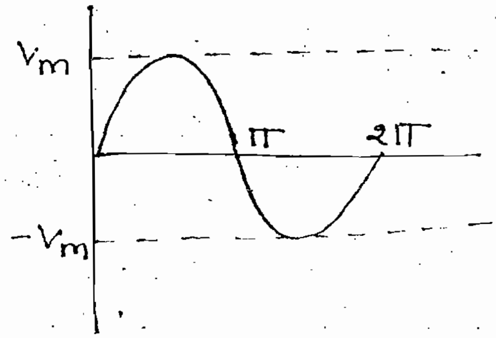
Steady State AC Circuits :-

$v(t) = V_m \sin\omega t$



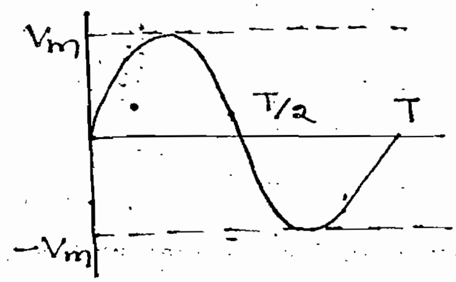
V_m = Peak or Max. value

ω = Angular frequency → rad/sec

ωt = Argument – rad

$\omega T = 2\pi$

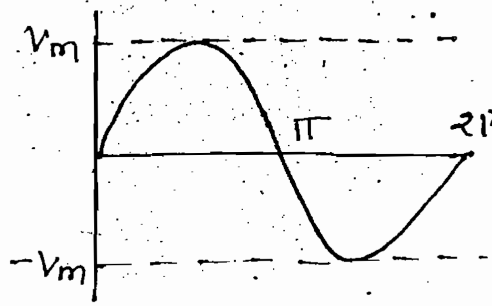
$\Rightarrow \boxed{T = \frac{2\pi}{\omega}}$ sec.



$\boxed{f = \frac{1}{T}}$

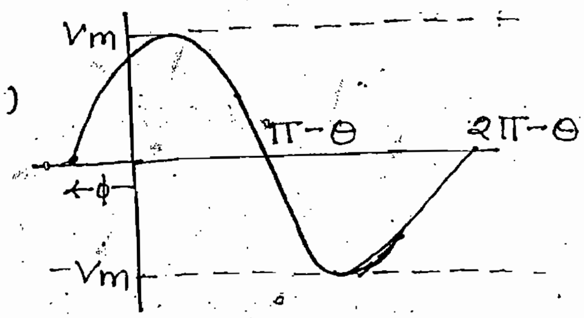
$\boxed{f = \frac{\omega}{2\pi}}$ Hz or cycles/sec.

$v(t) = V_m \sin \omega t$



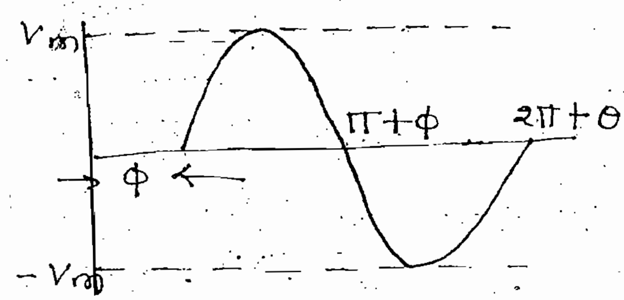
$v(t) = V_m \sin(\omega t + \theta)$

$\longrightarrow$ leading



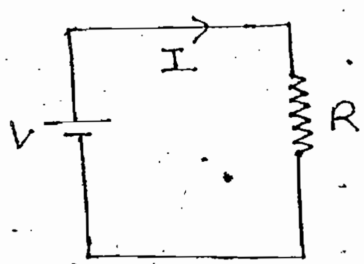
$v(t) = V_m \sin(\omega t - \theta)$

$\longrightarrow$ lagging



RMS Value :-

t

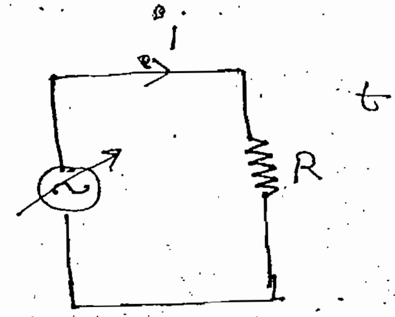


$P = I^2R$

$W = I^2Rt$

$\downarrow$ DC

Heat



t

$P = i^2R$

$W = i^2Rt$

$\downarrow$ AC

Heat

$$\boxed{W_{AC} = W_{DC}}$$

→ RMS value is defined based on heating effect of the waveform

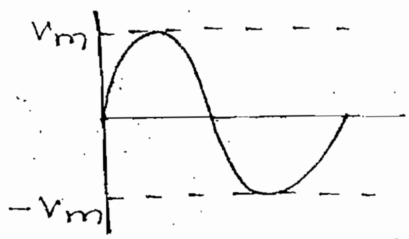
→ The voltage at which heat dissipation in A.C circuit is equal to heat dissipation in DC circuit is called as V_{RMS} provided both AC and DC circuit having equal value of resistance and operated for same time

$$\boxed{\begin{aligned} V_{RMS} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 \omega t} \\ V_{RMS} &= \sqrt{\frac{1}{T}\int_0^{T} V^2 dt} \end{aligned}}$$

Lecture - 4

ques:- Find RMS value of following waveforms

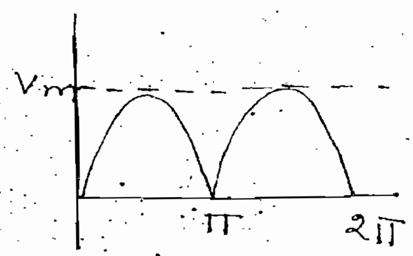
(1) $V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2 d\omega t}$



$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2 dt}$$

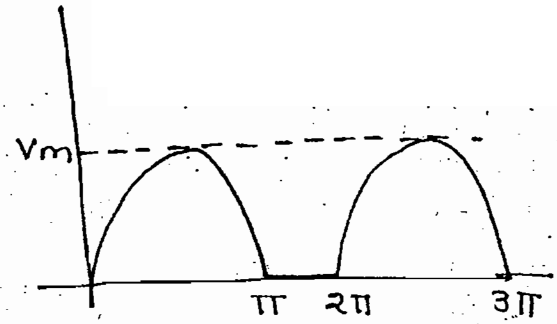
$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \left(\frac{1-\cos 2\omega t}{2}\right) dt}$$

$$\Rightarrow \boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$$



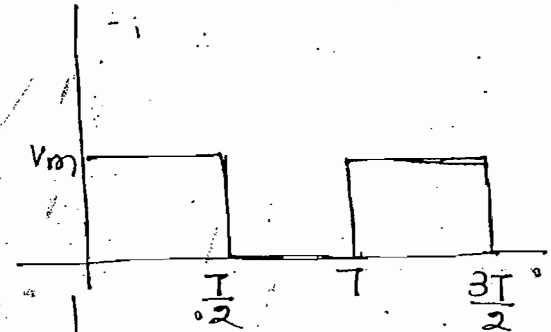
(II) $\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$

→ $\boxed{V_{RMS} = \frac{V_m}{2}}$



→ $\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$

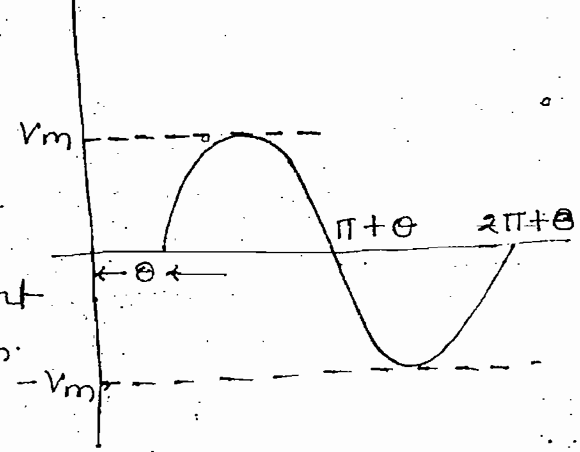
$$\left[V_{RMS} = \sqrt{\frac{1}{T}\left[\int_0^{T/2} V_m^2\,dt + \int_{T/2}^{T} 0\,dt\right]} \right]$$



→ $\boxed{V_{RMS} = \frac{V_m}{\sqrt{2}}}$

Note:

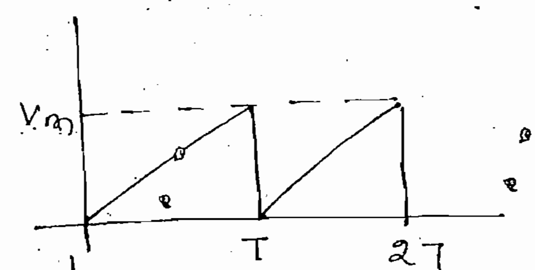
RMS value is independent on the position of starting of waveform. But it depends on the shape of waveform



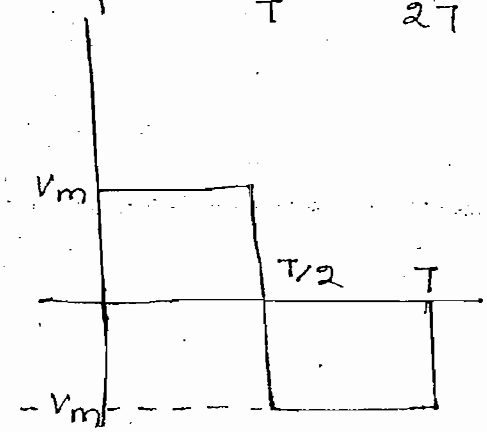
→ $\boxed{V_{RMS} = \frac{V_m}{\sqrt{3}}}$

$0 < t < T \quad y = mx$

$\Rightarrow \; V = \frac{V_m}{T} t$



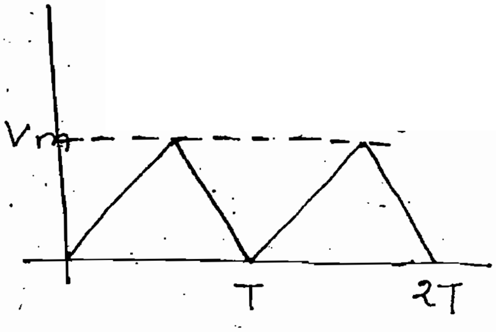
→ $\boxed{V_{RMS} = V_m}$



$\rightarrow$ $V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V^2 dt}$

$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T}t\right)^2 dt}$

$\Rightarrow$ $\boxed{V_{RMS} = \frac{V_m}{\sqrt{3}}}$



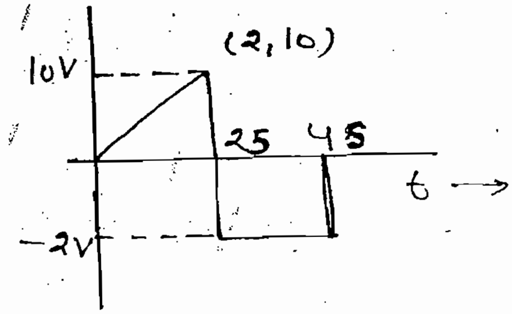
$\rightarrow$ $0 < t < 2$

$y = mx$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-0}{2-0}$

$= 5$

$V = 5t$

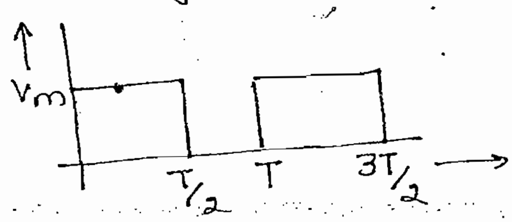


$V_{RMS} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-2)^2 dt\right.}$

$\Rightarrow$ $V_{RMS} = \sqrt{\frac{1}{4}\left[25\left(t^3/3\right)^2 + (4t)_2^4\right.}$

$\Rightarrow$ $V_{RMS} = 4$ ---------

Ques:- Find power dissipation in the resistor for the given voltage waveform



(a) $P_{av} = \frac{P_{peak}}{\sqrt{2}}$

(b) $P_{av} = \frac{P_{peak}}{2}$

(c) $P_{av} = P_{peak}$

(d) $P_{av} = \frac{P_{peak}}{\sqrt{3}}$

Soln:- B. $P_{peak} = \frac{V_m^2}{R}$

$P_{av} = \frac{V^2_{RMS}}{R} = \frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{2R} = \frac{P_{peak}}{2}$

Note:-

→ $$P_{RMS} = \frac{V_{RMS}^2}{R}$$

** → $$\boxed{\frac{P_{DC}}{P_{AC}} = \frac{I_{av}^2 R}{I_{RMS}^2 R}}$$

Ques:- Find RMS value for the following function

$$V(t) = 3 + \sin 3t + \cos t$$

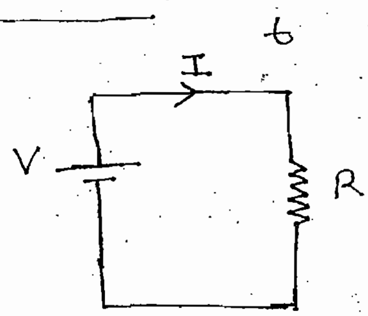
Soln:- $$V_{RMS} = \sqrt{V_{RMS_1}^2 + V_{RMS_2}^2 + V_{RMS_3}^2 + \cdots + V_{RMS_n}^2}$$

