Chapter – 7

Properties of Matter

Multiple Choice Questions

Question 1.
Consider two wires X and Y. The radius of wire X is 3 times the radius of Y. If they are stretched by the same load then the stress on Y is ……….
(a) equal to that on X
(b) thrice that on X
(c) nine times that on X
(d) Half that on X

Answer:
(c) nine times that on X

Solution:

$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress on X} = \frac{F_x}{A} = \frac{F_x}{\pi r_x^2} \quad ; \text{Stress on Y} = \frac{F_y}{A} = \frac{F_y}{\pi r_y^2}$$

If $F_x = F_y = F$, So $r_x = 3r_y$

$$\frac{(\text{Stress})_x}{(\text{Stress})_y} = \frac{\pi r_y^2}{\pi r_x^2} = \frac{r_y^2}{9r_y^2}$$

$$(\text{Stress})_y = 9(\text{stress})_x$$

Question 2.
If a wire is stretched to double of its original length, then the strain in the wire is ………
(a) 1
(b) 2
(c) 3
(d) 4

Answer:
(d) 4

Solution:

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{(2L - L)}{L}$$

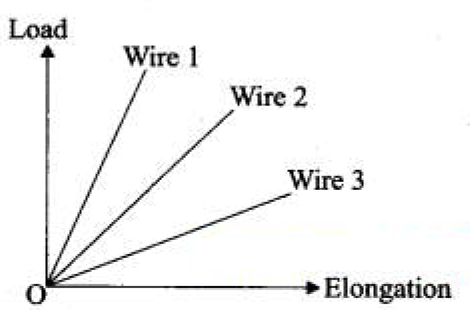
$$\text{Strain} = \frac{L}{L} = 1$$

In this case Young's modulus is equal to stress

Question 3.
The load-elongation graph of three wires of the same material are shown in figure. Which of the following wire is the thickest?
(a) wire 1
(b) wire 2
(c) wire 3
(d) all of them have same thickness

Answer:
(a) wire 1

Solution:



Wire 1 is the thickest compared to other wires.
Because the elongation of the wire 1 is minimum.

Question 4.
For a given material, the rigidity modulus is $\left(\frac{1}{3}\right)^{rd}$ of Young's modulus. Its Poisson's ratio is
(a) 0

(b) 0.25
(c) 0.3
(d) 0.5

Answer:
(d) 0.5

Solution:
The relationship of Poisson's ratio, rigidity and Young's modulus is

$$\eta = \frac{Y}{2(1-\sigma)} \qquad \sigma - \text{Poisson's ratio}$$

Here,
$$\eta = \frac{1}{3}y \Rightarrow 3\eta = y$$

Substituting this value, we get $\boxed{\sigma = 0.5}$

Question 5.
A small sphere of radius 2 cm falls from rest in a viscous liquid. Heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity is proportional to ……… [NEET model 2018]
(a) 2^2
(b) 2^3
(c) 2^4
(d) 2^5

Answer:
(d) 2^5

Solution:
Rate of heat produced

$$\frac{dQ}{dt} = F_V \times v_t \qquad F_V = 6\pi\eta r v_t$$

$$= 6\pi\eta r v_t \times v_t \qquad v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)}{\eta}g$$

$$\frac{dQ}{dT} \propto r v_T^2 \qquad v_T \propto r^2$$

$$\frac{dQ}{dT} \propto r(r^2)^2 \Rightarrow \frac{dQ}{dT} \propto r^5$$

Here radius of the sphere is 2 cm, so $\frac{dQ}{dT} \propto 2^5$

Question 6.
Two wires are made of the same material and have the same volume. The area of cross sections of the first and the second wires are A and 2A respectively. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount? (NEET model 2018)
(a) 2
(b) 4
(c) 8
(d) 16

Answer:
(b) 4

Solution:
Since the two wires have same volume
Length of wire 1 = l, Area of wire 1 = A
Length of wire 2 = l/2, Area of wire 2 = 2A

Wire 1: $Y_1 = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$ Wire 2: $Y_2 = \frac{\frac{F'}{A}}{\frac{\Delta l}{(l/2)}}$

$$Y_1 = Y_2$$

$$\frac{F}{A} \times \frac{l}{\Delta l} = \frac{F'}{2A} \times \frac{l}{2\Delta l}$$

$$F = \frac{F'}{4} \Rightarrow F' = 4F$$

Question 7.
With an increase in temperature, the viscosity of liquid and gas, respectively will

(a) increase and increase
(b) increase and decrease
(c) decrease and increase
(d) decrease and decrease

Answer:
(c) decrease and increase

Question 8.
The Young's modulus for a perfect rigid body is ……..
(a) 0
(b) 1
(c) 0.5
(d) infinity

Answer:
(d) infinity

Question 9.
Which of the following is not a scalar?
(a) viscosity
(b) surface tension
(c) pressure
(d) stress

Answer:
(d) stress

Question 10.
If the temperature of the wire is increased, then the Young's modulus will …….
(a) remain the same
(b) decrease
(c) increase rapidly
(d) increase by very a small amount

Answer:
(b) decrease

Solution:
As temperature is increased, the strain i.e., change dimensions of the body is increased, as a result, the stiffness of the material is reduced. Which causes decrease in magnitude of modulus of elasticity.

Question 11.
Copper of fixed volume V is drawn into a wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. If Y represents the Young's modulus, then which of the following graphs is a straight line? [NEET 2014 model]
(a) Δl versus V
(b) Δl versus Y
(c) Δl versus F
(d) Δl versus 1/l

Answer:
(c) Δl versus F

Solution:
From Young's modulus,

$$Y = \frac{Fl}{A\Delta l} = \frac{Fl}{\left(\frac{V}{l}\right)\Delta l} = \frac{Fl^2}{V \cdot \Delta l}$$

$$\Delta l = \frac{F \cdot l^2}{V \cdot Y}$$

Question 12.
A certain number of spherical drops of a liquid of radius R coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then

(a) energy $= 4V\ T\left(\frac{1}{r} - \frac{1}{R}\right)$ is released
(b) energy $= 3V\ T\left(\frac{1}{r} + \frac{1}{R}\right)$ is absorbed
(c) energy $= 3V\ T\left(\frac{1}{r} - \frac{1}{R}\right)$ is released
(d) energy is neither released nor absorbed

Answer:
(c) energy $= 3V\ T\left(\frac{1}{r} - \frac{1}{R}\right)$ is released

Question 13.
The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?
(a) length = 200 cm, diameter = 0.5 mm
(b) length = 200 cm, diameter = 1 mm
(c) length = 200 cm, diameter = 2 mm
(d) length = 200 cm, diameter = 3 mm

Answer:
(a) length = 200 cm, diameter = 0.5 mm

Solution:
From Young's modulus,
$$\Delta l = \frac{Fl}{\pi r^2 Y} \; ; \; \Delta l \propto \frac{l}{r^2}$$

