# **Chapter**Algebraic Expression

## **Learning Objectives**

- Introduction
- Identification of Terms of the Algebraic Expressions
- Operation on Algebraic Expressions

#### Algebraic Expressions TALENT & OLYMPIAD SUCCESS PACKAGE

#### Introduction

An equation with variable is known as an algebraic expression. The value of unknown variable is obtained by simplification of the expression.

All even numbers cannot be represented because there are infinite number of even numbers, therefore, it can be represented by an equation called algebraic expression.

Even numbers  $= 2 \times n$  whereas, n is a variable and the value of n may have 1, 2, 3, 4,..., etc.

#### Identification of Terms of the Algebraic expression

#### Literals or Variables

Alphabetical symbols are used in mathematics called variables or literals, *a*, *b*, *c*, *d*, *m*, *n*, *x*, *y*, *z* ....., etc. are some common letters used for variables.

#### Constant terms

The symbols which itself indicate a permanent value is called constant. All numbers are called constant.

 $6,10,\frac{10}{11},15,-6,\sqrt{3}$ ...... etc. are constant because, the value of the number does not change or remains unchanged. Therefore it is called constant.

#### Variable Terms

A term which contains various numerical values is called variable term. Product of 4 and X=4×X=4X Product of 2,X,Y<sup>2</sup> and  $Z = 2 \times X \times Y^2 \times Z = 2XY^2$ Product of -3, m and  $n = -3 \times m \times n = -3mn$ Thus,  $4X, 2XY^2Z - 3mn$ , are variable terms We also know that  $1 \times X = X, 1 \times Y \times z = YZ, -1 \times a^2 \times b \times c = -a^2 bc$  Thus X,YZ,-a<sup>2</sup> bc are variable terms

#### Types of Terms

There are two types of terms, like and unlike. Terms are classified by similarity of their variables.

#### Like Term

The terms having same variables are called like terms.

 $6X, X, -2X, \frac{4}{6}X,$ , are like terms, ab,-ab,4ab,9ab, ab, -ab,4ab,9ab, are like terms.

 $2X^2$ ,  $3X^2Y$ ,  $X^2Y$ ,  $\frac{10}{7}X^2Y$  are like terms.

#### Unlike Term

The terms having different variables are called unlike terms.  $6X, 2Y^2, -9X^2YZ, 4XY$ , are unlike terms.  $9a, -b, 3a^2, 4ab$ , are unlike terms.  $6X^2, 7ab, 4a^2b$ , are unlike terms.

#### Coefficient

The coefficient of every term is multiplied with the term. In term,  $-6m^2 np$ , coefficient of  $-6 = mn^2 p$  because  $mn^2 p$  is multiplied with -6 to form  $-6mn^2 p$  similarly. Coefficient of  $m^2 = -6np$ , coefficient of  $n = -6m^2 p$ Coefficient of  $m^2 n p = -6$  and Coefficient of  $-6 = m^2 np$ .

#### Variable or Literal Coefficient

The variable part of the term is called its variable or literal coefficient.

In term  $-\frac{5}{4}$  abc, variable coefficient is abc.

Constant Coefficient

The constant part of the term is called constant coefficient.

In term  $-\frac{5}{4}$  abc, constant coefficient is  $-\frac{5}{4}$ .

#### Polynomials

An expression having two or more terms is known as polynomials.

The expression 3+5x is a polynomial and degree of the polynomial is the highest power of variable which presents in the term. In the expression, 3+5x, x is the variable and its power is 1 therefore, the degree of the polynomial is 1.

 $5x^2 + 3y^3$  (It is a polynomial in x and y and its degree is 3)

 $5x^2 + 3y^{-3}$  (It is not a polynomial as exponent if y is negative integer)

#### Monomials

An expression which has one term is called monomials, ie.  $4y, 3b^2$ 

#### Binomials

An expression which has two terms is called binomials, ie.  $3b^2 - 4ac$ .

#### Trinomials

An expression which has three terms is called trinomials, ie.  $x^2 - ac + 3z$ 

#### Quadrinomials

An expression which has four terms is called Quadrinomials. ie.  $a^2 - bc + x - 5$ 

#### Operations on Algebraic Expressions

When constant and variables are linked with any of the following fundamental arithmetic operations, addition, subtraction, multiplication and division. The solution of the expression is obtained by simplification of the expression.



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Simplify, 2X^2-5X^2+6?

