Chapter 13

ALGEBRAIC EXPRESSION

13.1 We studied about the terms containing variables and constants e.g. x, $x+1$, $2p-1$, $y-5$, $3y+4$ in previous class. We have seen that by these terms, the problems can be expressed in a simple and general manner. Algebraic expressions can be represented in the form of general necessity in Algebra and by assuming this general concept as base, these are used in solving problems by operations with algebraic expressions.

13.2 Algebraic Expression

We made patterns from the game of matchsticks in the previous class.

Example 1 According to diagram place three sets of 2–2 matchsticks of shape with one matchstick ( )

In this figure, number of matchsticks are 3, 5, 7 respectively which can be written as $2\times1+1$, $2\times2+1$, $2\times3+1$ etc.

If a set of matchsticks can be expressed by "n" then generally number of matchsticks can be expressed by $2\times n+1$ means $2n+1$.

In this way a combination of variables and constants is known as 'algebraic expression'. Look at some examples of algebraic expressions.

(1) Addition of 3 in any number can be expressed by $(x+3)$.

(2) Subtraction of 5 from four times of any number can be expressed by $(4x-5)$.

(3) One less from half of any number can be expressed by $\left(\frac{x}{2}-1\right)$.
Here unknown number is written by x.

On combining algebraic terms like this, one can get 'algebraic expression'.
$(x+3)$, $(4x-5)$ $\left(\frac{x}{2}-1\right)$. Here we'll study about their properties.

There is necessarily at least one variable in algebraic expression.

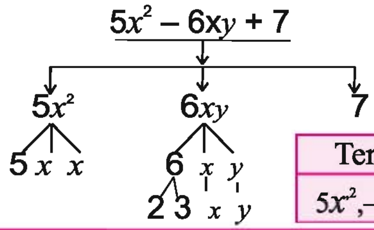
13.3 Terms of Algebraic Expression

Any algebraic expression has smaller parts. Consider $5x+3$ First we form $5x$ by multiplying 5 and x and add 3 to it. Similarly in $2x^2+3y$ we formed $2x^2$ by multiplying 2, x and x and then $3y$ by multiplying 3 and y separately. After forming $2x^2$ and $3y$ we add these two.
Thus expression $2x^2+3y$ is formed.

These small – small parts of an expression, which are first formed separately and then added are known as terms of expression. Expression $9y^2-4xy$ contains two terms, first term is $9y^2$ which is the product of 9, y and y respectively. Second term $-4xy$ is the product of -4, x and y respectively.
Then add these two terms as $9y^2 + (-4xy)$ and get an expression $9y^2-4xy$.

13.3.1 Factors of single term

Single term of algebraic expression can be the product of several variables and constants. We can represent the factors of expression and term by a tree diagram in a simple and attractive manner e.g.



Terms	Variable	Constant
$5x^2, -6xy, +7$	x & y	5, -6, 7

Do and Learn Fill the table given below.

Expression	No. of terms	Term	Factor of Term	Variable	constant
$3x^2 + 6xy + 7y^2$	3	$3x^2, 6xy, 7y^2$	$3x^2 = 3 \times x \times x$	x, y	3, 6, 7
			$6xy = 2 \times 3 \times x \times y$		
			$7y^2 = 7 \times y \times y$		
$a^2 - b^2$	2				
$8p^2 - 3p + 7$					

13.4 Coefficient

Coefficients of factors of any term are equal to the product of remaining factors of that term. Coefficient can be of both types-algebraic and numerical.

> When coefficient of any term is +1 then we don't write it eg. Coefficient of $x^3 y^2$ in $x^3 y^2$ is +1. Similarly coefficient of $x^2 y^2$ in $-x^2 y^2$ is (-1)

Example 1 What is the coefficient of x in following expression?
$8x - 3y$, $5 - x + z$, $y^2x - z^2$, $2z - 5xp$

Solution

	Expression	Terms with factor	Coefficient
(i)	$8x - 3y$	$8x$	8
(ii)	$5 - x + z$	$-x$	-1
(iii)	$y^2x - z^2$	y^2x	y^2
(iv)	$2z - 5xp$	$-5xp$	$-5p$

