CHAPTER :- 09

— Pressure Vessels —

Pressure vessels is defined as a closed cylinderical or spherical container designed to store fluids at a pressure substantially different from ambient pressure:

Pressure Vessels

- Thin P.V. $\frac{D}{t} \geq 20$
- Thick P.V. $\frac{D}{t} \leq 20$

- Cylindrical P.V. $\left(\frac{D}{t} \geq 20\right)$
- Spherical P.V. $\left(\frac{D}{t} < 20\right)$

D = Inner dia of shell

t = thickness

Thin C.P.V.

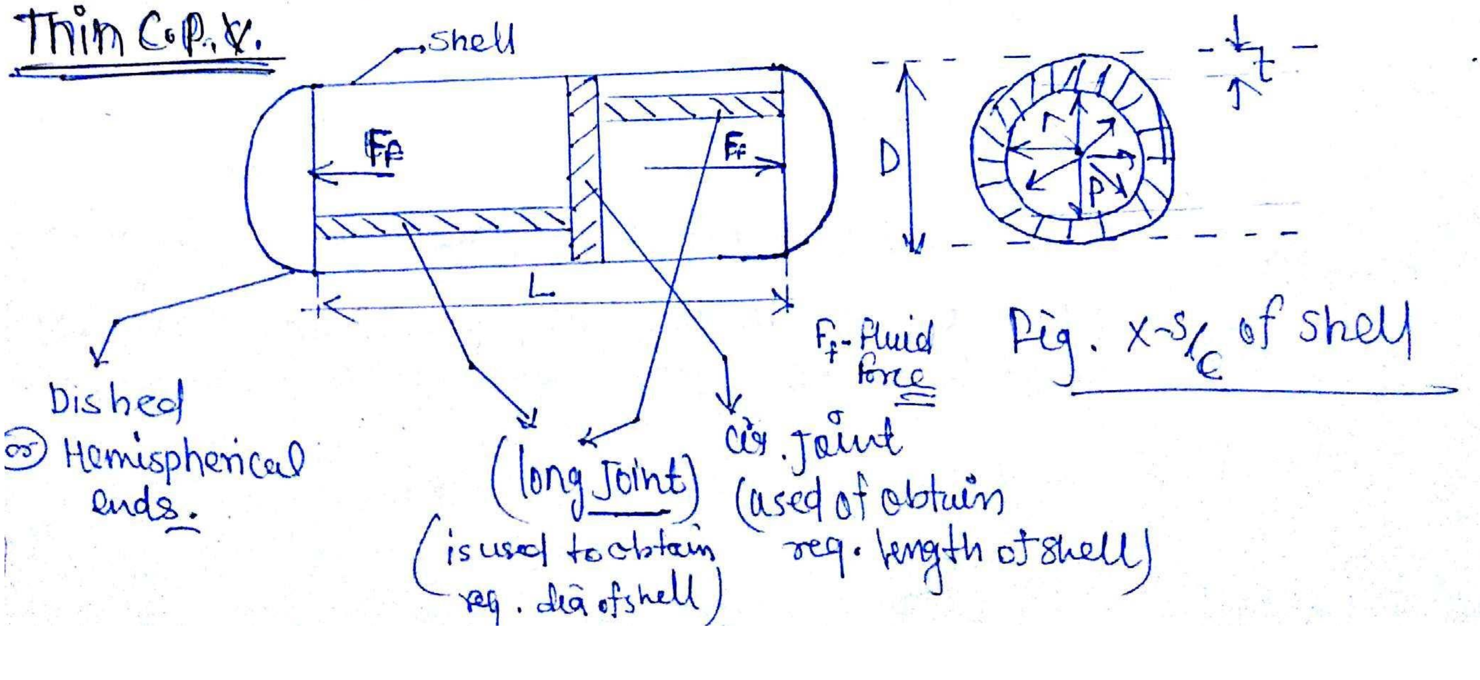


Fig. X-S/C of shell

$p \uparrow \longrightarrow F_f \uparrow$ $\qquad \sigma_H = \frac{Pd}{2t}$

$\Rightarrow L \uparrow$ $\qquad \sigma_L = \frac{Pd}{4t}$

$\Rightarrow \epsilon_{long} (\uparrow)$

$\Rightarrow \sigma_{long} (\uparrow)$

At a particular pressure (P)

$\sigma_{long} > S_{ut}$ $\longrightarrow$ bursting of P.V. occurs circumferentially.

