CHAPTER 13

Heights and Distances

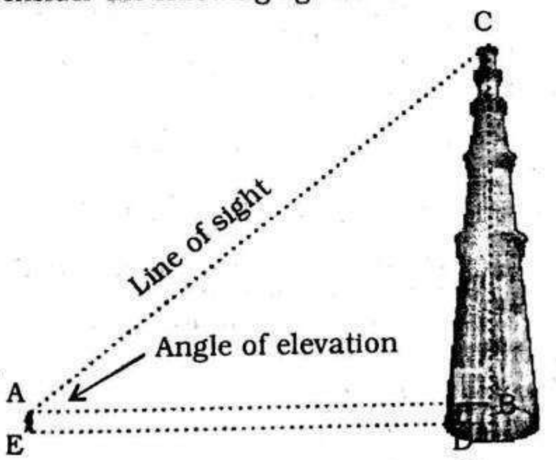
Trigonometry was invented because its need arose in astronomy. Since then the astronomers have used it, for instance, to calculate distances from the Earth to the planets and stars. Trigonometry is also used in geography and in navigation. The knowledge of trigonometry is used to construct maps, determine the position of an island in relation to the longitudes and latitudes.

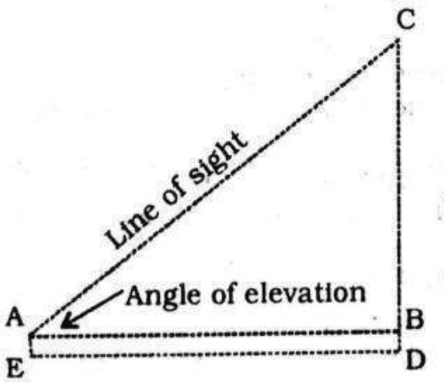
Surveyors have used trigonometry for centuries. One such large surveying project of the nineteenth century was the 'Great Trigonometric Survey' of British India for which the two largest-ever theodolites were built. During the survey in 1852, the highest mountain in the world was discovered. From a distance of over 160 km, the peak was observed from six different stations. In 1856, this peak was named after Sir George Everest, who had commissioned and first used the giant theodolites. The theodolites are now on display in the Museum of the Survey of India in Dehradun.

Trigonometry is also used for finding the heights and distances of various objects, without actually measuring them.

Heights and Distances

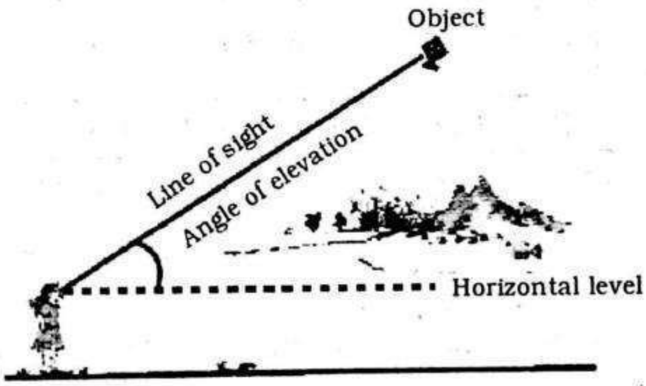
Consider the following figures :





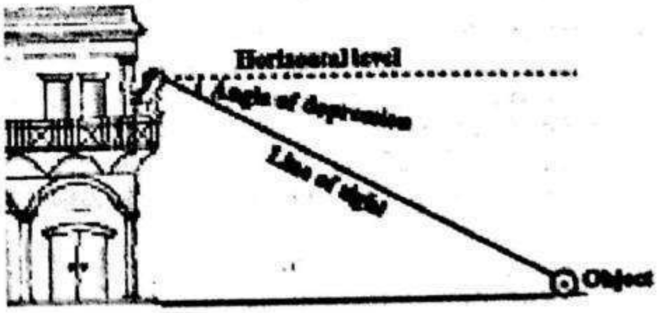
In this figure, the line AC drawn from the eye of the student to the top of the minar is called the line of sight. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the angle of elevation of the top of the minar from the eye of the student.

Thus, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer. The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object



Now, consider the situation given in the following figure. The girl sitting on the balcony is looking down at a flower pot placed on a stair of the temple. In this case, the line of sight is below the horizontal level. The angle so formed by the line of sight with the horizontal is called the **angle of depression**.

Thus, the **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed



If you want to find the height CD of the minar without actually measuring it, what information do you need? You would need to know the following :

(i) The distance DE at which the student is standing from the foot of the minar.

(ii) the angle of elevation, ∠BAC, of the top of the minar.

(iii) the height AE of the student.

Assuming that the above three conditions are known, how can we determine the height of the minar?

In the figure, CD = CB + BD. Here, BD = AE, which is the height of the student. To find BC, we will use trigonometric ratios of ∠BAC or ∠A.

In ΔABC, the side BC is the opposite side in relation to the known ∠A. Now, which of the trigonometric ratios can we use? Which one of them has the two values that we have and the one we need to determine? Our search narrows down to using either tan A or cot A, as these ratios involve AB and BC. Therefore, $\tan A = \frac{BC}{AB}$ or $\cot A = \frac{AB}{BC}$, which on solving would give us BC. By adding AE to BC, you will get the height of the minar.

SOLVED OBJECTIVE QUESTIONS

1. A person of height 2m wants to get a fruit which is on a pole of height $\left(\frac{10}{3}\right)$m. If he stands at a distance of $\left(\frac{4}{\sqrt{3}}\right)$ m from the foot of the pole, then the angle at which he should throw the stone, so that it hits the fruit is

(1) 60° (2) 45°
(3) 90° (4) 30°

2. A straight highway leads to the foot of a 50m tall tower. From the top of the tower the angles of depression of two cars on the highway are 30° and 60°. What is the distance between the two cars ?

(1) $\frac{100}{\sqrt{3}}$ m (2) $100\sqrt{3}$ m
(3) 86.50m (4) None of these

3. From the top of a pillar of height 20m the angles of elevation and depression of the top and bottom of another pillar are 30° and 45° respectively. The height of the second pillar (in metre) is :

(1) $\frac{20}{\sqrt{3}}\left(\sqrt{3}-1\right)$ m (2) $\frac{20}{\sqrt{3}}\left(\sqrt{3}+1\right)$ m
(3) $20\sqrt{3}$ m (4) $\frac{20}{\sqrt{3}}$ m

4. At a point P on the ground, the angle of elevation of the top of a 10m tall building and of a helicopter hovering some distance over the top of the building are 30° and 60° respectively. Then, the height of the helicopter above the ground is

(1) $\frac{10}{\sqrt{3}}$ m (2) $10\sqrt{3}$ m
(3) $\frac{20}{\sqrt{3}}$ m (4) 30 m

5. There is a small island in the middle of a 100 m wide river. There is a tall tree on the island. Points P and Q are points directly opposite to each other on the two banks and in line with the tree. If the angles of elevation of the top of the tree at P and Q are 30° and 45°, then the height of tree is:

(1) $50(\sqrt{3}-1)$ m (2) $50\sqrt{3}$ m
(3) $50(\sqrt{3}+1)$ m (4) $\frac{100}{\sqrt{3}-1}$ m

6. If the angles of a tower from two points distant a and b ($b > a$) from its foot and in the same straight line from it are 60° and 30°, then the height of the tower is:

(1) $\sqrt{a-b}$ (2) $\sqrt{b-a}$
(3) $\sqrt{ab}$ (4) $\sqrt{\frac{a}{b}}$

7. The angles of depression of two ships from the top of the light house are 45° and 30° towards east. If the ships are 100 m apart. then the height of the light house is :

(1) $50(\sqrt{3}-1)m$ (2) $50(\sqrt{3}+1)m$
(3) $50(\sqrt{3})m$ (4) $\frac{50}{\sqrt{3}-1}m$

8. A balloon of radius r makes an angle α at the eye of an observer and the angle of elevation of its centre is β. The height of its centre from the ground level is given by :

(1) $r \sin\beta \operatorname{cosec} \alpha/2$
(2) $r \operatorname{cosec} \alpha/2 \sin \alpha$
(3) $r \operatorname{cosec} \alpha \sin \beta$
(4) None of these

9. The angle of elevation 'θ' of the top of a light house at a point 'A' on the ground is such that $\tan\theta = \frac{5}{12}$. When the point is moved 240m towards the light house, the angle of elevation becomes ϕ such that $\tan\phi = \frac{3}{4}$. Then the height of light house is :

(1) 225 m (2) 200 m
(3) 215 m (4) 235 m

10. At the foot of a mountain the elevation of its summit is 45°; after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain:

(1) $\frac{\sqrt{3}+1}{2}$ km (2) $\frac{\sqrt{3}-1}{2}$ km

(3) $\frac{\sqrt{3}}{2}$ km (4) $\frac{1}{\sqrt{3}}$ km

11. The angles of elevation of the top of a building and the top of the chimney on the roof of the building from a point on the ground are x and 45° respectively. The height of building is h metre. Then the height of the chimney, in metre, is :

(1) $h\cot x + h$ (2) $h\cot x - h$
(3) $h\tan x - h$ (4) $h\tan x + h$

12. At the foot of a mountain the elevation of its summit is 45°. After ascending 1km towards the mountain upon an incline of 30°, the elevation changes to 60°. The height of the mountain is $\left(\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}\right)$:

(1) 1.455 km (2) 1.766 km
(3) 1.366 km (4) 2.366 km

13. At a distance a from the foot of a tower AB of height b, a flagstaff BC and the tower AB of subtend equal angles. Then the height of the flagstaff is :

(1) $\frac{b(a^2+b^2)}{a^2-b^2}$ (2) $\frac{b(a^2-b^2)}{a^2+b^2}$

(3) $\frac{b(a^2-2b^2)}{a^2-b^2}$ (4) $\frac{b(a+b)}{a-b}$

14. A tower on horizontal ground leans towards the north. At two points due south at distance a and b respectively from the foot, the angular elevations of the top of the tower are α and β. Find the inclination θ of the tower to the horizon.

(1) $\frac{b\cot\alpha + a\cot\beta}{a-b}$ (2) $\frac{b\sin\alpha + b\cos\beta}{b-a}$

(3) $\frac{b\cot\alpha - a\cot\beta}{b-a}$ (4) None of these

15. A pole, 15 m long rests against a vertical wall at an angle of 60° with the ground. How high up the wall will the pole reach ?

