Real Number

OLYMPIAD EXCELLENCE BOOK **MATHEMATICS**

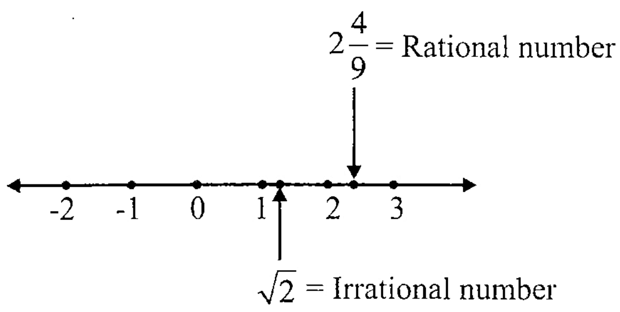
NOTES

FUNDAMENTALS

- Rational numbers: Numbers which can be written in the form of $\frac{p}{q}(q \neq 0)$, where p and q are integers, are called rational numbers.

  Note: Every terminating decimal and non-terminating repeating decimal can be expressed as a rational number,

- **Irrational numbers:** Numbers which cannot be written in the form of $\frac{p}{q}$ where p and q are integers and q ^ 0 are called irrational, numbers. Numbers which are not rational are called irrational numbers.
- **Real numbers:** The rational numbers and the irrational numbers together are called real numbers. Both rational & irrational numbers real line on number line.



  Note: Any number that can be represented on a number line is called a real number.

- **Lemma:** A proven statement which is used to prove another statement is called a lemma.
- **Euclid's division lemma:** For any two positive integers 'a' and 'b', there exist whole numbers 'q' and 'r' such that $a = bq + r, 0 \le r < b$

  This is an extension of the idea:

Dividend =	Divisor x	quotient +	Remainder
(a)	(b)	(q)	(r)

  Remainder 'r' is always less than divisor (b) (This is basic principle of mathematics).

  Note: Euclid's division algorithm is stated only for positive integers, but can be extended/or all negative integers.

- **Algorithm:** An algorithm is a process of solving particular problems.
- **Euclid's division algorithm** is used to find the Highest Common Factor (H.C.F.) of two numbers.

- **Following is the procedure for finding H.C.F. using Euclid's division algorithm:** Suppose the two positive numbers are 'a' and 'b', such that a > b. Then the H.C.F. of 'a' and 'b' can be found by following the steps given.

(a) Apply the division lemma to find 'q' and 'r' where $a = bq + r, 0 \le r < b$.

(b) If **r = 0,** then *H.C.F. is b. If* $r \ne 0$, then apply Euclid's lemma to find 'b' and 'r'.

- Continue steps (a) and (b) till r = 0. The divisor at this state will be H.C.F. (a, b). Also, H.C.F. (a, b) = H.C.F. (b, r).

- **Fundamental theorem of Arithmetic:** Every composite number can be expressed as a unique product of prime numbers. This is also called the prime factorization theorem.

  Note: (i) The order in which the prime factors occur is immaterial.

  In general, any composite number x, can be expressed as a product of prime numbers

Elementary Question: 1

Find HCF of 6 and 16.

Also verify that HCF of 18 and 48 is 3 times HCF of 6 and 16.

Sol.: 6 and 16: $6 = 2 \times 3$

$16 = 2 \times 2 \times 2 \times 2 \quad \therefore (HCF)_1 = 2$

18 and 48: $18 = 2 \times 3 \times 3; \quad 48 = 2 \times 2 \times 2 \times 2 \times 3$

$\therefore (HCF)_2 = 2 \times 3 = 6; \quad \therefore (HCF)_2 = 3 \times (HCF)_1$

Elementary Question: 2

Do the above problem by Euclid is division algorithm.

- L.C.M. of $\frac{a}{b}$ and $\frac{c}{d} = \frac{L.C.M.\ of\ a\ and\ c}{H.C.F.\ of\ b\ and\ d} = \frac{L.C.M.\ (a, c)}{H.C.F.\ (b, d)}$.
- **Some Important Result on Natural Numbers**
- $1 + 2 + 3 + \text{-----------} + n = \frac{n(n+1)}{2}$
- $1^2 + 2^2 + 3^2 + \text{-----------} + n^2 = \frac{n(n+1)(2n+1)}{6}$
- $1^3 + 2^3 + 3^3 + \text{-----------} + n^3 = \left[\frac{n(n+1)}{2}\right]^2$
- $x^n + y^n$ is divisible by $(x + y)$ if n is odd.
- $x^n - y^n$ is divisible by $(x - y)$ for all values of n.
- If a number is divisible by m and n, then it is always divisible by the LCM of m and n.
- $x^n - y^n$ is divisible by $(x + y)$ if n is even.