(a) 2X^2-5X^2 (b) 5X^2+6

(c) -3x^2+6 (d) All of these

(e) None of these

Answer: (c)

Explanation 2X^2-5X^2+6=-3X^2+6.
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#### Addition of Algebraic Expression

Addition is possible even if terms are like. The addition of two unlike terms is possible and their addition is in the same form. Addition of 2x+3x=5x but the addition of 2x+3y=2x+3y.

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Add the following polynomials,  $X^3 - 3X^2 - 6X + 10$  and  $4X^3 + 10X^2 + 15X - 20$ ?

(a)  $5X^3 + 7X^2 + 9X - 10$  (b)  $5X^2 + 6 + 45$ (c)  $5X^2 + 3X^2 + 6$  (d) All of these

(e) None of these

Answer: (a)

 $\begin{array}{r} \text{Explanation} & \frac{X^3 - 3X^2 - 6X + 10}{4X^3 + 10X^2 + 15X - 20} \\ \hline & \frac{4X^3 + 10X^2 + 15X - 20}{5X^3 + 7X^2 + 9X - 10} \end{array}$ 

#### **Alternative Method**

 $(X^{3} - 3X^{2} - 6X + 10 + (4X^{3} + 10X^{2} + 15X - 20) = X3 - 3X^{2} - 6X + 10 + 4X^{3} + 10X^{2} + 15X - 20)$ = X<sup>3</sup> + 4X<sup>3</sup> - 3X<sup>2</sup> + 10X<sup>2</sup> - 6X + 15X + 10 - 20 = 5X<sup>3</sup> + 7X<sup>2</sup> + 9X - 10

#### Subtraction of Algebraic Expression

Subtraction of two like terms is same as the subtraction of 2 mangoes from 4 mangoes. Number of mangoes are constant and the name, mangoes are like terms for both the numbers 2 and 4. The subtraction of 2 bananas from 4 mangoes is not possible.

#### Illustrative EXAMPLE

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Subtract:  $4X^2Y - 3XY + 5X$  from  $10X^26XY + 15X - 25$  ? (a)  $8x^3 + 2x^2 + 9x$  (b)  $6x^2y - 3xy + 10x - 25$ (c)  $5x - 3X^2 + 6$  (d) All of these (e) None of these

Answer: (b)

Explanation =  $(10X^2Y - 6XY + 15X - 25) - (4X^2Y - 3XY + 5X)$ = $10X^2Y - 6XY + 15X - 25 - 4X^2Y + 3XY - 5X$ = $10X^24X^2Y - 6XY + 3XY + 15X - 5X - 25$  $6X^2Y - 3XY + 10X - 25$ 

#### Multiplication of Algebraic Expression

The following steps are used to perform the multiplication of algebraic expression. I<sup>st</sup>: Write the sign of the resulting product according to the following rules,

$$\left(\frac{+x+=+,+x-=-}{-x-=+,-x+=-}\right)$$

II<sup>nd</sup> : Write the product of constant.

III<sup>rd</sup> : Write the product of variable according to the following rule,  $(a^m \times a^n = a^{m+n})$ 

#### Illustrative EXAMPLE

Multiply,  $(a^2 + ab + b^2)(a^2 - ab - b^2)$ (a)  $a^4a^2b^2 - 2ab^3 - b^4$  (b)  $6a^2b - 3ab + 10a - 25$ (c)  $5a - 3a^2 + 6$  (d) All of these (e) None of these

Answer: (a) Explanation  $(a^{2} + ab + b^{2})(a^{2} - ab - b^{2})$   $= a^{2}(a^{2} - ab - b^{2}) + ab(a^{2} - ab - b^{2}) + b^{2}(a^{2} - ab - b^{2})$   $= a^{4} - a^{3}b - a^{2}b^{2} + a^{3}b - a^{2}b^{2} - ab^{3} + a^{2}b^{2} - ab^{3} - b^{4}$   $= a^{4} - a^{2}b^{2} - ab^{3} - ab^{3} - b^{4}$  $= a^{4} - a^{4}b^{2} - 2ab^{3} - b^{4}$ 

#### Division of Algebraic Expression

The following steps are used to perform the division of the algebraic expression I<sup>st</sup>: First keep the polynomials which is to be divided in division form. II<sup>nd</sup>: Divide first term of dividend by 1st term of divisor and write quotient. III<sup>rd</sup>: Write the product of quotient x divisor, below the dividend and subtract it from dividend. IV <sup>th</sup>: Repeat this process until the degree of remainder becomes less than divisor

Example Divide:  $2x^2 + 3x + 1$  by (x+1)? (a) 3x+2 (b) 2x+1(c) 5x-3 (d) All of these (e) None of these Answer: (b) Explanation  $x+1\sqrt{\frac{2x+1}{2x^2+3x+1}}$  $\pm 2x^2 \pm 2x$ 

 $\frac{x+1}{\pm x\pm 1}$ 

0

Quotient of the division = 2x + 1 is the solution of the expression.