(1) 10.8 m (2) 12.9 m
(3) 11.9 m (4) 12 m

16. The shadow of a vertical tower AB on level ground is increased by 10m, when the altitude of the sun changes from 45° to 30°. The height of the tower is :

(1) 13.7 m (2) 14.7 m
(3) 12.7 m (4) 15.7 m

17. A boy is flying a kite with a string of length 100 m. If the string is taut and the angle of elevation of the kite is 30°, the height of the kite is :

(1) 40 m (2) 15 m
(3) 29 m (4) 50 m

18. From the top of a tower a guard saw a prisoner on the ground at an angle of depression of 60°. If the prisoner is 80 m away from the foot of the tower, the distance between the guard and the prisoner is :

(1) 120 m (2) 160 m
(3) 170 m (4) 150 m

19. A circus artist is climbing a 20 m long rope which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

(1) 11 m (2) 15 m
(3) 10 m (4) 18 m

20. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. The height of the tree is :

(1) $8\sqrt{3}$ m (2) $7\sqrt{3}$ m

(3) $6\sqrt{3}$ m (4) $4\sqrt{3}$ m

21. From a point on the ground the angles of elevation of the bottom and top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. The height of the tower is :

(1) 24.64 m (2) 10.64 m
(3) 9.64 m (4) 14.64 m

22. From the top of a building 30m high the angle of depression of a stone X on the side of the road was 60°. The angle of depression of another stone Y in line with the line joining the base of the building and X, is 45. The distance between the two stones is:

(1) $10(3-\sqrt{3})$ m

(2) $9(3+\sqrt{3})$ m

(3) $8(3-\sqrt{3})$ m

(4) $8(3+\sqrt{3})$ m

23. AB and CD are the outer walls of two buildings which are at a distance of AC from each other. If AB = 30m and the angles of depression of A,C from D, B are 60° and 30° respectively. Then the height of CD is:

(1) 80m (2) 90m
(3) 70m (4) 60m

24. A man stands at a point A on the bank of a river and looks at the top of a tree which is exactly opposite to him on the other bank. The angle of elevation is 45°. He then walks 200 m at right angles to the bank and away from it to the point B. From B he looks at the top of the tree and the angle of elevation as 30°. The height of the tree is:

(1) $10(\sqrt{3}+1)$m

(2) $100(\sqrt{3}-1)$m

(3) $88(\sqrt{3}+1)$m

(4) $100(\sqrt{3}+1)$m

25. From an aeroplane vertically above a straight road the angles of depressions of two consecutive km stones on the same side are 45° and 60°. The height of the aeroplane is :

(1) 2.366 km (2) 3.366 km
(3) 4.366 km (4) 1.366 km

26. From the top of a hill 200m, high the angles of depression of the top and the bottom of a pillar are 30° and 60° respectively. The height of the pillar is :

(1) $\frac{200}{3}$ m (2) $\frac{100}{3}$ m

(3) $\frac{400}{3}$ m (4) $\frac{250}{3}$ m

27. A vertical pole fixed to the ground is divided in the ratio 1 : 9 by a mark on it, the two parts subtend equal angles at a place on the ground, 15m from the base of the pole. If the lower part be shorter than the upper one, the height of the pole is:

(1) $40\sqrt{5}$ m (2) $60\sqrt{5}$ m

(3) $50\sqrt{5}$ m (4) $30\sqrt{5}$ m

28. A vertical pole on one side of a street subtends a right angle at a window exactly on the opposite side. If the angle of elevation of the window from the base of a pole be 60° and the width of the street be 30m. The height of the window is :

(1) $10\sqrt{3}$ m (2) $20\sqrt{3}$ m

(3) $40\sqrt{3}$ m (4) $30\sqrt{3}$ m

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

29. A boy standing in the middle of a field, observes a flying bird in the north at an angle of elevation of 30° and after 2 minutes, he observes the same bird in the south at an angle of elevation of 60°. If the bird flies all along in a straight line at a height of $50\sqrt{3}$ m, then its speed in km/h is :

(1) 4.5 (2) 3
(3) 9 (4) 6

[SSC Graduate Level Tier-I Exam, 2012]

30. A pole broken by the storm of wind and its top struck the ground at an angle of 30° and at a distance of 20 m from the foot of the pole. the height of the pole before it was broken was

(1) $20\sqrt{3}$ m (2) $\frac{40\sqrt{3}}{3}$ m

(3) $60\sqrt{3}$ m (4) $\frac{100\sqrt{3}}{3}$ m

[SSC Graduate Level Tier-I Exam, 2012]

31. The length of a shadow of a vertical tower is $\frac{1}{\sqrt{3}}$ times its height. The angle of elevation of the Sun is

(1) 30° (2) 45°
(3) 60° (4) 90°

[SSC Graduate Level Tier-II Exam, 2011]

32. When the angle of elevation of the sun increases from 30° to 60°, the shadow of a post is diminished by 5 metres. Then the height of the post is

(1) $\frac{5\sqrt{3}}{2}$ m (2) $\frac{2\sqrt{3}}{5}$ m

(3) $\frac{2}{5\sqrt{3}}$ m (4) $\frac{4}{5\sqrt{3}}$ m

[SSC Graduate Level Tier-I Exam, 2012]

33. A man is climbing a ladder which is inclined to the wall at an angle of 30°. If he ascends at a rate of 2 m/s, then he approaches the wall at the rate of

(1) 1.5 m/s (2) 1 m/s
(3) 2 m/s (4) 2.5 m/s

[SSC Graduate Level Tier-I Exam, 2012]

34. P and Q are two points observed from the top of a building $10\sqrt{3}$ m high. If the angles of depression of the points are complementary and PQ = 20 m, then the distance of P from the building is

(1) 25 m (2) 45 m
(3) 30 m (4) 40 m

[SSC Graduate Level Tier-I Exam, 2012]

35. A tree is broken by the wind. If the top of the tree struck the ground at an angle of 30° and at a distance of 30 m from the root, then the height of the tree is

(1) $25\sqrt{3}$ m (2) $30\sqrt{3}$ m

(3) $15\sqrt{3}$ m (4) $20\sqrt{3}$ m

[SSC Graduate Level Tier-I Exam, 2012]

36. From the top of a cliff 90 metre high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. The height of the tower is :

(1) 45 m (2) 60 m

(3) 75 m (4) 30 m

[SSC Graduate Level Tier-I Exam, 2012]

37. The angles of elevation of the top of a tower from two points at a distance x and y from the foot of the tower are complementary. The height of the tower is

(1) $\sqrt{\frac{x}{y}}$ (2) $\sqrt{x+y}$

(3) $\sqrt{xy}$ (4) $\frac{x}{y}$

[SSC CPO SI & Assistant Intelligence Officer Exam, 2012]

38. At a point on a horizontal line through the base of a monument, the angle of elevation of the top of the monument is found to be such that its tangent is $\frac{1}{5}$. On walking 138 metres towards the monument the secant of the angle of elevation is found to be $\frac{\sqrt{193}}{12}$. The height of the monument (in metre) is

(1) 35 (2) 49

(3) 42 (4) 56

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

39. The angles of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively 15° and 30°. If A and B are on the same side of the tower and AB = 48 metre, then the height of the tower is :

(1) $24\sqrt{3}$ metre (2) 24 metre

(3) $24\sqrt{2}$ metre (4) 96 metre

[SSC FCI Assistant Grade-III Exam, 2012]

40. The angles of elevation of the top of a building from the top and bottom of a tree are x and y respectively. If the height of the tree is h metre, then, in metre, the height of the building is

(1) $\frac{h\cot x}{\cot x+\cot y}$ (2) $\frac{h\cot y}{\cot x+\cot y}$

(3) $\frac{h\cot x}{\cot x-\cot y}$ (4) $\frac{h\cot y}{\cot x-\cot y}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

41. If the angle of elevation of the Sun changes from 30° to 45°, the length of the shadow of a pillar decreases by 20 metres. The height of the pillar is

(1) $20(\sqrt{3}-1)$ m (2) $20(\sqrt{3}+1)$ m

(3) $10(\sqrt{3}-1)$ m (4) $10(\sqrt{3}+1)$ m

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

42. There are two vertical posts, one on each side of a road, just opposite to each other. One post is 108 metre high. From the top of this post, the angles of depression of the top and foot of the other post are 30° and 60° respectively. The height of the other post, in metre, is

(1) 36 (2) 72

(3) 108 (4) 110

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

43. Two poles of equal heights are standing opposite to each other on either side of a road which is 100m wide. From a point between them on road, angles of elevation of their tops are 30° and 60°. The height of each pole in metre, is

(1) $25\sqrt{3}$ (2) $20\sqrt{3}$

(3) $28\sqrt{3}$ (4) $30\sqrt{3}$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

44. The angles of elevation of the top of a building and the top of the chimney on the roof of the building from a point on the ground are x and 45° respectively. The height of building is h metre. Then the height of the chimney, in metre, is:

(1) $h\cot x+h$ (2) $h\cot x-h$

(3) $h\tan x-h$ (4) $h\tan x+h$

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

45. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60°. The height of the tower is

(1) $\sqrt{3}$ m (2) $5\sqrt{3}$ m

(3) $10\sqrt{3}$ m (4) $20\sqrt{3}$ m

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

46. A telegraph post is bent at a point above the ground due to storm. Its top just meets the ground at a distance of $8\sqrt{3}$ metres from its foot and makes an angle of 30°, then the height of the post is :

(1) 16 metres (2) 23 metres
(3) 24 metres (4) 10 metres

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

47. A ladder is resting against a wall at a height of 10m. If the ladder is inclined with the ground at an angle of 30°, then the distance of the foot of the ladder from the wall is

(1) $\frac{10}{\sqrt{3}}$ m (2) $\frac{20}{\sqrt{3}}$ m
(3) $10\sqrt{3}$ m (4) $20\sqrt{3}$ m

[SSC (10+2) Higher Secondary Level Data Entry Operator and LDC Exam, 2012]

48. One flies a kite with a thread 150 metre long. If the thread of the kite makes an angle of 60° with the horizontal line, then the height of the kite from the ground (assuming the thread to be in a straight line) is

(1) 50 metre (2) $75\sqrt{3}$ metre
(3) $25\sqrt{3}$ metre (4) 80 metre

[SSC FCI Assistant Grade-III Exam, 2012]

49. The angle of elevation of a cloud at a height h above the level of water in a lake is α and the angle of the depression of its image in the lake is β. Then, the height of the cloud above the surface of the lake is

(1) $h \cot \beta$ (2) $h (\cot \alpha + \cot \beta)$
(3) $h \cot \alpha$ (4) $h \left(\frac{\cot \alpha + \cot \beta}{\cot \alpha - \cot \beta}\right)$

[SSC FCI Assistant Grade-III Exam, 2012]

50. The angle of elevation of an aeroplane from a point on the ground is 60°. After 15 seconds flight, the elevation changes to 30°. If the aeroplane is flying at a height of $1500\sqrt{3}$ m, find the speed of the plane.

(1) 300 m/sec (2) 200 m/sec
(3) 100 m/sec (4) 150 m/sec

[SSC Delhi Police S.I. Exam, 19.08.2012]

51. Two posts are x metres apart and the height of one is double that of the other. If from the mid-point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary, then the height (in metres) of the shorter post is

(1) $\frac{x}{2\sqrt{2}}$ (2) $\frac{x}{4}$
(3) $x\sqrt{2}$ (4) $\frac{x}{\sqrt{2}}$

52. An aeroplane when flying at a height of 5000m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. The vertical distance between the aeroplanes at that instant is

(1) $5000(\sqrt{3}-1)$ m (2) $5000(3-\sqrt{3})$ m
(3) $5000\left(1-\frac{1}{\sqrt{3}}\right)$ m (4) 4500 m

[SSC Graduate Level Tier-II Exam, 16.09.2012]

53. The shadow of a tower is $\sqrt{3}$ times its height. Then the angle of elevation of the top of the tower is

(1) 45° (2) 30°
(3) 60° (4) 90°

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (Ist Sitting)]

54. A man 6 ft tall casts a shadow 4 ft long at the same time when a flag pole casts a shadow 50 ft long. The height of the flag pole is

(1) 80 ft (2) 75 ft
(3) 60 ft (4) 70 ft

[SSC FCI Asstt. Grade-III Exam, 11.11.2012 (IInd Sitting)]

55. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30°. The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60°. If the height of the tower is 30 m, then the distance he moves is

(1) 22 m (2) $22\sqrt{3}$ m
(3) 20 m (4) $20\sqrt{3}$ m

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (Ist Sitting)]

56. The distance between two vertical poles is 60 m. The height of one of the poles is double the height of the other. The angles of elevation of the top of the poles from the middle point of the line segment joining their feet are complementary to each other. The heights of the poles are :

(1) 10 m and 20 m (2) 20 m and 40 m
(3) 20.9 m and 41.8 m (4) $15\sqrt{2}$ m and $30\sqrt{2}$ m

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

57. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. The length of the temple is :

(1) 18 m (2) 36 m

(3) $36\sqrt{3}$ m (4) $18\sqrt{3}$ m

[SSC (10+2) Level Data Entry Operator and LDC Exam, 21.10.2012 (IInd Sitting)]

58. The shadow of the tower becomes 60 metres longer when the altitude of the sun changes from 45° to 30°. Then the height of the tower is

(1) $20(\sqrt{3}+1)$ m (2) $24(\sqrt{3}+1)$ m

(3) $30(\sqrt{3}+1)$ m (4) $30(\sqrt{3}-1)$ m

59. An aeroplane when flying at a height of 3125m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. The distance between the two planes at that instant is

(1) 6520 m (2) 6000 m

(3) 5000 m (4) 6250 m

[SSC (10+2) Level Data Entry Operator and LDC Exam, 28.10.2012 (Ist Sitting)]

60. A vertical post 15 ft high is broken at a certain height and its upper part, not completely separated, meets the ground at an angle of 30°. Find the height at which the post is broken.

