SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	1(1)	–	1(3)	–	**2(4)**
2.	Inverse Trigonometric Functions	2(2)	1(2)	–	–	**3(4)**
3.	Matrices	2(2)#	1(2)	–	–	**3(4)**
4.	Determinants	1(1)	–	–	1(5)*	**2(6)**
5.	Continuity and Differentiability	–	1(2)*	2(6)	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)*	1(3)	–	**3(9)**
7.	Integrals	2(2)#	1(2)	1(3)*	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)	1(2)	1(3)*	–	**3(6)**
10.	Vector Algebra	3(3)#	1(2)*	–	–	**4(5)**
11.	Three Dimensional Geometry	4(4)#	–	–	1(5)*	**5(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	1(4)	2(4)	–	–	**3(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. If a matrix A is both symmetric and skew-symmetric, then show that A is a zero matrix.

OR

If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then find the value of k.

2. Evaluate : $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx$

3. If the direction ratios of a line are 1, –3, 2, then find its direction cosines.

OR

The coordinates of a point P are (3, 12, 4) w.r.t. origin O, then find the direction cosines of OP.

4. Let R be a relation defined on the set of natural numbers N as follow :
$R = \{(x, y) \mid x \in N, y \in N \text{ and } 2x + y = 24\}$
Find the domain and range of the relation R.

5. Find the distance from the origin to the plane $x + 3y - 2z + 1 = 0$.

OR

Find the foot of the perpendicular from (0, 0, 0) to $3x + 4y - 6z = 0$.

6. Construct a matrix $A = [a_{ij}]_{2\times2}$, where $a_{ij} = i + j$.

7. If $\vec{p} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are the position vectors of points P, Q respectively and point $R(\vec{r})$ divides the line PQ internally in the ratio 2 : 1, then find the coordinates of R.

OR

If $\vec{a} = \hat{i} + 3\hat{j}$, $\vec{b} = 2\hat{i} + 5\hat{j}$, $\vec{c} = 4\hat{i} + 2\hat{j}$ and $\vec{c} = t_1\vec{a} + t_2\vec{b}$, then find the value of t_1 and t_2.

8. Find the principal value of $\tan^{-1}(-\sqrt{3})$.

9. Evaluate : $\int_0^{\pi/4} \tan x\, dx$

OR

Evaluate : $\int x \cot^{-1} x\, dx$

10. Find the value of $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{i}\times\hat{k}) + \hat{k}\cdot(\hat{i}\times\hat{j})$.

11. If (2, 4, –3) is the foot of the perpendicular drawn from the origin to a plane, then find the equation of the plane.

12. Find the domain of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$.

13. Find the equation of the line in symmetric form which passes through the points $A(-2, -1, 5)$ and $B(1, 3, -1)$.

14. Find the value of x for which the matrix $A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular.

15. The cartesian equations of a line are $6x - 2 = 3y + 3 = 2z - 4$. Find the direction ratios of the line.

16. Find the integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{\sqrt{1-x^2}} = x$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. In a family there are four children. All of them have to work in fields to earn their livelihood at the age of 15.

Based on the above information, answer the following questions :

(i) Probability that all children working in fields are boys if it is given that elder child working in fields is a boy, is

(a) 3/8 (b) 1/8 (c) 5/8 (d) none of these

(ii) Probability that all children working in fields are grass, if first two children working in fields are girls, is

(a) 1/4 (b) 3/4 (c) 1/2 (d) none of these

(iii) Find the probability that two middle child working in fields are boys if it is given that first child working in fields is a girl.

(a) 0 (b) 3/4 (c) 1/4 (d) none of these

(iv) Find the probability that all children working in fields are girls if it is given that at least one of the children working in fields is a girl.

(a) 0 (b) 1/15 (c) 2/15 (d) 4/15

(v) Find the probability that all children working in fields are boys if it is given that at least three of the children working in fields are boys.

(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

18. In a street two lamp posts are 300 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\dfrac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\dfrac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source).

The combined light intensity is the sum of the two light intensities coming from both lamp posts.
Based on the above information, answer the following.

(i) If you are in between the lamp posts, at distance x feet from the stronger light, then the formula for the combined light intensity coming from both lamp posts as function of x, is

(a) $\dfrac{1000}{x^2}+\dfrac{125}{x^2}$ (b) $\dfrac{1000}{(300-x^2)}+\dfrac{125}{x^2}$ (c) $\dfrac{1000}{x^2}+\dfrac{125}{(300-x)^2}$ (d) None of these

(ii) The maximum value of x can not be

(a) 100 (b) 200 (c) 300 (d) None of these

(iii) The minimum value of x can not be

(a) 0 (b) 100 (c) 200 (d) None of these

(iv) If $I(x)$ denote the combined light intensity, then $I(x)$ will be minimum when $x =$

(a) 100 (b) 200 (c) 300 (d) 150

(v) The darkest spot between the two lights is

(a) at a distance of 100 feet from the weaker lamp post.
(b) at distance of 100 feet from the stronger lamp post.
(c) at a distance of 200 feet from the weaker lamp post.
(d) None of these

PART – B

Section - III

19. Evaluate $\displaystyle\int_2^4 \frac{x}{x^2+1}dx$ by using substitution method.

20. If $y^2 = ax^2 + bx + c$, then find the value of $\dfrac{d}{dx}\left(y^3 y_2\right)$.

OR

Differentiate $e^x \log(\sin 2x)$ w.r.t. x.

21. A bag contains 6 red, 5 blue and 7 white balls. If three balls are drawn one by one (without replacement), then what is the probability that all three balls are blue?

22. Determine the area enclosed between the curve $y = \cos^2 x,\ 0 \le x \le \dfrac{\pi}{2}$ and the axes.

23. Find a unit vector perpendicular to the plane of $\vec{a} = 2\hat{i}-6\hat{j}-3\hat{k}$ and $\vec{b} = 4\hat{i}+3\hat{j}-\hat{k}$.

