m = -1
3. **(c)** $\frac{7}{9}$

   Explanation: Let the fraction be $\frac{x}{y}$.

   According to question
   $\frac{x+2}{y+2}=\frac{9}{11}$
   ⇒ 11x + 22 = 9y + 18
   ⇒ 11x - 9y = -4 ... (i)
   And $\frac{x+3}{y+3}=\frac{5}{6}$
   ⇒ 6x + 18 = 5y + 15
   ⇒ 6x - 5y = -3 ... (ii)
   On solving eq. (i) and eq. (ii), we get
   x = 7, y = 9
   Therefore, the fraction is $\frac{7}{9}$
4. **(b)** 15 cm

   Explanation: 

   $\triangle$DEF $\sim$ $\triangle$ABC
   AB = 3CM, BC = 2 CM, CA = 2.5 CM, EF = 4CM
   Since $\triangle'$s are similar, we have
   $\frac{DE}{AB}=\frac{EF}{BC}=\frac{FD}{CA}$

$\Rightarrow \frac{\mathrm{DE}}{3} = \frac{4}{2} = \frac{\mathrm{FD}}{2.5}$

Now $\frac{\mathrm{DE}}{3} = \frac{4}{2}$

$\Rightarrow \mathrm{DE} = \frac{3\times4}{2} = 6\mathrm{cm}$

and $\mathrm{FD} = \frac{4}{2} \Rightarrow \mathrm{FD} = \frac{4\times2.5}{2} = 5\mathrm{cm}$

perimeter of $\triangle DEF$

= 6 + 4 + 5 = 15cm

5. **(a)** $\frac{1}{2}$

Explanation: Number of numbers between 2 and 6 on a dice = {3, 4, 5}, = 3

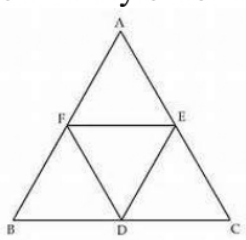
Number of possible outcomes = 3

Number of Total outcomes = 6

$\therefore$ Required Probability = $\frac{3}{6} = \frac{1}{2}$

6. **(d)** $\Delta FDE \sim \Delta CAB$

Explanation: Since $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}$, $then$ as sides are in proportion, then by SSS similarity criteria, $\Delta\mathrm{FDE} \sim \Delta\mathrm{CAB}$



Also, FEDC is a parallelogram, then $\angle F = \angle C$

And, AFDF is a parallelogram, then $\angle D = \angle A$

And, BDEF is a parallelogram, then $\angle E = \angle B$

Therefore, by AAA similarity criteria $\Delta FDE \sim \Delta CAB$

7. **(a)** 11.2

Explanation: The first 10 composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

$\therefore$ Mean = $\frac{\text{Sum of first 10 composite numbers}}{10}$

$= \frac{4+6+8+9+10+12+14+15+16+18}{10}$

$= \frac{112}{10}$

= 11.2

8. **(a)** 23

Explanation: Given, tan θ + cot θ = 5

Now squaring both sides,

$(\tan\theta + \cot\theta)^2 = 5^2$

$\Rightarrow \tan^2\theta + 2\tan\theta\cot\theta + \cot^2\theta = 25$

$\Rightarrow \tan^2\theta + 2\tan\theta\left(\frac{1}{\tan\theta}\right) + \cot^2\theta = 25$

$\Rightarrow \tan^2\theta + 2 + \cot^2\theta = 25$

$\Rightarrow \tan^2\theta + \cot^2\theta = 25 - 2$

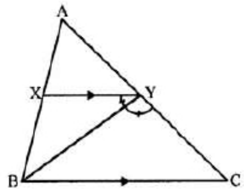
$\Rightarrow \tan^2\theta + \cot^2\theta = 23$

$\therefore (\tan^2\theta + \cot^2\theta) = 23$

9. **(a)** BC = CY

Explanation: In $\triangle$ ABC, XY || BC

Also BY is the bisector $\angle XYC$



$\angle XYB = \angle CYB$...... (i)

$XY \| BC$

$\angle XYB = \angle YBC$ (Alternate angles are equal).......(ii)

$\angle CYB = \angle YBC$

BC = CY

10. **(c)** 30 and 35

Explanation: Let one multiple of 5 be x then the next consecutive multiple will be (x + 5)

