Chapter :- 1
Real Numbers
Exercise :- 1.1

Ques:- Use Euclid's division algorithm to find the HCF of:-

(i) 135 and 225

Ans:- $225 > 135$

$225 = 135 \times 1 + 90$

$135 = 90 \times 1 + 45$

$90 = 45 \times 2 + 0$

H.C.F of 135 and 225 is → 45

$$135\overline{)225}(1$$
$$\underline{135}$$
$$90$$

$$90\overline{)135}(1$$
$$\underline{90}$$
$$\times$$

$$45\overline{)90}(2$$
$$\underline{90}$$
$$\times$$

(ii) 196 and 38220

$38220 > 196$

Ans:- $38220 = 196 \times 195 + 0$

H.C.F of 196 and 38220 is → 196

$$196\overline{)38220}(195$$
$$\underline{196}$$
$$1862$$
$$\underline{1764}$$
$$980$$
$$\underline{980}$$
$$\times$$

(iii) 867 and 255

Ans:- $867 = 255 \times 3 + 102$

$255 = 102 \times 2 + 51$

$102 = 51 \times 2 + 0$

H.C.F of 867 and 255 is → 51

$$255\overline{)867}(3$$
$$\underline{765}$$
$$102$$

$$102\overline{)255}(2$$
$$\underline{204}$$
$$51$$

$$51\overline{)102}(2$$
$$\underline{102}$$
$$\times$$

Ques(2) :- Show that any positive odd integer is of the form $6q+1$, or $6q+3$ or $6q+5$, where q is some integer.

Ans :- ~~Let~~ a is a positive odd integer
a and $b = 6$

by Euclid's division algorithm
$a = bq + r$, $0 < r < 6$
Possible remainders $= 0, 1, 2, 3, 4, 5$
$\therefore$ a can be $6q$, $6q+1$, $6q+3$, $6q+5$

Ques(3) An army contigent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the max. number of columns in which they can march?

Ans :- 32 and 616
$616 > 32$

$616 = 32 \times 19 + 8$
$32 = 8 \times 4 + 0$

HCF of 616 and 32 is $\rightarrow 8$

$$\begin{array}{r} 32 \overline{)616} (19 \\ 32\downarrow \\ \hline 296 \\ 288 \\ \hline 08 \end{array}$$

$$\begin{array}{r} 8 \overline{)32} (4 \\ 32 \\ \hline \times \end{array}$$

(4.) Use Euclid's division lemma to show that the squares of any positive integer is either of the form $3m$ or $3m+1$ for some integer m.
Hint :- Let x be any positive integer then it is of the form $3q$, $3q+1$ or $3q+2$. Now square each

of these and show that they can be rewritten in the form 3m or 3m+1.]

Ans:- by Euclid's division lemma

$a = bq + r \quad , \quad 0 < r < b$

let $b = 3$

$a = 3q + r \quad , \quad 0 < r < 3$

Where a is any positive integer

Case I $\Rightarrow$ $a = 3q$

$a^2 = (3q)^2$

$a^2 = 9q^2$

$a^2 = 3(3q^2)$

$\therefore$ a^2 is in the form of $3m$

Where m is $3q^2$

Case II $\Rightarrow$ $a = 3q + 1$

$a^2 = (3q+1)^2$

$a^2 = 9q^2 + 1 + 6q$

$a^2 = 3(3q^2 + 2q) + 1$

$\therefore$ a^2 is in the form of $3m+1$

Where m is $3q^2 + 2q$

Case III $\Rightarrow$ $a = (3q+2)$

$a^2 = (3q+2)^2$

$a^2 = 9q^2 + 4 + 12q$

$a^2 = 3(3q^2 + 4q + 1) + 1$

$\therefore$ a^2 is in the form of $3m+1$

Where m is $3q^2 + 4q + 1$

Ques5:) Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m+1 or 9m+8.