OR

Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{i}+\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+4\hat{k}$.

24. Solve the differential equation $\dfrac{dy}{dx}+1=e^{x+y}$.

25. An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B be the event 'odd number on the second throw'. Check the independence of the events A and B.

26. Evaluate : $\sin\left[2\cos^{-1}\left(\dfrac{-3}{5}\right)\right]$

27. If $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, then show that $A^2+I \neq A(A^2-I)$.

28. Find the values of x if $f(x)=6(x^2-5x-24)$ is an increasing function.

OR

A rod 108 metres long is bent to form a rectangle. Find its dimensions, if its area is maximum.

Section - IV

29. If the tangent at $P(1, 1)$ on $y^2=x(2-x)^2$ meets the curve again at Q, then find the point Q.

30. Show that the function $f: R \rightarrow R$ given by $f(x)=x^3+x$ is bijective.

31. If $\frac{xdy}{dx}+2y=\ln x$, then find the value of $e^2\, y(e)-y(1)$.

OR

Solve the initial value problem $2xy+y^2-2x^2\frac{dy}{dx}=0, y(1)=2$.

32. Find the value of k, for which $f(x)=\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\leq x<0\\ \frac{2x+1}{x-1}, & \text{if } 0\leq x<1\end{cases}$ is continuous at $x=0$.

33. Find the area bounded by the circle $x^2+y^2=8x$ and the line $x=2$.

34. Evaluate : $\int \frac{x^2+9}{x^4-2x^2+81}dx$

OR

Evaluate : $\int_0^{\pi/2} x^2 \sin^2 x\, dx$

35. Find the second order derivative of $a \sin^3 t$ with respect to $a \cos^3 t$ at $t=\pi/4$.

Section - V

36. Solve the following problem graphically :
Maximize $Z=22x+18y$
Subject to constraints :
$x+y\leq 20$
$36x+24y\leq 576$
$x, y\geq 0$

OR

Find the maximum value of $z=3x+5y$ subject to $x+4y\leq 24$, $3x+y\leq 21$, $x+y\leq 9$, $x\geq 0$, $y\geq 0$.

37. Find the equation of the plane containing the lines $\vec{r}=\hat{i}+\hat{j}+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=\hat{i}+\hat{j}+\mu(-\hat{i}+\hat{j}-2\hat{k})$. Find the distance of this plane from origin and also from the point (1, 1, 1).

OR

Find the length of the perpendicular drawn from the point (2, 4, –1) to the line $\vec{r}=\hat{i}+\lambda(2\hat{i}+\hat{j}+2\hat{k})$.

38. If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, then find A^{-1} and hence solve the system of linear equations $x+2y+z=4$, $-x+y+z=0$, $x-3y+z=2$.

OR

Solve the following system of equations :
$3x-y+z=5$
$2x-2y+3z=7$
$x+y-z=-1$

SOLUTIONS

1. Given, A is a symmetric matrix.

$\therefore\ A^T = A$ ...(i)

A is also a skew-symmetric matrix.

$\therefore\ A^T = -A$ ...(ii)

From (i) and (ii), $A = -A \Rightarrow A = O$

Hence, A is a zero matrix.

OR

Given, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1\cdot3+2\cdot2 & 1\cdot1+2\cdot5 \\ 3\cdot3+4\cdot2 & 3\cdot1+4\cdot5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix} \Rightarrow k = 17$

2. Let $I = \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

$= \int \frac{2\cos^2 x - 1 + 2\sin^2 x}{\cos^2 x} dx$

$= \int \frac{2(\cos^2 x + \sin^2 x) - 1}{\cos^2 x} dx = \int \sec^2 x\, dx = \tan x + c$

3. We have, $\sqrt{(1)^2 + (-3)^2 + (2)^2} = \sqrt{14}$

$\therefore$ Direction cosines are $\frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$.

OR

Direction ratios of OP are $(3 - 0, 12 - 0, 4 - 0)$ or $(3, 12, 4)$.

Also, $\sqrt{3^2 + 12^2 + 4^2} = 13$

$\therefore$ Direction cosines are $\left(\frac{3}{13}, \frac{12}{13}, \frac{4}{13}\right)$ or $\left(\frac{-3}{13}, \frac{-12}{13}, \frac{-4}{13}\right)$.

4. Here, $R = \{(x, y) \mid x \in N, y \in N \text{ and } 2x + y = 24\}$

$\therefore\ R = \{(1, 22), (2, 20), (3, 18), (4, 16), (5, 14), (6, 12), (7, 10), (8, 8), (9, 6), (10, 4), (11, 2)\}$

Domain of $R = \{1, 2, 3, 4, ..., 11\}$

Range of $R = \{2, 4, 6, 8, 10, 12, ..., 22\}$

5. Required distance,

$$d = \left|\frac{0+0-0+1}{\sqrt{(1)^2+(3)^2+(-2)^2}}\right| = \frac{1}{\sqrt{14}}$$

OR

Given plane is $3x + 4y - 6z = 0$ ...(i)

The d.r.'s of normal to plane (i) are $3, 4, -6$.

D.c.'s of normal are $\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$

Here, $d = \frac{0}{\sqrt{61}} = 0$

$\therefore$ Foot of perpendicular is (ld, md, nd) *i.e.*, $(0, 0, 0)$

6. Here, $a_{11} = 1 + 1 = 2$, $a_{12} = 1 + 2 = 3$, $a_{21} = 2 + 1 = 3$ and $a_{22} = 2 + 2 = 4$

Hence, $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$.