(1) 10ft (2) 5ft

(3) $15\sqrt{3}(2-\sqrt{3})$ ft (4) $5\sqrt{3}$ ft

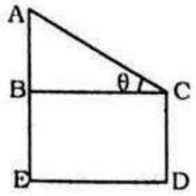
[SSC (10+2) Level Data Entry Operator and LDC Exam, 04.11.2012 (IInd Sitting)]

ANSWERS

1. (4)	**2.** (1)	**3.** (2)	**4.** (4)	**5.** (1)
6. (3)	**7.** (2)	**8.** (1)	**9.** (1)	**10.** (1)
11. (2)	**12.** (3)	**13.** (1)	**14.** (3)	**15.** (2)
16. (1)	**17.** (4)	**18.** (2)	**19.** (3)	**20.** (1)
21. (4)	**22.** (1)	**23.** (2)	**24.** (4)	**25.** (1)
26. (3)	**27.** (2)	**28.** (4)	**29.** (4)	**30.** (1)
31. (3)	**32.** (1)	**33.** (2)	**34.** (3)	**35.** (3)
36. (2)	**37.** (3)	**38.** (3)	**39.** (2)	**40.** (3)
41. (4)	**42.** (2)	**43.** (1)	**44.** (2)	**45.** (3)
46. (3)	**47.** (3)	**48.** (2)	**49.** (4)	**50.** (2)
51. (1)	**52.** (3)	**53.** (2)	**54.** (2)	**55.** (4)
56. (4)	**57.** (2)	**58.** (3)	**59.** (4)	**60.** (2)

EXPLANATIONS

1. (4)



BE = CD = height of man = 2m

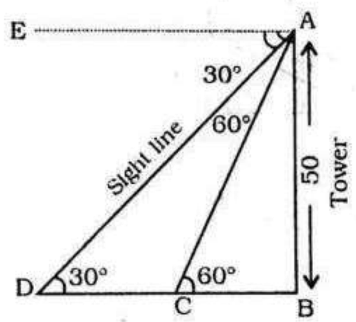
$$\tan\theta = \frac{AB}{BC} = \frac{\frac{10}{3}-2}{\frac{4}{\sqrt{3}}} = \frac{4}{3}\times\frac{\sqrt{3}}{4} = \frac{1}{\sqrt{3}}$$

$= \tan 30° \Rightarrow \theta = 30°$

2. (1) Here, $\theta_1 = 60°$, $\theta_2 = 30°$

$h = 50$m

DC = x

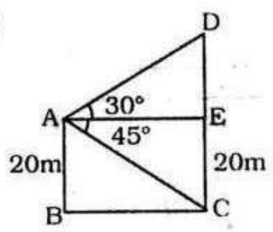


$$h = \frac{x}{\cot\theta_2 - \cot\theta_1}$$

$$x = h(\cot\theta_2 - \cot\theta_1)$$

$$= 50\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right) = \frac{50(2)}{\sqrt{3}} = \frac{100}{\sqrt{3}}\text{ m}$$

3. (2)



Let AB and CD are pillars.

Let DE = h

From ΔADE, tan 30°

$$= \frac{h}{AE} = \frac{1}{\sqrt{3}} \Rightarrow AE = h\sqrt{3} \quad .....(i)$$

From ΔACE,

$$\tan 45° = \frac{20}{AE} \Rightarrow AE = 20$$

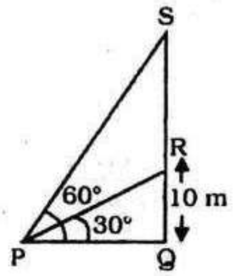
From equation (i),

$$20 = h\sqrt{3} \Rightarrow h = \frac{20}{\sqrt{3}} \text{ m}$$

$$\therefore \text{ Required height} = 20 + \frac{20}{\sqrt{3}} = \frac{20}{\sqrt{3}}(\sqrt{3}+1) \text{ m}$$

4. (4) Let RQ be the height of building, then RQ = 10m, S be the position of helicopter. Then
In Δ PQR,

$$\frac{RQ}{PQ} = \tan 30° \Rightarrow PQ = \frac{RQ}{\tan 30°} = 10\sqrt{3}$$

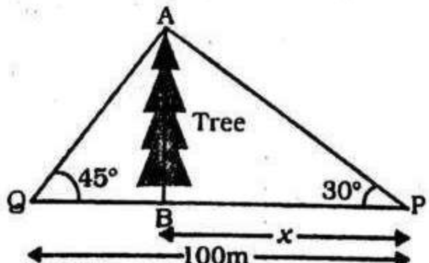


$$\therefore \text{ In } \Delta SPQ, \tan 60° = \frac{SQ}{PQ}$$

$$\Rightarrow \frac{SQ}{PQ} = \sqrt{3}$$

$$\Rightarrow SQ = PQ \times \sqrt{3} = 10\sqrt{3} \times \sqrt{3} = 30 \text{ m}$$

5. (1) Here height of tree = AB
In ΔAPB



$$\tan 30° = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{x}$$

or $x = \sqrt{3}$ AB ....(i)

In ΔAQB, $\tan 45° = \frac{AB}{BQ}$

$$\Rightarrow \frac{AB}{100-x} = 1$$

$\Rightarrow x = 100 - AB$ ....(ii)

So from (i) and (ii)

$$\sqrt{3}\ AB = 100 - AB \Rightarrow AB(\sqrt{3}+1) = 100$$

$$\Rightarrow AB = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 50(\sqrt{3}-1)$$

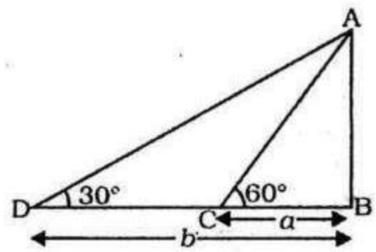
$\therefore$ Height of tree = $50(\sqrt{3}-1)$ metre.

6. (3) Let AB be the tower such that
CB = a and BD = b
In ΔABC

$$\tan 60° = \frac{AB}{BC} = \frac{AB}{a}$$

$\Rightarrow AB = a\sqrt{3}$ ...(i)

In ΔABD,



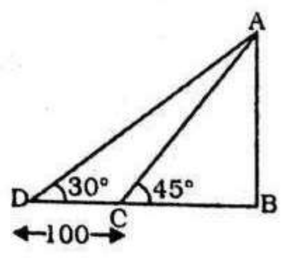
$$\tan 30° = \frac{AB}{BD}$$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{b}$ ....(ii)

From equations (i) and (ii)
$(AB)^2 = ab$

$AB = \sqrt{ab}$

7. (2) Here let AB be the height of light house and D, C are the ships such that CD = 100m.
Applying short cut method.



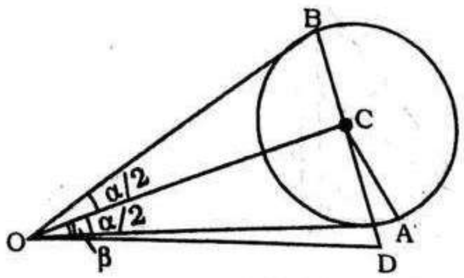
$$AB = \frac{100}{\cot 30° - \cot 45°}$$

$$\Rightarrow AB = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$AB = \frac{100(\sqrt{3}+1)}{2} = 50(\sqrt{3}+1)\text{m}$$

8. (1) Let O be the position of the man's eye and C be the centre of the balloon.

In ΔCOA



$\angle BOC = \angle COA = \frac{\alpha}{2}$

Here $CA = BC = r$

In right angled ΔCOD

$\sin\beta = \frac{CD}{OC}$

$\therefore CD = OC \sin\beta$ ....(i)

In right angled ΔCOA, we have

$\sin\beta = \frac{CA}{OC}$ $\therefore$

$OC = \frac{r}{\sin\frac{\alpha}{2}} = r\,\text{cosec}\frac{\alpha}{2}$

$\therefore$ So (i) become

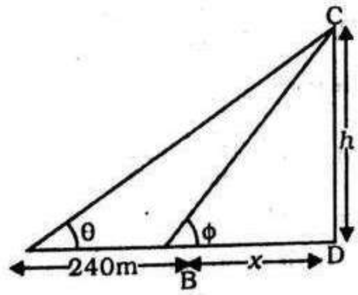
$CD = r\,\text{cosec}\frac{\alpha}{2}\sin\beta$

$\therefore$ Height of centre of the balloon is

$r\sin\beta\,\text{cosec}\frac{\alpha}{2}$

9. (1) Let CD be the height = h

In ΔADC $\tan\theta = \frac{h}{240+x}$



$\frac{5}{12} = \frac{h}{240+x}$ $\left\{\because \tan\theta = \frac{5}{12}\right\}$

$\therefore$ $12h = 5(240+x)$ ....(i)

In ΔBDC, we have

$\tan\phi = \frac{h}{x} \Rightarrow \frac{h}{x} = \frac{3}{4} \Rightarrow x = \frac{4}{3}h$

So (i) becomes $12h = 5\left(240 + \frac{4}{3}h\right)$

$\Rightarrow 12h = 1200 + \frac{20h}{3}$

$\Rightarrow 16h = 3600$

$\Rightarrow h = \frac{3600}{16} = 225\,m$

$\therefore$ Height of the light house = 225 m

10. (1) Suppose P be the summit of the mountain and Q be the foot. Here BN and BM are perpendiculars from B to PQ and AQ respectively.

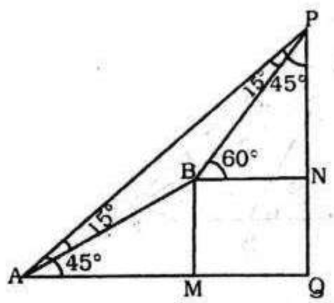
Here AB = 100 m = 1 km

$\angle MAB = 30^\circ$, $\angle MAP = 45^\circ$,

$\angle NBP = 60^\circ$, $\angle BAP = 15^\circ$

$\angle APB = 15^\circ$

$\therefore \Delta ABP$ is isosceles and AP = BP



But AB = 1km = PB

In ΔPBN

$PN = BP \sin 60^\circ$

In ΔABM

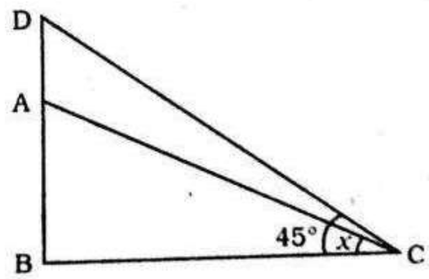
$BM = AB \sin 30^\circ$

$PQ = PN + NQ = PN + BM$

$= BP\sin 60^\circ + AB\sin 30^\circ = 1\frac{\sqrt{3}}{2} + 1.\frac{1}{2} = \frac{\sqrt{3}+1}{2}$

$\therefore$ Height of the mountain is $\frac{\sqrt{3}+1}{2}$ km.

11. (2)



AB = Buidling = h metre
AD = Chimney = y metre
From ΔBCD,

$\tan 45° = \frac{BD}{BC} \Rightarrow 1 = \frac{h+y}{BC}$

$\Rightarrow BC = h + y$ ... (i)

From ΔABC,

$\tan x = \frac{AB}{BC}$

$\Rightarrow \tan x = \frac{h}{BC}$

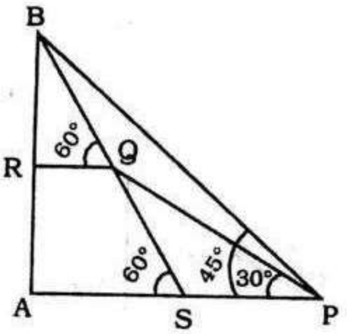
$\Rightarrow BC = h \cot x$ ...(ii)

From equations (i) and (ii),

$h + y = h \cot x$

$\Rightarrow y = (h \cot x - h)$ metre

12. (3) Let AB be the height of the mountain.