Question 14.
The wettability of a surface by a liquid depends primarily on
(a) viscosity
(b) surface tension
(c) density
(d) angle of contact between the surface and the liquid

Answer:
(d) angle of contact between the surface and the liquid

Question 15.
In a horizontal pipe of non-uniform cross section, water flows with a velocity of 1 ms^{-1} at a point where the diameter of the pipe is 20 cm. The velocity of water 1.5 (ms^{-1}) at a point where the diameter of the pipe is.
(a) 8
(b) 16
(c) 24
(d) 32

Answer:
(b) 16

Solution:
From equation of continuity,

$$a_1 v_1 = a_2 v_2 \qquad v_1 = 1\ \text{ms}^{-1}$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2 \qquad v_2 = 1.5\ \text{ms}^{-1}$$

$$(10 \times 10^{-2})^2 \times 1 = r_2^2 \times 1.5 \qquad d_1 = 20\ \text{cm}$$

$$r_2^2 = \frac{100 \times 10^{-4}}{1.5} \qquad r_1 = \frac{d_1}{2} = 10\ \text{cm}$$

$$r^2 = 8\ \text{cm} \qquad r_2 = ?$$

$$\therefore \quad d_2 = 16\ \text{cm}$$

Short Answer Questions

Question 1.
Define stress and strain.

Answer:
Stress: The restoring force per unit area of a deformed body is known as stress.

Strain: Strain produced in a body is defined as the ratio of change in size of a body to the original size.

Question 2.
State Hooke's law of elasticity.

Answer:
The stress is proportional to the strain in the elastic limit.

Stress $\propto$ Strain

$$\alpha \propto \varepsilon \Rightarrow \frac{F}{A} \propto \frac{\Delta L}{L}$$

Question 3.
Define Poisson's ratio.

Answer:

Poisson's ratio, which is defined as the ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Question 4.

Explain elasticity using intermolecular forces.

Answer:

Elastic behaviour of solid. In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. When deforming force is applied on a body so that its length increases, then the molecules of the body go far apart.

Question 5.

Which one of these is more elastic, steel or rubber? Why?

Answer:

Steel is more elastic than rubber. If an equal stress is applied to both steel and rubber, the steel produces less strain. So the Young's modulus is higher for steel than rubber. The object which has higher young's modulus is more elastic.

Question 6.

A spring balance shows wrong readings after using for a long time. Why?

Answer:

When a spring balance has been used for a long time, it develops an elastic fatigue, the spring of such a balance take longer time to recover its original configuration and therefore it does not give correct measurement.

Question 7.

What is the effect of temperature on elasticity?

Answer:

As the temperature of substance increases, its elasticity decreases.

Question 8.
Write down the expression for the elastic potential energy of a stretched wire.

Answer:
The work done in stretching the wire by dl,
dW = F.dl
The total work done in stretching the wire from 0 to l is

$$W = \int_0^l F \cdot dl \quad \text{...(1)}$$

From Young's modulus of elasticity, force becomes,

$$F = \frac{YAl}{L} \quad \text{...(2)}$$

Substituting equation (2) in (1) we get,

$$W = \int_0^l \frac{YAl'}{L} dl = \frac{YA}{L}\left[\frac{l'^2}{2}\right]_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left[\frac{YAl}{L}\right]l = \frac{1}{2}Fl$$

$$W = \frac{1}{2}Fl$$

This work done is known as the elastic potential energy of a stretched wire.

Question 9.
State Pascal's law in fluids.

Answer:
If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.

Question 10.
State Archimedes principle.

Answer:
If states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

Question 11.
What do you mean by upthrust or buoyancy?

Answer:
The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called upthrust or buoyant force.

Question 12.
State the law of floatation.

Answer:
The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body.

Question 13.
Define coefficient of viscosity of a liquid.

Answer:
The coefficient of viscosity of a liquid is the viscous force acting tangentially per unit area of a liquid layer having a unit velocity gradient in a direction perpendicular to the direction of flow of the liquid.

Question 14.
Distinguish between streamlined flow and turbulent flow.

Answer:

Streamlined flow	Turbulent Flow
When a liquid flow such that each particle of the liquid passing a point moves along the same path and has the same velocity as its predecessor than the flow of liquids is said to be streamlined flow.	During the flow of fluid, when the critical velocity is exceeded by the moving fluid, the motion becomes turbulent.

Question 15.
What is Reynold's number? Give its significance.

Answer:
It is a dimensionless number which determines the nature of the flow of fluid through a pipe. Reynold's number is given by,

$$R_c = \frac{\rho v D}{\eta}$$

If,

$$R_c < 1000 \text{ – stream line}$$
$$R_c > 2000 \text{ – turbulent}$$
$$1000 < R_c < 2000 \text{ – unsteady}$$

Question 16.
Define terminal velocity.

Answer:
The maximum constant velocity acquired by a body while falling freely through a viscous medium is called the terminal velocity V_T.

Question 17.
Write down the expression for the Stoke's force and explain the symbols involved in it.

Answer:
The viscous force F acting on a spherical body of radius r depends directly on:
(i) radius (r) of the sphere
(ii) velocity (v) of the sphere and
(iii) coefficient of viscosity η of the liquid

$$[MLT^{-2}] = k\,[ML^{-1}T^{-1}]^x \times [L]^y \times [LT^{-1}]^z$$

On solving, we get x = 1, y = 1 and z = 1. Therefore, F = kηrv
Experimentally, Stoke found that the value of k = 6π
F = 6πηrv This relation is known as Stoke's law.

Question 18.
State Bernoulli's theorem.

Answer:

It states that the sum of pressure energy, kinetic energy and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined, irrotational flow remains constant along
a streamline.

i.e., $\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant}$

Question 19.
What are the energies possessed by a liquid? Write down their equations.

Answer:
A liquid in motion possesses following three types of energy:

(i) Kinetic energy: It is the energy possessed by a liquid by virtue of its motion.

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$\text{K.E. per unit mass} = \frac{\frac{1}{2}mv^2}{m} = \frac{v^2}{2}$$

(ii) Potential energy: It is the energy possessed by a liquid by virtue of its height above the ground level.
P.E. = mgh

$$\text{P.E. per unit mass} = \frac{mgh}{m} = gh$$

(iii) Pressure energy: It is the energy possessed by a liquid by virtue of its pressure. Pressure energy per unit mass = Pm/v

Question 20.
Two streamlines cannot cross each other. Why?

Answer:
If two streamlines cross each other, there will be two directions of flow at the point of intersection which is impossible.

Question 21.
Define surface tension of a liquid. Mention its S.I. Unit and dimension.

Answer:
The surface of a liquid is defined as the, force per unit length of a liquid (or) the energy per unit area of the surface of a liquid. T = F/l
SI unit and dimensions of T are Nm^{-1} and MT^{-2}

Question 22.
How is surface tension related to surface energy?