According to question,

x(x + 5) = 1050

$\Rightarrow x^2 + 5x - 1050 = 0$

$\Rightarrow x^2 + 35x - 30x - 1050 = 0$

$\Rightarrow$ x(x + 35) -30(x + 35) = 0

$\Rightarrow$ (x - 30)(x + 35) = 0

$\Rightarrow$ x - 30 = 0 and x + 35 = 0

$\Rightarrow$ x = 30 and x = -35

x = -35 is not possible therefore x = 30

Then the other multiple of 5 is

= x + 5

= 30 + 5 = 35

Then the number are 30 and 35

11. **(b)** p^3q^2

Explanation: We know that LCM = product of the highest powers of all the prime factors of the numbers pq^2, p^3q^2

LCM = p^3q^2

12. **(b)** 8

Explanation: Mode of a series = Its mean + 12

Mean = mode - 12

Also we know that

Mode = 3 median - 2 Mean

$\Rightarrow$ Mode = 3 median - 2(mode - 12)

$\Rightarrow$ Mode = 3 median - 2 mode + 24

$\Rightarrow$ Mode + 2 mode - 3 median = 24

$\Rightarrow$ 3 mode - 3 median = 24

$\Rightarrow$ 3(mode - median) = 24

$\Rightarrow$ Mode - median = $\frac{24}{3}$ = 8

13. **(a)** isosceles triangle

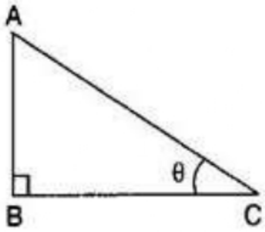
Explanation: $AB^2 = (4 + 4)^2 + (0 - 0)^2 = 8^2 + 0^2 = 64 + 0 = 64$

$\Rightarrow AB = \sqrt{64} = 8$ units

$BC^2 = (0-4)^2 + (3-0)^2 = (-4)^2 + 3^2 = 16 + 9 = 25$
$\Rightarrow BC = \sqrt{25} = 5$ units.
$AC^2 = (0+4)^2 + (3-0)^2 = 4^2 + 3^2 = 16 + 9 = 25$
$\Rightarrow AC = \sqrt{25} = 5$ units.
$\therefore \triangle ABC$ is isosceles.

14. **(b)** $90\sqrt{3}$ m

Explanation:



Let Height of the tower = AB = h meters,
Length of the shadow = BC = 90 m
And angle of elevation $\theta = 60^\circ$
$\therefore \tan 60^\circ = \frac{AB}{BC}$
$\Rightarrow \sqrt{3} = \frac{h}{90}$
$\Rightarrow h = 90\sqrt{3}$ meters

15. **(a)** $\frac{m^2-n^2}{m^2+n^2}$

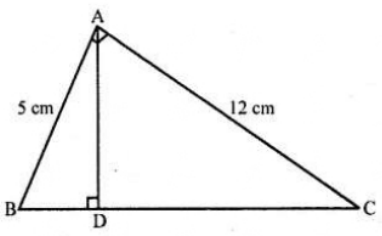
Explanation: Given: $\tan\theta = \frac{m}{n}$
Dividing all the terms of $\frac{m\sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta}$ by $\cos\theta$,
$= \frac{m\tan\theta - n}{m\tan\theta + n}$
$= \frac{m\times\frac{m}{n} - n}{m\times\frac{m}{n} + n}$
$= \frac{m^2-n^2}{m^2+n^2}$

16. **(c)** 2

Explanation: Number of tangents drawn from an external point to a circle is 2.

17. **(b)** $\frac{60}{13}$cm

Explanation:
In $\triangle ABC$ $\angle A = 90^\circ$, AB = 5 cm, AC = 12 cm



AD ⊥ BC

$BC^2 = AB^2 + AC^2$ (Pythagoras Theorem)
$=(4)^2 + (12)^2$
$= 25 + 144 = 169 = (13)^2$
$\therefore BC = 13$cm
Now area of $\Delta ABC = \frac{1}{2} AB \times AC$
$= \frac{1}{2} \times 5 \times 12 = 30\text{cm}^2$
and also area of of $\Delta ABC = \frac{1}{2} BC \times AD$