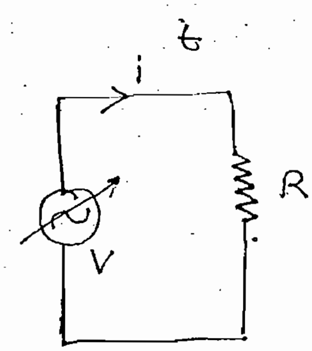
[Used when different ω are present]

$$\Rightarrow V_{RMS} = \sqrt{(3)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \boxed{V_{RMS} = \sqrt{10}}$$

Average Value:-





$$I = \frac{V}{R}$$

$$Q = It \rightarrow DC$$

$$i = \frac{V}{R}$$

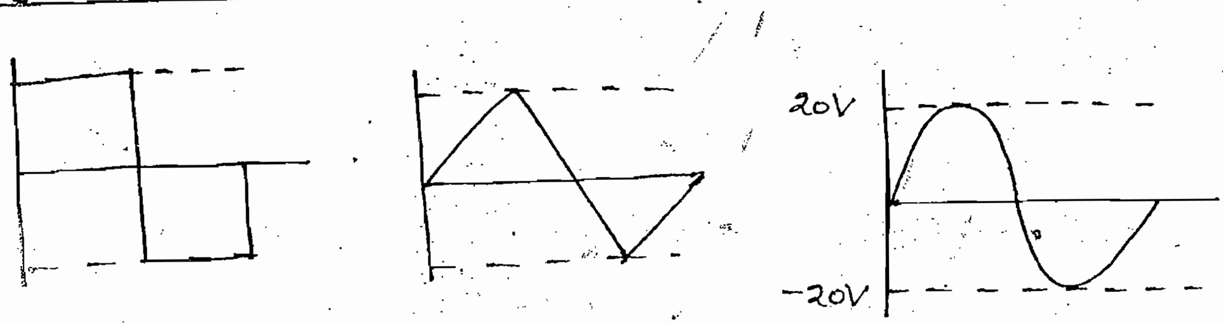
$$Q = it \rightarrow AC$$

$$\boxed{Q_{A.C} = Q_{DC}}$$

→ Average value is defined based on charge transfer in the circuit

→ The voltage at which charge transfer in AC circuit is equal to charge transfer in DC circuit is called as V_{avg} provided both AC and DC circuit having equal value of resistance and operated for same time.

Symmetrical Wave :-



$$\text{Form factor} = \frac{V_{RMS}}{V_{av}}$$

⟹ Avg value for complete symmetrical wave = 0

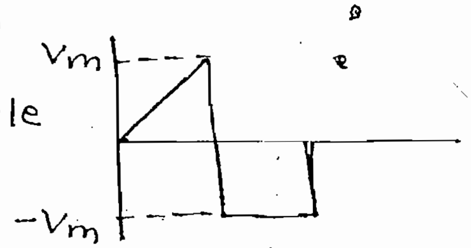
→ Hence we can find Average value only for half cycles for symmetrical waveform

$$\boxed{V_{av} = \frac{1}{\pi}\int_0^{\pi} V\,d\omega t}$$

[For current and voltage waveform

Unsymmetrical Wave :-

→ Finding average value of unsymmetrical wave angle of complete cycle is considered



$$\boxed{V_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} V\,d\omega t + \int_{\pi}^{2\pi} V\,d\omega t\right]}$$

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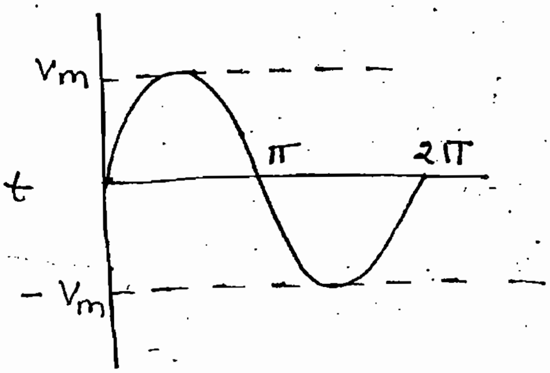
Ques:- Find Avg. value of following waveforms

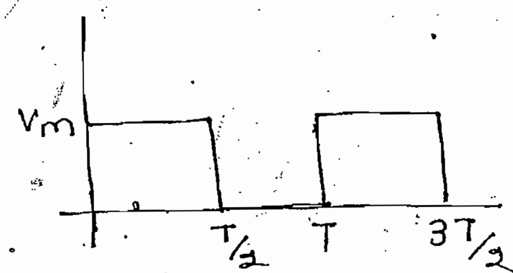
→ $$V_{av} = \frac{1}{\pi}\int_0^{\pi} v \, d\omega t$$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t$$

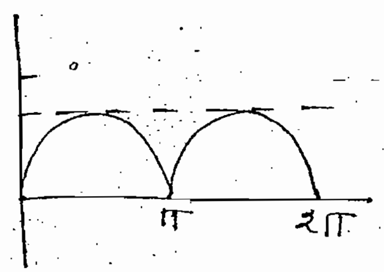
$$\Rightarrow \boxed{V_{av} = \frac{2V_m}{\pi}}$$



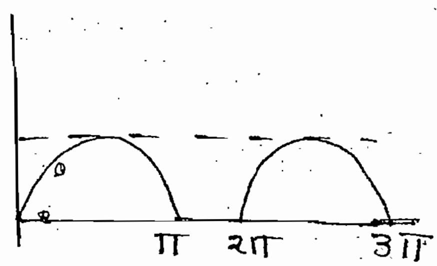
→ $$V_{av} = \frac{V_m}{2}$$



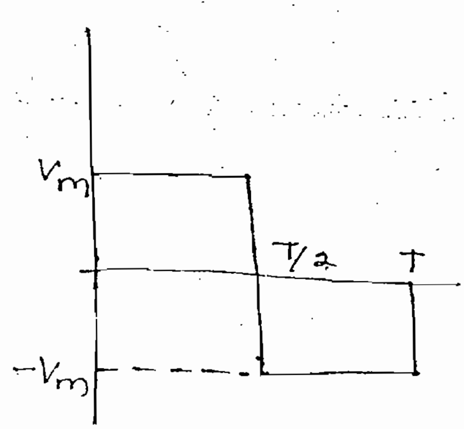
→ $$V_{av} = \frac{2V_m}{\pi}$$



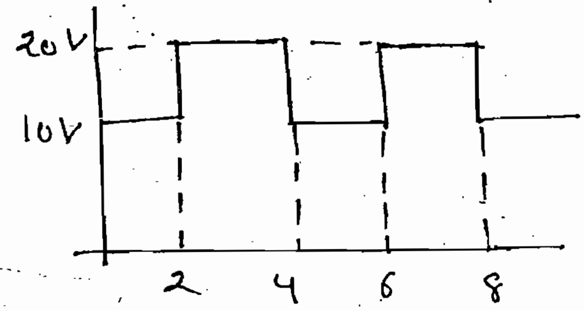
→ $$V_{av} = \frac{V_m}{\pi}$$



→ $$V_{RMS} = V_{av} = V_m$$

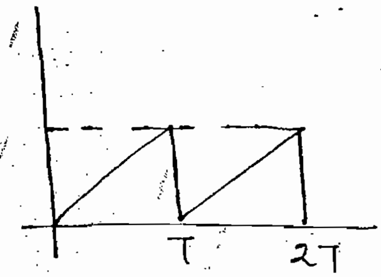


$\rightarrow V_{av} = \frac{1}{4}\left[\int_0^2 10\,dt + \int_2^4 20\,dt\right]$

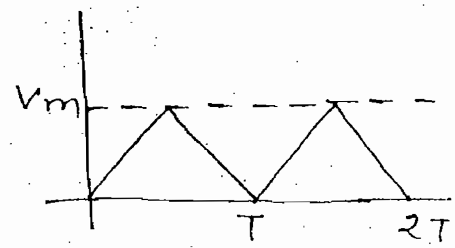


$$\boxed{V_{av} = 15}$$

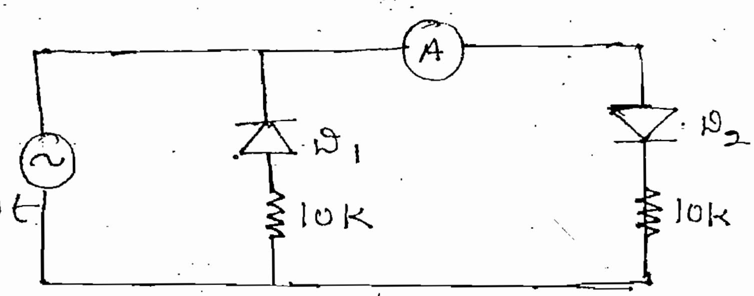
$\rightarrow$ $\boxed{V_{av} = \frac{V_m}{2}}$



$\rightarrow$ $\boxed{V_{av} = \frac{V_m}{2}}$



Ques:- When the circuit is having ideal diodes and avg. value of indicating ammeter. Find reading of ammeter



Soln:- $V_{av} = \frac{V_m}{\pi} = \frac{4}{\pi}$

$$I_{av} = \frac{V_{av}}{10\times10^3} = \frac{4/\pi}{10\times10^3}$$

$$\Rightarrow I_{av} = \frac{0.4}{\pi}\ mA$$

Note:-

For sine wave

$$\text{Form factor} = \frac{V_{RMS}}{V_{av}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = 1.11$$

$$\text{Peak factor} = \frac{V_m}{V_{RMS}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

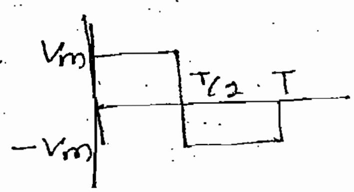
Power System :-

11 kV, 33 kV, 66 kV, 132 kV, 220 kV] → basis of form factor

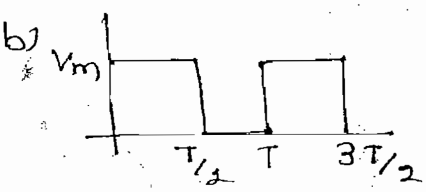
→ To justify above shape of waveform form factor and peak factor concepts are introduced

Ques:- Which of the following waveforms have form factor = peak factor ?

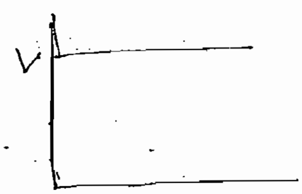
(a)



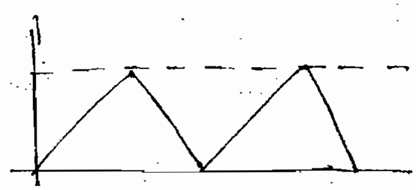
(b)



(c)



(d)



Soln:- (a) $V_{RMS} = V_{av} = V_m$

Form factor = 1

Peak factor = 1

(b) $V_{RMS} = \frac{V_m}{\sqrt{2}}$, $V_{av} = \frac{V_m}{2}$

$$\text{Form factor} = \frac{V_{RMS}}{V_{av}} = \sqrt{2}$$

$$\text{Peak factor} = \frac{V_m}{V_{RMS}} = \sqrt{2}$$

(c) $V_{RMS} = V_m = V_{av}$
$= V$

Form factor = 1

Peak factor = 1

(d) $V_{rms} = \frac{V_m}{\sqrt{3}}$

$V_{av} = V_m/2$

Form factor $= 2/\sqrt{3}$

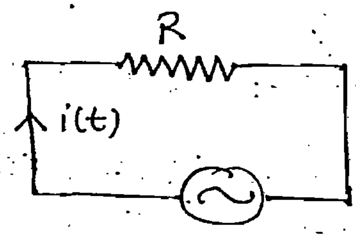
Peak factor $= \sqrt{3}$

AC Source Across Resistor :-

$$i(t) = \frac{V(t)}{R}$$

$$i(t) = \frac{V_m \sin\omega t}{R}$$

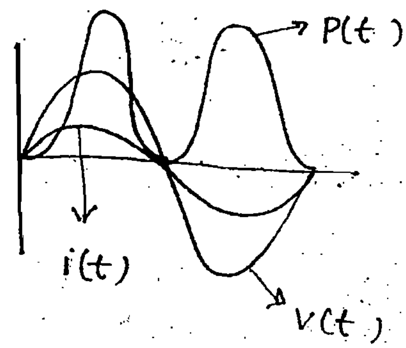
$$\Rightarrow \boxed{i(t) = I_m \sin\omega t}$$



$$P(t) = V(t)\, i(t)$$

$$P(t) = V_m \sin\omega t \; I_m \sin\omega t$$

$$\boxed{P(t) = \frac{V_m I_m}{2}(1 - \cos 2\omega t)}$$



$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\, d\omega t$$

$$\Rightarrow \boxed{P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS}}$$

eg:- $f = 50\,Hz$ or C/sec

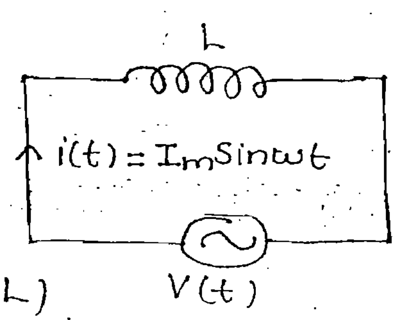
$\Rightarrow f_p = 100\,Hz$ (Power of frequency)

→ When voltage or current completes one cycle then power completes two cycle.