Do and Learn

Match the coefficient in the following algebraic expression $4x^2y^2 - 3xy + 15$

Coefficient x^2y^2	x^2
Coefficient xy	$-3y$
Coefficient x^2	$-y$
Coefficient $4y^2$	-3
Coefficient x	$4y^2$
Coefficient $3x$	4

Exercise 13.1

1. Find the factor of terms of expression by making tree diagram.
   (i) $9x^2-8$ (ii) $12x^2y + 8xy^2 - 15y^3$ (iii) $a^3 - b^3$
2. Find the coefficient in the given terms.
   (i) x in $4x$ (ii) y^2, x^2 and 9 in $9x^2y^2$
   (iii) x^3, y^3 and x^3y^3 in $\frac{-8}{5}x^3y^3$ (iv) a^2 and b^2 in $\frac{9a^2b^2}{13}$

13.5 Like and Unlike Terms

When algebraic factors of terms are same then they are known as like terms. When algebraic factors of terms are different then they are known as unlike terms eg. Consider $5xy$ and $3xy$ in $5xy - 6x + 3xy - 9$. Factors of $5xy$ are 5, x and y and factors of $3xy$ are 3, x and y. Thus their algebraic factors are same (by means of variables). Hence these are like terms.

> $3xy, 5yx$ are like terms as there is no effect on multiplication of variables in these terms because $xy = yx$.

Contrary to it, there are different algebraic factors in $5xy$ and $-6x$. These are unlike terms, similarly $5xy$ and -9 are unlike terms and $3xy$ and -9 are unlike terms.

Do and Learn

Select the like terms in the following:

3pq, -5p, 6q + 5, - 8pq, p^2 + q, qp

Example 2 Determine with reasons which pair are of like terms and which pair are of unlike terms in the given following pairs?

S. No.	Pair of term	Product	Algebraic term	Reason
1.	3ab 3b	3 x a x b 3 x b	Unlike	Variable a is not present in the second term.
2.	17a -6a	17 x a -6 x a	Like	Both the algebraic factors are equal.
3.	$5a^2b$ $5ab^2$	5 x a x a x b 5 x a x b x b	Unlike	Variables in both are same but their powers are different.
4.	-4ab 7ab	-4 x a x b 7 x b x a	Like	Both the algebraic factors are equal.

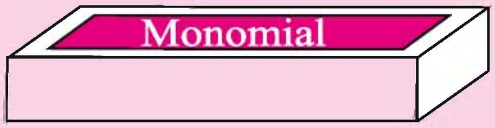
13.6 Polynomial Expression

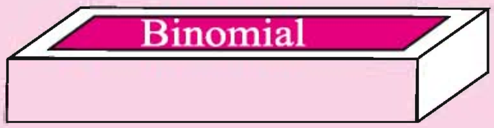
Monomial	Which has only single terms like	$7xy$, $-3m$, y^2, x^2y^2
Binomial	Which has only two terms like	$x+y$, $x-5$, $pq+5$, m^2n^2+5m
Trinomial	Which has only three terms like	$x+y+2$, $3x^2-5x+7$, $ab+ab^2+b^2$

Expression containing one or more than one terms are known as polynomial expression.

Do and Learn Write in appropriate box by selecting monomial, binomial and trinomial expression from the following;

1. $2a^2 + b$
2. $4x^2y^3$
3. $3m - 2n + 1$
4. $2mn-3$
5. $\frac{7}{8}xy^2z$
6. $\frac{1}{3}x^2 + \frac{2}{3}xy + xy^2$
7. $ab+bc+ca$
8. $ax^2 + bx + c$
9. $5xy - 7 + 3n$
10. $3x + 1$
11. $\frac{9}{17}a^2 + b^2 - \frac{1}{2}$
12. $\frac{8}{19}p^2r^2q^2$