Answer:
The surface energy per unit area of a surface is numerically equal to the surface tension.

Question 23.
Define angle of contact for a given pair of solid and liquid.

Answer:
The angle between tangents drawn at the point of contact to the liquid surface and solid surface inside the liquid is called the angle of contact for a pair of solid and liquid. It is denoted by θ.

Question 24.
Distinguish between cohesive and adhesive forces.

Answer:
The force between the like molecules which holds the liquid together is called 'cohesive force'. When the liquid is in contact with a solid, the molecules of the these solid and liquid will experience an attractive force which is called 'adhesive force'.

Question 25.
What are the factors affecting the surface tension of a liquid?

Answer:

1. The presence of any contamination or impurities.
2. The presence of dissolved substances.
3. Electrification
4. Temperature

Question 26.
What happens to the pressure inside a soap bubble when air is blown into it?

Answer:
When air is blown into a soap bubble, the pressure inside a bubble is decreased $P = 4T/R$

Question 27.
What do you mean by capillarity or capillary action?

Answer:
In a liquid whose angle of contact with solid is less than 90°, suffers capillary rise. On the other hand, in a liquid whose angle of contact is greater than 90°, suffers capillary fall. The rise or fall of a liquid in a narrow tube is called capillarity or capillary action.

Question 28.
A drop of oil placed on the surface of water spreads out. But a drop of water placed on oil contracts to a spherical shape. Why?

Answer:
The surface tension of the water is more than that of oil. Therefore, when oil is poured over water, the greater value of surface tension of water pulls oil in all directions, and as such it spreads on the water. On the other hand, when water is poured over oil, it does not spread over it because the surface tension of oil being less than that of water, it is not able to pull water over it.

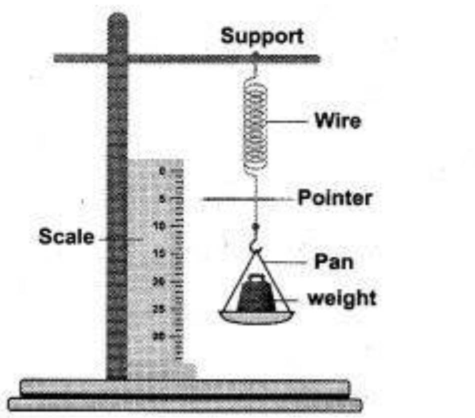
Question 29.
State the principle and usage of venturi meter.

Answer:
This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem.

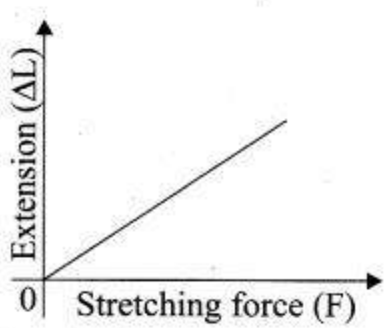
Long Answer Questions

Question 1.
State Hooke's law and verify it with the help of an experiment.

Answer:
Hooke's law is for a small deformation, when the stress and strain are proportional to each other. It can be verified in a simple way by stretching a thin straight wife (stretches like spring) of length L and uniform cross sectional area A suspended from a fixed-point O. A pan and a pointer are attached at the free end of the wire.



(a) Experimental verification of Hooke's law



(b) Variation of ΔL with F

The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL. It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X-axis and ΔL on the Y-axis.

Therefore $\Delta L = (\text{slope})F$

Multiplying and dividing by volume,

$V = AL$

$$F(\text{slope}) = \frac{AL}{AL}$$

Rearranging, we get $\frac{F}{A} = \left(\frac{L}{A(\text{slope})}\right)\frac{\Delta L}{L}$

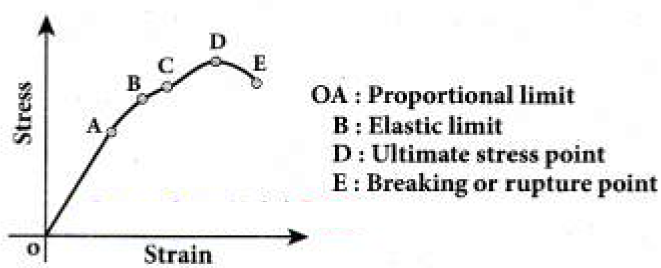
Therefore, $\frac{F}{A} \propto \left(\frac{\Delta L}{L}\right)$

Comparing with stress and strain equations,

$\sigma \propto \varepsilon$

i.e., the stress is proportional to the strain in the elastic limit.

Stress-strain profile curve: The stress versus strain profile is a plot in which stress and strain are noted for each load and a graph is drawn taking strain along the X-axis and stress along the Y-axis. The elastic characteristics of the materials can be analyzed from the stress-strain profile.



Stress-Strain profile

(a) Portion OA: In this region, stress is very small such that stress is proportional to strain, which means Hooke's law is valid. The point A is called limit of proportionality because above this point Hooke's law is not valid. The slope of the line OA gives the Young's modulus of the wire.

(b) Portion AB: This region is reached if the stress is increased by a very small amount. In this region, stress is not proportional to the strain. But once the stretching force is removed, the wire will regain its original length. This behaviour ends at point B and hence, the point B is known as yield point (elastic limit). The elastic hebaviour of the material (here wire) in stress-strain curve is OAB.

(c) Portion BC: If the wire is stretched beyond the point B (elastic limit), stress increases and the wire will not regain its original length after the removal of stretching force.

(d) Portion CD: With further increase in stress (beyond the point C), the strain increases rapidly and reaches the point D. Beyond D, the strain increases even when the load is removed and breaks (ruptures) at the point E. Therefore, the maximum stress (here D) beyond which the wire breaks is called breaking stress or tensile strength. The corresponding point D is known as fracture point. The region BCDE represents the plastic behaviour of the material of the wire.

Question 2.
Explain the different types of modulus of elasticity.

Answer:
From Hooke's law, the stress in a body is proportional to the corresponding strain, provided the deformation is very small. Here we shall define the elastic modulus of a given material. There are three types of elastic modulus.
(a) Young's modulus
(b) Rigidity modulus (or Shear modulus)
(c) Bulk modulus

Young's Modulus: When a wire is stretched or compressed, then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modules.

$$\text{Young modulus of a material} = \frac{\text{Tensile stress or compressive stress}}{\text{Tensile strain or compressive strain}}$$

$$Y = \frac{\sigma_t}{\varepsilon_t} \text{ or } Y = \frac{\sigma_c}{\varepsilon_c}$$

S.I. unit of Young modulus is Nm^{-2} or pascal.
Bulk modulus: Bulk modulus is defined as the ratio of volume stress to the volume strain.

$$\text{Bulk modulus, } K = \frac{\text{Normal (Perpendicular) stress or Pressure}}{\text{Volume strain}}$$

$$\text{The normal stress or pressure is } \sigma_n = \frac{F_n}{\Delta A} = \Delta P$$

$$\text{The volume strain is } \varepsilon_v = \frac{\Delta V}{V}$$

$$\text{Therefore, Bulk modulus is } K = -\frac{\sigma_n}{\varepsilon_v} = \frac{\Delta P}{\frac{\Delta V}{V}}$$

The negative sign indicates when pressure is applied on the body, its volume decreases. Further, the equation implies that a material can be easily compressed if it has a small value of bulk modulus. In other words, bulk modulus measures the resistance of solids to change in their volume. For an example, we know that gases can be easily compressed than solids, which means, gas has a small value of bulk modulus compared to solids. The S.I. unit of K is the same as that of pressure i.e., N m^{-2} or Pa (pascal).