$\Rightarrow 30 = \frac{1}{2} \times 13 \times \mathrm{AD}$

$\Rightarrow \mathrm{AD} = \frac{30\times2}{13} = \frac{60}{13}\mathrm{cm}$

18. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Both A and R are true and R is the correct explanation of A.
19. **(a)** Rs.8
Explanation: Let the original price of 1 kg sugar = Rs. x
$\therefore$ In Re. 1, the weight of sugar can be bought = $\frac{1}{x}$ kg
$\therefore$ In Rs. 56, the weight of sugar can be bought = $\frac{56}{x}$ kg
New price = Rs. (x - 1)
$\therefore$ In Rs. 56, the weight of sugar can be bought = $\frac{56}{x-1}$ kg
According to question, $\frac{56}{x-1} - \frac{56}{x} = 1$
$\Rightarrow \frac{56x-56x+56}{x(x-1)} = 1$
$\Rightarrow \frac{56}{x^2-x} = 1$
$\Rightarrow x^2 - x - 56 = 0$
$\Rightarrow x^2 - 8x + 7x - 56 = 0$
$\Rightarrow$ x(x - 8) +7(x - 8) = 0
$\Rightarrow$ (x + 7)(x - 8) = 0
$\Rightarrow$ x + 7 = 0 and x - 8 = 0
$\Rightarrow$ x = -7 and x = 8 [x = -7 is not possible]
Therefore, the original price of 1 kg of sugar is Rs. 8
20. **(d)** A is false but R is true.
Explanation: A is false but R is true.

Section B

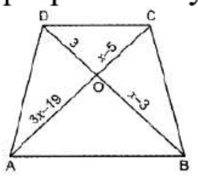
21. $\overset{K:1}{\overline{A(-3,10)\quad \underset{(-1,y)}{P} \quad B(6,-8)}}$

Let point P divides the line segment AB in the ratio K:1.
Applying section formula,
$(-1, y) = \left(\frac{6k-3}{k+1}, -\frac{8k+10}{k+1}\right)$
$\therefore \frac{6k-3}{k+1}$ = -1
6k-3= - k - 1
7k = 2
k = $\frac{2}{7}$
$\therefore$ Required Ratio is 2:7
Also, y = -$\frac{8k+10}{k+1}$
$= \frac{-8\left(\frac{2}{7}\right)+10}{\frac{2}{7}+1}$
$= \frac{-16+70}{2+7}$
$= \frac{54}{9}$
$\therefore$ y = 6
22. We have given that $2x^2 + x + 4 = 0$
Comparing it with standard form of quadratic equation,

$ax^2 + bx + c$
we get, a = 2, b = 1, c = 4
The roots are given as x $= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$= \frac{[-1\pm\sqrt{1-4(2)(4)}]}{2\times2} = \frac{-1\pm\sqrt{1-32}}{4} = \frac{-1\pm\sqrt{-31}}{4}$
This is not possible, Hence the roots do not exists.

23. LCM of (4,7,13) = 364
Largest 4 digit number = 9999
On dividing 9999 by 364 we get remainder as 171
Greatest number of 4 digits divisible by 4, 7 and 13 = (9999 – 171) = 9828
Hence, required number = (9828 + 3) = 9831
Therefore 9831 is the number.

24. We know that the diagonals of a trapezium divide each other proportionally.Therefore,we have,



$\frac{AO}{OC} = \frac{BO}{OD}$ ...(iii)
$\Rightarrow \frac{3x-19}{x-5} = \frac{x-3}{3}$
⇒ 3 (3x -19) = (x - 5)(x - 3)
⇒ 9x - 57 = x^2 - 8x + 15
⇒ x^2 - 17x + 72 = 0
⇒ (x - 8) (x - 9) = 0
⇒ x - 8 = 0 or, x - 9 = 0 ⇒ x = 8 or, x = 9

