# Cubes and Cube Roots

## Introduction

The word cube is used in geometry. In geometry the word cube refers to the solid having equal sides. Thus cube of a natural number is the multiple of three prime factors of each number. A given number is said to be a perfect cube if it can be expressed as a product of triplets of equal factors.

## Cube of a Real Number

According to arithmetic and algebra, the cube of a number n is its third power. If a number multiplied three times by itself the resultant number is called cube of that number.

$n^3 = n \times n \times n$. In this expression if $n \times n \times n = m$ then we can say that m is cube of n.

This is also the formula for volume of a geometric cube with sides of length "n". The inverse operation of finding a number whose cube is 'n' is called finding the cube root of "n". It determines the side of the cube of a given volume.

### Cubes of Certain Numbers which are Perfect Cube

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|---|---|---|
| $1^3 = 1$ | $2^3 = 8$ | $3^3 = 27$ |
| $4^3 = 64$ | $5^3 = 125$ | $6^3 = 216$ |
| $7^3 = 343$ | $8^3 = 512$ | $9^3 = 729$ |
| $10^3 = 1000$ | $11^3 = 1331$ | $12^3 = 1728$ |
| $13^3 = 2197$ | $14^3 = 2744$ | $15^3 = 3375$ |
| $16^3 = 4096$ | $17^3 = 4913$ | $18^3 = 5832$ |
| $19^3 = 6859$ | $20^3 = 8000$ | |

### Cube of a Negative Number

We know that the cube of a negative number is always negative.

$(-1)^3 = -1 \times -1 \times -1 = -1$

$(-2)^3 = -2 \times -2 \times -2 = -8$

$(-3)^3 = -3 \times -3 \times -3 = -27$

### Cube Roots

The inverse operation of the cube of a number is called its cube root. It is normally denoted by. $\sqrt[3]{n}$ or $(n)^{\frac{1}{3}}$.

The cube root of a number can be found by using the prime factorization method.

A number is called cube root of its cube.

**Illustrative EXAMPLE**

The cube root of 8 is 2 because $2^3 = 2 \times 2 \times 2 = 8$

In symbolic form, the cube root of 8 is written as $\sqrt[3]{8}$ Likewise:

$\sqrt[3]{27} = 3$ $(\because 3^3 = 3 \times 3 \times 3 = 27)$

$\sqrt[3]{64} = 4$ $(\because 4^3 = 4 \times 4 \times 4 = 64)$

$\sqrt[3]{125} = 5$ $(\because 54^3 = 5 \times 5 \times 5 = 125)$

## Cube Root of a Negative Number

The cube root of a negative number is always negative i.e. $(-n)^{\frac{1}{3}} = -(n)^{\frac{1}{3}}$

The cube root of -1000 is -10 because $(-10)^3 = -10 \times -10 \times -10 = -1000$.

In symbolic form, the cube root of -1000 is written as $\sqrt[3]{-1000}$ So, $\sqrt[3]{-1000} = -10$

$\because (-10)^3 = -10 \times -10 \times -10 = -1000$

**From the above, we can infer that:**

- The cube root of a positive number is a positive number.
- The cube root of a negative number is a negative number. In general:
- If $\sqrt[3]{x} = a$ then, $a^3 = x$ where represents the cube root of $x$.

## You Must KNOW

- If a number is divisible by 3, then its cube has digital root 9.
- If the remainder of the number is 1 when divide by 3, then its cube has digital root 1.
- If a number when divided by 3 leaves remainder 2, then its cube has digital root 8.
- Every positive rational number can be expressed as the sum of three positive rational cubes.

## SUMMARY

- For any positive integer $\sqrt[3]{-a} = -\sqrt[3]{a}$ .
- The cube root of a number m is the number whose cube is m.
- The cube of a number is always raised to the power of three of that number.
- The cube of a even number is always even.
- The cube of odd number is always odd.
- The cube root of a number can be found by using the prime factorization methods.

## Commonly Asked QUESTIONS

**Find the unit digit in the cube of the number 3331.**

(a) 1 (b) 8
(c) 4 (d) 9
(e) None of these

**Answer: (a)**

**Explanation**

We know that,

$(3331)^3 = 3331 \times 3331 \times 3331 = 36959313691$.

**The smallest number by which 2560 must be multiplied so that the product will be a perfect cube.**

(a) 35 (b) 25
(c) 8 (d) 5
(e) None of these

**Answer: (b)**
**Explanation**
The factors of 2560 is given by
$2560 = 5 \times 8 \times 8 \times 8$
In this factors there are three 8 and one 5. So in order to make the number 5 perfect cube we have to multiply it by 25. Therefore/ 25 is the least number by which it must be multiplied so that it becomes a perfect cube.

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**The smallest number by which we must divide 8788 so that it becomes a perfect cube.**

(a) 2 (b) 169
(c) 4 (d) 13
(e) None of these

**Answer: (c)**

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**Find the value of** $\left[(5^2 + 12^2)^{\frac{1}{2}}\right]^3$ **is given by:**

(a) 2197 (b) 169
(c) 1693 (d) 289
(e) None of these

**Answer: (a)**

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**Find the cube root of 42875.**

(a) 35 (b) 25
(c) 15 (d) 20
(e) 32

**Answer: (a)**
**Explanation**
The factors of $42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$
$\sqrt[3]{42874} = \sqrt[3]{\underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}} = 5 \times 7 = 35$
Make the factors of number by taking three identical numbers. Now multiply each number of the factors.