#### **Commonly Asked**

UESTIONS

Add the following expression, $6x^2 - 3by + 4cz$ , 7by - 8ax - 5cz and9cz2by + 2ax ?(a)  $x^2 - 2by + 8cz - 6ax$ (b)  $6x^2 + 2by + 8cz - 6ax$ (c)  $6x^2 + 2by + 8cz - 6ax$ (d) All of these(e) None of these

Answer: (c) Explanation  $(6x^2 - 3by + 4cz) + (7by - 8ax - 5cz) + (9cz - 2by + 2ax)$  $= 6x^2 - 3by + 4cz + 7by - 8ax - 5cz + 9cz - 2by + 2ax$  $= 6x^2 + 2by + 8cz - 6ax$ 



Subtract the following,  $x^3 - 3x^2 - 5x + 4$  from  $3x^3 - x^2 2x - 4$ ? (a)  $2x^3 + 2x^2 + 7x - 8$  (b)  $-2x^5 - 7x + 8$ (c)  $2x^3 - 2x^2 - 4x - 8$  (d) All of these (e) None of these

Answer: (a) Explanation  $3x^3 - 2 + 2x - 4 - (x^3 - 3x^2 - 5x + 4)$ =  $3x^3 - x^2 + 2x - 4 - x^3 + 3x^2 + 5x - 4$  $2x^3 + 2x^2 + 7x - 8$ 



A Find the product of (2x+4y)(3x-2y). (a)  $6x^2 + 8xy - 8y2$  (b)  $x^2 - 8xy - 8y^2$ (c)  $8x^2 - xy - 8y^2$  (d) All of these (e) None of these

Answer: (a)

**Explanation**  $(2x+4y)(3x-2y) = 6x^2 - 4xy + 12xy - 8y^2 = 6x^2 + 8xy - 8y^2$ 



Simplify the following,  $x^o - xy - 8y^o$ . (a) (xy+7) (b) (xy-7)(c) -(xy+7) (d) All of these (e) None of these

Answer: (c) Explanation  $x^{o} - xy - 8y^{o} = 1 - xy - 8 = -xy - 7 = -(xy + 7)$ 



#### Simplify the following, $6x^2 - 4x \div x$ . (a) 6x - 4 (b) 6x + 4(c) 4 + 6x (d) All of these (e) None of these

#### Answer: (a)

Explanation  $\frac{6x-4}{6x^2-4x}$   $\frac{76x^2}{-4x}$   $\frac{74x}{-4x} = (6x-4)$ 



Answer: (c) Explanation Numbers are the constant term.



The alternate name of variable of a term other than variable is?

- (a) Literal(c) Coefficient
- (b) Constant(d) All of these
- (e) None of these

Answer: (a) Explanation Variables are also known as literal.

If a term has one constant and one variable. They are linked to each other by which one of the following arithmetic operations if there is no sign of any operations between the term?

- (a) Multiplication
- (b) Addition
- (c) Subtraction
- (d) All of these
- (e) None of these

Answer: (a)

Explanation No sign between the term represent the product sign.



#### Consider the following two statements.

**Statement 1**: The division of two unlike terms, without constant can not be reduced to its lowest term. **Statement 2**: The product of two unlike terms is square of each of the term.

(a) Statement 1 and 2 are true

- (b) Statement 1 is true and 2 is false
- (c) Statement 1 is false and 2 is true
- (d) All of these
- (e) None of these

#### Answer: (b) Explanation

Let two unlike terms without constant term are x and y. The division of the terms  $=\frac{x}{y}$  and it cannot be

reduced.



#### Answer: (d)

**Explanation** The coefficient of 4 in the term 6abc + 4xyz is xyz.



- There can be various values for a variable in an expression.
- Constant has a fixed value.
- Variables are also known as literals.
- Elementary algebra deals with the properties of operations on real numbers.
- The geometrical study of algebra is known as the algebraic geometry.