Resisting Area ($\pi D t$)

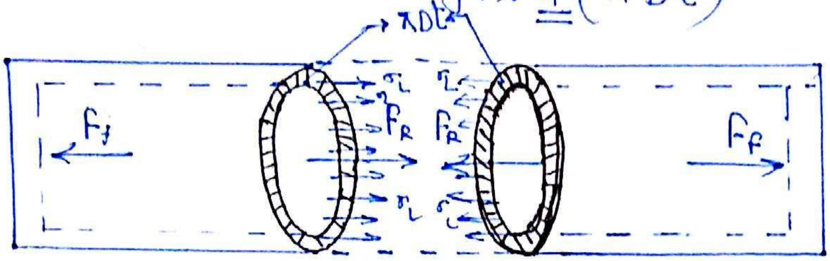


Fig bursting of P.V. circumferentially
ie ($\sigma_L > S_{ut}$) or ($F_f > F_R$)

$$F_f = F_R$$

$$p \times \frac{\pi}{4} D^2 = \sigma_L \times \pi D t$$

$$\boxed{\sigma_L = \frac{PD}{4t} \text{ or } \frac{PD}{4t\,\eta_{C.J.}}}$$

Auto Frettage - is a oldest method of prestressing the Cylinder
(method of increasing pressure capacity
also ~~increases~~ improve endurance strength)

$P \uparrow \rightarrow F_f \uparrow$

$\rightarrow$ Dia ($\uparrow$) $\rightarrow$ Circumference ($\uparrow$)

$\rightarrow \epsilon_{hoop} (\uparrow)$

$\rightarrow \sigma_{hoop} (\uparrow)$

At a particular pre. (P)

$\sigma_{hoop} > S_{ut}$

$\Rightarrow$ bursting of P.V. occur ~~circumferentially~~ longitudinally

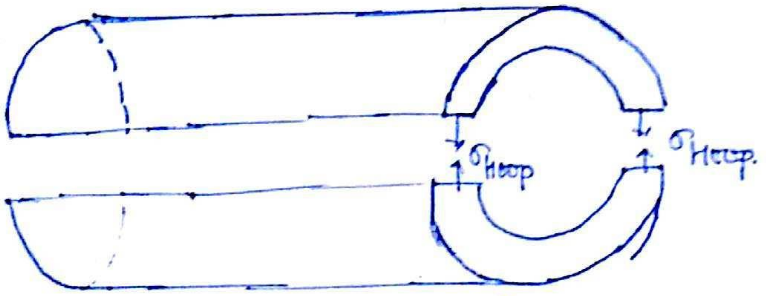
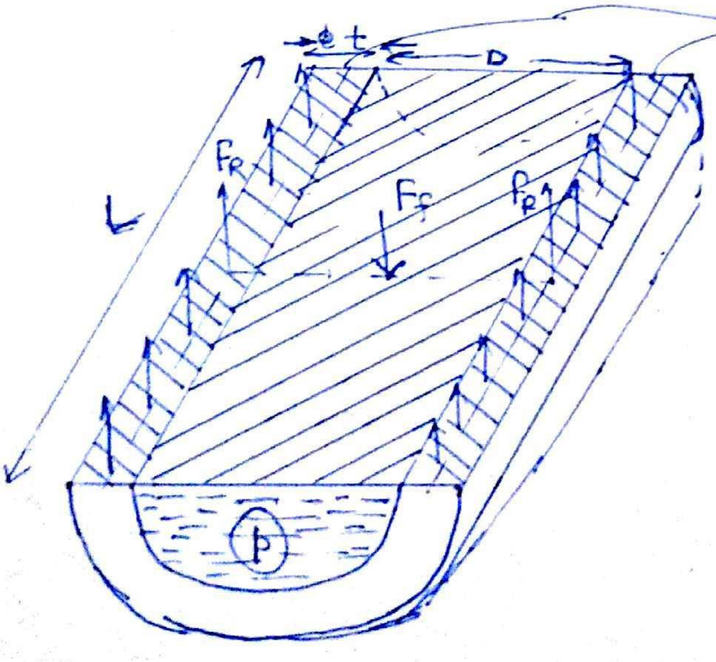


fig bursting of P.V. longitudinally [i.e. $\sigma_h > \sigma_{ut}$]