The rigidity modulus or shear modulus: The rigidity modulus is defined as the ratio of the shearing stress to shearing strain,

$$\eta_R = \frac{\text{shearing stress}}{\text{angle of shear or shearing strain}}$$

$$\text{The shearing stress is } \sigma_s = \frac{\text{Tangential force}}{\text{area over which it is applied}} = \frac{Ft}{\Delta A}$$

$$\text{The angle of shear or shearing strain } \varepsilon_s = \frac{x}{h} = \theta$$

$$\text{Therefore, Rigidity modulus is } \eta_R = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{\frac{F_t}{\Delta A}}{\theta}$$

Further, the above implies, that a material can be easily twisted if it has small value of rigidity modulus. For example, consider a wire, when it is twisted through an angle θ, a restoring torque is developed, that is
$\tau \propto \theta$.

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since, rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small. The S.I. unit of ηR is the same as that of pressure i.e., N m^{-2} or pascal.

Question 3.
Derive an expression for the elastic energy stored per unit volume of a wire.

Answer:
When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy.

Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension 1 and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.
The work done in stretching the wire by dl, dW = Fdl
The total work done in stretching the wire from 0 to l is

$$W = \int_0^l F\,dl \qquad ...(1)$$

From Young's modulus of elasticity

$$Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{YAl}{L} \qquad ...(2)$$

Substituting equation (2) in equation (1), we get

$$W = \int_0^l \frac{YAl}{L}\,dl$$

Since, l is the dummy variable in the integration, we can change l to l' (not in limits), therefore

$$W = \int_0^l \frac{YAl'}{L}\,dl' = \frac{YA}{L}\left(\frac{l'^2}{2}\right)_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right)l = \frac{1}{2}Fl$$

$$W = \frac{1}{2}Fl = \text{Elastic potential energy}$$

Energy per unit volume is called energy density

$$u = \frac{\text{Elastic potential energy}}{\text{Volume}} = \frac{\frac{1}{2}Fl}{AL}$$

$$\frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}(\text{Stress} \times \text{Strain})$$

Question 4.
Derive an equation for the total pressure at a depth 'A' below the liquid surface.

Answer:

In order to understand the increase in pressure with depth below the water surface, consider a water sample of cross-sectional area in the form of a cylinder. Let h_1 and h_2 be the depths from the air-water interface to level 1 and level 2 of the cylinder, respectively. Let F_1 be the force acting downwards on level 1 and F_2 be the force acting upwards on level 2, such that, $F_1 = P_1 A$ and $F_2 = P_2A$. Let us assume the mass of the sample to be m and under equilibrium condition, the total upward force (F_2) is balanced by the total downward force ($F_1 + mg$), in other words, the gravitational force will act downward which is being exactly balanced by the difference between the force $F_2 – F_1$.

$F_2 – F_1 = mg$ ...... (1)

where m is the mass of the water available in the sample element. Let ρ be the density of the water then, the mass of water available in the sample element is

$m = \rho V = \rho A(h_2 – h_1)$

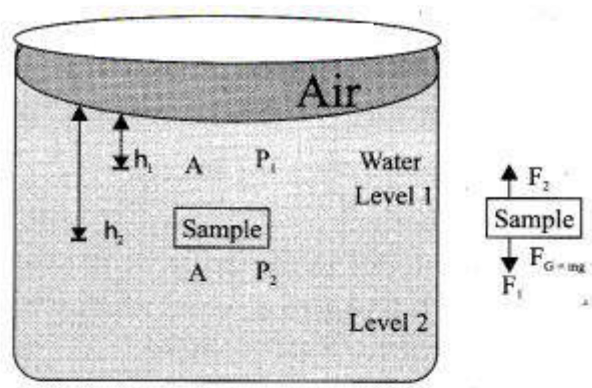
$V = A(h_2 – h_1)$

Hence, gravitational force, $F_G = \rho A (h_2 – h_1)g$

On substituting the W value in equation (1)

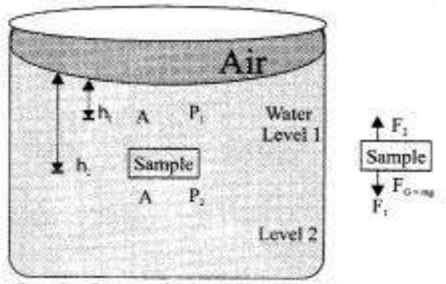
$F_2 = F_1 + mg \Rightarrow P_2A = P_1A + \rho A(h_2 – h_1)g$

Cancelling out A on both sides,



Sample of water with base area A in a static fluid with its forces in equilibrium

$P_2 = P_1 + \rho(h_2 – h_1)g$ ...(2)



Sample of water with base area A in a static fluid with its forces in equilibrium

If we choose the level 1 at the surface of the liquid (i.e., air-water interface) and the level 2 at a depth 'h' below the surface, then the value of h_1 becomes zero ($h_1 = 0$) and in turn P_1 assumes the value of atmospheric pressure (say P_a). In addition, the pressure (P_2) at a depth becomes P. Substituting these values in equation, we get

$P = P_a + \rho gh$ …… (3)

which means, the pressure at a depth h is greater than the pressure on the surface of the liquid, where Pa is the atmospheric pressure which is equal to 1.013×10^5 Pa.

If the atmospheric pressure is neglected or ignored then $P = \rho gh$ ….. (4)
for a given liquid, ρ is fixed and g is also constant, then the pressure due to the fluid column is directly proportional to vertical distance or height of the fluid column. This implies, the height of the fluid column is more important to decide the pressure and not the cross sectional or base area or even the shape of the container.