7. Here $\vec{r} = \frac{2\vec{q} + \vec{p}}{2+1} = \frac{2\hat{i} + 8\hat{j} - 4\hat{k} + \hat{i} - 2\hat{j} + \hat{k}}{3}$

$= \frac{3\hat{i} + 6\hat{j} - 3\hat{k}}{3} = \hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow R \equiv (1, 2, -1)$

OR

Here, $\vec{c} = t_1\vec{a} + t_2\vec{b}$

$\Rightarrow 4\hat{i} + 2\hat{j} = t_1(\hat{i} + 3\hat{j}) + t_2(2\hat{i} + 5\hat{j})$

$\Rightarrow t_1 + 2t_2 = 4$ ...(i)

and $3t_1 + 5t_2 = 2$ ...(ii)

Solving (i) and (ii), we get

$t_1 = -16, t_2 = 10$

8. Let $\tan^{-1}(-\sqrt{3}) = \alpha \Rightarrow \tan\alpha = -\sqrt{3} = -\tan\frac{\pi}{3}$

$= \tan\left(-\frac{\pi}{3}\right) \Rightarrow \alpha = \frac{-\pi}{3} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Principal value of $\tan^{-1}(-\sqrt{3})$ is $\left(\frac{-\pi}{3}\right)$.

9. We have $\int_0^{\pi/4} \tan x\, dx = [\log|\sec x|]_0^{\pi/4}$

$= \log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0| = \log|\sqrt{2}| - \log|1| = \frac{1}{2}\log 2$

OR

Let $I = \int x \cot^{-1} x\, dx$

$= \frac{x^2}{2}\cot^{-1} x + \int \frac{x^2/2}{1+x^2} dx$

$= \frac{x^2}{2}\cot^{-1} x + \frac{1}{2}\int \frac{x^2+1}{x^2+1} dx - \frac{1}{2}\int \frac{1}{x^2+1} dx$

$= \frac{x^2}{2}\cot^{-1} x + \frac{x}{2} + \frac{\cot^{-1} x}{2} + C$

10. $\hat{i}\cdot\hat{i}+\hat{j}(-\hat{j})+\hat{k}\cdot\hat{k}=1-1+1=1$

11. Foot of perpendicular from (0, 0, 0) to the plane is (2, 4 –3), then $\vec{a}=2\hat{i}+4\hat{j}-3\hat{k}$

Normal to plane is, $\vec{n}=2\hat{i}+4\hat{j}-3\hat{k}$

$\therefore$ Equation of plane is, $\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})\cdot(2\hat{i}+4\hat{j}-3\hat{k})=4+16+9=29$

$\Rightarrow 2x+4y-3z-29=0$

12. Given $f(x)=\sin^{-1}x+\tan^{-1}x+\sec^{-1}x$

Domain of $\sin^{-1}x=[-1,1]$

Domain of $\tan^{-1}x=(-\infty,\infty)$

Domain of $\sec^{-1}x=(-\infty,\infty)-(-1,1)$

Domain of $f(x)=[-1,1]\cap(-\infty,\infty)\cap[(-\infty,\infty)-(-1,1)]$
$=\{-1,1\}$

13. The symmetric form of the equation of line passing through $A(-2,-1,5)$ and $B(1,3,-1)$ is

$$\frac{x-(-2)}{1-(-2)}=\frac{y-(-1)}{3-(-1)}=\frac{z-5}{-1-5}\Rightarrow\frac{x+2}{3}=\frac{y+1}{4}=\frac{z-5}{-6}$$

14. A is singular $\therefore |A|=0$

$$\Rightarrow\begin{vmatrix}3-x & 2 & 2\\ 2 & 4-x & 1\\ -2 & -4 & -1-x\end{vmatrix}=0$$

$\Rightarrow (3-x)(-4-4x+x+x^2+4)-2(-2-2x+2)$
$+2(-8+8-2x)=0$

$\Rightarrow (3-x)(x^2-3x)+4x-4x=0$

$\Rightarrow (x-3)^2x=0\Rightarrow x=0,3$

15. The equations of the given line are

$6x-2=3y+3=2z-4$

$$\Rightarrow\frac{x-\frac{1}{3}}{1}=\frac{y+1}{2}=\frac{z-2}{3}$$

Clearly, the direction ratios of the given line are 1, 2, 3.

16. The given equation is $\dfrac{dy}{dx}+\dfrac{y}{\sqrt{1-x^2}}=x$,

where $P=\dfrac{1}{\sqrt{1-x^2}}$ and $Q=x$

$$\therefore \text{I.F.}=e^{\int Pdx}=e^{\int\frac{dx}{\sqrt{1-x^2}}}=e^{\sin^{-1}x}$$

17. Let B and G denote the boy and girl respectively.

If a family has 4 children each of four children can either boy or girl.

Sample space is given by

$S=\{BBBB, BBBG, BBGB, BGBB, BBGG, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG\}$

(i) (b) : Let A = All children are boys.

$\therefore A=\{BBBB\}$ *i.e.*, $n(A)=1$

B = Elder child is a boy

$\therefore B=\{BBBB, BBBG, BBGB, BGBB, BBGG, BGBG, BGGB, BGGG\}$ *i.e.*, $n(B)=8$

Now, $n(A\cap B)=1$

$$\therefore P(A/B)=\frac{(A\cap B)}{n(B)}=\frac{1}{8}$$

(ii) (a) : Let A = All are girls.

$\therefore A=\{GGGG\}$ *i.e.*, $n(A)=1$

B = First two children are girl

$\therefore B=\{GGBB, GGBG, GGGB, GGGG\}$ *i.e.*, $n(B)=4$

Now, $n(A\cap B)=1$

$$\therefore P(A/B)=\frac{n(A\cap B)}{n(B)}=\frac{1}{4}$$

(iii) (c) : Let A = Two middle child are boys.