Ans:- by Euclid's division Lemma

$a = bq + r, \quad 0 < r < b$

Let $b = 3$

$a = 3q + r$

where a is any positive integer

Case I => $a = 3q$

$a^3 = (3q)^3$

$a^3 = 27q^3$

$a^3 = 9(3q^3)$

$\therefore$ a^3 is in the form of $9m$

where $m = 3q^3$

Case II => $a = 3q + 1$

$a^3 = (3q+1)^3$

$a^3 = 27q^3 + 1 + 27q^2 + 9q$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$\therefore$ a^3 is in the form $9m+1$

where $m = 3q^3 + 3q^2 + q$

Case III => $a = 3q + 2$

$a^3 = (3q+2)^3$

$a^3 = 27q^3 + 8 + 54q^2 + 36q$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$\therefore$ a^3 is in the form of $9m+8$ where $m =$
$3q^3 + 6q^2 + 4q$

Hence proved that cube of any positive integer is in the form $9m$, $9m+1$, $9m+8$.

Exercise :- 1.2

Ques 1 Express each number as a product of its prime factors:-

(i) 140

Ans:-

2	140
2	70
7	35
5	5
	1

$140 = 2 \times 2 \times 5 \times 7$
$= 2^2 \times 5 \times 7$

(ii) 156

Ans:-

2	156
2	78
3	39
13	13
	1

$156 = 2 \times 2 \times 3 \times 13$
$= 2^2 \times 3 \times 13$

(iii) 3825

Ans:-

3	3825
3	1275
5	425
5	85
17	17

$3825 = 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$

(iv) 5005

Ans:-

5	5005
7	1001
11	143
13	13
	1

$5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

Ans-

17	7429
19	437
23	23
	1

$7429 = 17 \times 19 \times 23$

Ques(2) Find the L.C.M and HCF of the following pairs of integers and verify that L.C.M × HCF = product of the two No.

(i) 26 and 91

Ans:- $91 > 26$

$91 = 26 \times 3 + 13$

$26 = 13 \times 2 + 0$

H.C.F of 26 and 91 is $\rightarrow 13$

$26 \overline{)91} (3$
78
13

$13 \overline{)26} (2$
26
0

L.C.M =>

13	26 - 91
2	2 - 7
7	1 - 7
	1 - 1

$L.C.M = 13 \times 2 \times 7$
$= 182$

$26 \times 91 = L.CM \times HCF$
$26 \times 91 = 182 \times 13$
$2366 = 2366$

(vii) 510 and 92

Ans:- $510 = 92 \times 5 + 50$
$92 = 50 \times 1 + 42$
$50 = 42 \times 1 + 8$
$42 = 8 \times 5 + 2$
$8 = 2 \times 4 + 0$

H.C.F ⇒ 2

$$92\overline{)510}(5 \quad 460 \quad 50$$
$$50\overline{)92}(1 \quad 50 \quad 42$$
$$42\overline{)50}(1 \quad 42 \quad 8$$
$$4\overline{)8}(2 \quad 8 \quad \times$$

L.CM ⇒

2	510
5	255
3	51
17	17
	1

2	92
2	46
23	23
	1

$510 = \boxed{2} \times 5 \times 3 \times 17$
$92 = \boxed{2} \times 2 \times 23$

$$L.CM = \frac{\text{Product of two No.}}{H.C.F}$$

$$= \frac{510 \times \cancel{92}^{46}}{\cancel{2}}$$

$= 23460$

$510 \times 92 = L.C.M \times H.CF$
$46920 = 23460 \times 2$
$46920 = 46920$

(iii) 336 and 54

Ans:-

2	336
2	168
2	84
2	42
3	21
7	7
	1

2	54
3	27
3	9
3	3
	1

$336 = 2 \times 2 \times 2 \times 3 \times 7$
$54 = 2 \times 3 \times 3 \times 3$

H.C.F => $2 \times 3 = 6$

L.C.M => $\frac{\text{Product of two No.}}{\text{H.C.F}}$

$= \frac{336 \times \cancel{54}^{9}}{\cancel{6}}$

$= 3024$

$336 \times 54 = \text{L.C.M} \times \text{H.C.F}$
$18144 = 3024 \times 6$
$18144 = 18144$