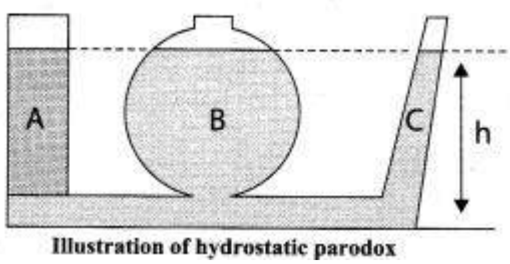
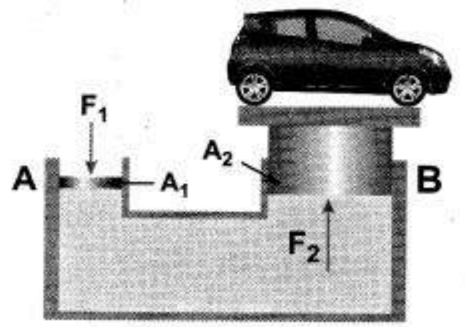


Illustration of hydrostatic parodox

Let us consider three vessels of different shapes A, B and C as shown in figure. These vessels are connected at the bottom by a horizontal pipe. When they are filled with a liquid (say water), it occupies the same level even though the vessels hold different amounts of water. It is true because the liquid at the bottom of each section of the vessel experiences the same pressure.

Question 5.
State and prove Pascal's law in fluids.

Answer:
Statement of Pascal's law: If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.



Hydraulic lift

Application of Pascal's law: Hydraulic lift: A practical application of Pascal's law is the hydraulic lift which is used to lift a heavy load with a small force. It is a force Hydraulic lift multiplier. In consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid. They are fitted with frictionless pistons of cross-sectional areas A_1 and A_2 ($A_2 > A_1$). Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increase to $P\left(\text{where, } P = \frac{F_1}{A_1}\right)$. But according to Pascal's law, this increased pressure P is transmitted undiminished in all directions. So a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2 \Rightarrow F_2 = \frac{A_2}{A_1} \times F_1$$

Therefore by changing the force on the smaller piston A, the force on the piston B has been increased by the factor A2/A1 and this factor is called the mechanical advantage of the lift.

Question 6.
State and prove Archimedes principle.

Answer:
Archimedes Principle: It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the

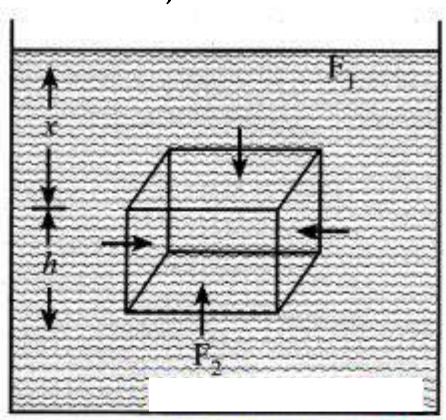
fluid displaced by it and its upthrust acts through the centre of gravity of the liquid displaced.

Upthrust (or) buoyant force = Weight of liquid displaced
Proof: Consider a body of height 'h' lying inside a liquid of density ρ, at a depth x below the free surface of the liquid. Area of cross section of the body is 'a'.

The forces on the sides of the body cancel out.
Pressure at the upper face of the body, $P_1 = x\rho g$
Pressure at the lower face of the body, $P_2 = (x + h)\rho g$
Thrust acting on the upper face of the body is $F_1 = P_1 a = x\rho ga$ acting vertically downwards,

Thrust acting on the lower face of the body is $F_2 = P_2 a = (x + h)\ \rho ga$ acting vertically upwards.

The resultant force $(F_2 – F_1)$ is acting on the body direction and is called upthrust (U).
$U = F_2 – F_1 = (x + h)\ \rho ga – x\rho ga = ah\rho g$
But ah = V, Volume of the body = Volume of liquid



Buoyant force on a body

$U = V\rho g = Mg$
i.e., Upthrust or buoyant force = Weight of liquid displaced.
This proves the Archimedes principle.

Question 7.
Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.

Answer:

Expression for terminal velocity: Consider a sphere of radius r which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.
Gravitational force acting on the sphere,

$$F_G = mg = \frac{4}{3}\pi r^3 \rho g \text{ (downward force)}$$

$$\text{Up thrust, } U = \frac{4}{3}\pi r^3 \sigma g \text{ (upward force)}$$

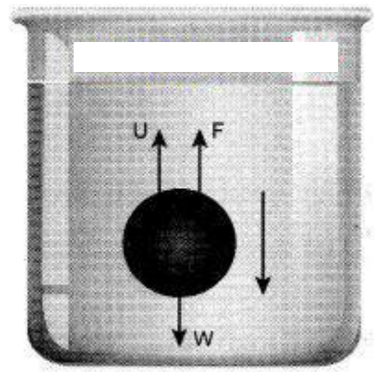
Viscous force $F = 6\pi\eta r v_t$

At terminal velocity v_t,

Downward force = upward force

$$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\sigma r^3 \sigma g = 6\pi\eta r v_t$$

$$v_t = \frac{2}{9} \times \frac{r^2(\rho - \sigma)}{\eta} g \Rightarrow v_t \propto r^2$$



Forces acting on the sphere when it falls in a viscous liquid

Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius. If a is greater than ρ, then the term (ρ – σ) becomes negative leading to a negative terminal velocity.

Question 8.

Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.

Answer:

Consider a liquid flowing steadily through a horizontal capillary tube. Let v = (v/t) be the volume of the liquid flowing out per second through a capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient (P/l).

Then, $$v \propto \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$v = k\eta^a r^b \left(\frac{P}{l}\right)^c \quad \text{...(1)}$$

where, k is a dimensionless constant.

Therefore, $[v] = \dfrac{\text{Volume}}{\text{Time}} = [L^3T^{-1}]; \left[\dfrac{dP}{dx}\right] = \dfrac{\text{Pressure}}{\text{Distance}} = [ML^{-2}T^{-2}]$

$$[\eta] = [ML^{-1}T^{-1}] \text{ and } [r] = [L]$$

Then, $$v \propto \eta^a r^b \left(\frac{P}{l}\right)^c$$

$$v = k\eta^a r^b \left(\frac{P}{l}\right)^c \qquad \ldots(1)$$

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Therefore, $[v] = \dfrac{\text{Volume}}{\text{Time}} = [L^3T^{-1}]; \left[\dfrac{dP}{dx}\right] = \dfrac{\text{Pressure}}{\text{Distance}} = [ML^{-2}T^{-2}]$

$$[\eta] = [ML^{-1}T^{-1}] \text{ and } [r] = [L]$$

Substituting in equation (1)

$$[L^3T^{-1}] = [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$$

$$M^0L^3T^{-1} = M^{a+c}L^{-a+b-2c}T^{-a-2c}$$

So, equating the powers of M, L and T on both sides, we get

a + c = 0, -a + b – 2c = 3, and – a – 2c = – 1

We have three unknowns a, b and c. We have three equations, on solving, we get

a = – 1, b = 4 and c = 1

Therefore, equation (1) becomes,

$$v = k\eta^{-1}r^4\left(\frac{P}{l}\right)^1$$

Experimentally, the value of k is shown to be π/8, we have

$$v = \frac{\pi r^4 P}{8\eta l}$$

The above equation is known as Poiseuille's equation for the flow of liquid through a narrow tube or a capillary tube. This relation holds good for the fluids whose velocities are lesser than the critical velocity (v_c).