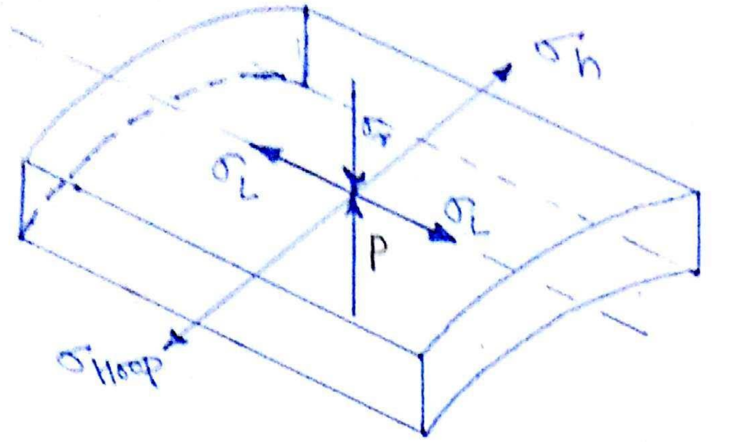
If $F_f > 2 F_R$

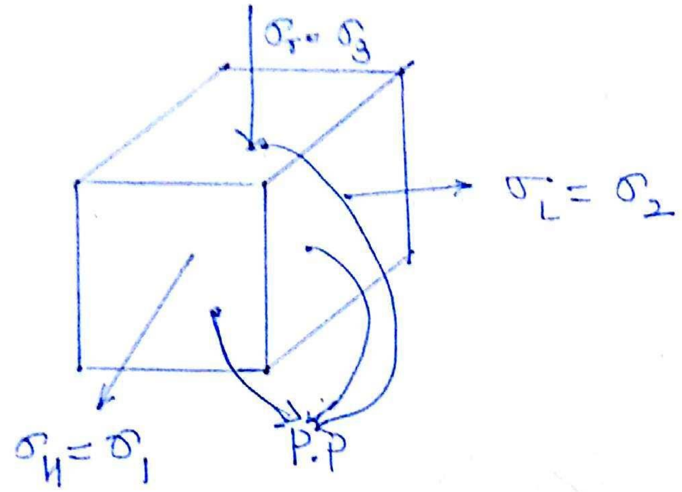
Safe Cond^n

$F_f = 2 F_R$

$P \times L \times D = 2 \times \sigma_h \times L \times t$

$$\boxed{\sigma_h = \frac{PD}{2t} \text{ or } \frac{PD}{2t\,\eta_{L.j.}}}$$





$$\sigma_1 = \sigma_H = \frac{PD}{2t}$$

$$\sigma_2 = \sigma_L = \frac{PD}{4t} = \frac{\sigma_1}{2}$$

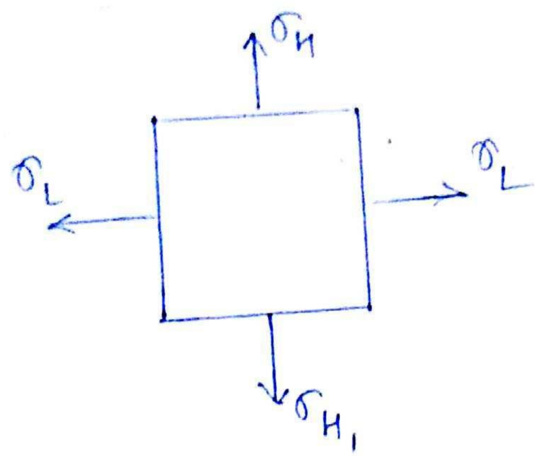
$$\sigma_3 = -\sigma_r = -P$$ (Neglect whe P less)

e.g. $p = 10\,MPa$, $D = 1\,m$; $t = 20\,mm$

$$\sigma_H = \frac{10 \times 1000}{2 \times 20} = 250\,MPa, \quad \sigma_L = 125\,MPa, \quad \sigma_r = 10\,MPa$$