Ques (3) Find the LCM and HCF of the following integers by applying the prime factorisation method

(i) 12, 15 and 21

Ans:-

2	12
2	6
3	3
	1

3	15
5	5
	1

3	21
7	7
	1

$12 = 2 \times 2 \times 3$
$15 = 5 \times 3$
$21 = 7 \times 3$

H.C.F = 3
L.C.M = 2×2×5×7×3
= 420

(ii) 17, 23 and 29

Ans-

17	17
	1

23	23
	1

29	29
	1

$17 = 17 \times 1$
$23 = 23 \times 1$
$29 = 29 \times 1$

H.C.F = 1
L.C.M = 17×23×29×1
= 11339

(iii) 8, 9, 25

Ans-

2	8
2	4
2	2
	1

3	9
3	3
	1

5	25
5	5
	1

8 ⇒ 2×2×2
9 ⇒ 3×3
25 ⇒ 5×5

H.C.F = 1

L.C.M = 2×2×2×3×3×5×5

Ques(4) Given that HCF (306, 657) = 9. find LCM (306, 657)

Ans:- Given that H.C.F of (306, 657) is → 9

$$L.C.M = \frac{\text{Product of two No.}}{H.C.F}$$

$$= \frac{306 \times \cancel{657}^{73}}{\cancel{9}}$$

306×73

$= 22338$

Ques(5) There is a circular path around a sports field. Sonia takes 18 min. to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go the same direction. After how many minutes will they meet again at the starting point?

Ans:- Sonia takes = 18 min.
Ravi takes = 12 min

$$\begin{array}{r|l} 3 & 18 \\ \hline 3 & 6 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$18 = 3 \times 3 \times 2$

$12 = 3 \times 2 \times 2$

$H.C.F = 3 \times 2 = 6$

$L.C.M = 3 \times 2 \times 3 \times 2$

$= 36$

Excercise :- 1.3

Ques(1) Show that $\sqrt{3}$ is irrational.

Ans: Let $\sqrt{5} = \frac{p}{q}$ be a rational No.

where p and q are co-prime $q \neq 0$

Then $\sqrt{5}q = p \Rightarrow 5q^2 = p^2$

$p^2 - 5q^2$ —(i)

Since 5 divides p^2, so it will divide p also

Let $p = 5r$

Then $p^2 = 25r^2$ [Squaring both sides]

$\rightarrow 5q^2 = 25r^2$ (from (i)]

$\rightarrow q^2 = 5r^2$

Since 5 divides q^2 so it will divide q also. Thus 5 is common factor of both p and q. This contradicts our ass that $\sqrt{5}$ is rational

Hence, $\sqrt{5}$ is irrational.

Ques(2) Show that $3+2\sqrt{5}$ is irrational

Ans: Let us suppose that $(3+2\sqrt{5})$ is rational

$\frac{a}{b} = 3+2\sqrt{5} = \frac{a}{b} - 3 = 2\sqrt{5}$

$= \frac{a-3b}{b} = 2\sqrt{5}$

$= \frac{a-3b}{2b} = \sqrt{5}$ ---(1)

a and b are integers.

its means L.H.S of (i) is rational but we know that $\sqrt{5}$ is irrational. It is not possible. Therefore our supposition is wrong $(3+2\sqrt{5})$ cannot be rational

Hence, $3+2\sqrt{5}$ is irrational.