OR

Given: $\triangle ABC$ and $\triangle DEF$ are similar triangles such that AB = 3 cm, BC = 2 cm CA = 2.5 cm and EF = 4 cm
To find: Perimeter of $\triangle DEF$
We know that it two triangles are similar then their corresponding sides are proportional
Hence, $\frac{\mathrm{AB}}{\mathrm{DE}} = \frac{\mathrm{BC}}{\mathrm{EF}} = \frac{\mathrm{CA}}{\mathrm{FD}}$
Substituting the values, we get
$\frac{AB}{BC} = \frac{DE}{EF}$
$\frac{3}{2} = \frac{DE}{4}$
DE = 6 cm ...(i)
Similarly,
$\frac{CA}{BC} = \frac{DF}{EF}$
$\frac{2.5}{2} = \frac{DF}{4}$
DF = 5 cm ...(ii)
Perimeter of $\triangle DEF$ = DE + EF + DF
= 6 + 4 + 5
= 15 cm

25. W e have,

LHS = (cosec$^2\theta$ - 1) tan$^2\theta$

= (1 + cot$^2\theta$ - 1) tan$^2\theta$ [$\because$ cosec^2 θ = 1 + cot$^2\theta$]

$= \cot^2\theta \cdot \tan^2\theta$

$= \frac{1}{\tan^2\theta} \cdot \tan^2\theta \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$

= 1 = RHS

OR

Given: cos A = $\sqrt{\frac{1+\cos 2A}{2}}$, ...(1)

cos 60° = $\frac{1}{2}$

To find: cos 30°

By putting A = 30° in equation (1), we get the following:

cos 30° = $\sqrt{\frac{1+\cos 60^\circ}{2}}$

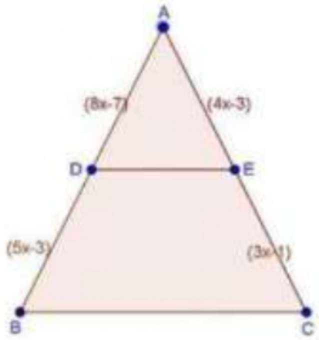
= $\sqrt{\frac{1+(1/2)}{2}}$

= $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\therefore$ cos 30° = $\frac{\sqrt{3}}{2}$

Section C

26. We have,



We are given that, DE || BC

Therefore, by thales theorem,

We have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

$\Rightarrow$ (8x - 7)(3x - 1) = (4x - 3)(5x - 3)

$\Rightarrow$ 24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9

$\Rightarrow$ 24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0

$\Rightarrow$ 4x^2 - 2x - 2 = 0

$\Rightarrow$ 2[2x^2 - x - 1] = 0

$\Rightarrow$ 2x^2 - x - 1 = 0

$\Rightarrow$ 2x^2 - 2x + 1x - 1 = 0

$\Rightarrow$ 2x(x - 1) + 1(x - 1) = 0

$\Rightarrow$ (2x + 1)(x - 1) = 0

$\Rightarrow 2x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -\frac{1}{2}$ or $x = 1$

$x = -\frac{1}{2}$ is not possible.

$\therefore x = 1$.

27. Given,

$(x-3)(x-4) = \frac{34}{33^2}$

$\Rightarrow x^2 - 4x - 3x + 12 = \frac{34}{33^2}$

$\Rightarrow \quad x^2 - 7x + 12 - \frac{34}{33^2} = 0$

$\Rightarrow \quad x^2 - 7x + \frac{13034}{33^2} = 0$

$\Rightarrow \quad x^2 - 7x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad x^2 - \frac{231}{33}x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad x^2 - \left(\frac{98}{33} + \frac{133}{33}\right)x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad x^2 - \frac{98}{33}x - \frac{133}{33}x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad \left(x^2 - \frac{98}{33}x\right) - \left(\frac{133}{33}x - \frac{98}{33} \times \frac{133}{33}\right) = 0$

$\Rightarrow \quad x\left(x - \frac{98}{33}\right) - \frac{133}{33}\left(x - \frac{98}{33}\right) = 0$

$\Rightarrow \quad \left(x - \frac{98}{33}\right)\left(x - \frac{133}{33}\right) = 0 \Rightarrow x = \frac{98}{33}$ or , $x = \frac{133}{33}$

28. Let $P(x) = 2\left(x^4 - y^4\right) = 2\left[\left(x^2\right)^2 - \left(y^2\right)^2\right]$

$= 2\left(x^2 + y^2\right)\left(x^2 - y^2\right)$

$= 2\left(\mathrm{x}^2 + \mathrm{y}^2\right)(\mathrm{x} + \mathrm{y})(\mathrm{x} - \mathrm{y})$ Using Identity $\mathrm{a}^2 - \mathrm{b}^2 = (\mathrm{a} + \mathrm{b})(\mathrm{a} - \mathrm{b})$

and $Q(x) = 3\left(x^3 + 2x^2y - xy^2 - 2y^3\right)$

$= 3\left[\mathrm{x}^2(\mathrm{x} + 2\mathrm{y}) - \mathrm{y}^2(\mathrm{x} + 2\mathrm{y})\right]$