★ $\sigma_r <<< \sigma_H$ & σ_L

σ_r can be neglected hence bi-axial state of stress is assumed.



$$\sigma_1 = \sigma_H = \frac{Pd}{2t}$$

$$\sigma_2 = \sigma_L = \frac{Pd}{4t} = \frac{\sigma_1}{2}$$

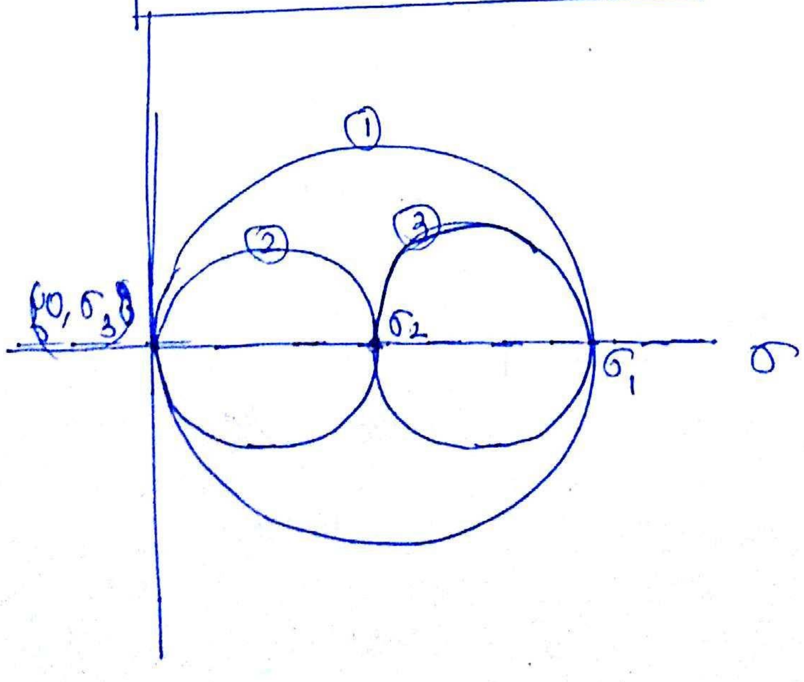
Fig:- Bi-axial state of stress at a point on shell of thin C.P.

$$\text{In plane } \tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1}{4} = \frac{PD}{8t}$$

$$\text{In plane } \tau_{max} = \frac{\sigma_2}{2} = \frac{PD}{8t}$$

$$\text{In } \cancel{\text{plane}} \text{ abs } \tau_{max} = \frac{\sigma_1}{2} = \frac{PD}{4t}$$

$$\boxed{\text{Abs. } \tau_{max} = \frac{PD}{4t}}$$



1. σ_1 & σ_3
2. σ_2 & σ_3
3. σ_1 & σ_2

$$\epsilon_{hoop} = \epsilon_1 = \frac{\delta D}{D} \quad -\text{(I)}$$

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \mu\sigma_2] \quad -\text{(II)}$$

$$\text{(I)} = \text{(II)}$$

$$\epsilon_{hoop} = \epsilon_1 = \frac{\delta D}{D} = \frac{1}{E}\left(\sigma_1 - \mu\frac{\sigma_1}{2}\right)$$

$$\epsilon_{hoop} = \epsilon_1 = \frac{\delta D}{D} = \frac{\sigma_1}{2E}(2-\mu)$$

$$\epsilon_{Hoop} = \epsilon_1 = \frac{\delta D}{D} = \frac{PD}{4tE}(2-\mu)$$

$$\epsilon_{long} = \frac{\delta L}{L} = \epsilon_2 \quad -\text{(III)}$$

$$\epsilon_2 = \frac{1}{E}(\sigma_2 - \mu\sigma_1) \quad -\text{(IV)}$$

$$\text{(III)} = \text{(IV)}$$

$$\epsilon_2 = \epsilon_{long} = \frac{\delta L}{L} = \frac{1}{E}\left[\frac{\sigma_1}{2} - \mu\sigma_1\right]$$

$$\epsilon_2 = \epsilon_{long} = \frac{\delta L}{L} = \frac{\sigma_1}{2E}[1-2\mu]$$