AC Source across Inductor :-

$$V = L\frac{di}{dt}$$

$$\therefore V(t) = L\frac{d}{dt}(I_m \sin\omega t)$$



$$V(t) = \omega L I_m \cos\omega t \quad (X_L = \omega L)$$

$$V(t) = V_m \sin(\omega t + 90)$$

w4

$$\boxed{\begin{aligned} P &= I^2R = VI\cos\theta \\ Q_L &= I^2X_L = VI\sin\theta \\ S &= I^2Z = VI^* \end{aligned}}$$ → conjugate

eg:- $V = 10\angle 40^\circ$ $\quad i = 5\angle 15^\circ$

$S = Vi$

$\Rightarrow S = 10\angle 40 \; 5\angle 15$

$\Rightarrow S = 50\angle 55$

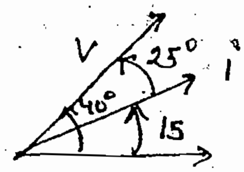
Wrong

$S = Vi^*$

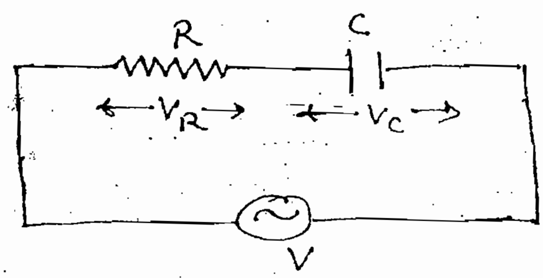
$\Rightarrow S = 10\angle 40 \; 5\angle -15$

$\Rightarrow S = 50\angle 25$

Correct



R-C series circuit :-

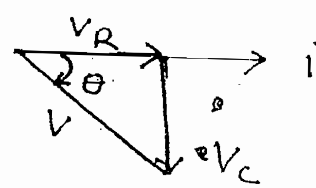


By KVL

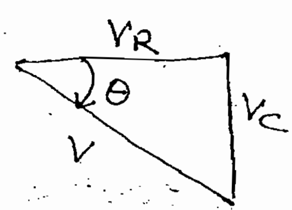
$V = V_R\angle 0 + V_C\angle -90^\circ$

$\Rightarrow IZ = IR - jIX_C$

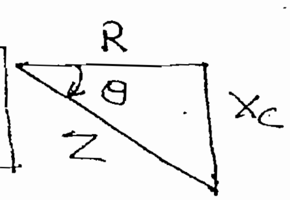
$\Rightarrow \boxed{Z = R - jX_C}$



$\boxed{V = \sqrt{V_R^2 + V_C^2}}$ $\quad \boxed{\theta = \tan^{-1}\left(\frac{-V_C}{R}\right)}$

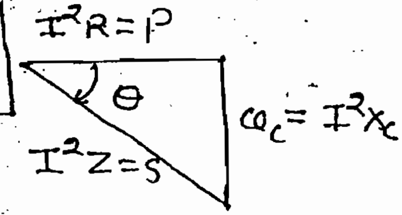


$\boxed{Z = \sqrt{R^2 + X_C^2}}$ $\quad \boxed{\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)}$



$$\boxed{S = \sqrt{P^2 + Q^2}}$$

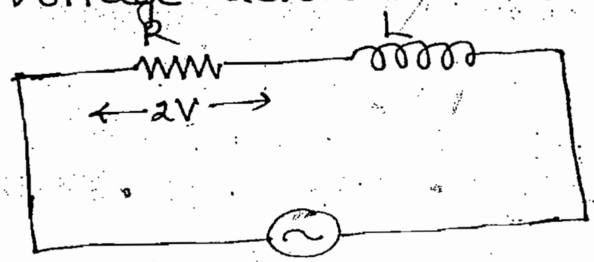
$$\boxed{\theta = \tan^{-1}\left(\frac{-Q_c}{P}\right)}$$



Power Factor :-

$$\boxed{\text{Power Factor} = \cos\theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S} \rightarrow \text{leading}}$$

Ques:- Find voltage across inductor

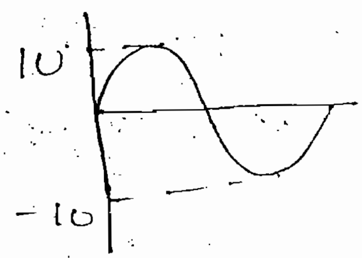


20V (P-P)

Solⁿ:-

$V_m = 10.$

$V_{RMS} = \frac{10}{\sqrt{2}} = V$



$V = \sqrt{V_R^2 + V_L^2}$

$\Rightarrow \frac{10}{\sqrt{2}} = \sqrt{2^2 + V_L^2} \Rightarrow \boxed{V_L = \sqrt{46}\ V}$

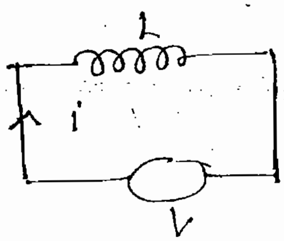
Ques:- Find circuit element for given voltage and current equations

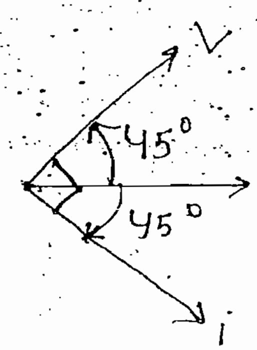
$V(t) = 9\sin(t + 45)$

$i(t) = 3\sin(t - 45°)$

Solⁿ:-

$X_L = \frac{V}{i}$





$X_L = \frac{9/\sqrt{2}}{3/\sqrt{2}}$

$\Rightarrow X_L = 3 = \omega L \Rightarrow \boxed{L = 3}$ $(\because \omega = 1)$

Ques:- Find circuit element for given voltage and current equations

$$V(t) = 9\sin(t + 30^\circ)$$
$$i(t) = 3\sin(2t + 60^\circ)$$

Note:-

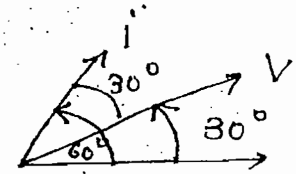
By using above equations it is not possible to design the network since frequency of voltage and current are unequal.

Ques:- Find active power, reactive power and apparent power by using following equations:-

$$V_0(t) = 9\sin(t + 30^\circ)$$
$$i(t) = 3\sin(t + 60)$$

Soln:-

$$P = VI\cos\theta$$



$$\Rightarrow P = \frac{9}{\sqrt{2}} \cdot \frac{3}{\sqrt{2}} \cos 30$$

$$\Rightarrow P = 27 \times \frac{\sqrt{3}}{2}$$

→ R-C Circuit (I leading)

$$Q_c = VI\sin\theta$$

$$\Rightarrow Q_c = \frac{9}{\sqrt{2}} \frac{3}{\sqrt{2}} \sin 30^\circ$$

$$\Rightarrow Q_c = \frac{27}{4}$$

$$S = \sqrt{P^2 + Q_c^2} =$$

Alternate Way:-

$$Z = V/I$$

$$Z = \frac{9/\sqrt{2}}{3/\sqrt{2}}$$

$$Z = 3$$

$$\cos\theta = \frac{R}{Z}$$

$$\Rightarrow \cos 30^\circ = R/3 \Rightarrow R = 3\cos 30^\circ$$

$$X_c = \sqrt{Z^2 - R^2}$$

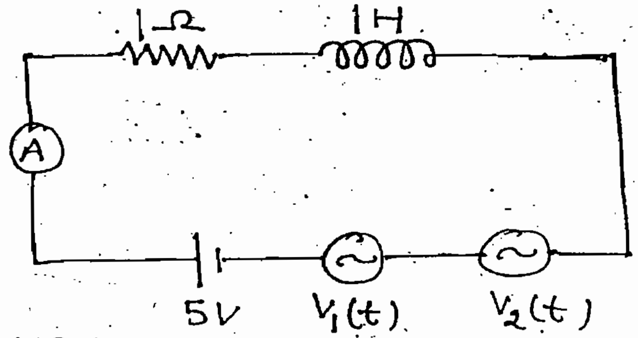
$$P = i^2 R = (3/\sqrt{2})^2 R$$

$$Q_c = I^2 X_c = (3/\sqrt{2})^2 X_c$$

Ques:- Find ammeter reading and power factor of the ckt shown

$V_1(t) = 10\sin t$

$V_2(t) = 10\sqrt{5}\sin 2t$



Soln:- When multiple sources are present then at one time only one source is activated.

For V_1 $\quad X_{L_1} = \omega_1 L = 1$

$$Z_1 = \sqrt{R^2 + X_{L_1}^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$i_1 = \frac{V_1}{Z_1} = \frac{10/\sqrt{2}}{\sqrt{2}} = 5$$

For V_2 $\quad X_{L_2} = \omega_2 L = 2$

$$Z_2 = \sqrt{1^2+2^2} = \sqrt{5}$$

$$i_2 = \frac{V_2}{Z_2} = \frac{10\sqrt{5}/\sqrt{2}}{\sqrt{5}} = \frac{10}{\sqrt{2}}$$

For V_3. $\quad$ DC $\rightarrow$ $\omega = 0$

$$i_3 = \frac{V_3}{R} = \frac{5}{1} \Rightarrow i_3 = 5$$

When same frequency are present then directly add them But above question different frequency are present Hence use general formula. $\left(i = \sqrt{i_1^2 + i_2^2 + i_3^2} = 10A\right)$ Ans

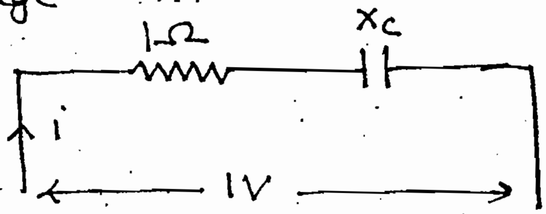
For power factor use power triangle not impedance triangle (same reason)

$$P.F = \cos\theta = P/S \Rightarrow \cos\theta = i^2R/V i = \frac{iR}{V}$$
$$= 0.55$$

$$V = \sqrt{V_1^2+V_2^2+V_3^2} = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{10\sqrt{5}}{\sqrt{2}}\right)^2 + 5^2}$$

$=$

Ques:- In the circuit shown power dissipation in the resistor is 500 mW. Find angle of current w.r.t source voltage



Soln:- $P = i^2 R \implies P = \left(\frac{V}{Z}\right)^2 R$

$$\implies P = \frac{V^2}{R^2 + X_c^2} R$$

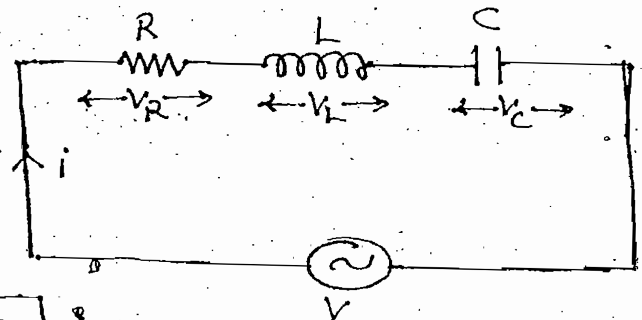
$$\implies \frac{500}{1000} = \frac{1^2}{1^2 + X_c^2}(1) \implies X_c = 1$$

$$Z = R - jX_c$$

$$Z = 1 - j1 \implies \theta = \tan^{-1}\left(\frac{-1}{1}\right) = -45^\circ$$

$$i = \frac{V\angle 0}{Z\angle -45} = \frac{V\angle +45}{Z} \implies \theta = 45^\circ \text{ Ans}$$

R-L-C Series Circuit :-

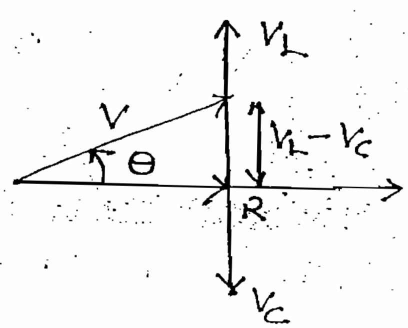


By KVL

$$V = V_R\angle 0 + V_L\angle 90 + V_c\angle -90$$

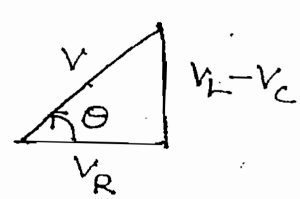
$$\implies IZ = IR + jIX_L - jX_c$$

$$\implies \boxed{Z = R + j(X_L - X_c)}$$



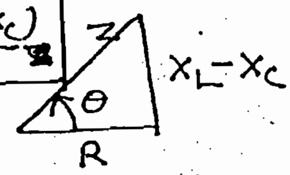
$$\boxed{V = \sqrt{V_R^2 + (V_L - V_c)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{V_L - V_c}{V_R}\right)}$$



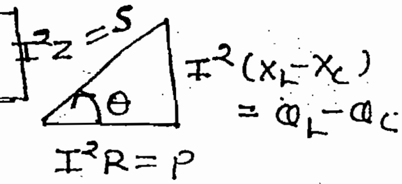
$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\boxed{\theta = \tan^{-1}\frac{(X_L - X_C)}{R}}$$



$$\boxed{S = \sqrt{P^2 + (Q_L - Q_C)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right)}$$



Power factor :-

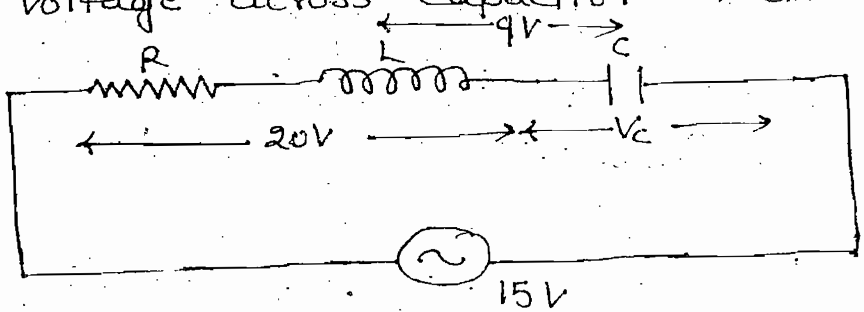
$$\cos\theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