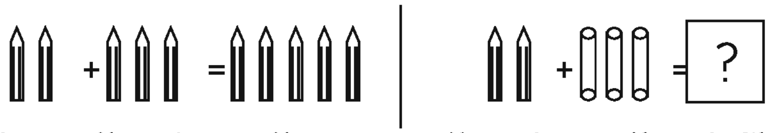




2. Match the like terms.

(a) $4a^2b$	(i) $\frac{8}{13}x^2y^2z^2$
(b) 5nm	(ii) $\frac{3p}{q}$
(c) $\frac{3}{4}x^2y^2z^2$	(iii) $\frac{5a^2}{7b^2}$
(d) $\frac{-1}{5}\frac{a^3b^3}{c^3}$	(iv) ga^2b
(e) $\frac{-22}{7}\frac{p}{q}$	(v) nm
(f) $\frac{a^2}{b^2}$	(vi) $\frac{a^3b^3}{c^3}$
(g) xyz	(vii) $\frac{8}{x^2y^2z^2}$
(h) $\frac{3}{x^2y^2z^2}$	(viii) $19xyz$

13.7 Addition and Subtractions of like terms



2 Pencils + 3 Pencils = 5 Pencils   2 Pencils + 3 Chalks

$$2x + 3x = (2+3)x$$
$$= 5x$$

We can add the pencils but cannot add pencils and chalks. Hence we can add or subtract the quantities of equal unit (equal variables).

eg.

$$5x^2y + 3x^2y = 8x^2y$$
$$9a^2b^2 - 4a^2b^2 = 5a^2b^2$$

Numerical coefficient of the term obtained by adding the like terms is equal to the sum of coefficients of all those terms. Similarly the result obtained by subtracting the two like terms is equal to the difference of numerical coefficients of those terms. It should be kept in mind that unlike terms cannot be added or subtracted in the way like terms are added or subtracted. For example on adding 5 to x the result comes out to be $x + 5$, similarly if $3xy$ is added to 7, the result is $3xy + 7$ and if 7 is subtracted from $3xy$, then the result is $3xy$-7.

Steps for addition, subtractions of algebraic expression

1. Identify the like and unlike terms.
2. Write down the like terms with their sign.
3. Add or subtract these like terms according to rules.
4. If there remains one or more unlike terms then write these by combining their signs.

Example 3 Add $3x + 8y$ and $8x + 5y$

Solutions $(3x + 8y) + (8x + 5y)$

$= 3x + 8x + 8y + 5y$ (on keeping equal algebraic terms altogether)

$= 11x + 13y$

We can add these two through column addition also.

$$\begin{array}{r} 3x + 8y \\ 8x + 5y \\ \hline 11x + 13y \\ \hline \end{array}$$

Example 4 Add $7ab + 4a$ and $2a + 5ba$

Solutions $(7ab + 4a) + (2a + 5ba)$

$= 7ab + 4a + 2a + 5ab$

$= 7ab + 5ab + 4a + 2a$

$= 12ab + 6a$

$$\begin{array}{r} 7ab + 4a \\ + \ 5ab + 2a \\ \hline 12ab + 6a \\ \hline \end{array}$$

Example 5 Subtract $3m^2 - 2xy$ from $11xy - 5m^2$

Solution $(11xy - 5m^2) - (3m^2 - 2xy)$

$= 11xy - 5m^2 - 3m^2 + 2xy$

$= 11xy + 2xy - 5m^2 - 3m^2$

$= 13xy - 8m^2$

Example 6 Solve $(3m + 2n - 7) + (2m^2 + 5m + n^2)$

Solution $3m + 2n - 7 + 2m^2 + 5m + n^2$

$= 3m + 5m + 2n - 7 + 2m^2 + n^2$

$= 8m + 2n - 7 + 2m^2 + n^2$

$= 2m^2 + n^2 + 8m + 2n - 7$

Do and Learn

Addition and subtraction of algebraic expression.