$$\epsilon_2 = \epsilon_{long} = \frac{PD}{4tE}[1-2\mu]$$

$$\epsilon_v = \frac{\delta V}{V} = \frac{\delta L}{L} + 2\left(\frac{\delta D}{D}\right)$$

$$\epsilon_v = \frac{\delta V}{V} = \epsilon_{long} + 2\,\epsilon_{Hoop}$$

$$\epsilon_v = \frac{\delta V}{V} =$$

$$\epsilon_v = \frac{\delta V}{V} = \frac{PD}{4tE}\left[5 - 4k\right]$$

For sphere (Thick) (under internal pressure)

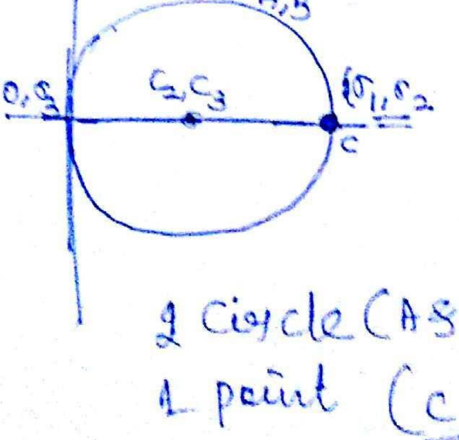
$$\sigma_1 = \sigma_2 = \sigma_H = \frac{PD}{4t} \quad ; \quad \sigma_3 = 0$$

In plane $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 0$

— " — $= \frac{\sigma_2}{2} = \frac{PD}{8t}$

— " — $= \sigma_1/2 = \frac{PD}{8t}$

Abs. $\tau_{max} = \frac{PD}{8t}$



2 circle (A & B)
1 point (C)

$$\epsilon_{Hoop} = \epsilon_1 = \frac{\delta D}{D} \quad -(1)$$

$$\epsilon_1 = \epsilon_2 = \frac{1}{E}[\sigma_1 - \mu\sigma_2] \quad -(2)$$

(1) = (2)

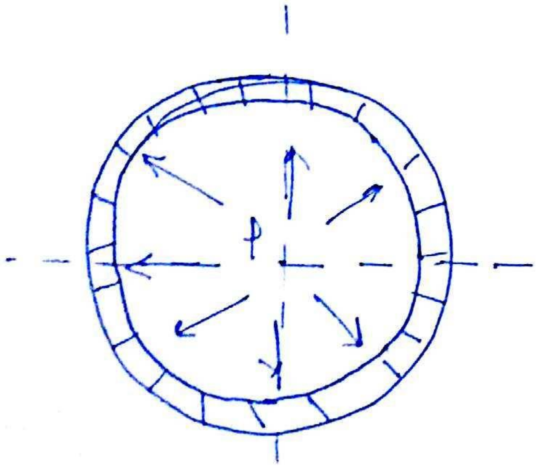
$$\epsilon_{Hoop} = \epsilon_1 = \frac{\delta D}{D} = \frac{1}{E}(\sigma_1 - \mu\sigma_2)$$

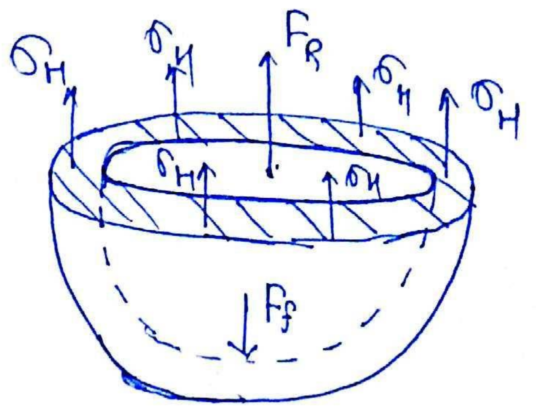
$$\epsilon_1 = \frac{\sigma_1}{E}(1-\mu)$$

$$\epsilon_{Hoop} = \epsilon_1 = \frac{\delta D}{D} = \frac{PD}{4tE}[1-\mu]$$

$$\epsilon_V = \frac{\delta V}{V} = 3\left(\frac{\delta D}{D}\right) = 3\,\epsilon_{Hoop}$$