$\therefore A=\{BBBB, BBBG, GBBB, GBBG\}$ *i.e.*, $n(A)=4$

B = First child is a girl = 8

$\therefore B=\{GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG\}$

i.e., $n(B)=8$

Now, $n(A\cap B)=2$

$$\therefore P(A/B)=\frac{2}{8}=\frac{1}{4}$$

(iv) (b) : Let A = All are girls.

$\therefore A=\{GGGG\}$ *i.e.* $n(A)=1$

B = At least one child is girl.

$\therefore B=\{BBBG, BBGB, BGBB, GBBB, BBGG, GGBB, GBGB, BGBG, BGGB, GBBG, GGGB, GGBG, GBGG, BGGG, GGGG\}$

i.e., $n(B)=15$

Now, $n(A\cap B)=1$

$$\therefore P(A/B)=\frac{1}{15}$$

(v) (a) : Let A = All are boys.

$\therefore A=\{B\,B\,B\,B\}$ *i.e.* $n(A)=1$

B = At least three of the children are boys.

$\therefore B=\{BBBB, BBBG, BBGB, BGBB, GBBB\}$ *i.e.*, $n(B)=5$

Now, $n(A\cap B)=1$

$$\therefore P(A/B)=\frac{1}{5}$$

18. (i) (c) : Since, the distance is x feet from the stronger light, therefore the distance from the weaker light will be $300-x$.

So, the combined light intensity from both lamp posts is given by $\dfrac{1000}{x^2}+\dfrac{125}{(300-x)^2}$.

(ii) (c) : Since, the person is in between the lamp posts, therefore x will lie in the interval (0, 300).
So, maximum value of x can't be 300.

(iii) (a) : Since, $0 < x < 300$, therefore minimum value of x can't be 0.

(iv) (b) : We have, $I(x) = \frac{1000}{x^2} + \frac{125}{(300-x)^2}$

$\Rightarrow \quad I'(x) = \frac{-2000}{x^3} + \frac{250}{(300-x)^3}$ and

$\Rightarrow \quad I''(x) = \frac{2000}{x^4} + \frac{750}{(300-x)^4}$

For maxima/minima, $I'(x) = 0$

$\Rightarrow \quad \frac{2000}{x^3} = \frac{250}{(300-x)^3} \quad \Rightarrow 8(300-x)^3 = x^3$

Taking cube root on both sides, we get

$2(300 - x) = x \Rightarrow 600 = 3x \Rightarrow x = 200$

Thus, $I(x)$ is minimum when you are at 200 feet from the strong intensity lamp post.

(v) (a) : Since, $I(x)$ is minimum when x = 200 feet, therefore the darkest spot between the two light is at a distance of 200 feet from stronger lamp post, *i.e.*, at a distance of 300 – 200 = 100 feet from the weaker lamp post.

19. Let $I = \int_2^4 \frac{x}{x^2+1}dx$

Put $x^2 + 1 = t \Rightarrow 2x\,dx = dt$

When $x = 2$, $t = 5$ and when $x = 4$, $t = 17$

$\therefore \quad I = \frac{1}{2}\int_5^{17}\frac{1}{t}dt = \frac{1}{2}[\log t]_5^{17} = \frac{1}{2}\log\left(\frac{17}{5}\right)$

20. Given, $y^2 = ax^2 + bx + c$

Differentiating both sides, we get $2yy_1 = 2ax + b$ ...(i)

Again differentiating, we get $2yy_2 + y_1(2y_1) = 2a$

$\Rightarrow \quad yy_2 = a - y_1^2 \quad \Rightarrow \quad yy_2 = a - \left(\frac{2ax+b}{2y}\right)^2$ (Using (i))

$= \frac{4y^2a - (4a^2x^2 + b^2 + 4abx)}{4y^2}$

$\Rightarrow \quad y^3y_2 = \frac{4a(ax^2+bx+c) - (4a^2x^2+b^2+4abx)}{4} = \frac{4ac-b^2}{4}$

$\Rightarrow \quad \frac{d}{dx}(y^3y_2) = 0$

OR

Let $y = e^x \log(\sin 2x)$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\{e^x \log(\sin 2x)\}$

$= e^x \cdot \frac{d}{dx}\{\log(\sin 2x)\} + \log(\sin 2x) \cdot \frac{d}{dx}(e^x)$

$= e^x \cdot \left\{\frac{1}{\sin 2x} \cdot \cos 2x \cdot 2\right\} + \log(\sin 2x) \cdot e^x$

$= 2e^x \cot 2x + e^x \log(\sin 2x)$

$= e^x\{2\cot 2x + \log(\sin 2x)\}$

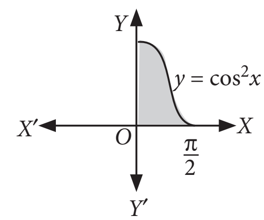
21. Let A, B and C be the events of drawing a blue ball in first, second and third draw respectively.

Then probability of getting blue ball in all three draws $= P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B)$

Now, $P(A) = \frac{5}{18}$, $P(B \mid A) = \frac{4}{17}$ and $P(C \mid A \cap B) = \frac{3}{16}$

$\therefore \quad P(A \cap B \cap C) = \frac{5}{18} \times \frac{4}{17} \times \frac{3}{16} = \frac{5}{408}$

22. Given curve is $y = \cos^2 x$, $0 \le x \le \frac{\pi}{2}$



Required area $= \int_0^{\pi/2} \cos^2 x\,dx$

$= \int_0^{\pi/2} \left[\frac{1+\cos 2x}{2}\right]dx$

$= \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/2}$

$= \left[\left(\frac{\pi}{4} - 0\right) - (0)\right] = \frac{\pi}{4}$ sq. units

23. We have, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$

$\Rightarrow \quad \vec{a} \times \vec{b} = \hat{i}(6+9) - \hat{j}(-2+12) + \hat{k}(6+24)$