In ΔABP,
$\angle BPA = 45°$, $\angle PAB = 90°$
$\therefore \angle ABP = 45°$
In ΔBQR,
$\angle BQR = 60°$, $\angle BRQ = 90°$
$\therefore \angle QBR = 30°$
$\therefore \angle QBP = 15°$, and $\angle QPB = 15°$, $\angle PQB = 150°$

Now in ΔPQB, $\frac{PQ}{\sin 15°} = \frac{BP}{\sin 150°}$

$\Rightarrow \frac{1}{\sin 15°} = \frac{BP}{\frac{1}{2}}$

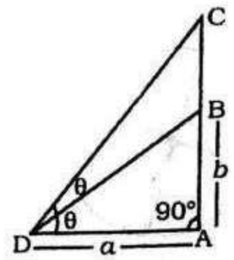
$\Rightarrow 2BP = \frac{2\sqrt{2}}{\sqrt{3}-1} \Rightarrow BP = \frac{\sqrt{2}}{\sqrt{3}-1}$

Now in right ΔAPB, $\sin 45° = \frac{AB}{PB}$

$\Rightarrow AB = \frac{PB}{\sqrt{2}} = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{\sqrt{3}+1}{3-1} = \frac{1.732+1}{2} = 1.366$ km

13. (1) Let AB be tower.
According to question AB = b, AD = a



Let $\angle ADB = \angle BDC = \theta$

In Δ ADB, $\tan\theta = \frac{AB}{AD} = \frac{b}{a}$

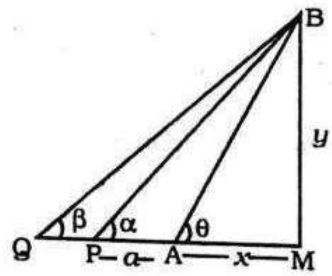
In Δ ADC, $\tan 2\theta = \frac{AC}{AD}$

$\therefore \frac{2\tan\theta}{1-\tan^2\theta} = \frac{AB+BC}{a}$

$\Rightarrow \frac{2b/a}{1-b^2/a^2} = \frac{b+BC}{a}$

$\Rightarrow BC = \frac{2a^2b}{a^2-b^2} - b = \frac{a^2b+b^3}{a^2-b^2} = \frac{b(a^2+b^2)}{a^2-b^2}$.

14. (3) $\angle BAM = \theta$, $\angle BPM = \alpha$,



$\angle BQM = \beta$,
Let BM = y, AM = x

In Δ BMP, $\cot\alpha = \frac{PM}{BM} = \frac{a+x}{y}$

$\Rightarrow a + x = y \cot\alpha$ ...(i)

In Δ BQM, $\cot\beta = \frac{QM}{BM} = \frac{b+x}{y}$

$\Rightarrow b + x = y \cot\beta$ ...(ii)

subtracting (i) from (ii)

$a - b = y(\cot\alpha - \cot\beta)$

$\therefore y = \frac{a-b}{\cot\alpha - \cot\beta}$

Now in equation (i),

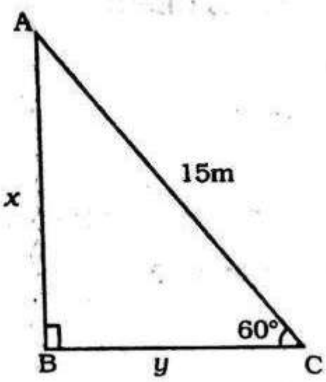
$x = y\cot\alpha - a = \frac{(a-b)\cot\alpha}{\cot\alpha - \cot\beta} = \frac{a\cot\beta - b\cot\alpha}{\cot\alpha - \cot\beta}$

In ΔABM, $\cot\theta = \frac{x}{y}$

$$= \frac{\frac{a\cot\beta - b\cot\alpha}{\cot\alpha - \cot\beta}}{\frac{a-b}{\cot\alpha - \cot\beta}}$$

$$= \frac{b\cot\alpha - a\cot\beta}{b-a}.$$

15. (2) Let AC represent the pole and AB the wall.



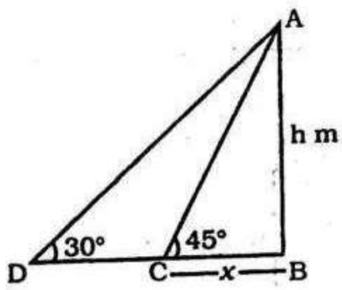
Let AB = x m

$\angle ACB = 60°$

In ΔABC, $\frac{x}{15} = \sin 60° = \frac{\sqrt{3}}{2}$

$\therefore x = 15 \times \frac{\sqrt{3}}{2}$ m $= \frac{15 \times 1.732}{2} \approx 12.9$ m.

16. (1) Let the height of the tower AB be hm and BC $= x$ m.



In ΔABC, $\tan 45° = \frac{h}{x}$

$\Rightarrow h = x$ ...(i)

In ΔADB, $\tan 30° = \frac{h}{x + 10}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h + 10}$

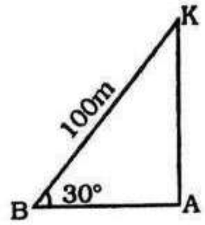
$\Rightarrow (\sqrt{3} - 1)h = 10$

$h = \frac{10}{\sqrt{3} - 1} = \frac{10(\sqrt{3} + 1)}{2}$

$= 5\ (1.732 + 2)$

$\therefore h = 13.7$ m.

17. (4) Let the kite be K. Let the boy be B.



Let $BA \perp AK$

Height of the kite = AK

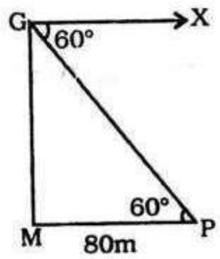
In ΔBAK, $\frac{AK}{BK} = \sin \angle KBA = \sin 30°$

$\Rightarrow \frac{AK}{100} = \frac{1}{2}$

$\therefore AK = 50$

Height of the kite = 50 m.

18. (2) Let the guard be G, P is prisoner and M the foot of the tower.



$\angle MPG = \angle PGX = 60°$

From ΔGMP, $\frac{MP}{GP} = \cos \angle MPG$

$\cos 60° \Rightarrow \frac{80}{GP} = \frac{1}{2}$

$\therefore GP = 160$

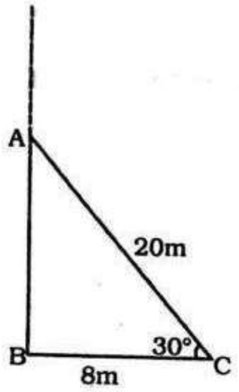
Hence, distance between the guard and the prisoner = 160 m.

19. (3) In right ΔABC, $\frac{AB}{AC} = \sin 30° = \frac{1}{2}$

$\Rightarrow AB = \frac{1}{2} \times 20$ m $= 10$ m.

Height of the pole is 10 m.

20. (1) In right ΔABC, $\frac{AB}{BC} = \tan 30° = \frac{1}{\sqrt{3}}$



$\therefore AB = \frac{8}{\sqrt{3}}$ ...(i)

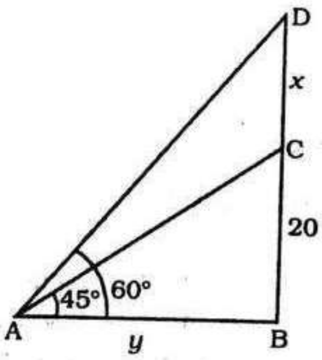
Again, $\frac{AC}{BC} = \sec 30° = \frac{2}{\sqrt{3}}$

$\Rightarrow AC = \frac{16}{\sqrt{3}}$ ...(ii)

Height of the tree = AB + AC

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = 8\sqrt{3}$ m.

21. (4) Let BC be the building of height 20 m and CD be the tower of height x m.



In ΔABC, we have

$\frac{BC}{AB} = \tan 45° = 1$

$\Rightarrow y = 20$ m.

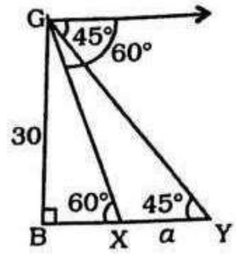
In ΔABD, $\frac{BD}{AB} = \tan 60°$

$\Rightarrow \frac{20 + x}{20} = \sqrt{3}$

$\Rightarrow 20 + x = 20\sqrt{3}$

$\therefore x = 20(\sqrt{3} - 1) = 14.64$ m.

22. (1) Let BG be the building, with B as the base.



Let X Y be a metre.

In ΔGBY, $\frac{GB}{BY} = \tan 45° \Rightarrow GB = BY$

$\therefore BX = (30 - a)$ m.

In ΔGBX, $\frac{BX}{BG} = \cot 60°$

$\Rightarrow \frac{30 - a}{30} = \frac{1}{\sqrt{3}} \Rightarrow a = \frac{30(\sqrt{3} - 1)}{\sqrt{3}} = 10(3 - \sqrt{3})$

Hence, the distance between X and Y $= 10(3 - \sqrt{3})$m.

23. (2) Let AC = x m.

In ΔABC, $\angle ACB = 30°$

$\Rightarrow \frac{AB}{AC} = \tan 30°$ $\quad \Rightarrow \frac{30}{x} = \frac{1}{\sqrt{3}}$

$\therefore x = 30\sqrt{3}$, so AC = $30\sqrt{3}$ m

In ΔADC, $\angle DAC = 60°$

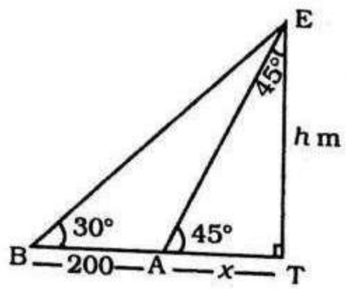
$\frac{DC}{AC} = \tan 60° \Rightarrow \frac{DC}{30\sqrt{3}} = \sqrt{3}$

$\therefore$ DC = 90 m.

24. (4) $\angle ATE = 90°$

$\therefore \angle AET = 45°$

Hence AT = TE, where TE represents the tree.



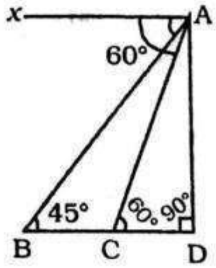
Let AT = TE = x m.

From ΔBTE, $\frac{ET}{BT} = \tan 30°$

$\Rightarrow \frac{x}{x + 200} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{200}{\sqrt{3} - 1} = \frac{200 \times \sqrt{3} + 1}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = 100(\sqrt{3} + 1)$

25. (1) $\angle XAB = 45° = \angle ABD$ and
$\angle XAC = 60° = \angle ACD$
Let AD = h and CD = x



In ΔABD, $\tan 45° = \frac{AD}{BD}$

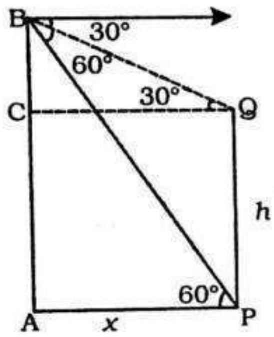
$\Rightarrow \frac{h}{BC+CD} = \frac{h}{1+x} \Rightarrow x = h-1$ ...(i)

In ΔACD, $\frac{AD}{CD} = \tan 60°$

$\Rightarrow \frac{h}{x} = \sqrt{3} \Rightarrow \frac{h}{h-1} = \sqrt{3}$ [From (i)]

$\Rightarrow h(\sqrt{3}-1) = \sqrt{3} = \frac{\sqrt{3}}{\sqrt{3}-1}$ = 2.366 km

26. (3) AB is a hill.