Question 9.

Obtain an expression for the excess of pressure inside a

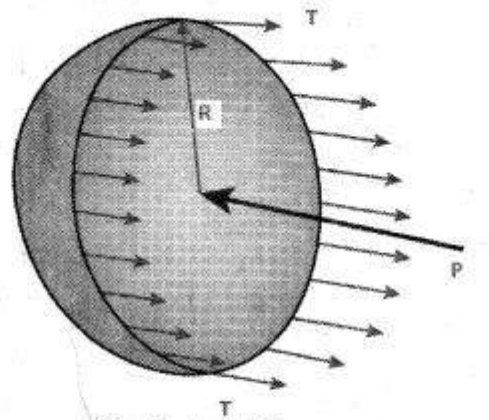
(i) liquid drop

(ii) liquid bubble

(iii) air bubble.

Answer:

1. Excess of pressure inside air bubble in a liquid:
Consider an air bubble of radius R inside a liquid having surface tension T. Let P_1 and P_2 be the pressures outside and inside the air bubble, respectively.
Now, the excess pressure inside the air bubble is $\Delta P = P_1 - P_2$
In order to find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble. For the hemispherical portion of the bubble, considering the forces acting on it, we get,

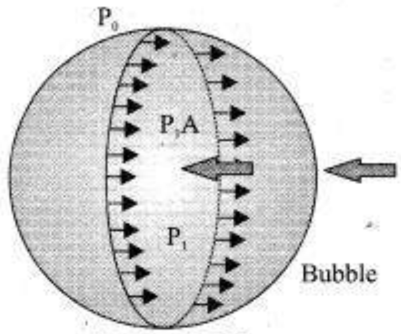


(a) Air bubble

(i) The force due to surface tension acting towards right around the rim of length $2\pi R$ is $F_T = 2\pi RT$
(ii) The force due to outside pressure P, is to the right acting across a cross sectional area of πR^2 is $P_1 \pi R^2$

(iii) The force due to pressure P_2 inside the bubble, acting to the left is
$F_{P_2} = P_2 \pi R^2$
As the air bubble is in equilibrium under the action of these forces,
$F_{P_2} = F_T + F_{P_1}$ Excess pressure is $\Delta P = P_2 - P_1 = 2T/R$

2. Excess pressure inside a soap bubble: Consider a soap bubble of radius R and the surface tension of the soap bubble be T. A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other outside the bubble. Therefore, the force on the soap bubble due to surface tension is $2 \times 2\pi RT$. The various forces acting on the soap bubble are,



(b) Soap bubble

(i) Force due to surface tension $F_T = 4\pi RT$ towards right.

(*ii*) Force due to outside pressure, $F_{P_1} = P_1\pi R^2$ towards right.

(*iii*) Force due to inside pressure, $F_{P_2} = P_2\pi R^2$ towards left.

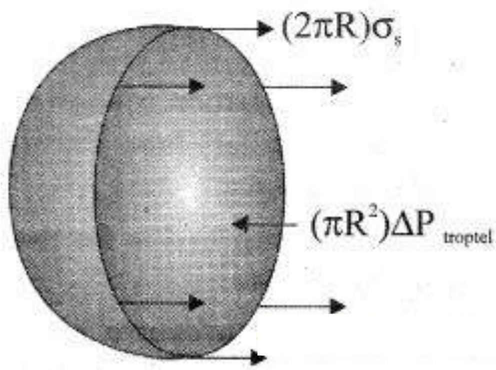
As the bubble is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 4\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 4\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{4T}{R}$

3. Excess pressure inside the liquid drop: Consider a liquid drop of radius R and the surface tension of the liquid is T.

The various forces acting on the liquid drop are:



(*i*) Force due to surface tension $F_T = 2\pi RT$ towards right.

(*ii*) Force due to outside pressure $F_{P_1} = P_1\pi R^2$ towards right.

(*iii*) Force due to inside pressure $F_{P_2} = P_2\pi R^2$ towards left.

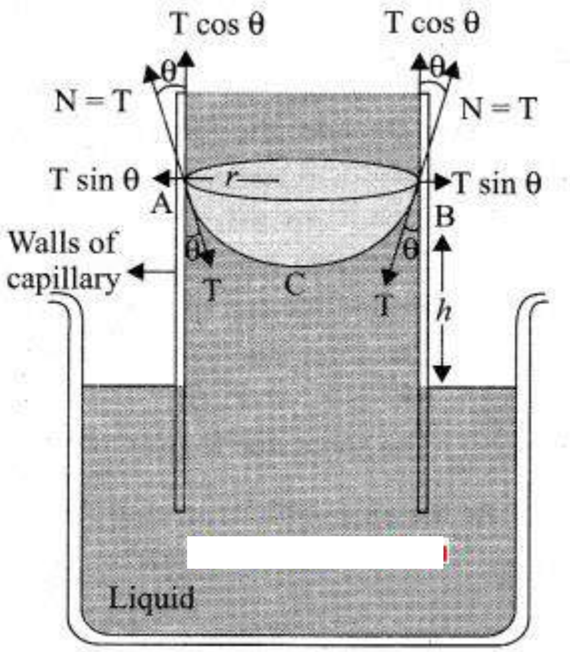
As the drop is in equilibrium, $F_{P_2} = F_T + F_{P_1}$

$$P_2\pi R^2 = 2\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$$

Excess pressure is $\Delta P = P_2 - P_1 = \dfrac{2T}{R}$

Question 10.
What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.

Answer:
In a liquid whose angle of contact with solid is less than 90°, suffers capillary rise. On the other hand, in a liquid whose angle of contact is greater than 90°, suffers capillary fall. The rise or fall of a liquid in a narrow tube is called capillarity or capillary action.



Capillary rise by surface tension

Practical application of capillarity
(i) Due to capillary action, oil rises in the cotton within an earthen lamp. Likewise, sap raises from the roots of a plant to its leaves and branches.
(ii) Absorption of ink by a blotting paper.
(iii) Capillary action is also essential for the tear fluid from the eye to drain constantly.
(iv) Cotton dresses are preferred in summer because cotton dresses have fine pores which act as capillaries for sweat.

Surface Tension by capillary rise method: The pressure difference across a curved liquid-air interface is the basic factor behind the rising up of water in a narrow tube (influence of gravity is ignored). The capillary rise is more dominant in the case of very fine tubes. But this phenomenon is the outcome of the force of surface tension. In order to arrive a relation between the capillary rise (h) and surface tension (T), consider a capillary tube which is held vertically in a beaker containing water, the water rises in the capillary tube to a height h due to surface tension.