- Numbers are the constant terms in the algebraic expressions.
- Letters are the variable terms in the algebraic expressions.
- In the algebraic expression variables are the coefficient of numericals and numericals are the coefficient of variables.
- The greatest power of all the terms in an algebraic expression is the degree of the expression.
- In the term, variables and constant are linked to each other by multiplication.

# Self Evaluation



1.	Find the expression which has degree 35?		
	(a) $x^{65} - 7x^{35} - 58$	(b) $44x^{30} + 7x^{35} - 0$	
	(c) $4x^{55} - 47x^{20} - 40$	(d) All of these	
	(e) None of these		
2.	From the options given below choose, which one of the following is not a polynomial?		
	(a) $4x^{65} - 7x^{35} = 58$	(b) $x + y = 60$	
	(c) $3x$	(d) All of these	
	(e) None of these		
3.	The remainder of the division, $2x^23x+1byx+1$ is?		
	(a) 0	(b) 1	
	(c) $X + 1$	(d) All of these	
	(e) None of these		
4.	The polynomial has degree three is called?		
	(a) Linear polynomial	(b) Quadratic polynomial	
	(c) Cubic polynomial (e) None of these	(d) All of these	
5.	If $4x^{84} + 7x^{42} + 4x^{70} - 7x^{35} = 200$ is divided by $x^{14} - 14$ . Which one of the following is the first term of the guotient?		
	(a) $-4x^{70}$	(b) $-4x^{60}$	
	(c) $4x^{70}$	(d) All of these	
	(e) None of these		
6	A quadratic polyopomial i	is divided by a linear polynomial, find the degree of the resulting quotient of the	

6. A quadratic polyonomial is divided by a linear polynomial, find the degree of the resulting quotient of the division.

- (a) 1
- (b) 2
- (c) 3
- (d) all of these
- (e) None of these

#### 7. The coefficient of $x^2y^6$ the term $-x^2y^6$ is ?

(a) 0

(b) -1

(c) 1 (d) All of these

(e) None of these

#### 8. Find the false statement from the options given below.

- (a) A polynomial contains constants and variables
- (b) A polynomial may have infinite number of terms
- (c) A monomial has only one term
- (d) All of these
- (e) None of these

9. The product of the Polynomials, 
$$(3u^2v - 5uv^2)$$
 and  $(\frac{1}{5}u^2 + \frac{1}{5}v^2)$  is :  
(a)  $\frac{3}{5}u^4v - u^3v^2 - uv^4 + \frac{3}{5}u^2v^3$  (b)  $\frac{u^4v}{5} - 2uv^2 + u^2v^3 - \frac{5uv^4}{3}$   
(c)  $\frac{3u^4v}{5}u^3v^2 + u^2v^3 - 5uv^4$  (d) All of these  
(e) None of these

### **10.** Two polynomials of degrees 8 and 4 have been multiplied with each other, find the degree of the resulting product of the polynomials?

(a) 32	(b) 4
(c) 12	(d) 16
(e) None of these	

**Answers – Self Evaluation Test** 1. В **2.** C 3. А 4. С 5. C 6. А 7. 8. В 9. А **10.** C В

## Self Evaluation



- **1.** The highest power of the polynomial is called degree of the polynomial. The highest power of the polynomial  $44x^{30} + 7x^{35} - 0$  is 35.
- **2.** The terms having two or more terms are called polynomials. 3x has one term therefore, this is not a polynomial.

5.  $x^{14} \times 4x^{70} = 4x^{(14+70)} = 4x^{84}$ 

- **6.** The degree of a quadratic polynomial is always 2. Hence, the degree of quotient of division of quadratic polynomial by a linear polynomial is always 1.
- **7.** The coefficient of  $x^2y^6$  in the term  $-x^2y^6$  is -1.
- **8.** A polynomial has finite number of terms. Therefore, option (B) is correct because this is a false statement. Hence, rest of the options is incorrect.

9. 
$$(3u^2v - 5uv^2)\left(\frac{1}{5}u^2 + \frac{1}{5}v^2\right) = \frac{3}{5}u^4v + \frac{3}{5}u^2v^3 - u^3v^2 - uv^4$$

10. The degrees of both the polynomials are 8 and 4.Hence, the degree of their product will be 8 + 4 = 12.