(1) $m - n$ and $m + n$

(2) $mn - 5 + 2n$ and $nm + 2m - 3$

(3) $\frac{xy}{5} + \frac{x}{3}$ and $\frac{xy}{2} - \frac{x}{3}$

Exercise 13.2

1. Add the following algebraic expression.
   - (i) t - 4tz, 2t + 6tz
   - (ii) $7xy$, $5xy$, $3xy$, $-2xy$
   - (iii) $5x - 7y$, $3y - 4x + 2$, $2x - 3xy - 5$
   - (iv) $m^2 - n^2 - 1$, $n^2 - 1 - m^2$, $1 - m^2 - n^2$
   - (v) $3x + 11 + 8z$, $5x - 7$
   - (vi) $a^2b + ab + ab^2$, $-a^2b + 2ba + 2a^2b^2$
   - (vii) $x - y$, $y - z$, $z - x$
2. Subtract the following algebraic expression.
   - (i) $-5x^2$ from x^2
   - (ii) (a-b) from (a +b)
   - (iii) $x^2 + 5x + 4$ from $4x^2 - 3xy + 8$
   - (iv) $5x^2 - 7xy + 5y^2$ from $3xy - 2x^2 - 2y^2$
   - (v) $4pq - 5q^2 - 3p^2$ from $5p^2 + 2q^2 - pq^2$
   - (vi) $x^2 + 10x - 5$ from $5x - 10$
3. What should be subtracted from $7x - 8y$ to get $x + y + z$?
4. What should be added to $2p + 6$ to get $3p - q + 6$?

13.8 Find the value of algebraic expression.

The value of an algebraic expression depends on the values of variables which make that expression. In several cases we examine that whether it satisfies the equation formed on putting the value of variable in any expression.

Example 7 Find the values of following expression for $x = 3$.

(i) $x + 5$ (ii) $9x - 3$ (iii) $25 - 3x^2$ (iv) $4x^2 + 5x - 51$

Solutions

(i) On putting 3 in place of x in $x + 5$

$= 3 + 5$

$= 8$

(ii) On putting 3 in place of x in $9x-3$

$= (9 \times 3) - 3$

$= 27 - 3 = 24$

(iii) $25 - 3x^2$

$= 25 - 3 \times (3)^2$

$= 25 - 3 \times 3 \times 3 = 25 - 27 = -2$

(iv) $4x^2 + 5x - 51$

$= 4 \times (3)^2 + 5(3) - 51$

$= 4 \times 9 + 5 \times 3 - 51$

$= 36 + 15 - 51 = 51 - 51 = 0$

Example 8: Find the values of following expression for

(i) $a + b$ (ii) $5a - 2b$ (iii) $a^2 - 2ab + b^2$ (iv) $a^3 - b^3$

Solutions: on putting a = 3 and b = 2 in given expressions

(i) $a + b = 3 + 2 = 5$

(ii) $5a - 2b = 5 \times 3 - 2 \times 2 = 15 - 4 = 11$

(iii) $a^2 - 2ab + b^2$
$= (3)^2 - 2 \times 3 \times 2 + (2)^2$
$= 9 - 12 + 4$
$= 13 - 12 = 1$

(iv) $a^3 - b^3$
$= (3)^3 - (2)^3$
$= 3 \times 3 \times 3 - 2 \times 2 \times 2$
$= 27 - 8$
$= 19$

Exercise 13.3

1. Find the value of the following if $x = 2$.

   (i) $x - 3$ (ii) $2x - 5$ (iii) $9 - 6x$ (iv) $3x^2 - 4x - 7$ (v) $\frac{5x}{2} - 4$

2. Find the value of the following if $p = -1$.

   (i) $4p + 5$ (ii) $-3p^2 + 4p + 8$ (iii) $3(p - 2) + 6$

3. Find the value of the following if a = 2 and b = -2.

   (i) $a^2 - b^2$ (ii) $a^2 - ab + b^2$ (iii) $a^2 + b^2$

4. Find the value of the following if $x = 1$ and $y = 0$.

   (i) $2x + 2y$ (ii) $2x^2 + y^2 + 1$ (iii) $2x^2y + 2x^2y^2 + y^2$ (iv) $x^2 + xy + 5$

We Learnt

1. Algebraic expressions are formed from variables and constants. Operations like +, -. ×, ÷ are to be carried out on variables and constants to make algebraic expressions.
2. Expression is composed from terms, by adding terms expression is formed.
3. Any term is a multiplication of its factors, factors of variables is said to be algebraic factor. Any one factor of term is known as coefficient of remainder of term.
4. An expression made up with one or more term is called polynomial It can be monomial (having single term), binomial (having two terms) and trinomial (having three terms).
5. The terms whose algebraic factors are same are known as like terms and the terms containing different algebraic factors are known as unlike terms.
6. Addition or subtraction of two like terms is again a like term, whose coefficient is equal to the sum or difference of coefficients of those like terms.
7. Two algebraic expression of like terms can be added or subtracted. The terms which are not like are left as such.
8. The value of algebraic expression depends on the value of variables.