The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, it resolved into two components

(i) Horizontal component T sin θ and
(ii) Vertical component T cos θ acting upwards, all along the whole circumference of the meniscus Total upward force = (T cos θ) (2πr) = 2πrT cos θ where θ is the angle of contact, r is the radius of the tube. Let ρ be the density of water and h be the height to which the liquid rises inside the tube. Then,

$$\begin{pmatrix}\text{The volume of liquid}\\ \text{column in the tube, V}\end{pmatrix} = \begin{pmatrix}\text{Volume of the liquid}\\ \text{column of radius } r \text{ height } h\end{pmatrix} + \begin{pmatrix}\text{Volume of liquid of radius } r \text{ and height } h\\ -\text{Volume of the hemisphere of radius } r\end{pmatrix}$$

$$\text{V} = \pi r^2 h + \left(\pi r^2 \times r - \frac{2}{3}\pi r^3\right)$$

$$\Rightarrow \quad \text{V} = \pi r^2 h + \frac{1}{3}\pi r^3$$

The upward force supports the weight of the liquid column above the free surface, therefore,

$$2\pi r \text{T} \cos\theta = \pi r^2\left(h + \frac{1}{3}r\right)\rho g \Rightarrow T = \frac{r\left(h + \frac{1}{3}r\right)\rho g}{2\cos\theta}$$

If the capillary is a very fine tube of radius (i.e., radius is very small) then r/3 can be neglected when it is compared to the height h. Therefore,

$$\text{T} = \frac{r\rho g h}{2\cos\theta}$$

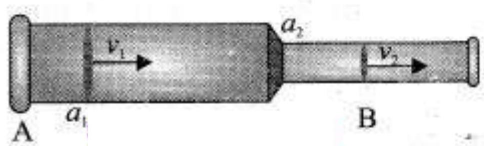
Liquid rises through a height h

$$h = \frac{2\text{T}\cos\theta}{r\rho g} \Rightarrow h \propto \frac{1}{r}$$

This implies that the capillary rise (h) is inversely proportional to the radius (r) of the tube, i. e., the smaller the radius of the tube greater will be the capillarity.

Question 11.
Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

Answer:
The mass flow rate through a pipe, it is necessary to assume that the flow of fluid is steady, the flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant with respect to time.



A streamlined flow of fluid through a pipe of varying cross sectional area

Under this condition, the path taken by the fluid particle is a streamline. Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous -and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 and a_2, respectively.

Let m_1 be the mass of fluid flowing through section A in time Δt, $m_1 = (a_1 v_1 \Delta t)\rho$.

Let m_2 be the mass of fluid flowing through section B in time Δt, $m_2 = (a_2 v_2 \Delta t)\rho$.

For an incompressible liquid, mass is conserved $m_1 = m_2$

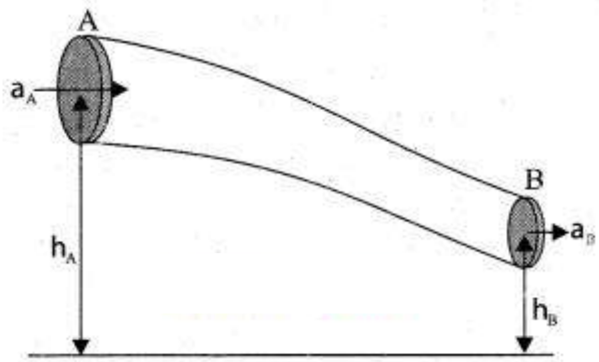
$$a_1 v_1 \Delta t \rho = a_2 v_2 \Delta t \rho$$
$$a_1 v_1 = a_2 v_2 \Rightarrow a\ v = \text{constant}$$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general, a v = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

Question 12.
State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.

Answer:
Bernoulli's theorem: According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant. Mathematically,



Flow of liquid through a pipe AB

$$\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}$$

This is known as Bernoulli's equation.
Proof: Let us consider a flow of liquid through a Flow of liquid through a pipe AB pipe AB. Let V be the volume of the liquid when it enters A in a time t which is equal to the volume of the liquid leaving B in the same time. Let a_A, v_A and P_A be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at A respectively.

Let the force exerted by the liquid at A is $F_A = P_A a_A$
Distance travelled by the liquid in time t is $d = v_A t$

Therefore, the work done is $W = F_A d = P_A a_A v_A t$

But $a_A v_A t = a_A d = V$, volume of the liquid entering at A.

Thus, the work done is the pressure energy (at A), $W = F_A d = P_A V$

Pressure energy per unit volume at A $= \frac{\text{Pressure energy}}{\text{Volume}} = \frac{P_A V}{V} = P_A$

$$\text{Pressure energy per unit mass at A} = \frac{\text{Pressure energy}}{\text{Mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$$

since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is

liquid at A is

$$E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$$

Potential energy of the liquid at A,

$$PE_A = mg\,h_A$$

Due to the flow of liquid, the kinetic energy of the liquid at A,

$$KE_A = \frac{1}{2} m\,v_A^2$$

Therefore, the total energy due to the flow of liquid at A,

$$E_A = E_{PA} + KE_A + PE_A$$

$$E_A = m\frac{P_A}{\rho} + \frac{1}{2}mv_A^2 + mg\,h_A$$

Similarly, let a_B, v_B and P_B be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at B. Calculating the total energy at E_B, we get

$$E_B = m\frac{P_B}{\rho} + \frac{1}{2}mv_B^2 + mg\,h_B$$

From the law of conservation of energy, $E_A = E_B$

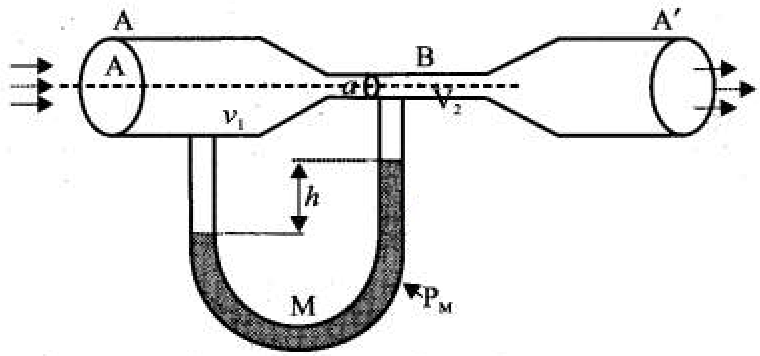
$$m\frac{P_A}{\rho} + \frac{1}{2}mv_A^2 + mgh_A = m\frac{P_B}{\rho} + \frac{1}{2}mv_B^2 + mg\,h_B$$

$$\frac{P_A}{\rho} + \frac{1}{2}v_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2}v_B^2 + gh_B = \text{constant}$$

Thus, the above equation can be written as

$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}$$

The above equation is the consequence of the conservation of energy which is true until there is no loss of energy due to friction. But in practice, some energy is lost due to friction. This arises due to the fact that in a fluid flow, the layers flowing with different velocities exert frictional forces on each other. This loss of energy is generally converted into heat energy. Therefore, Bernoulli's relation is strictly valid for fluids with zero viscosity or non-viscous liquids. Notice that when the liquid flows through a horizontal pipe, then



A schematic diagram of venturimeter

$$h = 0 \Rightarrow \frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$

Question 13.
Describe the construction and working of venturi meter and obtain an equation for the volume of liquid flowing per second through a wider entry of the tube.