$$\epsilon_V = \frac{\delta V}{V} = \frac{3PD}{4tE}(1-\mu)$$





→ Spheric Pressure Vassel Under Internal pressure

$$F_f = F_R$$

$$P \times \frac{\pi}{4} D^2 = \sigma_H \pi D t$$

$$\boxed{\sigma_H = \frac{PD}{4t}}$$

Thick Pressure Vassels $\left(\frac{D}{t} < 20\right)$

⇒ Hoop stress Varies parabolically

⇒ ~~long. stress dist linearly~~

⇒ Radial Compressive parabolically

⇒ long. tensile const along length

lame's eqⁿs :-

Cylinderical pressure Vassels

$$p_x = -a + \frac{b}{x^2} \quad -(I)$$

$$(\sigma_h)_x = a + \frac{b}{x^2} \quad -(II)$$

$$(\sigma_r)_x = -p_x = a - \frac{b}{x^2}$$

Spherical pressure Vassels

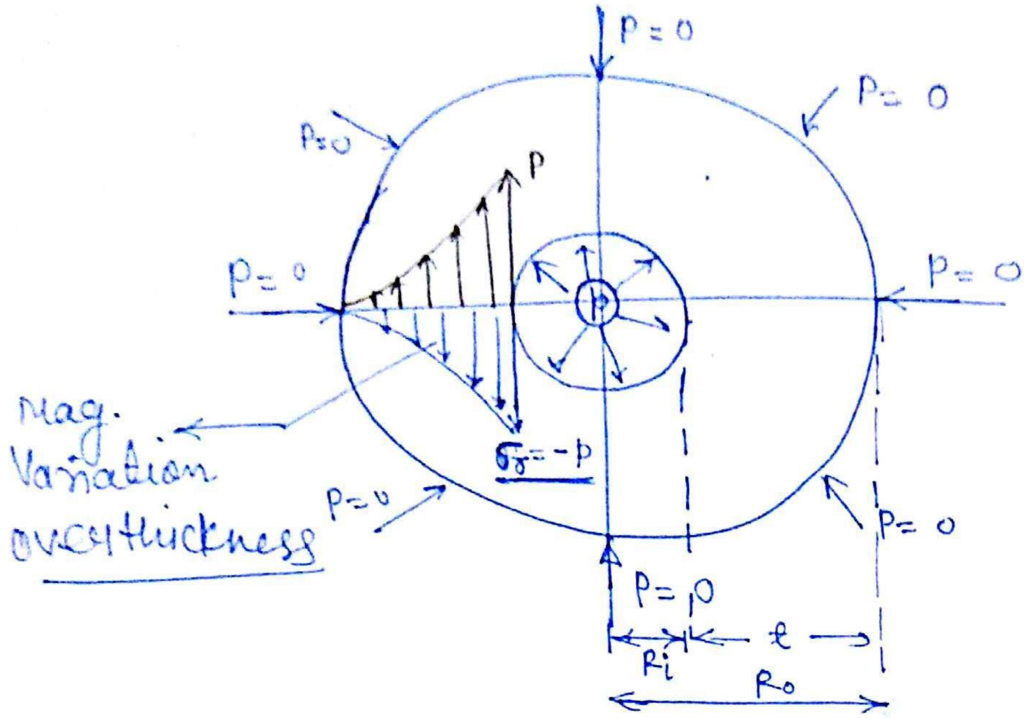
$$p_x = -a + \frac{2b}{x^3}$$

$$(\sigma_h)_x = a + b/x^3$$

where a & b are lame's Constant

⇒ det. by using B.C. of pressure.