(I) If $V_L > V_C$ $\longrightarrow$ lagging power factor

(II) If $V_L < V_C$ $\longrightarrow$ leading power factor

(III) If $V_L = V_C$ $\longrightarrow$ Unity power factor

Ques:- Find voltage across capacitor of circuit shown



(a) 7 (b) 25 (c) 7 or 25 (d) 20

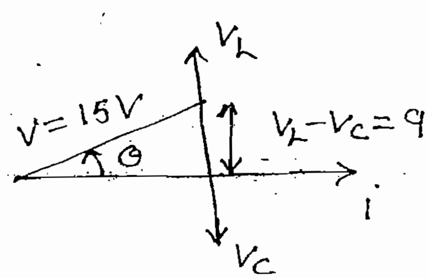
Soln:- $V_L - V_C = 9V$

$V = 15V$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow 15^2 = V_R^2 + 9^2$$

$$V_R = \sqrt{225 - 81} = 12V$$



$$20 = \sqrt{V_R^2 + V_L^2} = \sqrt{12^2 + V_L^2} \Rightarrow V_L = 16$$

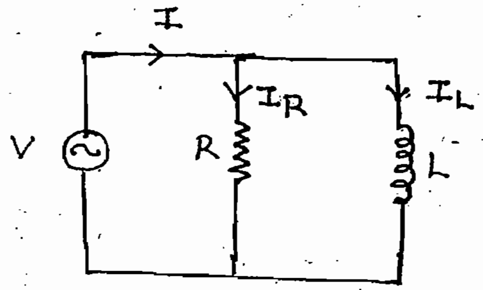
$$V_L - V_C = 9 \Rightarrow V_C = 7V, \text{ Ans}$$

d

R-L Parallel circuit :-

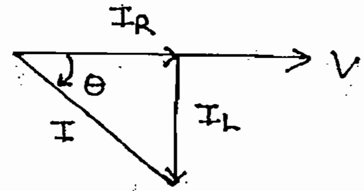
By KCL

$I = I_R \angle 0 + I_L \angle -90^\circ$



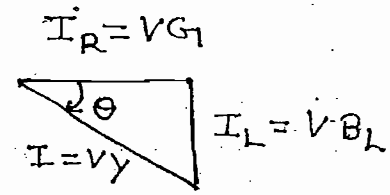
$\Rightarrow \frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L}$

$\Rightarrow VY = VG - jVB_L$

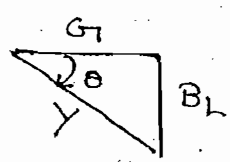


$\Rightarrow \boxed{\underbrace{Y}_{\text{mho}} = \underbrace{G}_{\text{mho}} - j\underbrace{B_L}_{\text{mho}}}$

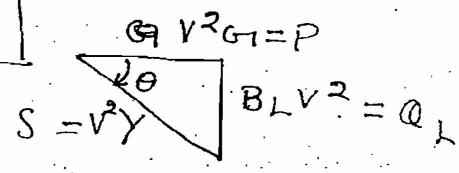
$\boxed{I = \sqrt{I_R^2 + I_L^2}}$ $\boxed{\theta = \tan^{-1}\left(\frac{-I_L}{I_R}\right)}$



$\boxed{Y = \sqrt{G^2 + B_L^2}}$ $\boxed{\theta = \tan^{-1}\left(-\frac{B_L}{G}\right)}$



$\boxed{S = \sqrt{P^2 + Q_L^2}}$ $\boxed{\theta = \tan^{-1}\left(\frac{-Q_L}{P}\right)}$



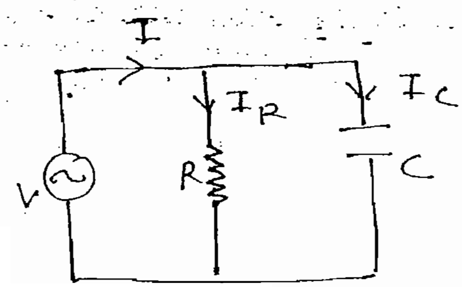
Power factor :-

$\rightarrow$ $\boxed{\cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S} = \text{Lagging}}$

R-C Parallel circuit :-

By KCL

$I = I_R \angle 0 + I_C \angle +90$

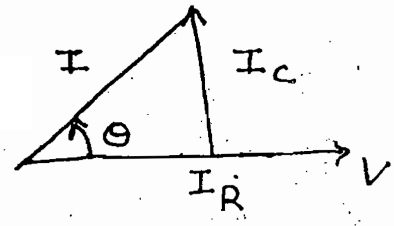


$\Rightarrow \frac{V}{Z} = \frac{V}{R} + j\frac{V}{X_C}$

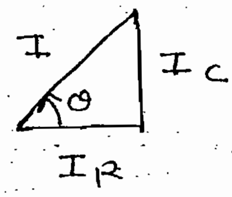
$$\Rightarrow \quad V.Y = VG + jVB_c$$

$$\Rightarrow \quad \boxed{Y = G + jB_c}$$

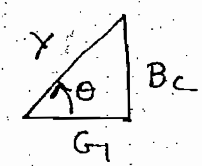
Mho Mho Mho



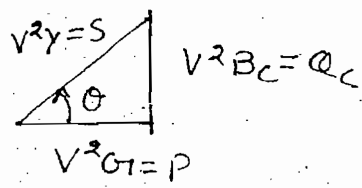
$$\boxed{I = \sqrt{I_R^2 + I_c^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{I_c}{I_R}\right)}$$



$$\boxed{Y = \sqrt{G^2 + B_c^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{B_c}{G}\right)}$$



$$\boxed{S = \sqrt{P^2 + Q_c^2}} \qquad \boxed{\theta = \tan^{-1}\left(\frac{Q_c}{P}\right)}$$

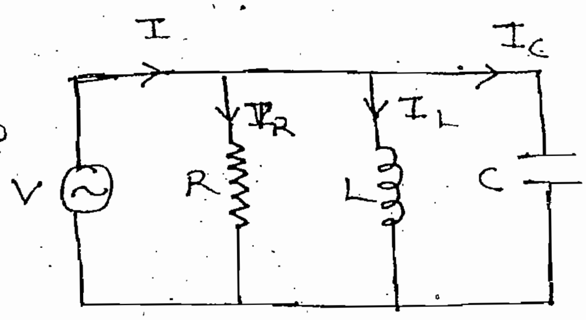


Power Factor $\Rightarrow$

$$\boxed{\cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S} = \text{leading}}$$

RLC Parallel Circuit :—

By KCL

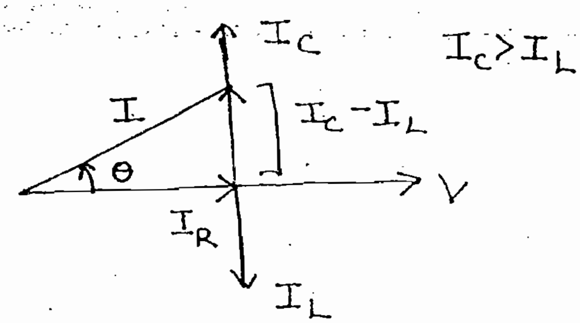


$$I = I_R\angle 0 + I_L\angle -90 + I_c\angle +90$$

$$\Rightarrow \quad \frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_c}$$

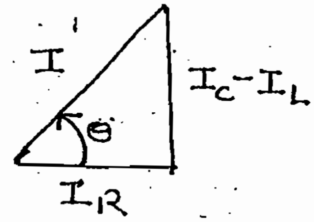
$$\Rightarrow \quad VY = V\left[G + j(B_c - B_L)\right]$$

$$\Rightarrow \quad \boxed{Y = G + j(B_c - B_L)}$$



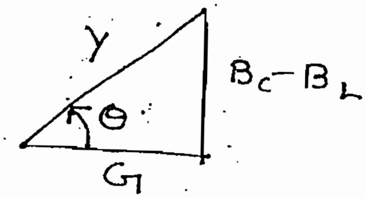
$$\boxed{I = \sqrt{I_R^2 + (I_C - I_L)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)}$$



$$\boxed{Y = \sqrt{G^2 + (B_C - B_L)^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)}$$



$$\boxed{\text{Power Factor} = \cos\theta = \frac{I_R}{I} = \frac{G}{Y} = \frac{P}{S}}$$

(i) $I_C > I_L \longrightarrow$ leading

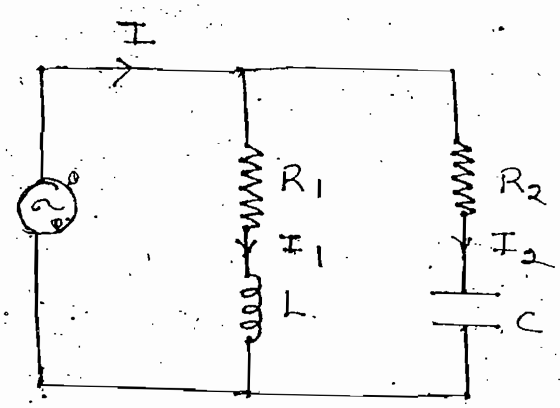
(ii) $I_C < I_L \longrightarrow$ lagging

(iv) $I_C = I_L \longrightarrow$ Unity Power factor

<u>Note:-</u>

When combinational elements are present then we can't directly find conductance, admittance etc. Then following procedure is used

$$I_1 = \left(\frac{V}{R_1 + jX_L}\right)\left(\frac{R_1 - jX_L}{R_1 - jX_L}\right)$$



$$\Rightarrow \frac{V}{Z_1} = V\left[\frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2}\right]V$$

$$\Rightarrow \boxed{Y_1 = G_1 - jB_L}$$

$$I_2 = \frac{V}{R_2 - jX_C}\;\frac{R_2 + jX_C}{R_2 + jX_C}$$

$$\Rightarrow \frac{V}{Z_2} = V\left[\frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2}\right]$$

$$\boxed{Y_2 = G_2 + jB_C}$$

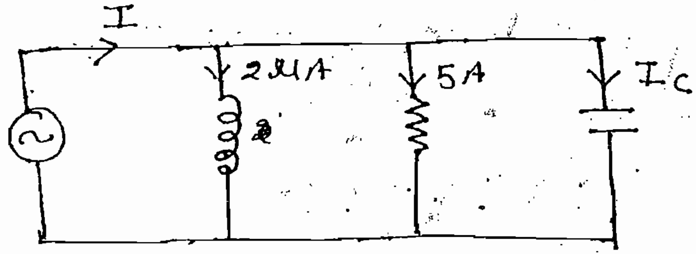
$$I = I_1 + I_2$$

$$\Rightarrow V Y_{eq} = V Y_1 + V Y_2$$

$$\Rightarrow Y_{eq} = Y_1 + Y_2$$

$$\Rightarrow \boxed{Y_{eq} = (G_1+G_2) + j(B_C - B_L)}$$

Ques:- Find I_C and I of the circuit shown



Soln:-

$$13 = \sqrt{5^2 + I_C^2} \quad \Rightarrow \quad I_C = 12A$$

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\Rightarrow I = \sqrt{5^2 + (12 - 2\mu A)^2} = 13 \text{ A}$$

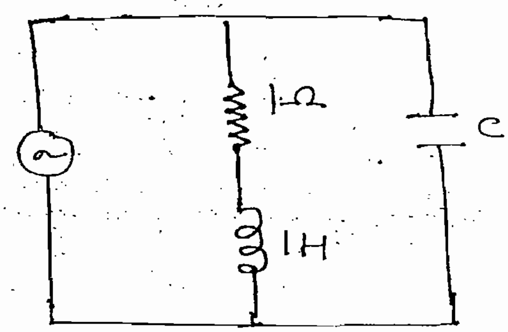
Note:-

A.C $\longrightarrow$ (KVL, KCL) $\longrightarrow$ Phasor sum

D.C $\longrightarrow$ (KVL, KCL) $\longrightarrow$ Arithmatic sum

Ques:- Find capacitance of the capacitor when power factor of the circuit is 0.8 lagging