Let height of PQ = h and AP = x
AB = 200 m (given)
$\angle CQB = \angle QBX = 30°$
and $\angle APB = \angle PBX = 60°$

In ΔAPB, $\frac{200}{x} = \tan 60° = \sqrt{3}$

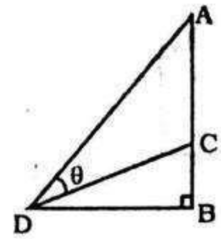
$\therefore x = \frac{200}{\sqrt{3}}$ m

In, ΔBCQ, $\frac{BC}{CQ} = \tan 30°$

$\Rightarrow BC = x \tan 30° = \frac{200}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{200}{3}$

$\therefore h = AB - BC = 200 - \frac{1}{3} \times 200 = \frac{400}{3}$ m.

27. (2) Let BC = x, then CA = $9x$



$\therefore$ AB = $10x$
According to question,
$\angle ADC = \angle CDB = \theta$ and BD = 15 m.

In ΔBDC, $\tan\theta = \frac{BC}{BD} = \frac{x}{15}$

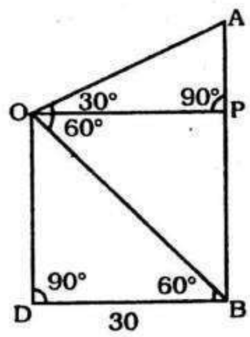
In ΔADB, $\tan 2\theta = \frac{AB}{BD} = \frac{10x}{15}$

or $\frac{2\tan\theta}{1-\tan^2\theta} = \frac{10x}{15} = \frac{2x}{3}$

$\Rightarrow \frac{\frac{2x}{15}}{\frac{1-x^2}{225}} = \frac{2x}{3}$ $\therefore x = 6\sqrt{5}$ m.

The height of the pole is $6\sqrt{5}$ m.

28. (4) Let AB be a pole and O be window.



Then $\angle AOB = 90°$, $\angle OBD = 60°$, BD=30 m.
$\therefore$ OP = DB = 30 m. and $\angle POB = \angle OBD = 60°$
$\therefore \angle AOP = \angle AOB - \angle POB = 90° - 60° = 30°$

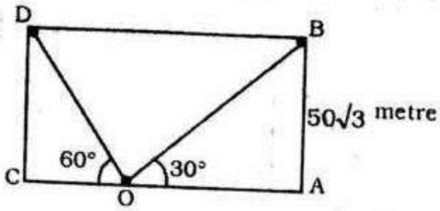
In ΔOBD, $\tan 60° = \frac{OD}{DB}$

$\therefore OD = 30\sqrt{3}$m

In ΔAOP, $\tan 30° = \frac{AP}{OP}$

$\therefore AP = 10\sqrt{3}$m
The height of pillar
$= AB = AP + PB = 10\sqrt{3}$m $+ OD = 10\sqrt{3} + 30\sqrt{3}$
$= 40\sqrt{3}$ m
$\therefore$ The height of the window = $30\sqrt{3}$ m.

29. (4)



$AB = CD = 50\sqrt{3}$ metre

From Δ OAB,

$\tan 30° = \frac{AB}{OA}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{OA}$

$\Rightarrow OA = 50\sqrt{3} \times \sqrt{3} = 150$ metre

From Δ OCD,

$\tan 60° = \frac{CD}{OC}$

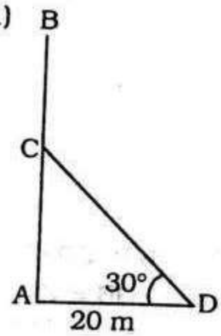
$\sqrt{3} = \frac{50\sqrt{3}}{OC} \Rightarrow OC = 50$ metre

$\therefore BD = AC = 150 + 50 = 200$ metre

$\therefore$ Speed of bird $= \frac{200}{2} = 100$ m/minute

$= \frac{100}{1000} \times 60$ kmph = 6 kmph

30. (1)



AB = Pole

BC = CD = broken part of pole

AD = 20 metre

In Δ ACD,

$\tan 30° = \frac{AC}{AD}$

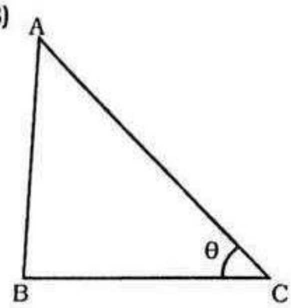
$\Rightarrow AC = AD. \tan 30° = \frac{20}{\sqrt{3}}$ metre

$\cos 30° = \frac{AD}{CD}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{20}{CD} \Rightarrow CD = \frac{40}{\sqrt{3}}$ metre

$\therefore AB = AC + CD = \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ metre

31. (3)



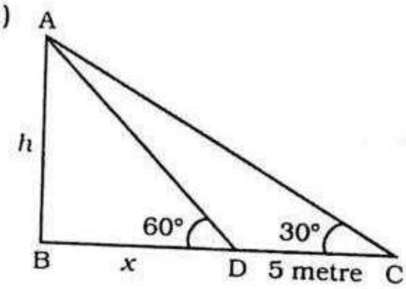
Let AB be tower and BC be its shadow.

If $AB = x$, then $BC = \frac{x}{\sqrt{3}}$

$\therefore \tan\theta = \frac{AB}{BC} = \frac{x}{\frac{x}{\sqrt{3}}} = \sqrt{3}$

$\therefore \tan\theta = \tan 60°$ $\Rightarrow \theta = 60°$

32. (1)



AB = Pole = h metre

BD = x metre

From Δ ABC,

$\tan 30° = \frac{h}{x+5}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+5}$

$\Rightarrow x + 5 = \sqrt{3}h$ ...(i)

From ΔABD,

$\tan 60° = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$ $\Rightarrow x = \frac{h}{\sqrt{3}}$

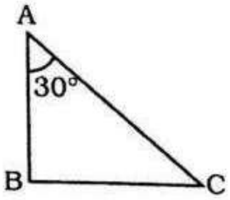
$\therefore x + 5 = \sqrt{3}h$ $\Rightarrow \frac{h}{\sqrt{3}} + 5 = \sqrt{3}h$

$\Rightarrow h + 5\sqrt{3} = 3h$

$\Rightarrow 2h = 5\sqrt{3}$

$\Rightarrow h = \frac{5\sqrt{3}}{2}$ metre

33. (2)



$\angle BAC = 30°$

$BC = x$; $AC = y$

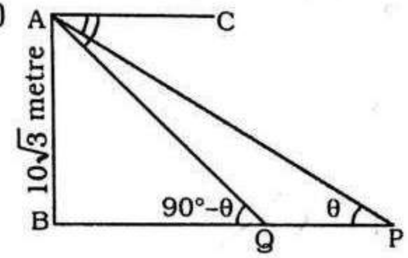
$\therefore \sin 30° = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{x}{y}$

$\Rightarrow y = 2x$

After 1 second

Required speed $= \frac{1}{2}y = 2 \times \frac{1}{2} = 1$ m/sec

34. (3)



AB = Building $= 10\sqrt{3}$ metre

PQ = 20 metre

BQ = x metre (let)

If $\angle APB = \theta$ then

$\angle AQB = 90° - \theta$

From ΔABP,

$\tan \theta = \frac{AB}{BP}$

$= \frac{10\sqrt{3}}{x + 20}$ ....(i)

From ΔABQ,

$\tan (90° - \theta) = \frac{AB}{BQ}$

$\Rightarrow \cot \theta = \frac{10\sqrt{3}}{x}$ ....(ii)

By multiplying both equations,

$\tan\theta . \cot\theta = \frac{10\sqrt{3}}{x+20} \times \frac{10\sqrt{3}}{x}$

$\Rightarrow x^2 + 20x = 10 \times 10 \times 3$

$\Rightarrow x^2 + 20x - 300 = 0$

$\Rightarrow x^2 + 30x - 10x - 300 = 0$

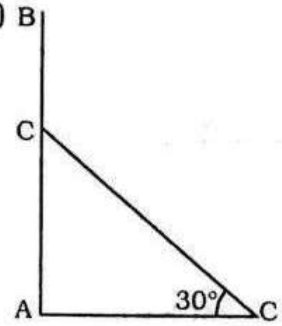
$\Rightarrow x(x + 30) - 10(x + 30) = 0$

$\Rightarrow (x - 10)(x + 30) = 0$

$\Rightarrow x = 10 \qquad x \neq -30$

$\therefore$ BP = 10 + 20 = 30 metre

35. (3)



AB = tree

BC = broken part

$\therefore$ BC = CD

AD = 30 metre

From ΔACD, $\tan 30° = \frac{AC}{AD}$

$\Rightarrow AC = AD \times \frac{1}{\sqrt{3}} = \frac{30}{3} = 10\sqrt{3}$ metre

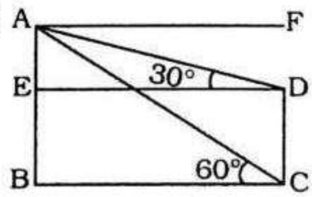
$CD = AC \sin 30°$

$= 10\sqrt{3} \times \frac{1}{2} = 5\sqrt{3} = BC$

$\therefore AB = AC + BC$

$= 10\sqrt{3} + 5\sqrt{3} = 15\sqrt{3}$ metre

36. (2)



AB = cliff = 90 metre

$\angle ADE = 30°$

$\angle ACB = 60°$

CD = tower = h metre

BC = x metre

From Δ ABC,

$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{90}{x}$

$\Rightarrow x = \frac{90}{\sqrt{3}} = 30\sqrt{3}$ metre

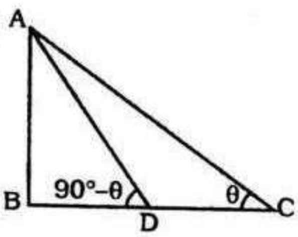
From Δ ADE,

$\tan 30° = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{90-h}{30\sqrt{3}}$

$\therefore 90 - h = 30$

$\Rightarrow h = 90 - 30 = 60$ metre

37. (3)



BD = x metre
BC = y metre
AB = h metre
$\angle ADB = 90° - \theta$ and $\angle ACB = \theta$
From ΔABC,

$\tan\theta = \frac{h}{y}$

From ΔABD,

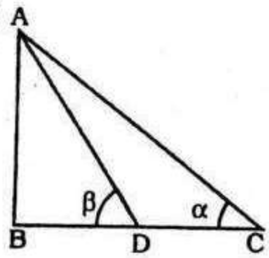
$\tan(90°-\theta) = \frac{h}{x}$

$\Rightarrow \cot\theta = \frac{h}{x}$

$\therefore \tan\theta . \cot\theta = \frac{h}{y} \times \frac{h}{x}$

$\Rightarrow \frac{h^2}{xy} = 1 \qquad \Rightarrow h = \sqrt{xy}$

38. (3)



AB = monument = h metre
DC = 138 metre
BD = x metre

$\tan\alpha = \frac{1}{5}$

$\sec\beta = \frac{\sqrt{193}}{12}$

$\therefore \tan\beta = \sqrt{\sec^2\beta - 1}$

$= \sqrt{\frac{193}{144} - 1} = \sqrt{\frac{193-144}{144}} = \sqrt{\frac{49}{144}} = \frac{7}{12}$

$\therefore$ From ΔABC,

$\tan\alpha = \frac{AB}{BC} \Rightarrow \frac{1}{5} = \frac{h}{x+138}$

$\Rightarrow h = \frac{x+138}{5}$

$\Rightarrow 5h = x + 138$ ... (i)

From ΔABD,

$\tan\beta \frac{h}{x} \Rightarrow \frac{7}{12} = \frac{h}{x}$

$\Rightarrow x = \frac{12h}{7}$ ... (ii)

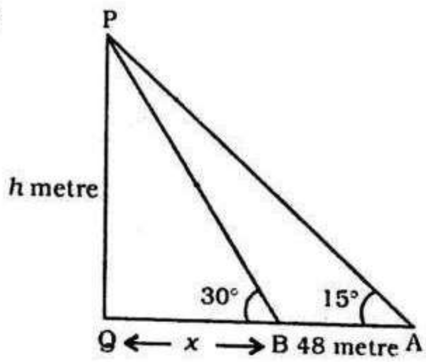
$\therefore 5h = \frac{12h}{7} + 138$

$\Rightarrow 35h - 12h = 138 \times 7$

$\Rightarrow 23\ h = 138 \times 7$

$\Rightarrow h = \frac{138 \times 7}{23} = 42$ metre

39. (2)