Ques (3.) Prove that they are irrationals :-

(i) $\frac{1}{\sqrt{2}}$

Ans:- Let us suppose that $\frac{1}{\sqrt{2}}$ is rational

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a} \quad \text{--- (1)}$$

R.H.S of (i) is rational but we know that $\sqrt{2}$ is irrational.
It is not possible which means our supposition is wrong.
Therefore, $\frac{1}{\sqrt{2}}$ is irrational

(ii) $7\sqrt{5}$

Ans:- Suppose $7\sqrt{5}$ is rational

$$7\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5} = \frac{a}{7b} \quad \text{--- (1)}$$

R.H.S of (i) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Ans. Let $6+\sqrt{2}$ be rational
therefore, $6+\sqrt{2} = \frac{a}{b}$

$$\sqrt{2} = \frac{a}{b} - 6 \quad \text{---(i)}$$

Since a and b are integers $\frac{a}{b} - 6$ is rational and $\sqrt{2}$ is also rational. This contradicts the fact that $\sqrt{2}$ is irrational.
Hence, $6+\sqrt{2}$ is irrational.

Exercise :→ 1.4

Ques(1)

(i) $\frac{13}{3125}$

Ans:- $3125 = 5 \times 5 \times 5 \times 5 \times 5$
$= 5^5$

5	3125
5	625
5	125
5	25
5	5
	1

The given rational No $\frac{13}{3125}$ will have terminating Expansion.

(ii) $\frac{17}{8}$

Ans:-

$8 = 2 \times 2 \times 2$
$= 2^3$

2	8
2	4
2	2
	1

terminating decimal Expansion.

(iii) $\frac{64}{455}$

Ans:- $455 = 5 \times 13 \times 7$

5	455
13	91
7	7
	1

Non-terminating decimal Expansion.

(iv) $\frac{15}{1600}$

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

Ans- $1600 = 2\times2\times2\times2\times2\times2\times5\times5$
$= 2^6\times5^2$

terminating decimal expansion

(v) $\frac{29}{343}$

7	343
7	49
7	7
	1

Ans- $343 = 7\times7\times7$
$= 7^3$

Non-terminating decimal Expansion

(vi) $\frac{23}{2^3 5^2}$

Ans- terminating decimal expansion

(vii) $\frac{129}{2^2 5^3 7^3}$

Ans- Non terminating decimal expansion

(viii) $\frac{6}{15}$

Ans:- $\frac{6}{5\times3}$

$= \frac{2\times\cancel{3}}{5\times\cancel{3}} = \frac{2}{5}$

5	15
3	3
	1

terminating decimal expansion.

(ix.) $\frac{35}{50}$

Ans:-

$\frac{35}{2\times5^2}$

2	50
5	25
5	5
	1

terminating decimal expansion

(x.) $\frac{77}{210}$

Ans:- $\frac{77}{2\times5\times7\times3}$

2	210
5	105
7	21
3	3
	1

Non-terminating decimal expansion.

Ques (2.) Write down the decimal expansion of those rational No. in ques 1 above which have terminating decimal expansion

Ans: (i), (ii), (iv), (vi), (viii), (ix)

(i) $\frac{13}{3125}$

$\Rightarrow \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{416}{100000}$

$= 0.00416$

(ii) $\frac{17}{8}$

Ans: $\frac{17 \times 5^3}{2^3 \times 5^3} = \frac{2125}{1000}$

$= 2.125$

(iv) $\frac{15}{1600}$

Ans: $\frac{15 \times 5^4}{2^6 \times 5^2 \times 5^4} = \frac{9375}{1000000}$

$= 0.009375$

(vi) $\frac{23}{2^3 5^2}$

Ans:

$\frac{23 \times 5}{2^3 5^2 \times 5} = \frac{115}{}$

(viii) $\frac{6}{15}$

Ans) $\frac{2 \times 2}{5 \times 2} = \frac{4}{10}$

$= 0.4$

(ix) $\frac{35}{50}$

Ans:- $\frac{35 \times 2}{2^1 5^2 \times 2^1} = \frac{70}{100}$

0.7

Ques (3)

(i) 43.123456789

Ans:- It is terminating so it is rational No. $R = \frac{p}{q}$ for q we can say that the P.F of q is in the form of $2^n \times 5^m$

(ii) 0.120112001200...

Ans!- It is irrational No.

(iii) $43.\overline{123456789}$

Ans:- It is non-terminating. It is a rational No

$R = \frac{p}{q}$

for q we can say that PF of q is in the form of $2^n \times 5^m$