$V(t) = \sin t$



Soln:- For Branch-1

$$X_L = \omega L = 1$$

$$G_1 = \frac{R_1}{R_1^2 + X_L^2} = \frac{1}{1^2+1^2} = \frac{1}{2}$$

$$B_L = \frac{X_L}{R_1^2 + X_L^2} = \frac{1}{1^2+1^2} = \frac{1}{2}$$

$$Y_1 = G_1 - jB_L$$

$$\Rightarrow Y = \frac{1}{2} - j\frac{1}{2}$$

<u>For Branch-2</u>

$$Y_2 = +jB_C$$

$$Y_2 = j\frac{1}{X_C} = j\omega C \quad \Rightarrow \quad Y_2 = \omega C$$

$$Y_{eq} = Y_1 + Y_2$$

$$\Rightarrow Y_{eq} = \frac{1}{2} + j\cdot\left(C - \frac{1}{2}\right)$$

$$\cos\theta = \frac{G_1}{\sqrt{G_1^2 + (B_C - B_L)^2}}$$

$$\Rightarrow 0.8 = \frac{1/2}{\sqrt{(1/2)^2 + (C - 1/2)^2}}$$

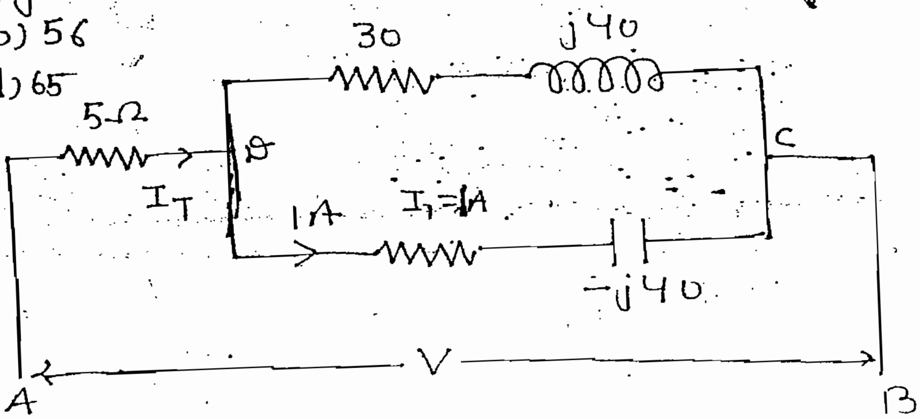
$$\Rightarrow C = \frac{7}{8} \text{ or } \frac{1}{8}$$

Since $B_L > B_C$

$\frac{1}{2} > C$ Hence, $\boxed{C = \frac{1}{8}}$ Ans

<u>Ques:-</u> Find voltage across A and B of the circuit shown (a) 55 (b) 56 (c) 60 (d) 65

<u>Soln:-</u>



Apply current division technique,

$$I_1 = I_T \frac{30 + j40}{30 + j40 + 30 - j40} =$$

$$\Rightarrow \quad 1 = I_T \cdot \frac{50 \angle \tan^{-1}(4/3)}{60}$$

$$\Rightarrow \quad I_T = \frac{60}{50 \angle \tan^{-1}(4/3)} = 1.2 \angle \tan^{-1}(-4/3)$$

$$V_{AD} = I_T \times 5$$

$$\Rightarrow \quad V_{AD} = (1.2 \times 5) \angle \tan^{-1}(-4/3)$$

$$\Rightarrow \quad V_{AD} = 6 \angle \tan^{-1} 4/3$$

$$V_{DC} = (30 - j40)\, 1$$

$$V_{DC} = 50 \angle \tan^{-1}(-4/3)$$

Angles are same hence they can be added

$$V_{AB} = V_{AC} = V_{AD} + V_{DC}$$

$$\Rightarrow \quad \boxed{V_{AB} = 56 \angle \tan^{-1}(-4/3)} \quad \text{Ans}$$

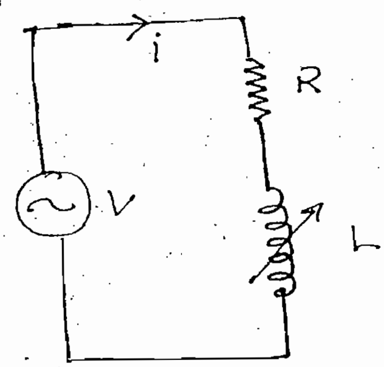
Locus Diagram :-

$$X_L = 2\pi f L$$

$$X_L = 0$$

$$Z = R \quad \Rightarrow I = \frac{V}{R}$$

$$\Rightarrow \quad \theta = 0$$

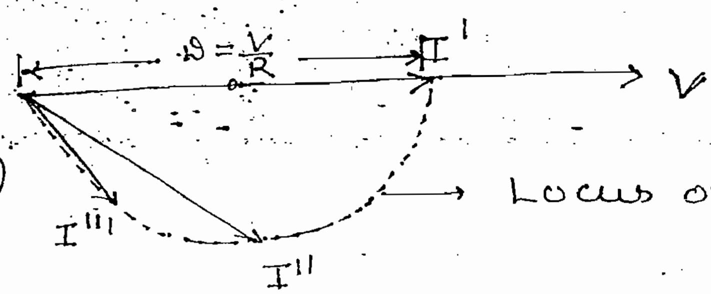


$X_L \uparrow \quad Z \uparrow \quad I \downarrow \qquad \boxed{\theta = \tan^{-1}\left(\frac{X_L}{R}\right)} \uparrow$

$X_L \simeq \infty \quad Z \simeq \infty \quad I = 0$

→ semicircle

→ centre $\left(\frac{V}{2R}, 0\right)$



→ Locus diagram are useful for analysis and designing of the circuit eg:- filters

→ With respect to practical application it is possible to develop the following locus diagram

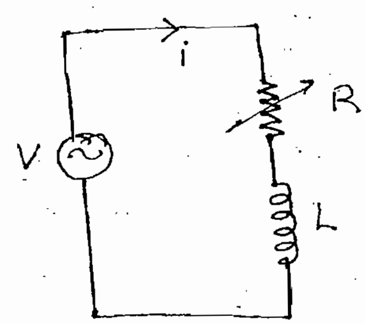
(I) current locus diagram
(II) Voltage locus diagram
(III) Impedance locus diagram
(IV) Admittance locus diagram

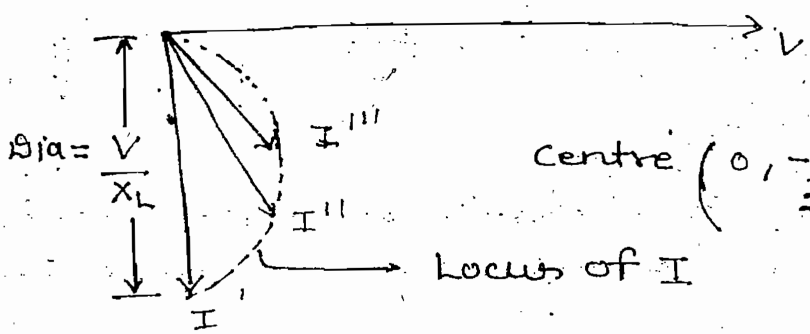
→ The path traced by terminous of current vectors by varying either any of the circuit elements or by varying source frequency is called as current locus

→ In the above circuit by keeping all the elements constant and by varying source frequency also same shape of the current locus diagram is obtained.

→ Develop current locus of the circuit shown

$R = 0$	$R\uparrow$
$Z = X_L$	$Z\uparrow$
$I = \frac{V}{X_L}$	$I\downarrow$
$\theta = 90°$	$\theta = \tan^{-1}\left(\frac{X_L}{R\uparrow}\right)\downarrow$

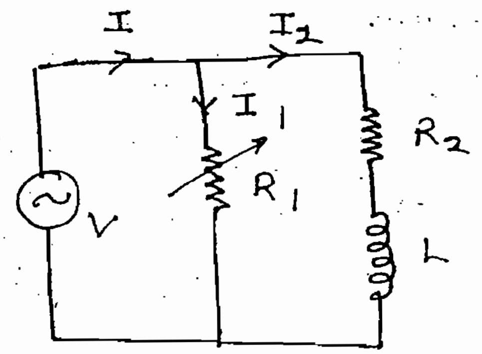




Centre $\left(0, \frac{-V}{2X_L}\right)$

→ Diameter is always due to constant elemen

Ques:- Develop current locus of I_1 and I of the circuit shown:-



Soln:-

$R_1 \simeq 0.1\,\Omega$

$I_1 = \dfrac{V}{R_1}$

$\theta_1 = 0$

$I = I_1 + I_2$

$R_1 \uparrow$

$I_1 \downarrow$

$\theta_1 = 0$

$I_2 =$ constant

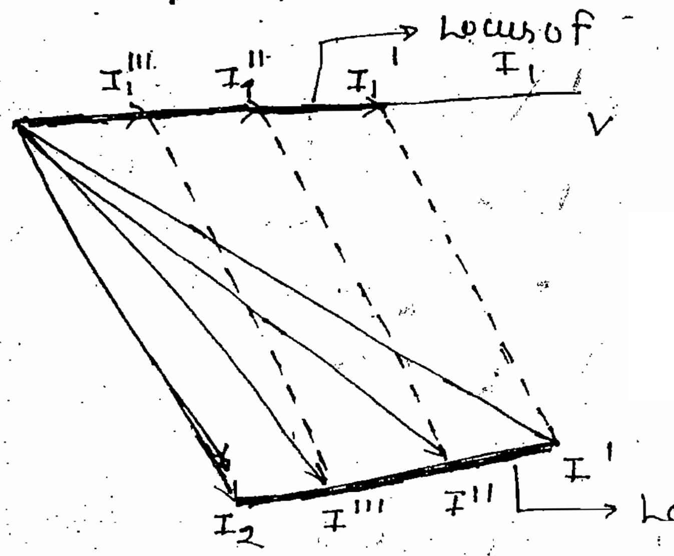
$I \downarrow$

$R_1 \simeq \infty$

$I_1 = 0$

$I_2 = I$

$I_2 = \dfrac{V}{R_2 + jX_L}$

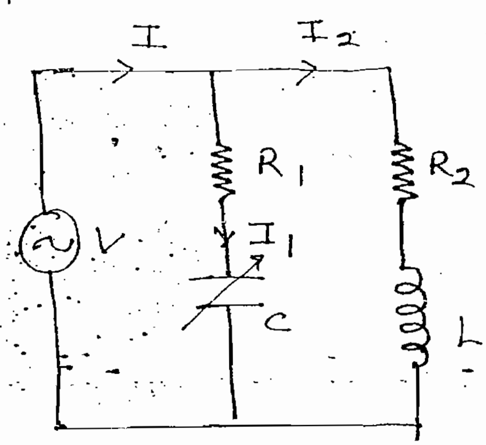


Note:-

→ When power factor angle is variable the shape of current locus is semi-circle

→ When power factor angle is constant the shape of the current locus diagram is straight line.

Ques:- Develop current locus of I_1 and I of the circuit shown



Soln:-

$X_C = \dfrac{1}{2\pi f C}$

$X_C \simeq 0$

$I_1 = \dfrac{V}{R_1}$

$\theta_1 = 0$

$I = I_1 + I_2$

$X_C \uparrow$

$Z_1 = \sqrt{R_1^2 + X_C^2} \uparrow$

$I_1 \downarrow$

$\theta_1 = \tan^{-1}\left(\dfrac{-X_C}{R}\right) \uparrow$

$I_2 =$ constant

$I_1 \downarrow$

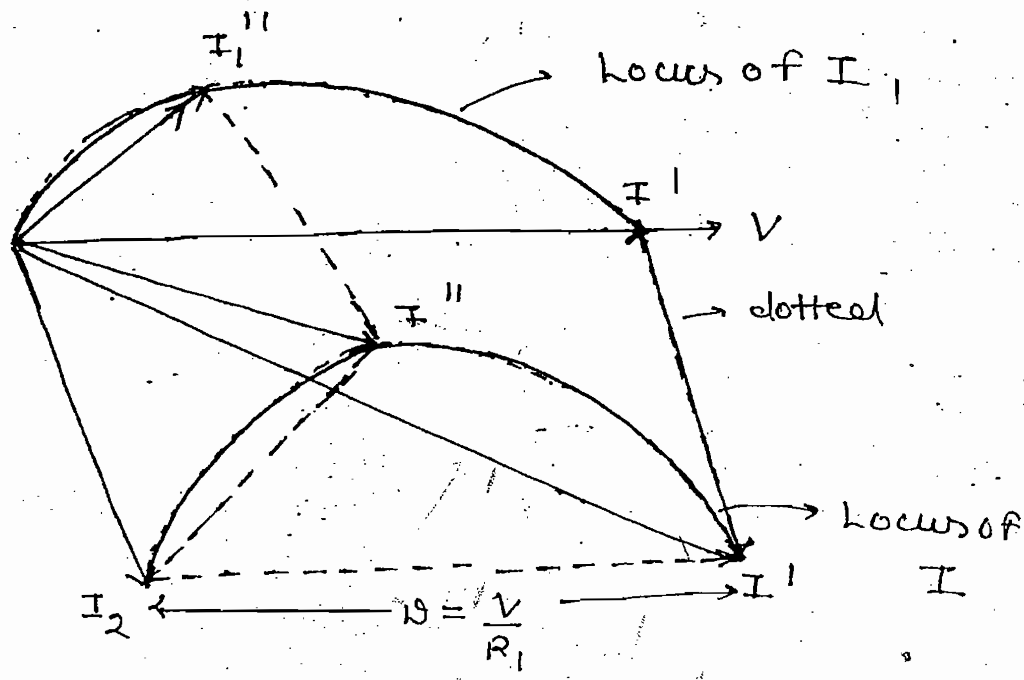
$X_C \simeq \infty$

$I_1 = 0$

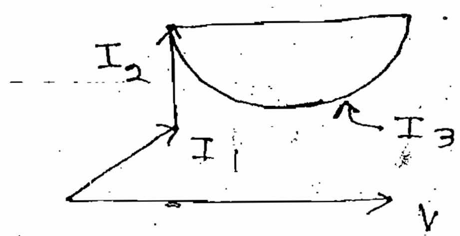
$I = I_2$

$I_2 = \dfrac{V}{R_2 + jX_L}$

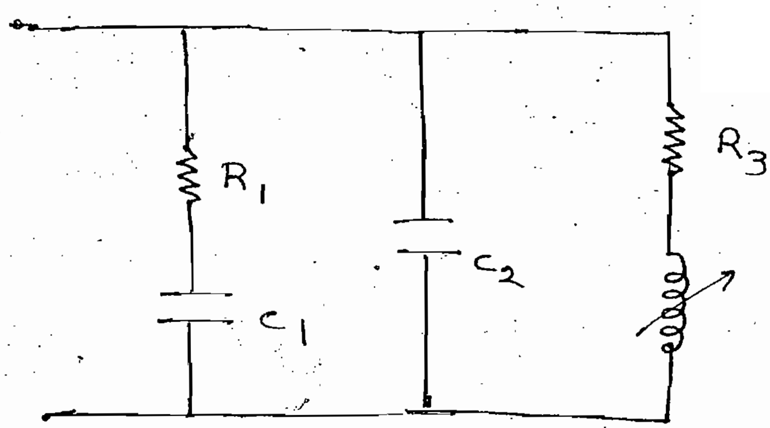
Focus only angles not sign but focus on magnitude