Case-I When Internal pressure acting alone :-



$$x = R_i \Rightarrow P_x = P_i = P$$

$$x = R_o \Rightarrow P_x = P_e = 0$$

by using pressure eq^n (1)

$$P = -a + \frac{b}{R_i^2} \quad \text{— (a)}$$

$$0 = -a + \frac{b}{R_o^2} \quad \text{— (b)}$$

$$\quad + \qquad -$$

$$p = 0 + b\left[\frac{1}{R_i^2} - \frac{1}{R_o^2}\right]$$

$$b = p\left[\frac{R_i^2 R_o^2}{R_o^2 - R_i^2}\right] = \text{+ve}$$

from eq^n (b) $\quad a = \frac{b}{R_o^2} = \frac{P R_i^2}{R_o^2 - R_i^2}$

by sub. values of a & b in $(\sigma_h)_x$ eqⁿ

$\underline{\underline{x = R_i}}$

$$(\sigma_h)_{x=R_i} = p\left[\frac{R_i^2}{R_o^2 - R_i^2}\right] + p\left[\frac{R_i^2 R_o^2}{R_o^2 - R_i^2}\right] \times \frac{1}{R_i^2}$$

$$(\sigma_h)_{x=R_i} = p\left[\frac{R_o^2 + R_i^2}{R_o^2 - R_i^2}\right] \underset{+ve}{=} (\sigma_h)_{max}$$

$\underline{\underline{x = R_o}}$

$$(\sigma_h)_{x=R_o} = p\left[\frac{R_i^2}{R_o^2 - R_i^2}\right] + p\left[\frac{R_i^2 R_o^2}{R_o^2 - R_i^2}\right] \times \frac{1}{R_o^2}$$

$$(\sigma_h)_{x=R_o} = p\left[\frac{2R_i^2}{R_o^2 - R_i^2}\right] \underset{+ve}{=} (\sigma_h)_{min}$$

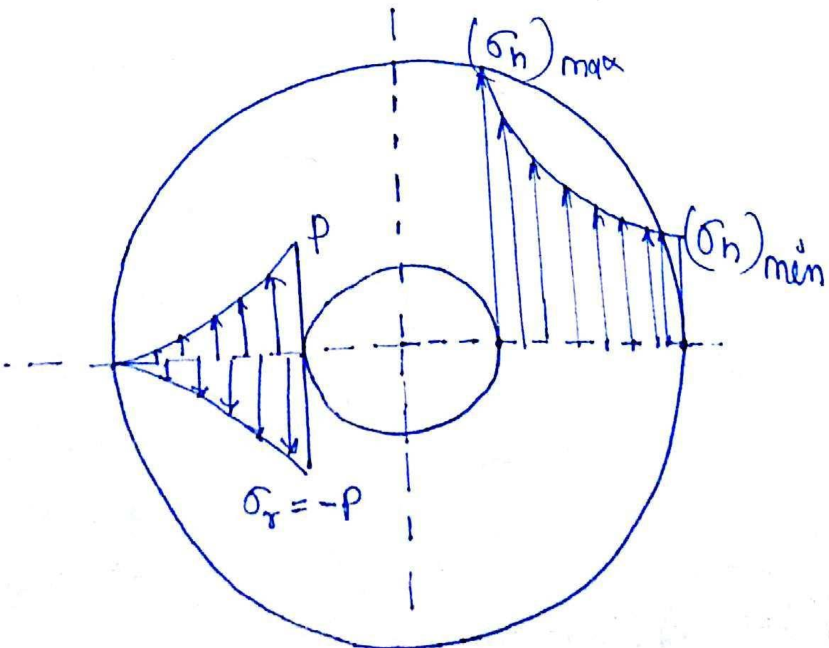
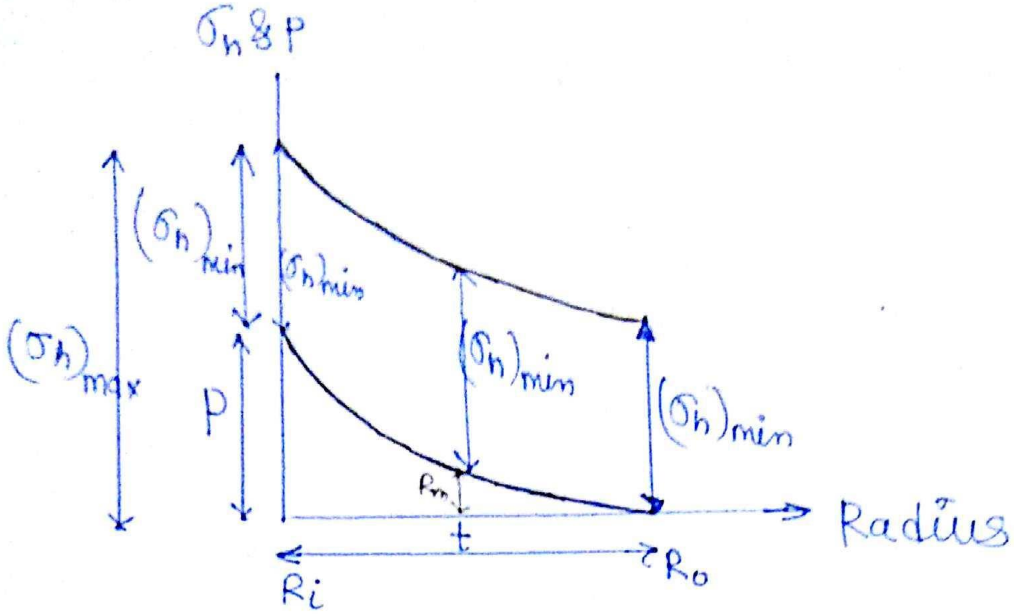


