TALENT & OLYMPIAD

Cubes and Cube Roots

Introduction

The word cube is used in geometry. In geometry the word cube refers to the solid having equal sides. Thus cube of a natural number is the multiple of three prime factors of each number. A given number is said to be a perfect cube if it can be expressed as a product of triplets of equal factors.

🍄 🛛 Cube of a Real Number

According to arithmetic and algebra, the cube of a number n is its third power. If a number multiplied three times by itself the resultant number is called cube of that number.

 $n^3 = n \times n \times n$. In this expression if $n \times n \times n = m$ then we can say that m is cube of n.

This is also the formula for volume of a geometric cube with sides of length "n". The inverse operation of finding a number whose cube is 'n' is called finding the cube root of "n". It determines the side of the cube of a given volume.

Cubes of Certain Numbers which are Perfect Cube

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$
$4^3 = 64$	$5^3 = 125$	$6^3 = 216$
$7^3 = 343$	$8^3 = 512$	$9^3 = 729$
$10^3 = 1000$	$11^3 = 1331$	$12^3 = 1728$
$13^3 = 2197$	$14^3 = 2744$	$15^3 = 3375$
$16^3 = 4096$	$17^3 = 4913$	$18^3 = 5832$
$19^3 = 6859$	$20^3 = 8000$	

Cube of a Negative Number

We know that the cube of a negative number is always negative.

 $(-1)^{3} = -1 \times -1 \times -1 = -1$ $(-2)^{3} = -2 \times -2 \times -2 = -8$ $(-3)^{3} = -3 \times -3 \times -3 = -27$

Cube Roots

The inverse operation of the cube of a number is called its cube root. It is normally denoted by. $\sqrt[3]{n}$ or $(n)^{\frac{1}{3}}$. The cube root of a number can be found by using the prime factorization method. A number is called cube root of its cube.

EXAMPLE

The cube root of 8 is 2 because $2^3 = 2 \times 2 \times 2 = 8$ In symbolic form, the cube root of 8 is written as $\sqrt[3]{27} = 3$ ($\because 3^3 = 3 \times 3 \times 3 = 27$) $\sqrt[3]{64} = 4$ ($\because 4^3 = 4 \times 4 \times 4 = 64$) $\sqrt[3]{125} = 5$ ($\because 54^3 = 5 \times 5 \times 5 = 125$)

Cube Root of a Negative Number

The cube root of a negative number is always negative i.e. $(-n)^{\frac{1}{3}} = -(n)^{\frac{1}{3}}$ The cube root of -1000 is -10 because $(-10)^3 = -10 \times -10 \times -10 = -1000$. In symbolic form, the cube root of -1000 is written as $\sqrt[3]{-1000}$ So, $\sqrt[3]{-1000} = -10$ $\therefore (-10)^3 = -10 \times -10 \times -10 = -1000$

From the above, we can infer that:

- The cube root of a positive number is a positive number.
- The cube root of a negative number is a negative number. In general:
- If $\sqrt[3]{x} = a$ then, $a^3 = x$ where represents the cube root of x.



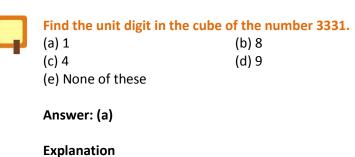
- If a number is divisible by 3, then its cube has digital root 9.
- If the remainder of the number is 1 when divide by 3, then its cube has digital root 1.
- If a number when divided by 3 leaves remainder 2, then its cube has digital root 8.
- Every positive rational number can be expressed as the sum of three positive rational cubes.

SUMMARY



- For any positive integer $\sqrt[3]{-a} = -\sqrt[3]{a}$.
- The cube root of a number m is the number whose cube is m.
- The cube of a number is always raised to the power of three of that number.
- The cube of a even number is always even.
- The cube of odd number is always odd.
- The cube root of a number can be found by using the prime factorization methods.

Commonly Asked



We know that, $(3331)^3 = 3331 \times 3331 \times 3331 = 36959313691$.

The smallest number by which 2560 must be multiplied so that the product will be a perfect cube. (b) 25

(d) 5

(a) 35 (c) 8

(e) None of these

Answer: (b)

Explanation

The factors of 2560 is given by

 $2560 = 5 \times 8 \times 8 \times 8$

In this factors there are three 8 and one 5. So in order to make the number 5 perfect cube we have to multiply it by 25. Therefore/ 25 is the least number by which it must be multiplied so that it becomes a perfect cube.



(e) None of these

Answer: (c)

Find the value of $\left[(5^2 + 12^2)^{\frac{1}{2}} \right]$	³ is given by:
(a) 2197	(b) 169
(c) 1693	(d) 289
(e) None of these	

Answer: (a)



Find the cube root of 42875. (a) 35

(a) 35	(b) 25
(c) 15	(d) 20
(e) 32	

Answer: (a) Explanation

The factors of $42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$ $\sqrt[3]{42874} = \sqrt[3]{5 \times 5 \times 5} \times 7 \times 7 \times 7 = 5 \times 7 = 35$

Make the factors of number by taking three identical numbers. Now multiply each number of the factors.