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**Find the least number by which 3087 must be multiplied to make it a perfect cube.**

(a) 3 (b) 4
(c) 9 (d) 7
(e) None of these

**Answer: (a)**

**Exploration**

The factors of $3087 = 3\times3\times7\times7\times7$

We note that the factor 3 appears only 2 times, so if we multiply 3087 by 3 we get $3084\times3 = (3)^3\times(7)^3 = (3\times7)^3$, so the smallest number is 3 which when multiplied to 3087, it gives a perfect cube.

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**Find the cube root of $\frac{-2197}{1331}$.**

(a) $\frac{-13}{11}$ (b) $\frac{13}{11}$

(c) $-\frac{13}{21}$ (d) $-\frac{17}{21}$

(e) None of these

**Answer: (a)**

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**Find the value of $\sqrt[3]{\frac{0.027}{0.008}} - \sqrt{\frac{0.09}{0.04}} - 1$**

(a) 1 (b) -1

(c) 0 (d) $\frac{3}{2}$

(e) None of these

**Answer: (b)**

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# Self Evaluation TEST

Duration 10 Minutes

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**1.** **Which one of the following is a perfect cube?**

(a) 1525 (b) 1728
(c) 1458 (d) 3993
(e) None of these

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**2.** **Which one of the following numbers is not a perfect cube?**

(a) 2197 (b) 512
(c) 2916 (d) 343
(e) None of these

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**3.** **What is the least number by which 1565109 must be multiplied so that the product becomes a prefect cube?**

(a) 11 (b) 12
(c) 13 (d) 14
(e) None of these

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**4.** **What is the least number by which 273375 must be divided so that the quotient will be a perfect cube?**

(a) 49 (b) 25
(c) 81 (d) 27
(e) None of these

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**5.** **If $y=\sqrt[3]{2\frac{93}{125}}$ ,then find the value of y.**

(a) $\frac{7}{5}$ (b) $\frac{5}{7}$
(c) $\frac{33}{7}$ (d) $\frac{13}{7}$
(e) None of these

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**6.** **Find the value of:** $\sqrt[3]{\frac{1728}{2744}}=$

(a) $-\frac{6}{11}$ (b) $-\frac{6}{7}$
(c) $-\frac{3}{4}$ (d) $-\frac{12}{14}$
(e) None of these

**7.** **Simplify:** $\sqrt[3]{64 \times 729} =$

(a) 72 (b) 18
(c) 36 (d) 27
(e) None of these

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**8.** **Find the smallest number by which 8788 must be divided so that the quotient will be a perfect cube.**

(a) 2 (b) 3
(c) 5 (d) 6
(e) None of these

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**9.** **What is the smallest number by which 1600 must be divided so that the quotient will be a perfect cube?**

(a) 15 (b) 20
(c) 25 (d) 30
(e) None of these

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**10.** **Find the smallest number by which 2560 must be multiplied so that the product will be a perfect cube.**

(a) 10 (b) 15
(c) 20 (d) 25
(e) None of these

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**Answers – Self Evaluation Test**

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | B | **2.** | C | **3.** | C | **4.** | C | **5.** | A | **6.** | D | **7.** | C | **8.** | E | **9.** | C | **10.** | D |

# Self Evaluation Test

## SOLUTIONS

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**1.** $1728 = 2\times2\times2\times2\times2\times2\times3\times3\times3$

$\sqrt[3]{1728} = \sqrt[3]{\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}} = 2\times2\times3 = 12$

**Incorrect Options:**

(a) $1525 = 5\times5\times61$

(c) $1428 = 2\times9\times9\times9$

(d) $3993 = 3\times11\times11\times11$

(e) Option (e) is incorrect

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**2.** **Correct Option:**

$2916 = 2\times2\times9\times9\times9$ It is not a perfect cube.

**Incorrect Options:**

(a) $2197 = 13\times13\times13$

(d) $343 = 7\times7\times7$

(b) $512 = 8\times8\times8$

(e) $1728 = 12\times12\times12$

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**6.** $\sqrt[3]{-\frac{1728}{2744}} = \sqrt[3]{\left[\frac{\underline{\underline{2\times2\times2}}\times\underline{2\times2\times2}\times\underline{3\times3\times3}}{\underline{2\times2\times2}\times\underline{7\times7\times7}}\right]} =$

$-\left[\frac{2\times2\times3}{2\times7}\right] = -\frac{12}{14}$

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**7.** $\sqrt[3]{64\times729} = \sqrt[3]{64}\times\sqrt[3]{729}$

$\sqrt[3]{2\times2\times2\times2\times2\times2}\times\sqrt[3]{9\times9\times9}$

$= 2\times2\times9 = 36$

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**8.** $8788 = 2\times2\times13\times13\times13$

Therefore, by above calculation we get that if 8788 is divided by 4 then it gives a perfect cube.

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**9.** $1600 = 2\times2\times2\times2\times2\times2\times5\times5$

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**10.** $2560 = 2\times2\times2\times2\times2\times2\times2\times2\times2\times5$

There are only one 5 so if we multiply the number by 25 then it becomes a perfect square.

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