$= 3(x + 2y)\left(x^2 - y^2\right)$

$= 3(x + 2y)(x + y)(x - y)$

$\therefore HCF = (x + y)(x - y) = x^2 - y^2$ Using identity $a^2 - b^2 = (a + b)(a - b)$

29. It can be observed that Niharika posted the green flag at $\frac{1}{4}$th of the distance AD i.e., $\frac{1}{4} \times 100 = 25m$ from the starting point of 2$^{\text{nd}}$ line. Therefore, the coordinates of this point G is (2, 25)

Similarly, Preet posted a red flag at $\frac{1}{5}$th of the distance AD i.e., $\frac{1}{5} \times 100 = 20m$ from the starting point of 8$^{\text{th}}$ line. Therefore, the coordinates of this point R are (8, 20)

Now we have the positions of posts by Preet and Niharika

According to distance formula, the distance between points A(x_1, y_1) and B(x_2, y_2) is given by

$D = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

Distance between these flags by using distance formula,

$D = \sqrt{(8-2)^2 + (25-20)^2}$

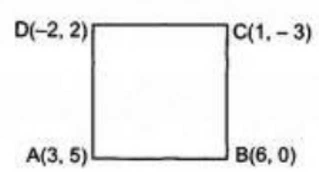
$= \sqrt{36 + 25}\mathrm{m}$

$= \sqrt{61}\,\mathrm{m}$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (X,Y)

Now by midpoint formula,
$(X, Y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$X = \left(\frac{2+8}{2}\right) = 5$
$Y = \left(\frac{25+20}{2}\right) = 22.5$
Hence, A (X,Y) = (5, 22.5)
Therefore, Rashmi should post her blue flag at 22.5m on the 5^{th} line.

OR



AB = $\sqrt{(6-3)^2+(0-5)^2}$
$= \sqrt{9+25} = \sqrt{34}$
BC = $\sqrt{(6-1)^2+(0+3)^2}$
$= \sqrt{25+9} = \sqrt{34}$
CD = $\sqrt{(1+2)^2+(-3-2)^2}$
$= \sqrt{9+25} = \sqrt{34}$
DA = $\sqrt{(-2-3)^2+(2-5)^2}$
$= \sqrt{25+9} = \sqrt{34}$
AC = $\sqrt{(1-3)^2+(-3-5)^2}$
$= \sqrt{4+64} = \sqrt{68}$
BD = $\sqrt{(6+2)^2+(0-2)^2}$
$= \sqrt{64+4} = \sqrt{68}$
AB = BC = CD = DA = $\sqrt{34}$
Diagonal AC = diagonal BD =$\sqrt{6}8$
Hence A, B, C and D are vertices of a square.

30. We have

Class interval	Frequency (f_i)	Class mark (x_i)	$f_i x_1$
0 - 20	5	10	50
20-40	8	30	240
40-60	x	50	50x
60-80	12	70	840
80-100	7	90	630
100-120	8	110	880
Total	$\Sigma f_i = 40 + x$		$\Sigma f_i x_i = 2640 + 50x$

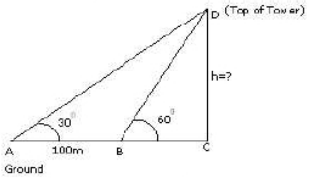
Here, $\Sigma f_i x_i = 2640 + 50x$, $\Sigma f_i = 40 + x$, $\bar{X} = 62.8$
$\therefore \quad \text{Mean}(\bar{X}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$
$\Rightarrow \quad 62.8 = \frac{2640+50x}{40+x}$
⇒ 2512 + 62.8x = 2640 + 50x

$\Rightarrow$ 62.8x - 50x = 2640 - 2512

$\Rightarrow$ 12.8x = 128

$\therefore \quad x = \frac{128}{12.8} = 10$

Hence, the missing frequency is 10

31. 