Find the least number by which 3087 must be multiplied to make it a perfect cube.

(a) 3 (b) 4 (c) 9 (d) 7 (e) None of these

Answer: (a)

Exploration The factors of $3087 = 3 \times 3 \times 7 \times 7 \times 7$

We note that the factor 3 appears only 2 times, so if we multiply 3087 by 3 we get $3084 \times 3 = (3)^3 \times (7)^3 = (3 \times 7)^3$, so the smallest number is 3 which when multiplied to 3087, it gives a perfect

(b) $\frac{13}{11}$

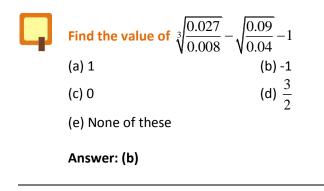
(d) $-\frac{17}{21}$

cube.

Find the cube root of $\frac{-2197}{1331}$.
(a) $\frac{-13}{11}$
(c) $-\frac{13}{21}$

(e) None of these

Answer: (a)



Self Evaluation



1.	Which one of the follow	ing is a perfect cube?							
	(a) 1525	(b) 1728							
	(c) 1458	(d) 3993							
	(e) None of these								
2.	Which one of the following numbers is not a perfect cube?								
	(a) 2197	(b) 512							
	(c) 2916	(d) 343							
	(e) None of these								
3.	What is the least numb cube?	er by which 1565109 must be multiplied so that the product becomes a prefect							
	(a) 11	(b) 12							
	(c) 13	(d) 14							
	(e) None of these								
4.	What is the least numbe	r by which 273375 must be divided so that the quotient will be a perfect cube?							
	(a) 49	(b) 25							
	(c) 81	(d) 27							
	(e) None of these								
5.	If $y = \sqrt[3]{2\frac{93}{125}}$, then find	the value of y.							
	(a)	(b) $\frac{5}{7}$							
	(c) $\frac{33}{7}$	(d) $\frac{13}{7}$							
	$(c) - \frac{7}{7}$	$(0) \frac{7}{7}$							
	(e) None of these								

6. Find the value of:
$$\sqrt[3]{\frac{1728}{2744}} =$$

(a) $-\frac{6}{11}$ (b) $-\frac{6}{7}$
(c) $-\frac{3}{4}$ (d) $-\frac{12}{14}$
(e) None of these

7.	Simplify: $\sqrt[3]{64 \times 729} =$	
	(a) 72	(b) 18
	(c) 36	(d) 27
	(e) None of these	
8.	Find the smallest number b	by which 8788 must be divided so that the quotient will be a perfect cube.
	(a) 2	(b) 3
	(c) 5	(d) 6
	(e) None of these	
9.		er by which 1600 must be divided so that the quotient will be a perfect cube?
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Э.	What is the smallest numb	er by which 1600 must be divided so that the quotient will be a perfect cube?
9.	What is the smallest numb (a) 15	er by which 1600 must be divided so that the quotient will be a perfect cube? (b) 20
9.	What is the smallest numb (a) 15 (c) 25 (e) None of these	er by which 1600 must be divided so that the quotient will be a perfect cube? (b) 20
	What is the smallest numb (a) 15 (c) 25 (e) None of these	er by which 1600 must be divided so that the quotient will be a perfect cube? (b) 20 (d) 30
	What is the smallest numb (a) 15 (c) 25 (e) None of these Find the smallest number t	er by which 1600 must be divided so that the quotient will be a perfect cube? (b) 20 (d) 30 by which 2560 must be multiplied so that the product will be a perfect cube.

Answers – Self Evaluation Test																			
1.	В	2.	С	3.	С	4.	С	5.	A	6.	D	7.	С	8.	E	9.	С	10. D	

Self Evaluation Test SOLUTIONS

 $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $\sqrt[3]{1728} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3} = 2 \times 2 \times 3 = 12$ Incorrect Options:
(a) $1525 = 5 \times 5 \times 61$ (c) $1428 = 2 \times 9 \times 9 \times 9$ (d) $3993 = 3 \times 11 \times 11 \times 11$ (e) Option (e) is incorrect

2. Correct Option:

1.

2916 = $2 \times 2 \times 9 \times 9 \times 9$ It is not a perfect cube. Incorrect Options: (a) 2197 = $13 \times 13 \times 13$ (d) $343 = 7 \times 7 \times 7$ (b) $512 = 8 \times 8 \times 8$ (e) $1728 = 12 \times 12 \times 12$

6.
$$\sqrt[3]{-\frac{1728}{2744}} = \sqrt[3]{\left[\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}\right]} = -\left[\frac{2 \times 2 \times 3}{2 \times 7}\right] = -\frac{12}{14}$$

- 7. $\sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$ $\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2} \times \sqrt[3]{9 \times 9 \times 9}$ $= 2 \times 2 \times 9 = 36$
- 8. $8788 = 2 \times 2 \times 13 \times 13 \times 13$ Therefore, by above calculation we get that if 8788 is divided by 4 then it gives a perfect cube.

 $9. 1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

10. $2560 = 2 \times 5$ There are only one 5 so if we multiply the number by 25 then it becomes a perfect square.