$\Rightarrow \quad \vec{a} \times \vec{b} = 15\hat{i} - 10\hat{j} + 30\hat{k}$

and $|\vec{a} \times \vec{b}| = \sqrt{15^2 + (-10)^2 + (30)^2} = 35$

$\therefore \quad$ Required vector $= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

OR

Given that, $\vec{a} = \hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 4\hat{k}$

$\therefore \quad \vec{a} + \vec{b} = 2\hat{i} + 8\hat{k}$ and $\vec{a} - \vec{b} = 2\hat{j}$

Let θ be the angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, then

$\cos\theta = \frac{(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}$

$= \frac{(2\hat{i}+0\hat{j}+8\hat{k}) \cdot (0\hat{i}+2\hat{j}+0\hat{k})}{\sqrt{2^2+0^2+8^2}\sqrt{0^2+2^2+0^2}} = \frac{0+0+0}{\sqrt{4+64}\sqrt{4}} = 0$

$\Rightarrow \quad \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

24. We have $\frac{dy}{dx}+1=e^{x+y}$

Let $x+y=u \Rightarrow 1+\frac{dy}{dx}=\frac{du}{dx}$

$\Rightarrow \frac{du}{dx}=e^u \Rightarrow \int e^{-u}du=\int dx \Rightarrow -e^{-u}=x+c_1$

$\Rightarrow e^{-(x+y)}+x=c$, where $-c_1=c$

25. If all the 36 elementary events of the experiment are considered to be equally likely, we have

$P(A)=\frac{18}{36}=\frac{1}{2}$ and $P(B)=\frac{18}{36}=\frac{1}{2}$

Also, $P(A\cap B)=P$(odd number on both throws)

$=\frac{9}{36}=\frac{1}{4}$

Now, $P(A)P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

Clearly, $P(A\cap B)=P(A)\times P(B)$

Thus, A and B are independent events.

26. Let $\cos^{-1}\left(\frac{-3}{5}\right)=\theta$, where $\theta\in[0,\pi]$

Then, $\cos\theta=\frac{-3}{5}$

Since $\theta\in[0,\pi]$, we have $\sin\theta>0$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

$\therefore\ \sin\left[2\cos^{-1}\left(\frac{-3}{5}\right)\right]=\sin 2\theta=2\sin\theta\cos\theta$

$=\left\{2\times\frac{4}{5}\times\left(\frac{-3}{5}\right)\right\}=\frac{-24}{25}$

27. Here, $A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

$\text{L.H.S.}=A^2+I=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}0&-1\\1&0\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$ ...(i)

$\text{R.H.S.}=A(A^2-I)$

$=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\left\{\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}0&-1\\1&0\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}$

$=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\left\{\begin{bmatrix}-1&0\\0&-1\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}\right\}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}-2&0\\0&-2\end{bmatrix}$

$=\begin{bmatrix}0&2\\-2&0\end{bmatrix}$ ...(ii)

From (i) and (ii), we get R.H.S. $\neq$ L.H.S.

28. $f(x)$ is increasing if $f'(x)>0$

$\Rightarrow 6(x^2-5x-24)>0$

$\Rightarrow x^2-5x-24>0 \Rightarrow x^2-5x>24$

$\Rightarrow x^2-5x+\frac{25}{4}>24+\frac{25}{4} \Rightarrow \left(x-\frac{5}{2}\right)^2>\frac{121}{4}$

$\Rightarrow x-\frac{5}{2}>\frac{11}{2}$ or $x-\frac{5}{2}<-\frac{11}{2}$

$\Rightarrow x>8$ or $x<-3$

$\therefore$ f is increasing, if $x<-3$ or $x>8$.

OR

Let x be the length and y be the breadth of the rectangle

$\therefore\ 2x+2y=108 \Rightarrow y=54-x$

Now, area of rectangle $=xy=x(54-x)$

Let $f(x)=54x-x^2$

$\therefore\ f'(x)=54-2x$ and $f''(x)=-2$

For extreme values, $f'(x)=0$

$\Rightarrow 54-2x=0 \Rightarrow x=27$

$f''(27)=-2<0$

$\therefore$ Area is maximum when $x=27$, $y=27$

29. Here, $y^2=x(2-x)^2 \Rightarrow y^2=x^3-4x^2+4x$ ...(i)

$\Rightarrow 2y\frac{dy}{dx}=3x^2-8x+4 \Rightarrow \frac{dy}{dx}=\frac{3x^2-8x+4}{2y}$

$\Rightarrow \left[\frac{dy}{dx}\right]_{(1,1)}=\frac{3-8+4}{2}=-\frac{1}{2}$

$\therefore$ Equation of tangent at P is

$y-1=-\frac{1}{2}(x-1) \Rightarrow x+2y-3=0$ ...(ii)

Using $y=\frac{3-x}{2}$ in (i), we get

$\left(\frac{3-x}{2}\right)^2=x^3-4x^2+4x$

$\Rightarrow 9+x^2-6x=4x^3-16x^2+16x$

$\Rightarrow 4x^3-17x^2+22x-9=0$ which has two roots 1, 1

(Because of (ii) being tangent at (1, 1))

Sum of 3 roots $=\frac{17}{4} \Rightarrow 3^{\text{rd}}$ root $=\frac{17}{4}-2=\frac{9}{4}$

Then, $y=\frac{3-\frac{9}{4}}{2}=\frac{3}{8}$

So, Q is $\left(\frac{9}{4},\frac{3}{8}\right)$.