In $\triangle BCD$, $\frac{h}{x} = \tan 60^\circ = \sqrt{3}$ (BC=x)

$h = \sqrt{3x}$..............(i)

In $\triangle ACD$, $\frac{h}{100+x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow h\sqrt{3} = 100 + x$

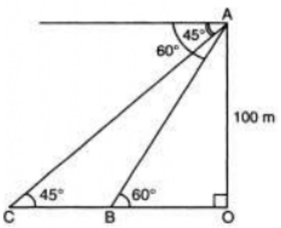
$\Rightarrow h\sqrt{3} = 100 + \frac{h}{\sqrt{3}}$

$\Rightarrow h\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 100$

$\Rightarrow h\left[\frac{3-1}{\sqrt{3}}\right] = 100$

$\Rightarrow h = \frac{100\sqrt{3}}{2} = 50\sqrt{3} = 50 \times 1.732 = 86.6\text{m}$

OR



In the given figure,

$\angle ACO = \angle CAX = 45^\circ$

and $\angle ABO = \angle XAB = 60^\circ$

Let A be a point and B, C be two objects.

In $\triangle$ AOC, $\frac{AO}{CO} = \tan 45^\circ$

$\Rightarrow \quad \frac{100}{\text{CO}} = 1$

$\Rightarrow CO = 100m$

Also in $\triangle$ ABO, $\frac{AO}{OB} = \tan 60^\circ$

$\Rightarrow \quad \frac{100}{OB} = \sqrt{3}$

$\Rightarrow \quad OB = \frac{100}{\sqrt{3}}$

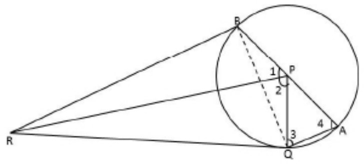
$\therefore \quad BC = CO - OB = 100 - \frac{100}{\sqrt{3}}$

$= 100\left(1 - \frac{1}{\sqrt{3}}\right)$ m

$100\frac{(\sqrt{3}-1)}{\sqrt{3}} = 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{100(3-\sqrt{3})}{3}$m

Section D

32. 

Given: A circle with centre P, AB is the diameter.

QA||RP, where RQ is the tangent to the circle.

To prove: RB is tangent to the circle i.e. ∠RBP = 90°.

Construction: Join BQ and PQ.

Proof: RQ is tangent to the circle and PQ is the radius at point of contact Q.

∴ PQ⊥RQ (radius of a circle is perpendicular to the tangent at point of contact)

⇒ ∠PQR = 90°....(1)

QA || RP and PQ is the transversal.

⇒ ∠2 = ∠3.....(2) (Alternate interior angles)

But, PQ = PA (radii of the circle)

∴ In ΔPQA, ∠3 = ∠4..... (3) (Angles opposite to equal sides are equal)

From (2) and (3) ⇒ ∠2 = ∠3 = ∠4

∠BPQ and ∠BAQ are the angles made by the arc BQ at the centre P and on the remaining part of the circle respectively.

∴ ∠BPQ = 2 ∠BAQ

i.e., ∠1 + ∠2 = 2∠4

⇒ ∠1 + ∠2 = ∠4 + ∠4

⇒ ∠1 = ∠4 (as ∠2 = ∠4)

So, ∠1 = ∠2 = ∠3 = ∠4

⇒ ∠1 = ∠2

In ΔBPR and ΔRPQ,

BP = PQ (radii of the circle)

∠1 = ∠2 (proved)

RP = RP (common)

∴ ΔBPR ≅ ΔQPR (SAS congruency)

⇒ ∠PBR = ∠PQR (corresponding angles)

But ∠PBR = 90° from equation (1)

i.e., PB ⊥ BR

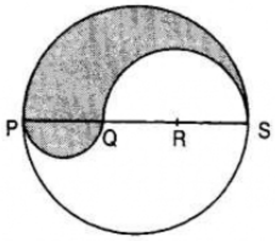
Therefore, RB is a tangent to the circle at point B.

33. PS = Diameter of a circle of radius 6 cm = 12 cm

∴ PQ = QR = RS = $\frac{12}{3}$ = 4cm, QS = QR + RS = (4 + 4)cm = 8 cm

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



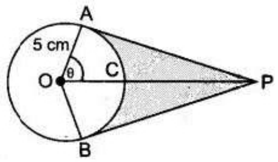
$\Rightarrow P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{cm} = 12\pi\text{cm}$

and, A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter - Area of semi-circle with QS as diameter.