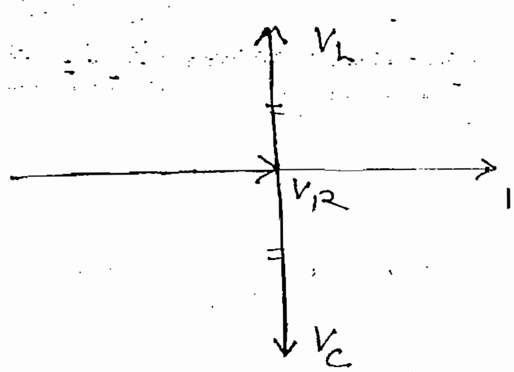
Ques:- Design a N/w for given current locus diagram

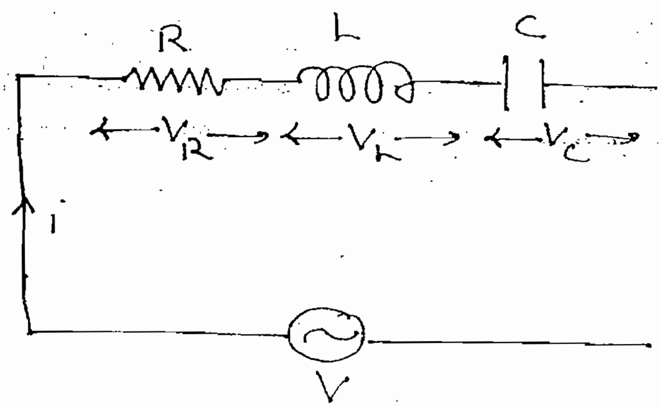


Sol^n:-



Resonance :-





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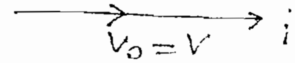


By KVL, $V = V_R\angle 0 + V_L\angle 90 + V_C\angle -90$

At resonance,

$$V_L = V_C$$

$$IX_L = IX_C$$

$$\Rightarrow \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \text{ rad/sec}$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz}$$

→ For occurence of resonance in any circuit two energies are required. In RLC circuit inductor is having energy in the form of magnetic field capacitor is having energy in the form of electric field. When these two energies are present at particular frequency wide variations are present in the system is called as resonance

→ The circuit is said to be resonance when source current is ~~influence~~ phase with source voltage

→ The frequency at which $X_C = X_L$ is called as resonant frequency

→ The resonant frequency indicates rate at which energy transformation is done b/w inductor and capacitor

1 → $$Z = R + j\underbrace{(X_L - X_C)}_{=0}$$

$$Z_{min} = R$$

2 → $$I_{max} = \frac{V}{Z_{min}} = \frac{V}{R}$$

3 → $\cos\theta = 1$

4 → $V_R = V$

5. Net Reactive.

6. Voltage across inductor or voltage across ~~capacit~~ capacitor greater than source. This phenomenon is called as <u>voltage magnification</u>

Application :-

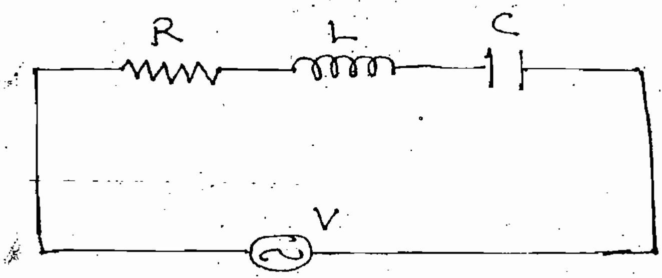
→ Oscillators

→ Filters (BP, BE)

→ Tuning circuits

→ Induction heating

Variation of voltage across inductor and voltage across capacitor w.r.t frequency :-



$V_C = I X_C$

$X_L = 2\pi f L \quad , \quad X_C = \dfrac{1}{2\pi f C}$

$\rightarrow f = 0 \ , \ X_L = 0 \ , \ X_C = \infty$

$\rightarrow \ X = |X_L - X_C| = \infty$

$\Rightarrow \ Z = \infty \ \& \ I = 0 \qquad \Rightarrow \boxed{V_C = V}$

~~For inductor~~:-

$\rightarrow f\uparrow \quad X_L\uparrow \quad X_C\downarrow \quad X = |X_L - X_C| \downarrow\downarrow \quad Z\downarrow\downarrow \quad I\uparrow\uparrow \ \boxed{V_C\uparrow}$

$\quad (10\Omega) \quad (100\Omega) \quad (90\Omega)$

$\rightarrow f\uparrow\uparrow \quad X_L\uparrow\uparrow \quad X_C\downarrow\downarrow \quad Z = R + j(X_L - X_C)\uparrow\uparrow\uparrow \quad I\downarrow\downarrow\downarrow$

$\quad \begin{pmatrix}\text{very}\\ \text{low}\end{pmatrix} \qquad V_C\downarrow$

$$\rightarrow \quad V_C = \frac{V X_C}{\sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow V_C = \frac{V \frac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{---(1)}$$

Differentiate eq-(1) w.r.t ω and equal to zero we get

$$\boxed{f_C = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \left(\frac{R^2 C}{2L}\right)^2}}$$

dL

For inductor :-

→ $V_L = IX_L$

→ $X_L = 2\pi f L$ & $X_C = \frac{1}{2\pi f C}$

→ $f = 0$, $X_L = 0$, $X_C = \infty$ $X = |X_L - X_C| = \infty$

$Z = \infty$, $I = 0$, $\boxed{V_L = 0}$

→ $f\uparrow$ $X_L \uparrow$ $X_C \downarrow$ $X = |X_L - X_C| \downarrow\downarrow$ $Z \downarrow\downarrow$

(10Ω) (100Ω)

$I\uparrow\uparrow$ $\boxed{V_L \uparrow}$

→ $f\uparrow\uparrow$ $X_L \uparrow\uparrow$ $X_C \downarrow\downarrow$

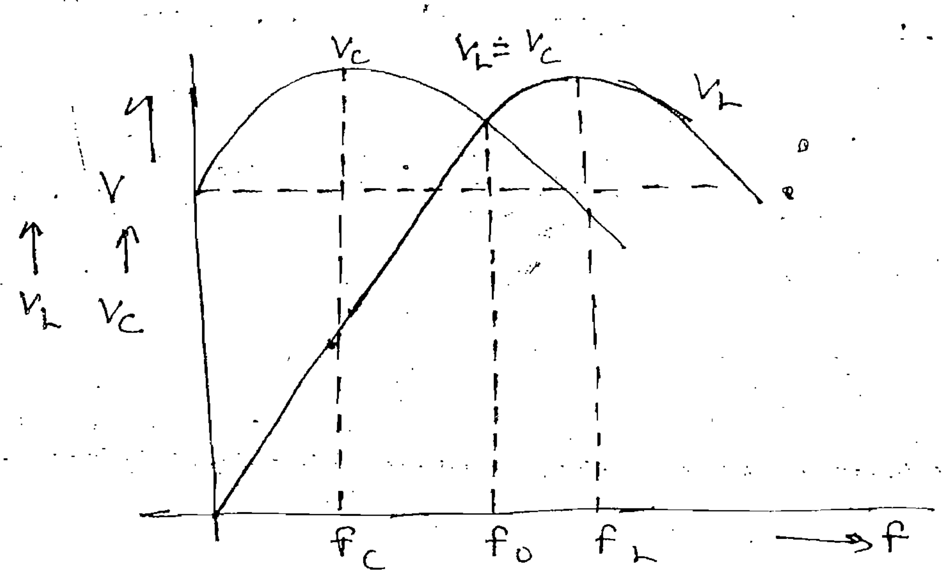
(Very low)

$Z = R + j(X_L - X_C)\uparrow\uparrow\uparrow$ $I_C \downarrow\downarrow\downarrow$ $\boxed{V_L \downarrow}$

→ $$V_L = \frac{VX_L}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V\omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})}} \quad -(11)$$

Differentiate eq-(11) w.r.t ω and equal to zero

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1 - \left(\frac{R^2 C}{2L}\right)}}}$$



Quality factor :-

→ Q-factor is a ratio of max. energy stored the circuit to power dissipation per cycle

$$Q =$$

$$i(t) = I_m \sin\omega t \quad -(1)$$

$$V_c(t) = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int I_m \sin\omega t\,dt$$

$$\Rightarrow V_c(t) = \frac{-I_m}{\omega C}\cos\omega t \quad -(11)$$

$$W_g = \frac{1}{2}Li^2 + \frac{1}{2}CV_c^2$$

$$\Rightarrow W_g = \frac{1}{2}L(I_m \sin\omega t)^2 + \frac{1}{2}C\left(-\frac{I_m}{\omega C}\cos\omega t\right)^2$$

$$\Rightarrow W_g = \frac{1}{2}LI_m^2\sin^2\omega t + \frac{1}{2}C\frac{I_m^2}{\omega^2 C^2}\cos^2\omega t$$

$$\Rightarrow W_g = \frac{1}{2}LI_m^2\sin^2\omega t + \frac{1}{2}LI_m^2\cos^2\omega t \quad \left[\omega^2 = \frac{1}{LC},\ L = \frac{1}{\omega^2 C}\right]$$

$$\Rightarrow \boxed{W_g = \frac{1}{2}LI_m^2} \qquad \boxed{W_g = \frac{1}{2}CV_{cm}^2}$$

$$\left(\because V_{cm} = \frac{I_m}{\omega C}\right)$$

$$Q = \frac{\frac{1}{2}LI_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 R\frac{1}{\omega}} \qquad I^2R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$$\Rightarrow \boxed{Q = \frac{\omega L}{R}} \qquad \left(\omega = \frac{1}{\sqrt{C}}\right)$$

$$\Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

→ $$\boxed{Q = \frac{\omega L}{R} = \frac{IX_L}{IR} = \frac{V_L}{V_R} = \frac{V_L}{V} \quad (V_R = V)}$$

d/\

→ $\boxed{Q = \frac{V_L \text{ or } V_C}{V}}$ $(\because V_L = V_C)$

→ $\boxed{Q = \frac{I^2 X_L}{I^2 R} = \frac{Q_L}{P}}$

→ $\boxed{Q = \frac{X_L \text{ or } X_C}{R}}$ $(\because X_L = X_C)$

→ $\boxed{Q = \frac{X_C}{R} = \frac{1}{\omega RC}}$

→ $Q > 1$, $X_L > R$, $X_C > R$

→ $Q \propto \frac{1}{\text{Power loss } (I^2R = P)}$

→ $Q \propto \frac{1}{BW}$

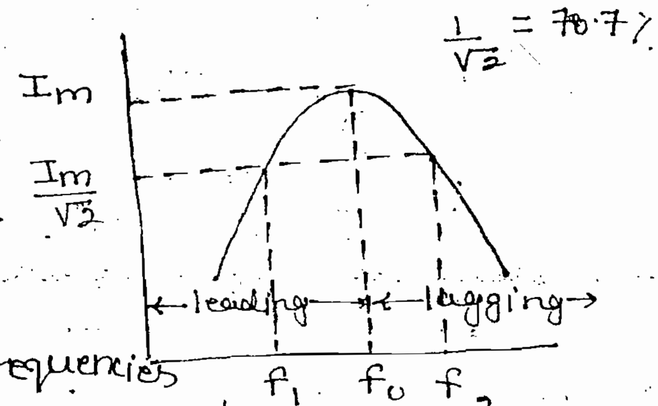
Note:-

→ To obtain high efficiency circuit is designed with high Q- factor

→ To obtain wide BW circuit is designed with low Q- factor

Bandwidth:-

→ When the curve is developed b/w current and frequency then curve is called as resonance curve



→ BW is the range of frequencies on either side of the resonant frequencies where the current falls from max. value to 70.7% of the max. value and it is given by

$$BW = f_2 - f_1$$

where f_2 = upper cut-off frequency

f_1 = lower " " "

f_1, f_2 = 3dB points or half power frequencies

1. $f_0 \rightarrow I_m$, $Z = R$

$f_1, f_2 \rightarrow \frac{I_m}{\sqrt{2}} \Rightarrow \boxed{Z = \sqrt{2} R}$

2. $f_0 \rightarrow \cos\theta = 1$

$f_1, f_2 \rightarrow Z = R \pm jX$

$Z = \sqrt{R^2 + X^2} = \sqrt{2} R$

$\Rightarrow \boxed{X = R}$

$X = X_L - X_C$

$X_L = 2\pi f L$

$X_C = \frac{1}{2\pi f C}$

$f_1 \rightarrow X_C > X_L$

$X \rightarrow -ve$

$f_2 \rightarrow X_L > X_C$

$X \rightarrow +ve$

w.r.t

$\underline{f_1} \rightarrow Z = R - jX$ $X = R$

Impedance Angle $= \tan^{-1}\left(\frac{-X}{R}\right)$

$= -45°$

$$I = \frac{V \angle 0}{Z \angle -45} = \frac{V}{Z} \angle +45$$

Power factor angle = 45

$\Rightarrow$ Power factor $= \cos 45 = \frac{1}{\sqrt{2}} \rightarrow$ leading

w.r.t f_2 :-

$f_2 \rightarrow Z = R + jX$ $X = R$

Impedance angle $= \theta = \tan^{-1}(X/R) = +45°$

Power factor angle $= -45°$

Power factor $= \cos(-45°) = \frac{1}{\sqrt{2}} \rightarrow$ lagging

3. $f_1 \rightarrow X_C > X_L$ $X = R$

$\frac{1}{\omega_1 C} - \omega_1 L = R$ —— (1)

d9

$f_2 \longrightarrow X_L > X_C \qquad X = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{--- (II)}$$