30. Here, $f:R\to R$, $f(x)=x^3+x$

One-One : $x_1,x_2\in R$ such that, $f(x_1)=f(x_2)$

$\Rightarrow x_1^3+x_1=x_2^3+x_2 \Rightarrow x_1^3-x_2^3+x_1-x_2=0$

$\Rightarrow (x_1-x_2)(x_1^2+x_1x_2+x_2^2+1)=0$

$\Rightarrow\ x_1 - x_2 = 0 \qquad \{\because x_1^2 + x_1x_2 + x_2^2 \geq 0\ \forall\, x_1, x_2 \in R\}$

$\Rightarrow\ x_1 = x_2$

Thus, $f(x_1) = f(x_2) \Rightarrow\ x_1 = x_2$

So, f is one-one.

Onto : Let y be any arbitrary element of R. Then,

$f(x) = y\ \Rightarrow\ x^3 + x = y$

$\Rightarrow\ y = x^3 + x\ \Rightarrow\ x^3 + x - y = 0$

We know that an odd degree equation has at least one real root. Therefore, for every real value of y, the equation $x^3 + x - y = 0$ has a real root a such that

$a^3 + a - y = 0\ \Rightarrow a^3 + a = y \Rightarrow f(a) = y.$

Thus, for every $y \in R$, there exists $a \in R$ such that $f(a) = y$.

So, f is onto.

Hence, $f: R \to R$ is a bijective function.

31. We have $\dfrac{dy}{dx} + \dfrac{2}{x}y = \dfrac{\ln x}{x}$.

It is a linear differential equation.

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = x^2$$

$$\therefore\ \text{Solution is, } yx^2 = \int x^2 \cdot \frac{\ln x}{x}dx = \int x \log x\, dx$$

$$\Rightarrow\ yx^2 = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x}\,dx$$

$$\Rightarrow\ yx^2 = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

$$\therefore\ x = e \Rightarrow e^2y(e) = \frac{e^2}{2} - \frac{e^2}{4} + c = \frac{e^2}{4} + c$$

and $x = 1 \Rightarrow y(1) = -\dfrac{1}{4} + c$

So, $e^2y(e) - y(1) = \dfrac{e^2+1}{4}$.

OR

We have $2xy + y^2 - 2x^2\dfrac{dy}{dx} = 0$

$$\Rightarrow\ \frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

Putting $y = vx\ \Rightarrow\ \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$$\therefore\ v + x\frac{dv}{dx} = \frac{2v+v^2}{2}\ \Rightarrow\ x\frac{dv}{dx} = \frac{2v+v^2}{2} - v$$

$$\Rightarrow\ 2\int \frac{1}{v^2}dv = \int \frac{1}{x}dx$$

$$\Rightarrow\ -\frac{2}{v} = \log|x| + c\ \Rightarrow\ -\frac{2x}{y} = \log|x| + c \qquad \text{...(i)}$$

It is given that $y(1) = 2$ *i.e.*, $y = 2$ when $x = 1$

$\therefore$ (i) becomes $-1 = 0 + c \Rightarrow c = -1$

Putting $c = -1$ in (i), we get

$$-\frac{2x}{y} = \log|x| - 1 \Rightarrow\ y = \frac{2x}{1 - \log|x|}$$

Clearly, y is defined if $x \neq 0$, and $1 - \log|x| \neq 0$.

Now, $1 - \log|x| = 0 \Rightarrow \log|x| = 1 \Rightarrow |x| = e \Rightarrow x = \pm e$.

Hence $y = \dfrac{2x}{1 - \log|x|}$, where $x \neq 0, \pm e$ gives the solution of the given differential equation.

32. Since, $f(x)$ is continuous at $x = 0$.

$$\therefore\ \lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x) \qquad \text{...(i)}$$

Now, $f(0) = \dfrac{2\times 0 + 1}{0 - 1} = -1$

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{2h+1}{h-1} = -1$$

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sqrt{1+kh} - \sqrt{1-kh}}{h}$$

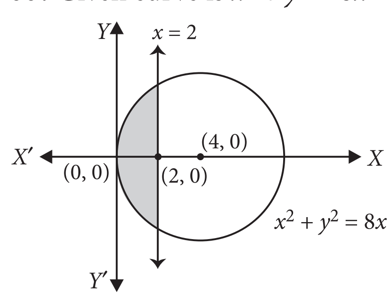
$$= \lim_{h\to 0}\frac{\sqrt{1+kh} - \sqrt{1-kh}}{h} \times \frac{\sqrt{1+kh} + \sqrt{1-kh}}{\sqrt{1+kh} + \sqrt{1-kh}}$$

$$= \lim_{h\to 0}\frac{(1+kh) - (1-kh)}{h\,[\sqrt{1+kh} + \sqrt{1-kh}]}$$

$$= \lim_{h\to 0}\frac{2k}{\sqrt{1+kh} + \sqrt{1-kh}} = \frac{2k}{2} = k$$

$\therefore$ From (i), we get $k = -1$

33. Given curve is $x^2 + y^2 = 8x$



$$\text{Required area} = 2\int_0^2 \sqrt{8x - x^2}\,dx$$

$$= 2\int_0^2 \sqrt{(4)^2 - (x-4)^2}\,dx$$

$$= 2\left[\left(\frac{x-4}{2}\right)\sqrt{4^2 - (x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_0^2$$

$$= 2\left[\left(\frac{-2}{2}\right)\sqrt{16-4} + 8\sin^{-1}\left(\frac{-1}{2}\right) - 8\sin^{-1}(-1)\right]$$

$$= 2\left[-\sqrt{12} - 8\times\frac{\pi}{6} + \frac{8\pi}{2}\right] = 2\left[-\sqrt{12} + \frac{16\pi}{6}\right]$$

$$= \left(\frac{16\pi}{3} - 2\sqrt{12}\right)\text{sq. units}$$

34. Let $I=\int\frac{x^2+9}{x^4-2x^2+81}dx$

$$\Rightarrow\ I=\int\frac{1+\frac{9}{x^2}}{x^2-2+\frac{81}{x^2}}dx=\int\frac{1+\frac{9}{x^2}}{x^2+\frac{81}{x^2}-2}dx$$

$$\Rightarrow\ I=\int\frac{1+\frac{9}{x^2}}{\left(x-\frac{9}{x}\right)^2+18-2}dx$$