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7}\left(6^2 + 2^2 - 4^2\right) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{cm}^2 = 37.71 \text{ cm}^2$

OR



$\cos\theta = \frac{1}{2}$ or, $\theta = 60^\circ$

Reflex $\angle AOB = 120^\circ$

$\therefore$ ADB = $\frac{2\times 3.14\times 5\times 240}{360}$ = 20.93 cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

a ($\triangle$OAP) = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

Area ($\triangle$OAP + $\triangle$OBP) = 2$\times$ $\frac{25\sqrt{3}}{2} = 25\sqrt{3}$ = 43.25 cm^2

Area of sector OACB = $\frac{\theta}{360} \times \pi r^2$

$= \frac{25\times 3.14\times 120}{360}$ = 26.16 cm^2

Shaded Area = 43.25 - 26.16 = 17.09 cm^2

34. The given system of equations is

$\frac{x}{7} + \frac{y}{3} = 5$............(i)

$\frac{x}{2} - \frac{y}{9} = 6$............(ii)

From (i), we get

3x + 7y = 5(21)

$\Rightarrow$ 3x + 7y = 105

$\Rightarrow$ 3x = 105 - 7y

$x = \frac{105-7y}{3}$ .... (iii)

From (ii), we get

$\frac{9x-2y}{18} = 6$

$\Rightarrow$ 9x - 2y = 18(6)

$\Rightarrow$ 9x - 2y = 108 ...(iv)

Substituting (iii) in (iv), we get

$9\left(\frac{105-7y}{3}\right) - 2y = 108$

$\Rightarrow \frac{945-63y}{3} - 2y = 108$

$\Rightarrow 945 - 63y - 6y = 108 \times 3$

$\Rightarrow 945 - 69y = 324$

$\Rightarrow 945 - 324 = 69y$

$\Rightarrow 69y = 621$

$\Rightarrow y = \frac{621}{69} = 9$

Putting y = 9 in (iii), we get

$x = \frac{105-7\times9}{3}$
$= \frac{105-63}{3}$
$\Rightarrow x = \frac{42}{3}$
$\therefore x = 14$
Hence, the solution of the given system of equations is x = 14, y = 9.

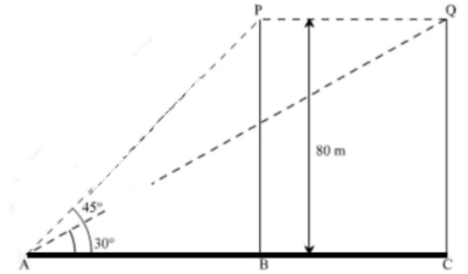
OR

Suppose the fixed charge be Rs. x and the extra charge per day be Rs y.
According to the question, Mona paid Rs 27 for a book kept for 7 days,
$\Rightarrow x + 4y = 27$........(i)
Tanvy paid Rs.21 for a book kept for 5 days,
$\Rightarrow x + 2y = 21$.........(ii)
Subtracting (ii) from (i),
$\Rightarrow 2y = 6$
$\Rightarrow y = 3$
Substituting $y = 3$ in (ii), we get $x = 15$
The fixed charge is Rs. 15 and the charge per day is Rs 3.

35. No. of cards removed= 3 face cards of heart +3 face cards of diamond=6
Remaining cards =52-6=46
So total No.of events are n=46
(i) No. of red card left = 13-6=7 so m =7
so P(E)=$\frac{7}{46}$
(ii) No. of queen left = 4 - 2 queens of heart and diamond = 2
So m = 2
P (E) = $\frac{2}{46} = \frac{1}{23}$
(iii) Total No. of aces = 4 so m=4
P(E)=$\frac{4}{46} = \frac{2}{23}$
(iv) No.of face cards left= 12 - total face cards removed =12 - 6 = 6
So m = 6
Hence P(E)=$\frac{6}{46} = \frac{3}{23}$

Section E

36. **Read the text carefully and answer the questions:**
A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.