Answer:
Venturi meter : This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem. It consists of two wider tubes A and A' (with cross sectional area A) connected by a narrow tube B (with cross sectional area a). A manometer in the form of U-tube is also attached between the wide and narrow tubes.

The manometer contains a liquid of density 'ρ_m'.
Let P_1 be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density 'ρ' flows from the pipe with speed 'v_1' and into the narrow region, its speed increases to 'v_2'. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure P_2 at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B' is noted by measuring the height difference ($\Delta P = P_1 - P_2$) between the surfaces of the manometer liquid.

From the equation of continuity, we can say that $Av_1 = av_2$ which means that

$$v_2 = \frac{A}{a} v_1$$

Using Bernoulli's equation, $P_1 + \rho\frac{v_1^2}{2} = P_2 + \rho\frac{v_2^2}{2} = P_2 + \rho\frac{1}{2}\left(\frac{A}{a}v_1\right)^2$

From the above equation, the pressure difference

$$\Delta P = P_1 - P_2 = \rho\frac{v_1^2}{2}\frac{(A^2 - a^2)}{a^2}$$

Thus, the speed of flow of fluid at the wide end of the tube A

$$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2 - a^2)} \Rightarrow v_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}}$$

The volume of the liquid flowing out per second is

$$V = Av_1 = A\sqrt{\frac{2(\Delta P)a^2}{\rho(A^2 - a^2)}} = aA\sqrt{\frac{2(\Delta P)}{\rho(A^2 - a^2)}}$$

Numerical Problems

Question 1.
A capillary of diameter dmm is dipped in water such that the water rises to a height of 30 mm. If the radius of the capillary is made of its previous value, then compute the height up to which water will rise in the new capillary?

Answer:
We know the height of the capillary rise $h = \frac{2T\cos\theta}{rdg}$

where T = surface tension, r = radius, d = density, g = acceleration due to gravity

Now, $h_1 r_1 = h_2 r_2 \Rightarrow \frac{h_2}{h_1} = \frac{r_1}{r_2}$

Here $r_2 = \frac{2}{3} r_1$ and $h_1 = 30$ mm

$$h_2 = \frac{r_1}{\left(\frac{2}{3}\right) r_1} \times 30 = \frac{3}{2} \times 30$$

$$\boxed{h_2 = 45 \text{ mm}}$$

Question 2.
A cylinder of length 1.5 m and diameter 4 cm is fixed at one end. A tangential force of 4×10^5 N is applied at the other end. If the rigidity modulus of the cylinder is 6×10^{10} Nm^{-2} then, calculate the twist produced in the cylinder.

Answer:

Torsion of a cylinder $\tau = \dfrac{\pi \eta r^4 \phi}{2l}$

Twist produced in the cylinder $\phi = \dfrac{\tau \times (2l)}{\pi \eta r^4} = \dfrac{(F \times l) \times (2l)}{\pi \eta r^4}$

$$= \frac{4 \times 10^5 \times 2 \times (1.5)^2}{3.14 \times 6 \times 10^{10} \times (2 \times 10^{-2})^4} = \frac{18 \times 10^5}{3.01 \times 10^{-6} \times 10^{10}}$$

$$\boxed{\phi = 59.80}$$

Question 3.
A spherical soap bubble A of radius 2 cm is formed inside another bubble B of radius 4 cm. Show that the radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is lesser than radius of both soap bubbles A and B.

Answer:
From the excess pressure inside a soap bubble

$$\Delta P = \frac{4T}{R}$$

Here the two bubbles having the same pressure and temperature. So the radius of the combined bubbles,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$$

$$R = \frac{4}{3} = 1.33$$

$$\therefore \quad \boxed{R = 1.33 \text{ cm}}$$

Question 4.

A block of Ag of mass x kg hanging from a string is immersed in a liquid of relative density 0.72. If the relative density of Ag is 10 and tension in the string is 37.12 N then compute the mass of Ag block.

Answer:

From the terminal velocity condition, $F_G - U = F$ F = T, m = x

$$mg - mg\left(\frac{\rho_w}{\rho}\right) = T$$

$$mg\left[1 - \frac{0.72}{10}\right] = 37.12$$

$$9.8x\,[1 - 0.072) = 37.12$$

$$9.0944x = 37.12$$

$$\therefore \quad \boxed{x = 4\text{ kg}}$$

Question 5.

The reading of pressure meter attached with a closed pipe is 5×10^5 Nm^{-2}. On opening the valve of the pipe, the reading of the pressure meter is 4.5×10^5 Nm^{-2}. Calculate the speed of the water flowing in the pipe.

Answer:

Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = P_1 - P_2$$

Here initial velocity $V_1 = 0$ and density of water p = 1000 kg m^{-3}

$$\frac{1}{2} \times 10^3 \times v_2^2 = (5 - 4.5) \times 10^5$$

$$v_2^2 = \frac{0.5 \times 10^5 \times 2}{10^3} = 1 \times 10^2\ ;\ v_2^2 = 100$$

$$\boxed{v_2 = 10\text{ ms}^{-1}}$$

Conceptual Questions

Question 1.
Why coffee runs up into a sugar lump (a small cube of sugar) when one corner of the sugar lump is held in the liquid?

Answer:
Dip the comer of a sugar cube in coffee, and get the whole cube coffee-flavoured due to "capillary action".

Question 2.
Why two holes are made to empty an oil tin?

Answer:
When oil comes out through a tin with one hole, the pressure inside the tin becomes less than the atmospheric pressure, soon the oil stops flowing out. When two holes are made in the tin, air keeps on entering the tin through the other hole and maintains pressure inside.

Question 3.
We can cut vegetables easily with a sharp knife as compared to a blunt knife. Why?

Answer:
The area of a sharp edge is much less than the area of a blunt edge. For the same total force, the effective force per unit area is more for the sharp edge than the blunt edge. Hence, a sharp knife cuts easily than a blunt knife.

Question 4.
Why the passengers are advised to remove the ink from their pens while going up in an aeroplane?

Answer:
We know that atmospheric pressure decreases with height. Since ink inside the pen is filled at the atmospheric pressure existing on the surface of Earth, it tends to come out to equalise the pressure. This can spoil the clothes of the passengers, so they are advised to remove the ink from the pen.

Question 5.
We use straw to suck soft drinks, why?

Answer:
When we suck through the straw, the pressure inside the straw becomes less than the atmospheric pressure. Due to the pressure difference, the soft drink rises in the straw and we are able to take the soft drink easily.