Put $x-\frac{9}{x}=t\ \Rightarrow\left(1+\frac{9}{x^2}\right)dx=dt$

$$\therefore\ I=\int\frac{dt}{t^2+16}=\frac{1}{4}\tan^{-1}\left(\frac{t}{4}\right)+C$$

$$=\frac{1}{4}\tan^{-1}\left(\frac{x-\frac{9}{x}}{4}\right)+C=\frac{1}{4}\tan^{-1}\left(\frac{x^2-9}{4x}\right)+C$$

OR

$$\int_0^{\frac{\pi}{2}}x^2\sin^2x\,dx=\int_0^{\frac{\pi}{2}}x^2\cdot\left(\frac{1-\cos2x}{2}\right)dx$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}x^2dx-\frac{1}{2}\int_0^{\frac{\pi}{2}}x^2\cos2x\,dx$$

$$=\frac{1}{2}\left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}}-\frac{1}{2}\left\{\left[x^2\cdot\frac{\sin2x}{2}\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}2x\cdot\frac{\sin2x}{2}dx\right\}$$

$$=\frac{1}{2}\left[\frac{\pi^3}{24}\right]-\frac{1}{2}\left[\frac{\pi^2}{4}\times0-0\right]+\frac{1}{2}\int_0^{\frac{\pi}{2}}x\sin2x\,dx$$

$$=\frac{\pi^3}{48}+\frac{1}{2}\left\{\left[x\cdot\frac{(-\cos2x)}{2}\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}1\cdot\left(\frac{-\cos2x}{2}\right)dx\right\}$$

$$=\frac{\pi^3}{48}+\frac{1}{2}\left[\frac{-\pi}{4}\cos\pi-0\right]+\frac{1}{4}\int_0^{\frac{\pi}{2}}\cos2x\,dx$$

$$=\frac{\pi^3}{48}-\frac{\pi}{8}(-1)+\frac{1}{4}\left[\frac{\sin2x}{2}\right]_0^{\pi/2}$$

$$=\frac{\pi^3}{48}+\frac{\pi}{8}+\frac{1}{8}[0]=\frac{\pi^3}{48}+\frac{\pi}{8}$$

35. Let $y=a\sin^3t$ and $x=a\cos^3t$

On differentiating w.r.t. t, we get

$$\frac{dy}{dt}=3a\sin^2t\cos t\ \text{ and }\ \frac{dx}{dt}=3a\cos^2t(-\sin t)$$

$$\therefore\ \frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{3a\sin^2t\cos t}{3a\cos^2t(-\sin t)}=-\tan t$$

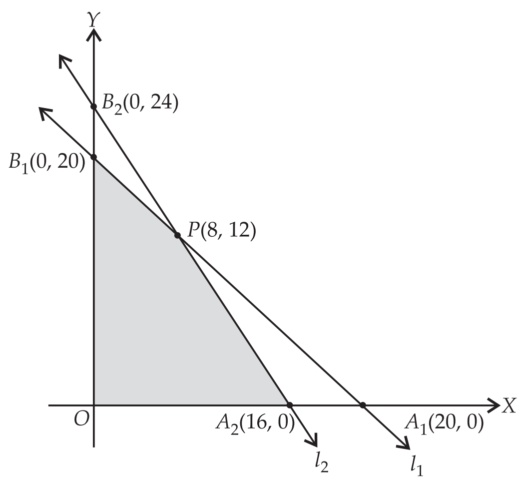
Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2}=-\sec^2t\frac{dt}{dx}=\frac{-\sec^2t}{3a\cos^2t(-\sin t)}=\frac{1}{3a}\left(\frac{\sec^4t}{\sin t}\right)$$

$$\therefore\ \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}}=\frac{1}{3a}\cdot\frac{4}{\frac{1}{\sqrt{2}}}=\frac{4\sqrt{2}}{3a}$$

36. To solve this LPP graphically, we first convert the inequations into equations $l_1 : x + y = 20$, $l_2 : 36x + 24y = 576$, $l_3 : x = 0$, $l_4 : y = 0$. Draw the corresponding lines.

The feasible region of the LPP is shaded in figure. The corner points of the feasible region OA_2PB_1 are $O(0, 0)$, $A_2(16, 0)$, $P(8, 12)$ and $B_1(0, 20)$.



The values of the objective function Z at corner-points of the feasible region are given in the following table.

Corner Points	Value of $Z = 22x + 18y$
$O(0, 0)$	0
$A_2(16, 0)$	352
$P(8, 12)$	392 (maximum)
$B_1(0, 20)$	360

$\therefore$ Z has maximum value 392 at $P(8, 12)$.

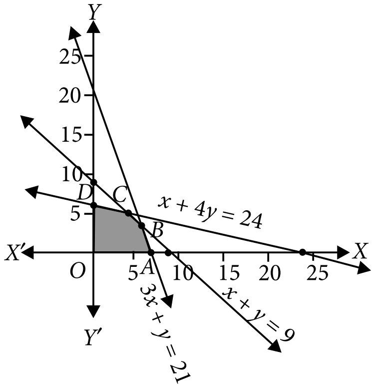
OR

Converting inequations into equations and drawing the corresponding lines.