From (I) & (II)

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{--- (III)}$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC} \quad \text{--- (IV)}$$

From (III) & (IV)

$$\omega_0^2 = \omega_1 \omega_2$$

$$\Rightarrow \boxed{\begin{aligned} \omega_0 &= \sqrt{\omega_1 \omega_2} \\ f_0 &= \sqrt{f_1 f_2} \end{aligned}}$$

Add eq- (I) & (II)

$$\frac{1}{C}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\Rightarrow \frac{1}{C}\left[\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\Rightarrow L\left[\omega_2 - \omega_1\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

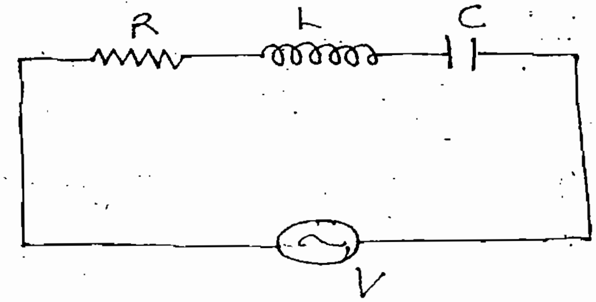
$$\boxed{\begin{aligned} BW &= \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec} \\ BW &= f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz} \end{aligned}}$$

4. $Q = \dfrac{\omega_0 L}{R} \Rightarrow Q = \dfrac{\omega_0}{R/L}$

$$\Rightarrow \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}}$$

Lecture-6

By KVL



$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Diff w.r.t t

$$V' = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

Dividing both sides of L

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{V'}{L}$$

$$\left.\begin{aligned} &\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 \\ &s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \end{aligned}\right\} \quad \frac{d}{dt} = D$$

$$2\xi\omega_n = \frac{R}{L}$$

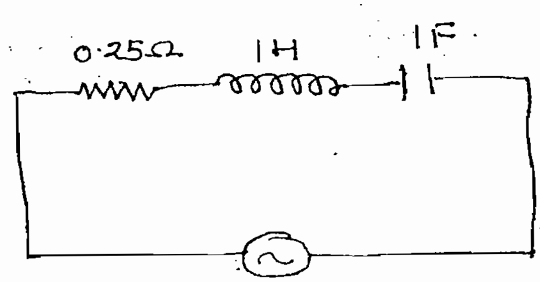
$$2\xi\frac{1}{\sqrt{LC}} = \frac{R}{L}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\text{Damping ratio} = \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\boxed{\xi = \frac{1}{2Q}}$$

Ques:- Find f_0, Q, ξ, B.W, f_1, f_2, I at f_0



$V(t) = 10\sin\omega t$

Soln:- (i) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi}$

(ii) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{0.25}\sqrt{\dfrac{1}{1}} = 4$

(iii) $\delta = \dfrac{1}{2Q} = \dfrac{1}{8}$

(iv) $\underset{(f_2-f_1)}{BW} = \dfrac{f_0}{Q} = \dfrac{1/2\pi}{4} = \dfrac{1}{8\pi}$

(v) $f_2 - f_1 = \dfrac{1}{8\pi}$

$f_1 f_2 = f_0^2 = \left(\dfrac{1}{2\pi}\right)^2$

$(f_2+f_1)^2 - (f_2-f_1)^2 = 4f_1f_2$

$f_1 =$

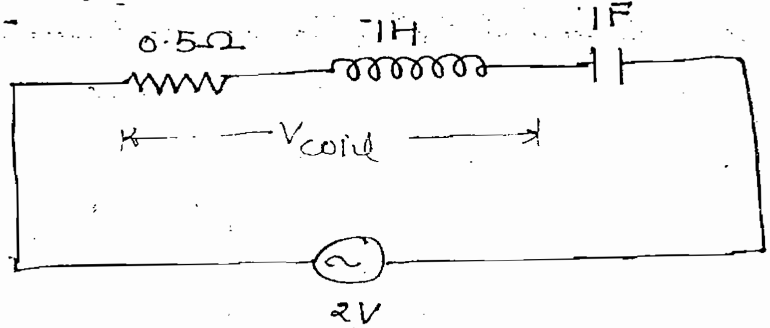
$f_2 =$

(vi) $I = \dfrac{V}{Z} = \dfrac{V}{R}$, (At Resonance)

$I = \dfrac{10/\sqrt{2}}{0.25}$

$I = \dfrac{40}{\sqrt{2}}$, Ans.

Ques:- Find voltage across the coil under resonance condition :-



Soln:— $V_R = V = 2$ [When nothing is given take it as RMS]

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{0.5}\sqrt{\frac{1}{1}} = 2$$

$$Q = \frac{V_L}{V}$$

$$\Rightarrow 2 = \frac{V_L}{2}$$

$$\Rightarrow V_L = 4$$

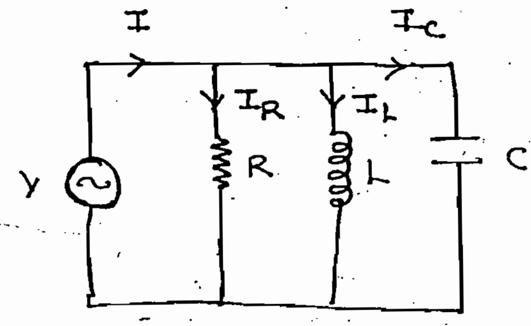
$$V_{coil} = \sqrt{V_R^2 + V_L^2}$$

$$\Rightarrow V_{coil} = \sqrt{2^2 + 4^2}$$

$$= \sqrt{20}$$, Ans.

Parallel Resonance :-

Case-(1) :-

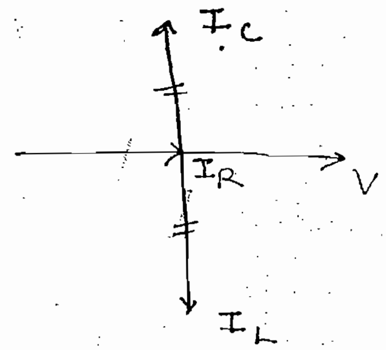


$$I_C = I_L$$

$$\frac{V}{X_C} = \frac{V}{X_L} \qquad X_L = X_L$$

$$\Rightarrow \boxed{B_C = B_L}$$

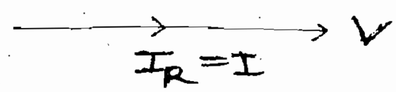
$$\omega C = \frac{1}{\omega L}$$



$$\boxed{\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \ \text{rad/sec} \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \ \text{Hz} \end{aligned}}$$

(I) $Y = G + j\underbrace{(B_C - B_L)}_{0}$

$Y_{min} = G$



(II) $Z_{max} = \frac{1}{Y_{min}}$

(III) $I_{min} = \frac{V}{Z_{max}}$

(IV) $\cos\theta = 1$

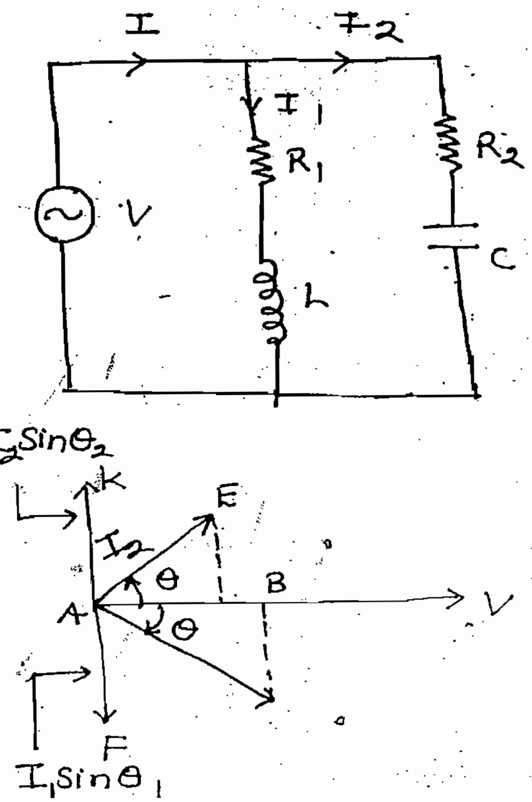
(V) $I_R = I$

(VI) Net Reactive current = 0

(VII) current in inductor or current in capacitor is greater than total current. This phenomenon is called as <u>current magnification</u>

viii) Parallel Resonance circuit is also called as Anti-Resonance circuit.

Case-(II):-



$$AB = I_1 \cos\theta_1$$

$$AF = BC = I_1 \sin\theta_1$$

$$AB = I_2 \cos\theta_2$$

$$AK = BE = I_2 \sin\theta_2$$

$$B_L = B_C$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega L}{R_1^2 + (\omega L)^2} = \frac{1/\omega C}{R_2^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - \frac{L}{C}}{R_2^2 - L/C}}}$$

$\longrightarrow V$
I

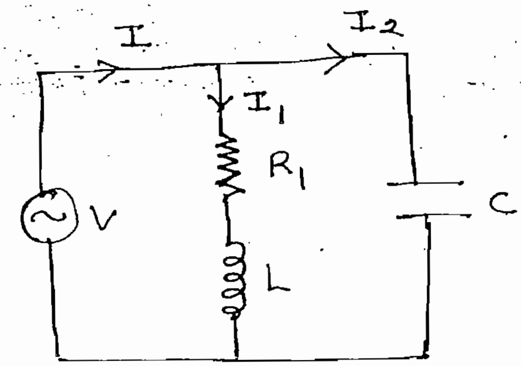
$$I = I_1 \cos\theta_1 + I_2 \cos\theta_2$$

$$I = VY$$

$$\Rightarrow I = V\left[(G_1 + G_2) + j\underbrace{(B_C - B_L)}_{0}\right]$$

$$\Rightarrow \boxed{I = V\left[\frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2}\right]}$$

Imp
Case-(III):-

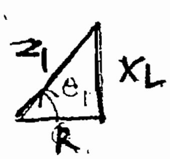


Tank ckt

$$AB = I_1 \cos\theta_1$$

$$AF = BC = I_1 \sin\theta_1$$

$$\cos\theta_1 = \frac{R_1}{Z_1}$$



$$\sin\theta_1 = \frac{X_L}{Z_1}$$

$$I_2 = I_1 \sin\theta_1$$

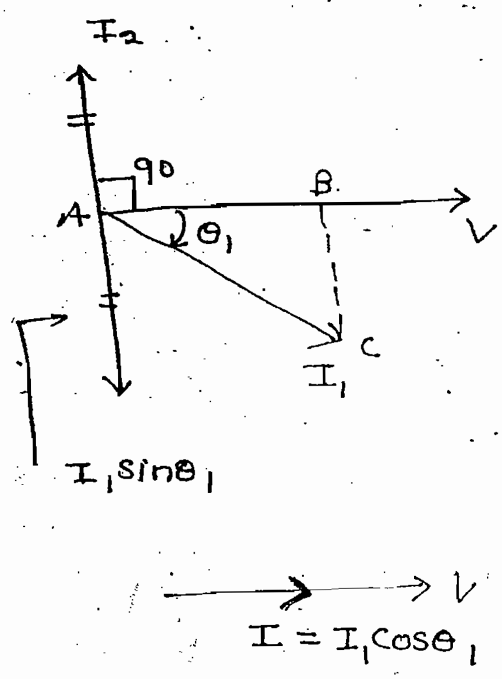
$$\frac{V}{X_C} = \frac{V}{Z_1}\,\frac{X_L}{Z_1}$$

$$\Rightarrow Z_1^2 = X_L X_C$$

$$\Rightarrow Z_1^2 = \frac{\omega L}{\omega C}$$

$$\Rightarrow Z_1^2 = \frac{L}{C}$$

$$\Rightarrow \boxed{Z_1 = \sqrt{\frac{L}{C}}}\ \Omega$$



$$\rightarrow \quad B_L = B_C$$

$$Z_1^2 = R_1^2 + X_L^2$$

$$\frac{L}{C} = R_1^2 + (2\pi f_0 L)^2$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_1^2}{L^2}}}$$

$$\rightarrow \quad I = I_1 \cos\theta_1$$

$$\Rightarrow I = \frac{V}{Z_1}\,\frac{R_1}{Z_1}$$

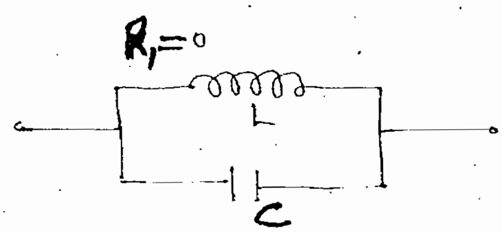
$$\Rightarrow I = \frac{VR_1}{Z_1^2} \Rightarrow I = \frac{VR_1}{L/C}$$

$$\Rightarrow \boxed{I = \frac{V}{\frac{L}{R_1 C}}}$$

$$\Rightarrow \boxed{Z_{Dyn} = \frac{L}{R_1 C}}\ \Omega$$

Ideal Tank Circuit :-

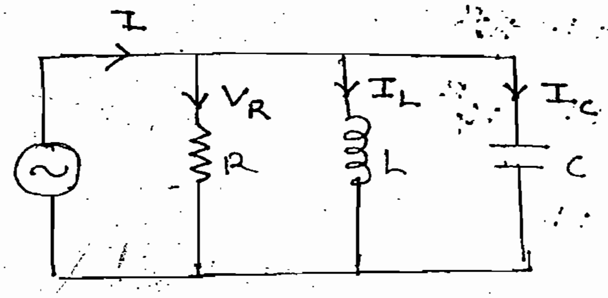
$$\boxed{\begin{aligned} Z_{Dyn} &= \frac{L}{R_1 C} = \infty \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}}$$