(i) Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree

In $\triangle$ABP

$\tan 45^{\circ} = \frac{BP}{AB}$

$\Rightarrow 1 = \frac{80}{AB}$

$\Rightarrow$ AB = 80 m

(ii) The speed of the bird

In $\triangle$AQC

$\tan 30^{\circ} = \frac{QC}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow$ AC = $80\sqrt{3}$ m

AC - AB = BC

$\Rightarrow$ BC = $80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$m

Speed of bird = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3} - 1)$

$\Rightarrow$ Speed of the bird = 29.28 m/sec

(iii) The distance between second position of bird and observer.

In $\triangle$AQC

$\sin 30^{\circ} = \frac{QC}{AQ}$

$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$

$\Rightarrow$ AQ = 160 m

OR

The distance between initial position of bird and observer.

In $\triangle$ABP

$\sin 45^{\circ} = \frac{BP}{AP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{80}{AP}$

$\Rightarrow$ AP = $80\sqrt{2}m$

37. **Read the text carefully and answer the questions:**

Kamla and her husband were working in a factory in Seelampur, New Delhi. During the pandemic, they were asked to leave the job. As they have very limited resources to survive in a metro city, they decided to go back to their hometown in Himachal Pradesh. After a few months of struggle, they thought to grow roses in their fields and sell them to local vendors as roses have been always in demand. Their business started growing up and they hired many workers to manage their garden and do packaging of the flowers.

In their garden bed, there are 23 rose plants in the first row, 21 are in the 2^{nd}, 19 in 3^{rd} row and so on. There are 5 plants in the last row.

(i) The number of rose plants in the 1^{st}, 2^{nd}, ... are 23, 21, 19, ... 5

$a = 23$, $d = 21 - 23 = -2$, $a_n = 5$

$\therefore a_n = a + (n - 1)d$

or, $5 = 23 + (n - 1)(-2)$

or, $5 = 23 - 2n + 2$

or, $5 = 25 - 2n$

or, $2n = 20$

or, $n = 10$

(ii) Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$

$S_{10} = 5[46 - 20 + 2]$

$S_{10} = 5(46 - 18)$

$S_{10} = 5(28)$

$S_{10} = 140$

OR

$S_n = 80$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 80 = \frac{n}{2}[2 \times 23 + (n-1) \times -2]$

$\Rightarrow 80 = 23n - n^2 + n$

$\Rightarrow n^2 - 24n + 80 = 0$

$\Rightarrow (n - 4)(n - 20) = 0$

$\Rightarrow n = 4$ or $n = 20$

$n = 20$ not possible

$a_{20} = 23 + 19 \times (-2) = -15$

Number of plants cannot be negative.

$n = 4$

(iii) $a_n = a + (n - 1)d$

$\Rightarrow a_6 = 23 + 5 \times (-2)$

$\Rightarrow a_6 = 13$

38. **Read the text carefully and answer the questions:**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.

(i) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The apparent capacity of the glass = Volume of cylinder

$= \pi r^2 h$

$= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10$

$= 3.14 \times \frac{25}{4} \times 10 = 196.25 \text{ cm}^3$

(ii) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The actual capacity of glass = Apparent capacity of glass - Volume of hemispherical part of the glass

The volume of hemispherical part $= \frac{2}{3}\pi r^2 h = \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 = 32.71 \text{ cm}^3$

Actual capacity of glass = 196.25 - 32.71 = 163.54 cm^3

OR

We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

Number of glasses $= \frac{\text{Volume of container}}{\text{Actual volume of one glass}}$

$\Rightarrow$ Number of glasses $= \frac{20 \times 3.14 \times 20 \times 50}{3.14 \times \frac{2\pi}{4} \times 10 - \frac{2}{3} \times 3.14 \times \frac{125}{8}}$

$\Rightarrow$ Number of glasses $= \frac{20000}{\frac{250}{4} - \frac{125}{12}} = \frac{20000 \times 12}{750 - 125} = \frac{240000}{625} = 384$

$\Rightarrow$ Number of glasses = 384

(iii)We have, inner diameter of the glass, d = 5 cm, height of the glass = 10 cm

Volume of container = V = $\pi r^2 h$

$\Rightarrow$ V = 3.14 $\times$ 20 $\times$ 20 $\times$ 50 = 62800 cm^3

$\Rightarrow$ V = 62.8 litre