Tower = PQ = h metre
QB = x metre
From ΔAPQ,

$\tan 15° = \frac{h}{x+48}$

$2 - \sqrt{3} = \frac{h}{x+48}$ ...(i)

From ΔPQB,

$\tan 30° = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow \sqrt{3}h = x$ ...(ii)

$\Rightarrow 2 - \sqrt{3} = \frac{h}{\sqrt{3}h + 48}$

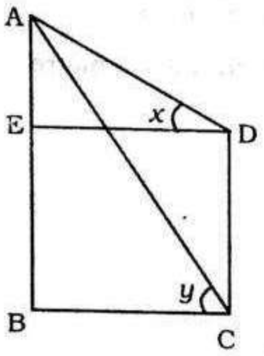
$\Rightarrow 2\sqrt{3}h - 3h + (2-\sqrt{3})48 = h$

$\Rightarrow h + 3h - 2\sqrt{3}\ h = \left(2-\sqrt{3}\right)\times 48$

$\Rightarrow 2h\left(2-\sqrt{3}\right) = 48 \times \left(2-\sqrt{3}\right)$

$\Rightarrow h = \dfrac{48}{2} = 24$ metre

40. (3)



CD = tree = h metre
AB = building = a metre
BC = ED = b metre
∴ From Δ AED,

$\tan x = \dfrac{AE}{ED} \Rightarrow \tan x = \dfrac{a-h}{b}$

$\Rightarrow b = (a-h)\cot x$ ...(i)

From Δ ABC,

$\tan y = \dfrac{AB}{BC}$

$\Rightarrow \tan y = \dfrac{a}{b}$

$\Rightarrow b = a\cot y$ ...(ii)

From equations (i) and (ii),

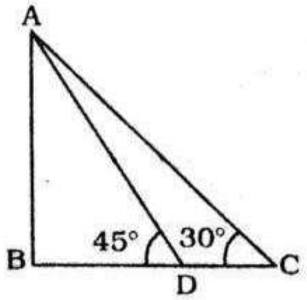
$(a-h)\cot x = a\cot y$

$\Rightarrow a\cot x - h\cot x = a\cot y$

$\Rightarrow h\cot x = a(\cot x - \cot y)$

$\Rightarrow a = \dfrac{h\cot x}{\cot x - \cot y}$

41. (4)



Let AB be a pillar of height h metre.
If BC = length of shadow
= x, then
BD = $(x + 20)$ metre
From Δ ABC,

$\tan 45° = \dfrac{h}{x} \Rightarrow h = x$ .....(i)

From Δ ABD,

$\tan 30° = \dfrac{AB}{BD} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+20}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{h+20}$

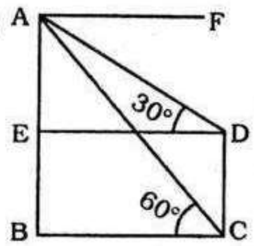
$\Rightarrow \sqrt{3}h = h + 20$

$\Rightarrow \left(\sqrt{3}-1\right)h = 20$

$\Rightarrow h = \dfrac{20}{\sqrt{3}-1} = \dfrac{20}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{20\left(\sqrt{3}+1\right)}{2} = 10\left(\sqrt{3}+1\right)$ metre

42. (2)



AB = 108 m
CD = x metre
From Δ ABC,

$\tan 60° = \dfrac{AB}{BC}$

$\Rightarrow \sqrt{3} = \dfrac{108}{BC}$

$\Rightarrow BC = \dfrac{108}{\sqrt{3}} = 36\sqrt{3}\,m$

From Δ AED,

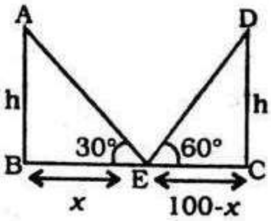
$\tan 30° = \dfrac{AE}{ED}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{108-x}{36\sqrt{3}}$

$\Rightarrow 108 - x = 36$

$\Rightarrow x = 108 - 36 = 72$ m

43. (1)



AB = CD = h metre (Height of pole)
From ΔABE,

$$\tan 30° = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow \sqrt{3}h = x \quad ...(i)$$

From ΔDEC,

$$\tan 60° = \frac{h}{100-x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{100-x}$$

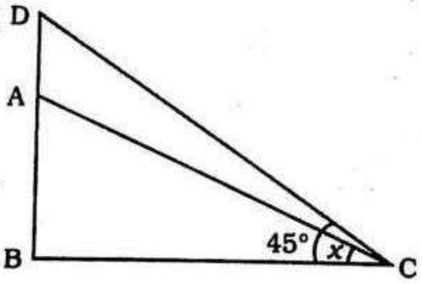
$$\Rightarrow \sqrt{3}(100-x) = h$$

$$\Rightarrow \sqrt{3}(100-\sqrt{3}h) = h \quad \text{[From equation (i)]}$$

$$\Rightarrow 100\sqrt{3} - 3h = h \Rightarrow 4h = 100\sqrt{3}$$

$$\Rightarrow h = 25\sqrt{3} \text{ metre}$$

44. (2)



AB = Buidling = h metre
AD = Chimney = y metre
From ΔBCD,

$$\tan 45° = \frac{BD}{BC} \Rightarrow 1 = \frac{h+y}{BC}$$

$$\Rightarrow BC = h + y \quad ...(i)$$

From ΔABC,

$$\tan x = \frac{AB}{BC}$$

$$\Rightarrow \tan x = \frac{h}{BC}$$

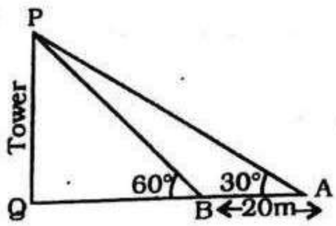
$$\Rightarrow BC = h \cot x \quad ...(ii)$$

From equations (i) and (ii),

$h + y = h \cot x$

$\Rightarrow y = (h \cot x - h)$ metre

45. (3)



Let PQ = h metre and BQ = x metre.
From Δ APQ,

$$\tan 30° = \frac{h}{x+20}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

$$\Rightarrow \sqrt{3}h = x + 20 \quad ......(i)$$

From Δ PQB,

$$\tan 60° = \frac{PQ}{BQ} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}h \quad .....(ii)$$

$$\therefore \sqrt{3}h = \frac{1}{\sqrt{3}}h + 20$$

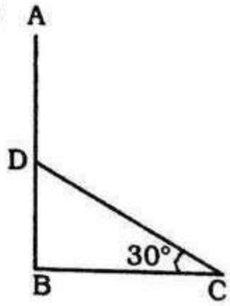
[From equations (i) and (ii)]

$$\Rightarrow 3h - h = 20\sqrt{3}$$

$$\Rightarrow 2h = 20\sqrt{3}$$

$$\therefore h = 10\sqrt{3} \text{ metre}$$

46. (3)



AB = Telegraph post = h metre
Telegraph post bends at point D.
DB = x metre
∴ AD = CD = $(h - x)$ metre

$BC = 8\sqrt{3}$ metre

From Δ DBC,

$$\sin 30° = \frac{DB}{DC}$$

$$\Rightarrow \frac{1}{2} = \frac{x}{h - x}$$

$$\Rightarrow 2x = h - x$$

$$\Rightarrow 3x = h \quad \text{..... (i)}$$

Again,

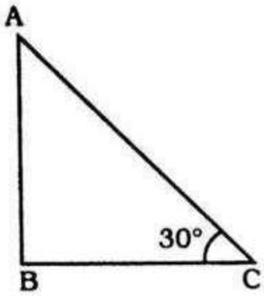
$$\tan 30° = \frac{DB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{8\sqrt{3}}$$

$$\Rightarrow x = 8 \text{ metre}$$

$$\therefore h = 3 \times 8 = 24 \text{ metre}$$

47. (3)



AB = wall = 10 metre

∠ACB = 30°

BC = x metre

∴ from ΔABC,

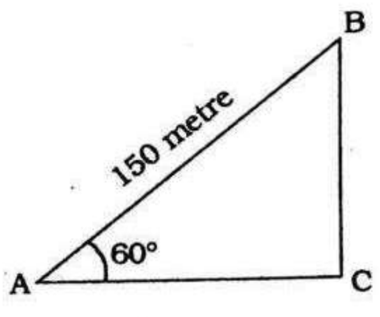
$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$\Rightarrow x = 10\sqrt{3} \text{ metre}$$

48. (2) AB = Length of the thred = 150 metre

∠BAC = 60°

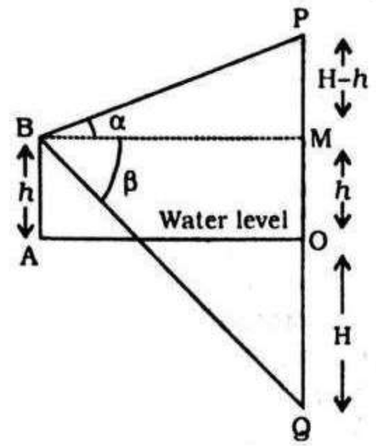


In ΔABC,

$$\sin 60° = \frac{BC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{BC}{150}$$

$$\Rightarrow BC = 150 \times \frac{\sqrt{3}}{2} = 75\sqrt{3} \text{ metre}$$

49. (4) Let P be the cloud at height H above the level of the water in the lake and Q its image in the water.



∴ OQ = OP = H

B is at a point at a height AB = h, above the water. Angle of elevation of P and depression of Q from B are α and β respectively.

In Δ PBM,

$$\tan \alpha = \frac{H - h}{BM}$$

$$\therefore BM = (H - h) \cot \alpha \quad \text{....... (i)}$$

In Δ QMB,

$$\tan \beta = \frac{QM}{BM}$$

$$\therefore BM = (H + h) \cot \beta \quad \text{...... (ii)}$$

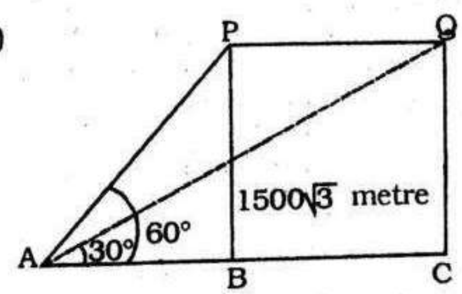
From equations (i) and (ii),

$$(H - h) \cot \alpha = (H + h) \cot \beta$$

$$\Rightarrow H (\cot \alpha - \cot \beta) = h (\cot \alpha + \cot \beta)$$

$$\therefore H = \frac{h(\cot \alpha + \cot \beta)}{\cot \alpha - \cot \beta}$$

50. (2)



P & Q are the positions of the plane.
$\angle PAB = 60°$; $\angle QAB = 30°$

$PB = 1500\sqrt{3}$ metre

In Δ ABP

$$\tan 60° = \frac{BP}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{AB}$$

$\Rightarrow$ AB = 1500 metre

In Δ ACQ

$$\tan 30° = \frac{CQ}{AC}$$

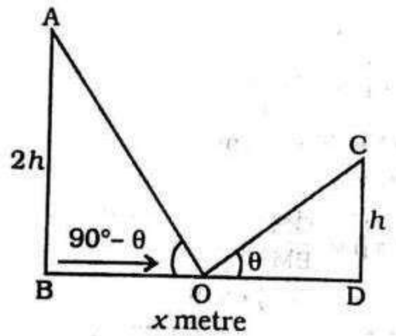
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AC}$$