Q - Factor :-

$$\boxed{Q = \frac{V_L \text{ or } V_C}{V}} \rightarrow \text{Series}$$

$$\boxed{Q = \frac{I_L \text{ or } I_C}{I}} \rightarrow \text{Parallel}$$



$$\boxed{Q = \frac{I_L}{I} = \frac{I_L}{I_R} = \frac{\text{Reactive component of current}}{\text{Active component of current}}}$$

This combination is valid for any combination of parallel circuit

$$\boxed{Q = \frac{I_L}{I_R} = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega L}} \qquad \left(\omega = \frac{1}{\sqrt{LC}}\right)$$

$$\boxed{Q = \frac{1/X_L}{1/R} = \frac{B_L \text{ or } B_C}{G}} \qquad (B_L = B_C)$$

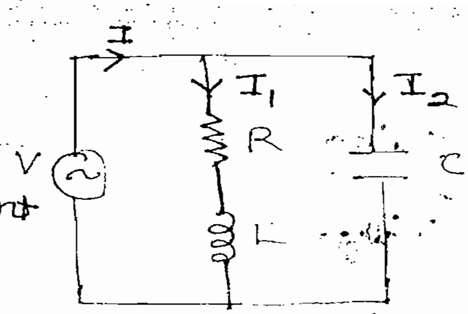
$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$

$$\boxed{Q = \frac{I_C}{I} = \frac{I_C}{I_R} = \frac{V/X_C}{V/R} = \frac{R}{X_C} = R\omega C}$$

For $\boxed{Q > 1 \quad , \quad R > X_L \quad , \quad R > X_C}$

Tank Circuit :-

$$Q = \frac{\text{Reactive component of current}}{\text{Active comp. of current}}$$

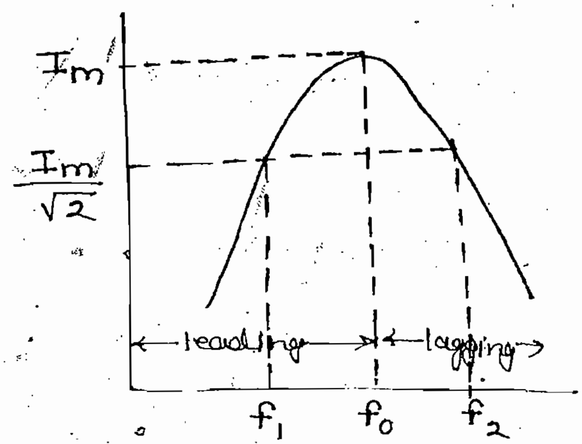


$$\boxed{Q = \frac{I_1 \sin\theta_1 \text{ or } I_2}{I}}$$

$$\boxed{Q = \frac{I_2}{I} = \frac{V/X_C}{\frac{V}{L/RC}} = \frac{\frac{1}{1/\omega C}}{\frac{1}{L/RC}} = \frac{\omega L}{R} = \frac{X_L}{R}}$$

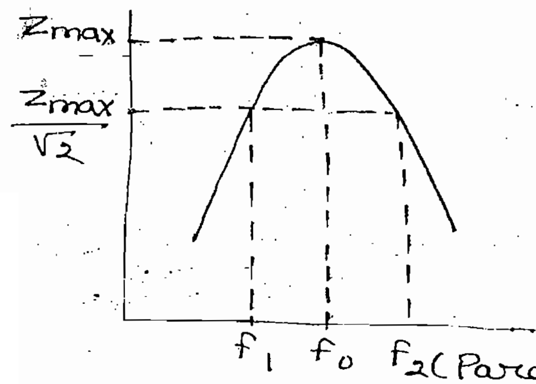
For $Q > 1$, $\boxed{X_L > R}$

$$\boxed{B \cdot W = f_2 - f_1}$$



(Series)

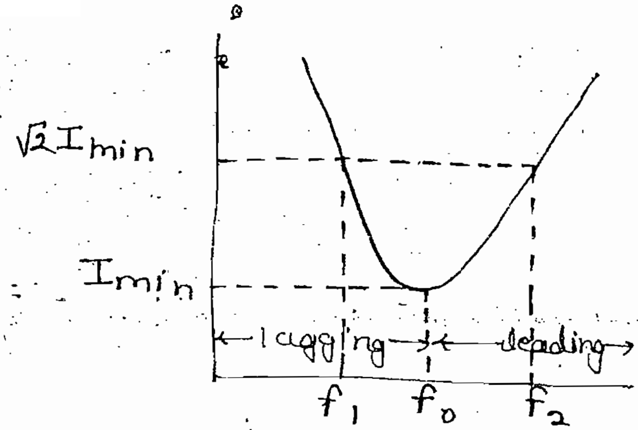
$$\boxed{B \cdot W = f_2 - f_1}$$



(Parallel)

$$\boxed{B \cdot W = f_2 - f_1}$$

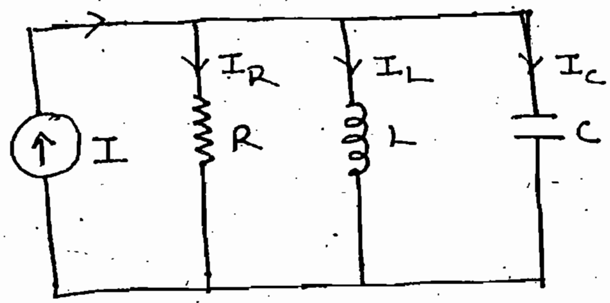
$$\boxed{\begin{aligned} B_L &= \frac{1}{2\pi f L} \\ B_C &= 2\pi f C \end{aligned}}$$



(Parallel)

For parallel circuit we concentrate on the value of B_L and B_C

By KCL



$$I = \frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int V dt$$

Diff. w.r.t t

$$I' = \frac{1}{R}\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{V}{L}$$

Dividing both sides by C, we get

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V}{LC} = \frac{I'}{C}$$

$$D = \frac{d}{dt}$$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V = 0$$

Compare with $s^2 + 2\delta\omega_n s + \omega_n^2 = 0$ we get

$$2\delta\omega_n = \frac{1}{RC}, \quad \omega_n^2 = \frac{1}{LC}$$

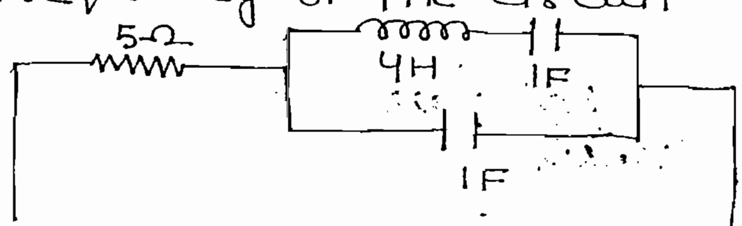
$$2\delta\frac{1}{\sqrt{LC}} = \frac{1}{RC}, \quad \omega_n = \frac{1}{\sqrt{LC}}$$

$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$

$$\boxed{\text{Damping Ratio} = \delta = \frac{1}{2R}\sqrt{\frac{L}{C}}}$$

$$\boxed{\delta = \frac{1}{2Q}}$$

Ques:- Find resonant frequency of the circuit shown



Soln:— At resonance in any of the case imag. part = 0

Note:—

To find resonant frequency for any combination of the network

(I) Find Z_{eq}

(II) Equate imag. part of $Z = 0$

$$Z_1 = j(X_L - X_C) = j\left(\omega L - \frac{1}{\omega C}\right) = j\left(4\omega - \frac{1}{\omega}\right)$$

$$Z_2 = -jX_C = -\frac{j}{\omega C} = -\frac{j}{\omega}$$

$$Z'_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\Rightarrow Z'_{eq} = \frac{j\left(4\omega - \frac{1}{\omega}\right)\left(-j/\omega\right)}{j\left(4\omega - \frac{1}{\omega}\right) - j/\omega}$$

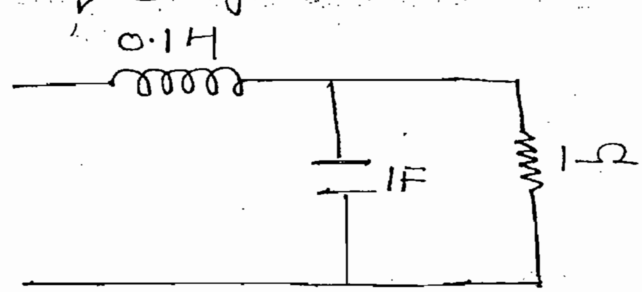
$$\Rightarrow Z'_{eq} = \frac{\left(4\omega - 1/\omega\right)\left(+1/\omega\right)}{j\left(4\omega - \frac{1}{\omega}\right) - j/\omega} \times \frac{j}{j}$$

$$\text{Im } Z'_{eq} = 0$$

$$\left(4\omega - \frac{1}{\omega}\right)\frac{1}{\omega} = 0$$

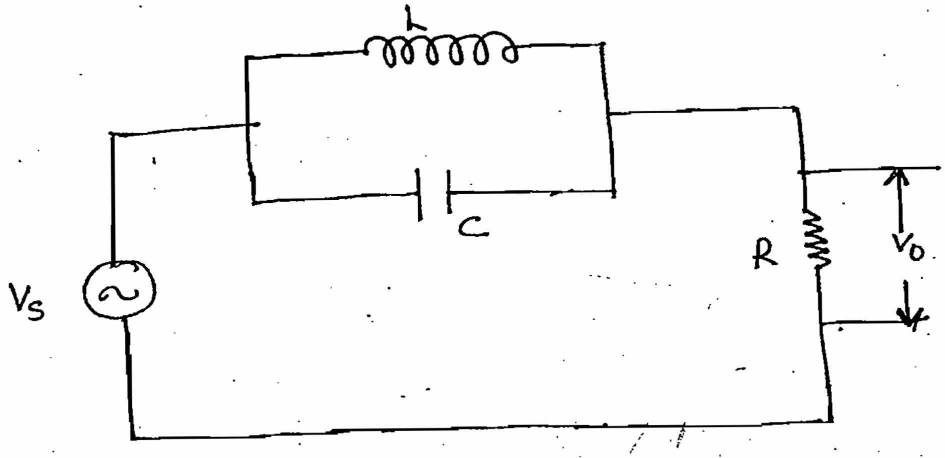
$$\Rightarrow \boxed{\omega_0 = 0.5 \text{ rad/sec}}$$

Ques:— Find resonant frequency of the circuit shown

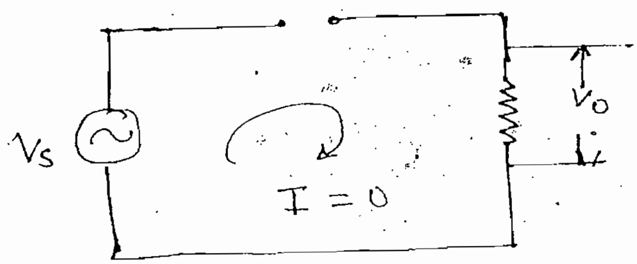


Ans:— $\omega_0 = 3$ rad/sec

Ques 1- Find V_o under resonance condition



Soln- Ideal tank circuit, $Z_{Dyn} = \infty$

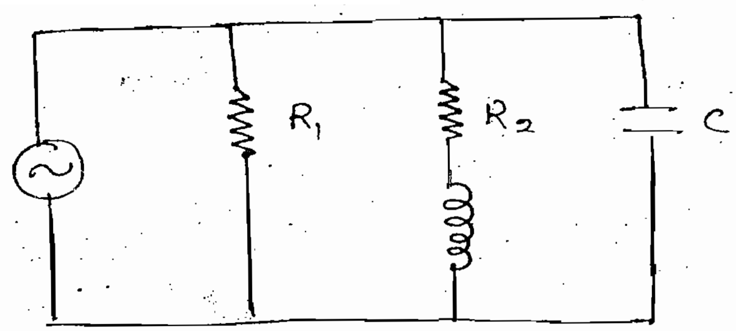


$V_o = 0$, Ans

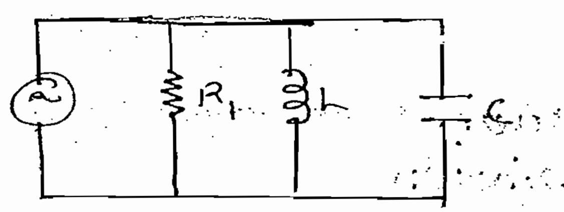
Note:-

$R_1 \uparrow \quad Q \uparrow$

$R_2 \uparrow \quad Q \downarrow$



$$Q = \frac{R_1}{\omega L} \leftarrow$$



$$Q = \frac{\omega L}{R_2} \leftarrow$$