Fig :- Variation of p, σ_r & σ_h mag. over the thickness of shell under I. P.



$$(\sigma_h)_{max} = P + (\sigma_h)_{min}$$

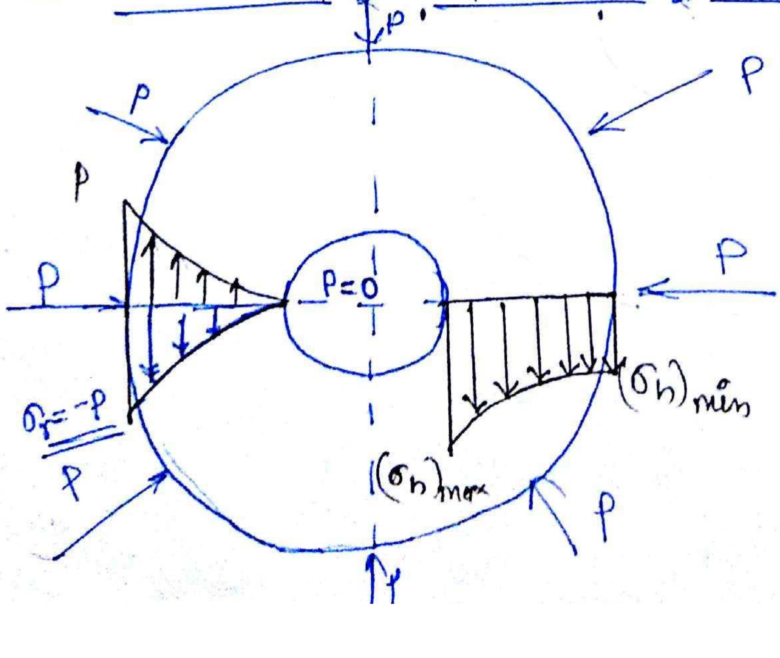
Eg $P = 50$ MPa ; $(\sigma_h)_{x=R_i} = 300$ MPa ; $(\sigma_h)_{x=R_o} = ?$

Solⁿ $(\sigma_h)_{x=R_i} = 300 = (\sigma_h)_{max}$

$P = 50$ MPa

$$(\sigma_h)_{\substack{min \\ x=R_o}} = 300 - \cancel{2}50 = 250 \text{ Min}$$

Case - II When External pressure acting alone :—



$x = R_i$

$\Rightarrow P_x = P_i = 0$

$x = R_o$

$\Rightarrow P_x = P_e = P$

$(\sigma_h)_{max}$ & $(\sigma_h)_{min}$ both are Compressive

Case-III When Internal & External pressure are acting simunteneously

$x = R_i \Rightarrow P_x = P_i$

$x = R_o \Rightarrow P_x = P_o$

when $P_i = P_o = P$

from lame's eqn

$$P = -a + \frac{b}{R_i}$$

$$P = -a + \frac{b}{R_i}$$

$$0 = 0 + b\left[\frac{1}{R_i^2} - \frac{1}{R_o^2}\right]$$

$$b = 0 \;;\; a = -P$$

$$(\sigma_r)_x = a \;\&\; (\sigma_h)_x = -P \text{ (Constant)}$$

When $P_i = P_o = P$