$x + 4y = 24$, $3x + y = 21$, $x + y = 9$

i.e., $\frac{x}{24}+\frac{y}{6}=1,\ \ \frac{x}{7}+\frac{y}{21}=1,\ \ \frac{x}{9}+\frac{y}{9}=1$

As $x \geq 0, y \geq 0$ solution lies in first quadrant



The feasible region of the LPP is shaded in figure. The corner points of the feasible region $OABCD$ are $O(0, 0)$, $A(7, 0)$, $B(6, 3)$, $C(4, 5)$ and $D(0, 6)$.

Corner Points	Value of $Z = 3x + 5y$
$O(0, 0)$	0
$A(7, 0)$	21
$B(6, 3)$	33
$C(4, 5)$	37 (Maximum)
$D(0, 6)$	30

$\therefore$ Z has maximum value 37 at $C(4, 5)$.

37. We are given the equation of lines as

$\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and

$\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$

We know that the equation of plane containing $\vec{r} = \vec{a} + \lambda\vec{b}_1$ and $\vec{r} = \vec{a} + \lambda\vec{b}_2$ is $(\vec{r} - \vec{a}) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

Here, $\vec{a} = \hat{i} + \hat{j}, \vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}, \vec{b}_2 = -\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = (-4 + 1)\hat{i} - (-2 - 1)\hat{j} + (1 + 2)\hat{k}$$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{j} + 3\hat{k}$

Therefore, the equation of plane is,

$$(\vec{r} - (\hat{i} + \hat{j})) \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k}) = 0$$

$\Rightarrow \vec{r} \cdot (-3\hat{i} + 3\hat{j} + 3\hat{k}) + 3 - 3 = 0 \Rightarrow \vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 0$

This is the equation of the required plane.

We know that the distance of a point P with position vector $\vec{a}$ from the plane $\vec{r} \cdot \vec{n} = d$ is given by $\dfrac{|\vec{a} \cdot \vec{n} - d|}{|\vec{n}|}$

Now, distance from origin is 0 and distance from the point (1, 1, 1) *i.e.,* $\hat{i} + \hat{j} + \hat{k}$ is

$$\frac{|(\hat{i} + \hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j} + \hat{k})|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{|-1 + 1 + 1|}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ units}$$

OR

Let M be the foot of the perpendicular drawn from $P(2\hat{i} + 4\hat{j} - \hat{k})$ on the line $\vec{r} = \hat{i} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$

Let the position vector of M be $\hat{i} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$

$= (1 + 2\lambda)\hat{i} + (\lambda)\hat{j} + (2\lambda)\hat{k}$ ($\because$ M lies on the line)

Then $\overrightarrow{PM} = [(1 + 2\lambda)\hat{i} + (\lambda)\hat{j} + (2\lambda)\hat{k}] - (2\hat{i} + 4\hat{j} - \hat{k})$

$\therefore$ $\overrightarrow{PM} = (-1 + 2\lambda)\hat{i} + (-4 + \lambda)\hat{j} + (1 + 2\lambda)\hat{k}$

Since $\overrightarrow{PM}$ is perpendicular to the given line which is parallel to $\vec{b} = (2\hat{i} + \hat{j} + 2\hat{k})$

$\therefore$ $\overrightarrow{PM} \cdot \vec{b} = 0$

i.e., $[(-1 + 2\lambda)\hat{i} + (-4 + \lambda)\hat{j} + (1 + 2\lambda)\hat{k}] \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 0$

$\Rightarrow 2(-1 + 2\lambda) + 1(-4 + \lambda) + 2(1 + 2\lambda) = 0$

$\Rightarrow 9\lambda - 4 = 0 \Rightarrow \lambda = \dfrac{4}{9}$

Putting the value of λ, we obtain the position vector of M as $\left(\dfrac{17}{9}\hat{i} + \dfrac{4}{9}\hat{j} + \dfrac{8}{9}\hat{k}\right)$

Now, $\overrightarrow{PM} = \dfrac{-1}{9}\hat{i} - \dfrac{32}{9}\hat{j} + \dfrac{17}{9}\hat{k}$

$\therefore$ Required length $|\overrightarrow{PM}| = \sqrt{\dfrac{1}{81} + \dfrac{1029}{81} + \dfrac{289}{81}}$

$$= \frac{\sqrt{1319}}{9} \text{ units}$$

38. We have, $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 10 \neq 0$

So, A is invertible.

$$\therefore \text{adj } A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \quad \text{... (i)}$$

Now, the given system of equations is expressible as

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

or $A'X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Now, $|A'| = |A| = 10 \neq 0$. So, the given system of equations has a solution given by

$X = (A')^{-1} B = (A^{-1})'B$ $\quad [\because (A')^{-1} = (A^{-1})']$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}'\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} \quad \text{[Using (i)]}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 9/5 \\ 2/5 \\ 7/5 \end{bmatrix}$$

$\Rightarrow x = 9/5,\ y = 2/5$ and $z = 7/5$.

OR

The given system of equations can be written as $AX = B$

where $A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & -2 & 3 \\ 1 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$

Now, $|A| = 3(2 - 3) - (-1)(-2 - 3) + 1(2 + 2) = -4 \neq 0$

$\Rightarrow A^{-1}$ exists and so the given system has a solution given by $X = A^{-1}B$.

Now, $\text{adj } A = \begin{bmatrix} -1 & 0 & -1 \\ 5 & -4 & -7 \\ 4 & -4 & -4 \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = -\frac{1}{4}\begin{bmatrix} -1 & 0 & -1 \\ 5 & -4 & -7 \\ 4 & -4 & -4 \end{bmatrix}$$

$$\therefore X = A^{-1}B = -\frac{1}{4}\begin{bmatrix} -1 & 0 & -1 \\ 5 & -4 & -7 \\ 4 & -4 & -4 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \Rightarrow x = 1, y = -1, z = 1.$$

Hence, the solution of the given system of equations is $x = 1$, $y = -1$ and $z = 1$.

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