= AC = 1500 × 3 = 4500 metre
PQ = BC = AC − AB
= 4500 − 1500 = 3000 metre

$$\therefore \text{Speed of plane} = \frac{3000}{15}$$

= 200 metre/second

51. (1) CD = h metre, AB = 2h metre



$$OB = OD = \frac{x}{2} \text{ metre}$$

From ΔOCD,

$$\tan\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x} \qquad .... (i)$$

From ΔOAB,

$$\tan(90° - \theta) = \frac{AB}{BO}$$

$$\Rightarrow \cot\theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x} \qquad .... (ii)$$

Multiplying both equations,

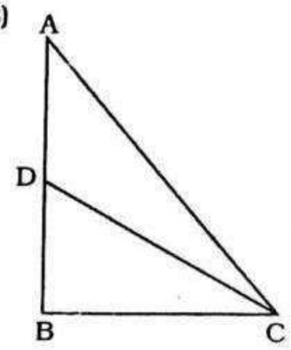
$$\tan\theta.\cot\theta = \frac{2h}{x} \times \frac{4h}{x}$$

$$\Rightarrow x^2 = 8h^2$$

$$\Rightarrow h^2 = \frac{x^2}{8}$$

$$\Rightarrow h = \frac{x}{2\sqrt{2}} \text{ metre}$$

52. (3)



$\angle ACB = 60°$
$\angle DCB = 45°$
AB = 5000 metre
AD = x metre
$\therefore$ From ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{5000}{BC}$$

$$\Rightarrow BC = \frac{5000}{\sqrt{3}} \text{ metre}$$

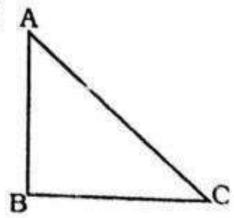
From ΔDBC,

$$\tan 45° = \frac{DB}{BC} \qquad \Rightarrow DB = BC = \frac{5000}{\sqrt{3}}$$

$$\therefore AD = AB - BD = 5000 - \frac{5000}{\sqrt{3}}$$

$$= 5000\left(1 - \frac{1}{\sqrt{3}}\right) = 5000\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \text{ metre}$$

53. (2)



AB = Tower = x units

BC = Shadow = $\sqrt{3}\,x$ units

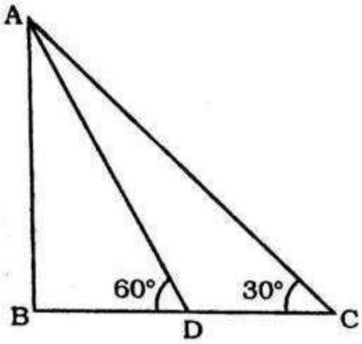
$\tan ACB = \frac{AB}{BC} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore \angle ACB = 30°$

54. (2) $\frac{6}{4} = \frac{h}{50}$

$\Rightarrow h = \frac{50 \times 6}{4} = 75$ feet

55. (4)



AB = Tower = 30 metre
CD = x metre
$\angle ACB = 30°$
$\angle ADB = 60°$
From Δ ABD,

$\tan 60° = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{30}{BD}$

$\Rightarrow BD = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ metre

From Δ ABC,

$\tan 30° = \frac{AB}{BC}$

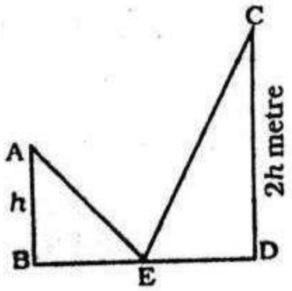
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{10\sqrt{3} + x}$

$\Rightarrow 10\sqrt{3} + x = 30\sqrt{3}$

$\Rightarrow x = 30\sqrt{3} - 10\sqrt{3}$

$= 20\sqrt{3}$ metre

56. (4)



BE = DE = 30 metre
$\angle AEB = \theta$
$\therefore \angle CED = 90° - \theta$
From Δ ABE,

$\tan \theta = \frac{AB}{BE}$

$\Rightarrow \tan \theta = \frac{h}{30}$

$\Rightarrow h = 30 \tan \theta$ ...(i)

From Δ CDE,

$\tan (90° - \theta) = \frac{2h}{30}$

$\Rightarrow \cot\theta = \frac{h}{15} \Rightarrow h = 15\cot\theta$ ...(ii)

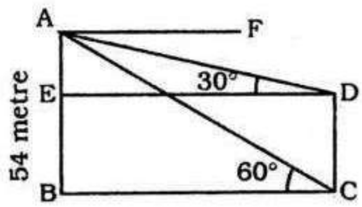
By multiplying both equations,
$h^2 = 30 \times 15 \times \tan\theta . \cot\theta$
$\Rightarrow h^2 = 30 \times 15$

$[\because \tan\theta . \cot\theta = 1]$

$\Rightarrow h = 15\sqrt{2}$ metre = AB

$\Rightarrow 2h = 30\sqrt{2}$ metre = CD

57. (2)



AB = temple = 54 metre
CD = temple = h metre
BC = width of river = x metre
From ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{54}{x}$

$\Rightarrow x = \frac{54}{\sqrt{3}} = 18\sqrt{3}$ metre

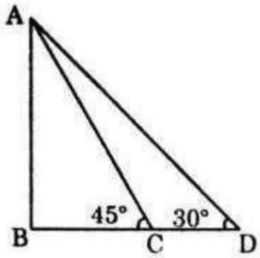
From ΔADE,

$\tan 30° = \frac{AE}{DE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{54 - h}{18\sqrt{3}}$

$\Rightarrow 54 - h = 18$
$\Rightarrow h = 54 - 18 = 36$ metre

58. (3)



AB = Tower = h metre

$\angle ADB = 30°$

$\angle ACB = 45°$

CD = 60 metre

BC = x metre

From ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{h}{x} \Rightarrow h = x$$

From ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+60}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{h+60}$$

$$\Rightarrow \sqrt{3}\,h = h + 60$$

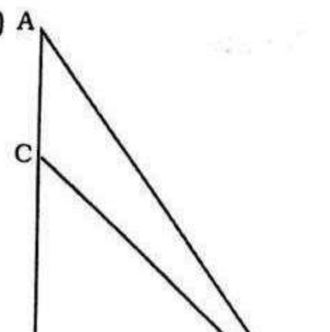
$$\Rightarrow \sqrt{3}\,h - h = 60$$

$$\Rightarrow h\left(\sqrt{3}-1\right) = 60$$

$$\Rightarrow h = \frac{60}{\sqrt{3}-1} = \frac{60\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$

$$= 30\left(\sqrt{3}+1\right) \text{ metre}$$

59. (4)



A and C $\Rightarrow$ position of planes

BC = 3125m

AC = x metre

In ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{3125+x}{BD}$$

$$\Rightarrow BD = \frac{3125+x}{\sqrt{3}}$$

In ΔBCD,

$$\tan 30° = \frac{BC}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3125}{\frac{3125+x}{\sqrt{3}}}$$

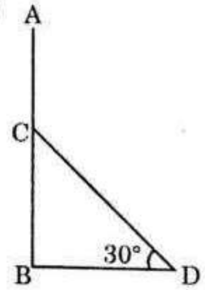
$\Rightarrow 3\ (3125) = 3125 + x$

$\Rightarrow 9375 = 3125 + x$

$\Rightarrow x = 9375 - 3125$

= 6250 metre

60. (2)



AB = Post = 15 feet

The post breaks at point C.

BC = x feet

$\Rightarrow$ AC = CD = $(15 - x)$ feet

$\angle CDB = 30°$

From Δ BCD,

$$\sin 30° = \frac{BC}{CD}$$

$$\Rightarrow \frac{1}{2} = \frac{x}{15-x}$$

$\Rightarrow 2x = 15 - x$

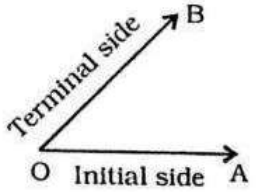
$\Rightarrow 3x = 15$

$\Rightarrow x = 5$ feet

IMPORTANT POINTS : AT A GLANCE

MEASUREMENT OF ANGLES

Angles : An angle is considered as the figure obtained by rotating a given ray about its end - point.



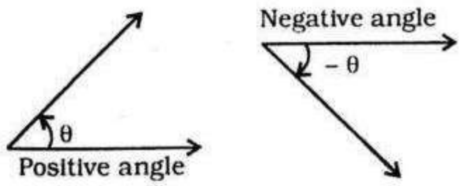
Consider a ray $\overrightarrow{OA}$. If this ray rotates about about its end point O and takes the position $\overrightarrow{OB}$, then we say that angle ∠AOB has been generated.

The revolving ray is called the generating line of the angle.

Measure of An Angle : The measure of an angle is the amount of rotation from the initial side to the terminal side.

If the revolving ray starting from the initial position to final position describes one quarter of a circle, then we say that the measure of the angle formed is a right angle.

Sense of An Angle : The sense of an angle is determined by the direction of rotation of the initial side into the terminal side. The sense of the angle is said to be positive if the initial side rotates in anticlockwise direction and it is negative if the initial side rotates in the clockwise direction to get to the terminal side.



Systems of Measurement of Angles : There are three systems for measuring angles :

(i) Sexagesimal or English System

(ii) Centesimal or French System

(iii) Circular System

Sexagesimal System : In this system, a right angle is divided into 90 equal parts, called degrees. The symbol 1° is used to denote one degree. Thus one degree is one ninetieth part of a right angle. Each degree is divided into 60 equal parts, called minutes. The symbol 1' is used to denote one minute. And each minute is divided into 60 equal parts, called second. The symbol 1" is used to denote one second.

Thus, 1 right angle = 90 degrees (=90°)

1° = 60 minutes (=60')

1' = 60 seconds (=60")

Centesimal System : In this system, a right angle is divided into 100 equal parts, called grades; each grade is subdivided into 100 minutes and each minute into 100 seconds.

Symbols 1^g, 1' and 1" are used to denote a grade, a minute and a second respectively.

Thus,

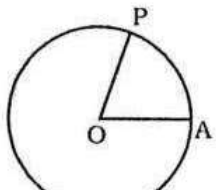
1 right angle = 100 grades (= 100^g)

1 grade = 100 minutes (= 100')

1 minute = 100 seconds (= 100")

Circular System : In this system, the unit of measurement is radian.

One radian, written as 1^c, is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

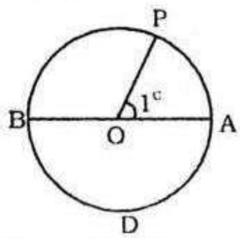


arc AP = radius r (OA) of the circle

∴ ∠AOP = 1 radian (= 1^c)

Theorem : Radian is a constant angle.

Proof : Consider a circle with centre O and radius r.



Arc AP = radius r.   ∴∠AOP = 1^c.

Produce AO to meet the circle at B so that ∠AOB = a straight angle = 2 right angles.

Since the angles at the centre of a circle are proportional to the arcs subtending them. Therefore,

$$\frac{\angle AOP}{\angle AOB} = \frac{arc\ AP}{arc\ APB}$$

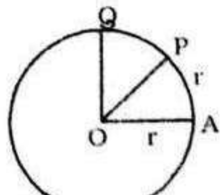
$$\Rightarrow \frac{\angle AOP}{\angle AOB} = \frac{r}{\pi r} \Rightarrow \angle AOP = \frac{1}{\pi}\angle AOB$$

$$\Rightarrow 1^c = \frac{\text{a straight angle}}{\pi} = \frac{180^\circ}{\pi}$$

$$\therefore\ 1^c = \frac{180^\circ}{\pi} \qquad \Rightarrow \pi^c = 180^\circ$$

• The number of radians in an angle subtended by an arc of a circle at the centre is equal to $\frac{arc}{radius}$ i.e. $\theta = \frac{s}{r}$

Proof : Consider a circle with centre O and radius r. Let ∠AOQ = 0^c and let arc AQ = s. Let P be a point on the arc AQ such that arc AP = r.



Then, $\angle AOP = 1^c$ Since angles at the centre of a circle are proportional to the arcs subtending them. Therefore,

$$\frac{\angle AOQ}{\angle AOP} = \frac{arc\ AQ}{arc\ AP}$$

$$\Rightarrow \angle AOQ = \left(\frac{arc\ AQ}{arc\ AP} \times 1\right)^c \qquad [\because \angle AOP = 1^c]$$

$$\Rightarrow \theta = \frac{s}{r} \text{ radians.}$$

Remarks :

Since $180° = \pi$ radians

Therefore, $1° = \frac{\pi}{180}$ radians

Hence, $30° = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$ radians

$45° = \frac{\pi}{180} \times 45 = \frac{\pi}{4}$ radians

$60° = \frac{\pi}{180} \times 60 = \frac{\pi}{3}$ radians

$90° = \frac{\pi}{180} \times 90 = \frac{\pi}{2}$ radians

Degree	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

- We have,
  π radians = 180°

  $\therefore 1 \text{ radian} = \frac{180°}{\pi} = \left(\frac{180}{22} \times 7\right)^°$

  = 57° 16' 22" (approx).
- We have,
  180° = π radians

  $\therefore 1^0 = \frac{\pi}{180}$ radians

  $= \left(\frac{22}{7 \times 180}\right)$ radian = 0.01746 radian.

Relation Between Three Systems of Measurement

Let D be the number of degrees, R be the number of radians and G be the number of grades in an angle θ.

Now, 90° = 1 right angle

$\Rightarrow 1° = \frac{1}{90}$ right angle

$\Rightarrow D° = \frac{D}{90}$ right angles

$\Rightarrow \theta = \frac{D}{90}$ right angles

Again, π radians = 2 right angles

$\Rightarrow$ R radians $= \frac{2R}{\pi}$ right angles

and, 100 grades = 1 right angle

$\Rightarrow$ G grades $= \frac{G}{100}$ right angles

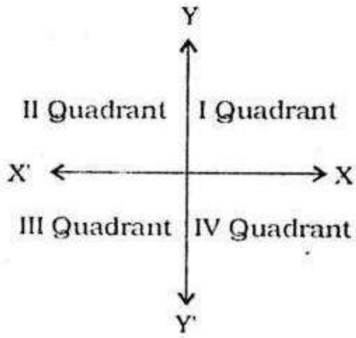
$$\therefore \frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Some Useful Points

- The angle between two consecutive digits in a clock is $30° \left(= \frac{\pi}{6} \text{ radians}\right)$.
- The hour hand rotates through an angle of 30° in one hour i.e. $\left(\frac{1}{2}\right)^°$ in one minute.
- The minute hand rotates through an angle of 6° in one minute.

TRIGONOMETRIC IDENTITIES

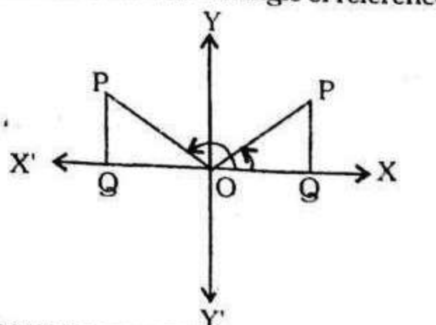
SOME USEFUL TERMS : Quadrants : Let *XOX'* and *YOY'* be two lines at right angles in the plane of paper. These two perpendicular lines divide the plane of the paper into four equal parts, these four parts are known as four quadrants and lines *XOX'* and *YOY'* are known as *x*-axis and *y*-axis respectively.



The parts *XOY*, *YOX'*, *X'OY'* and *Y'OX* are known as 1st, 2nd, 3rd and 4th quadrants respectively.

Angle in a Quadrant : An angle is said to be in a particular quadrant, if the terminal side of the angle in standard position lies in that quadrant.

Triangle of Reference : If from any point *P* on the terminal side of an angle in standard position a perpendicular *PQ* is drawn on *x*-axis, then the right angled triangle *OPQ* thus formed is called the triangle of reference of $\angle XOP$.

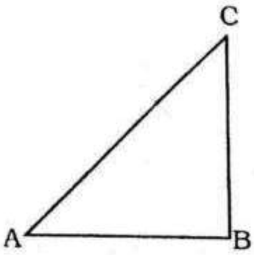


SOME BASIC FORMULAE

1. Relation between Angles, Radius and Arc length : If 's' is the length of a circle of radius *r*. If this arc subtends an angle θ radians at the centre of the circle then

$$s = r\theta \Rightarrow \theta = \frac{s}{r}$$

2. Trigonometric Ratios : If we consider a ΔABC right angled at B then the side opposite to $\angle B$ i.e., AC is called as hypotenuse and if AB is chosen an initial line then AB is called as base and BC will be perpendicular.



So the trigonometric ratios can be represented as

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

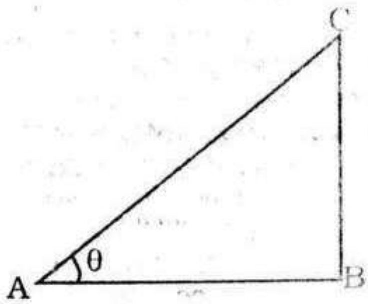
$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{1}{\cos\theta},$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$



Identities

(1) $\sin^2\theta + \cos^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $1 + \cot^2\theta = \text{cosec}^2\theta$

Note : $\sin^2 A + \cos^2 B$ can not be equal to 1 because the angles are different.

RELATION AMONG T-RATIOS						
	sin θ	**cos θ**	**tan θ**	**cot θ**	**sec θ**	**cosec θ**
sin θ	sin θ	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\text{cosec}\,\theta}$
cos θ	$\sqrt{1-\sin^2\theta}$	cos θ	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\text{cosec}^2\theta-1}}{\text{cosec}\,\theta}$
tan θ	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	tan θ	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\text{cosec}^2\theta-1}}$
cot θ	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	cot θ	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\text{cosec}^2\theta-1}$
sec θ	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	sec θ	$\frac{\text{cosec}\,\theta}{\sqrt{\text{cosec}^2\theta-1}}$
cosec θ	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	cosec θ

Sign of Trigonometric Ratios

Quadrant	sinθ	cosθ	tanθ	cotθ	secθ	cosecθ
I	+	+	+	+	+	+
II	+	–	–	–	–	+
III	–	–	+	+	–	–
IV	–	+	–	–	+	–

TRIGONOMETRIC RATIOS OF ALLIED ANGLES

Two angles are said to be allied when their sum or difference is either zero or a multiple of 90°.

The angles like – θ, 90°± θ, 180°± θ, 360°± θ etc. are angles allied to the angle θ if θ is measured in degrees.

Trigonometric ratios of (– θ) in terms of θ:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\text{cosec}(-\theta) = -\text{cosec}\,\theta$

- If θ is –ve when only cos and sec are positive as – θ lies in IV Quadrant.
- While T-ratio remains same
- All T-ratios are positive as 90° – θ lies in 1st quadrant.
- Sin↔cos, tan↔cot, sec↔cosec

Trigonometric ratios of (90° + θ) in terms of θ

$\sin(90° + \theta) = +\cos\theta$
$\cos(90° + \theta) = -\sin\theta$
$\tan(90° + \theta) = -\cot\theta$
$\cot(90° + \theta) = -\tan\theta$
$\sec(90° + \theta) = -\text{cosec}\,\theta$
$\text{cosec}(90° + \theta) = +\sec\theta$

•As (90° + θ) lies in IInd quadrant so only sin (90° + θ) and cosec (90° + θ) are positive

•Sin↔cos, tan↔cot, sec↔cosec

Trigonometric ratios of (180°–θ) in terms of θ

$\sin(180° - \theta) = +\sin\theta$
$\cos(180° - \theta) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$
$\cot(180° - \theta) = -\cot\theta$
$\sec(180° - \theta) = -\sec\theta$
$\text{cosec}(180° - \theta) = +\text{cosec}\,\theta$

•As (180°– θ) lies in IInd quadrant so only sin (180°+ θ) and cosec (180°+ θ) are +ve.

•In (180°– θ) T-ratio does not changes.

Trigonometric ratios of (180°– θ) in terms of θ

$\sin(180° + \theta) = -\sin\theta$
$\cos(180° + \theta) = -\cos\theta$
$\tan(180° + \theta) = +\tan\theta$
$\cot(180° + \theta) = +\cot\theta$
$\sec(180° + \theta) = -\sec\theta$
$\text{cosec}(180° + \theta) = -\text{cosec}\,\theta$

•As (180°+ θ) lies in IIIrd quadrants so only tan (180°+ θ) and cot (180 + θ) are +ve.

•T-ratio remain same as sin (180°+ θ) = – sin θ

Trigonometric ratios of (360°– θ) in terms of θ:

$\sin(360° - \theta) = -\sin\theta$
$\cos(360° - \theta) = +\cos\theta$
$\tan(360° - \theta) = -\tan\theta$
$\cot(360° - \theta) = -\cot\theta$
$\sec(360° - \theta) = +\sec\theta$
$\text{cosec}(360° - \theta) = -\text{cosec}\,\theta$

•As (360°– θ) lies in IV quadrant so only cos (360°– θ) and sec (360°– θ) are positive.

Trigonometric ratio of (360°+ θ) in terms of θ:

$\sin(360° + \theta) = \sin\theta$
$\cos(360° + \theta) = \cos\theta$
$\tan(360° + \theta) = \tan\theta$
$\cot(360° + \theta) = \cot\theta$
$\sec(360° + \theta) = \sec\theta$
$\text{cosec}(360° + \theta) = \text{cosec}\,\theta$

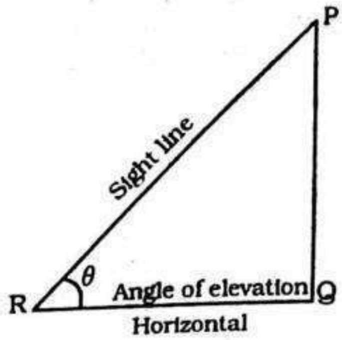
•As (360°+ θ) lies in Ist quadrant so all are +ve.

•T-ratio does not change.

HEIGHTS & DISTANCES

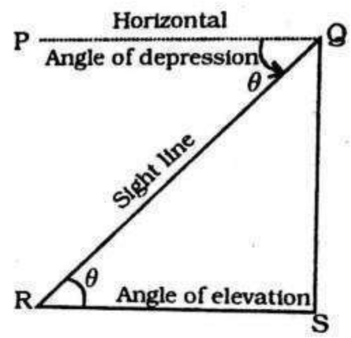
Trigonometry helps us in finding heights of objects and distances between the points without actually measuring them.

Angle of Elevation : Let PQ be a tower (pillar/ shell/minar/pole) etc. standing on a level ground and the observer is standing at any point R on the level ground, is viewing at P.



The angle, which the line RP makes with the horizontal line RQ is called angle of elevation. So, ∠ PRQ is angle of elevation.

Angle of Depression : If observer is at Q and is viewing an object R on the ground, then angle between PQ and QR is the angle of depression. So, ∠ PQR is angle of depression.

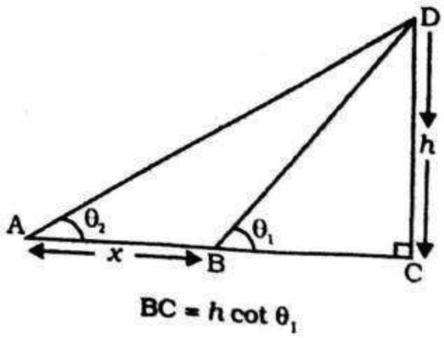


Note

- Numerically angle of elevation is equal to the angle of depression.
- The angle of elevation and the angle of depression both are measured with the horizontal.

SHORT CUT FORMULAE

1. If 'h' is height of building and the observer is x units away from base then.
   $h = x\tan\theta$ and $x = h\cot\theta$
2. If '$DC = h$' be the height of a building and θ_1 and θ_2 are elevations measured at B and A respectively along the same straight line which are x units apart then



$BC = h\cot\theta_1$

$$h = \frac{x}{\cot\theta_2 - \cot\theta_1}$$

- Here if θ_1 and θ_2 are complementary then $\theta_1 + \theta_2 = 90°$