Unit 11

Probability

Teaching points

● Conditional probability: If A and B are two events associated with any random experiment, thenP(A/B) repersents the probability of occurence of event-A knowing that event B has already occurred

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

P(B) ≠ 0, means that the events should not be impossible

P(A∩B) = P(Aand B) = P(B) × P(A/B)

Similarly P(A∩B∩C) = P(A) × P(B/A) × P(C/AB)

● **Multiplication Therem on Probability:**

If the event Aand Bare associated with any random experiment and the occurrence of one depends on the other

then P(A∩B) = P(A) × P(B/A) where P(A) ≠0

● when the occurrence of one does not depends on the other then these events are said to be independent events.

Here P(A/B) = P(A) or P(B/A) = P(B)

∴ P(A ∩ B) = P(A) × P(B)

● Theorem on total probability: If E_1, E_2, E_3,...$E_n$be a partition of sample space and E_1, E_2... $E_n$all has non-zero probability. A be any event associated with sample space S, then

$$P(A) = P(E_1).\ P(A/E_l) + P(E_2).\ P(A/E_2) + ... + P(E_n).\ P(A/E_n)$$

● Baye’s theorem: Let Sbe the sample space and E_1, E_2...$E_n$be nmutually exclusive and exhaustive events associated with a random experiment. If Ais any event which occurs with E_1, or E_2or ... E_n, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

● Random variable: It is real valued function whose domain is the sample space of a random experiment and whose range is a subset of real numbers.

● Probability distribution: It is a system of numbers of random variable (x), such that

X:	X_1	X_2	X_3	X_n
$P(X_1)$:	$P(X_1)$	$P(X_2)$	$P(X_3)$...	$P(X_n)$

Where $P(X_1) > 0$ and $\sum_{\lambda=1}^{n} P(X_i) = 1$

● Mean or expectation of a random variables (x) denoted by E(x)

$E(x) = \mu = \sum_{i=1}^{n} P(X_i) = 1$

● Variance of Xdenoted by var (x) or a $\sigma_x^2$and

$var(x) = \sigma_x^2 = \sum_{i=1}^{n} (X_i - \mu)^2 P(X_i) = \sum_{I=1}^{n} X_i P(X_i) = (\mu)^2$

● The non-negative number $\sigma_x = \sqrt{\text{var}(x)}$ is called standard deviation of random variable X.

● **Bernouli Trials:** Trials of random experiment are called Bernouli trials if

(i) No. of trials are finite.

(ii) Trials are independent

(iii) Each trial has exactly two outcomes either success or failure,

(iv) Probability of success remains same in each trial.

● Binomial Distribution:

$P(X= r) = {}^nC_r\, q^{n-r} p^r$, where r= 0, 1, 2,...n

p= prob. of success

q= prob. of Failure

n= total no. of trials

r= value of random variable.

● Recurrence formula for Binomal Distribution:

$P(r+1) = \frac{n-r}{r+1} . \frac{p}{q} p(r).$

Questions for Practice

Very Short Answer Type Questions (1 Mark)

Q1. Three coins are tossed once. What is the probability of getting at least one head?

Q2. If P(A) = 0.3, P(B) = 0.5 and P(A/B) = 0.4, then find P(B/A)?

Q3. A policeman tries three bullets on a dacoit. The probability that the dacoit will be killed by one bullet is 0.7. What is the probability that dacoit is still alive?

Q4. What is the probability that a leap year will have 53 Sundays?

Q5. If A and B are independent events with P(A) = 0.3 and P(B) = 0.5 then what is P (A ∩ B)?

Q6. Two students A and B solve a problem independently with probabilities $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that problem is not solved?

Q7. Two balls are drawn one by one without replacement from a bag containing 10 red and 5 black balls. What is the probability that both balls are black?

Q8. The random variable X has a probability distribution is

x	0	1	2
$P(x)$	K	$2K$	$3K$

What is the value of K.

Q9. Given P(A) = 1/4, P(B) = 2/3 and P(A ∪ B) = 3/4. Are the events independent?

Q10. A and B are two events such that P(A) = 0.3 and $P(A \cup \overline{B}) = 0.8$. If A and B are independent events. Find P(B)?

Short Answer Type Questions (4 Marks)

Q11. Two cards are drawn without replacement from a well shuffled pack of 52 cards. What is the probability that one card is a red queen and other is a king of black colour?

Q12. Two balls are drawn at random from a bag containing 2 white, 3 red, 5 green and 4 black balls one by one without replacement. Find the probability that both the balls are of different colours.

Q13. The probability that a student A can solve a question is 6/7 and that another students Bsolving a question 3/4. Assuming the two events “Acan solve the question” and “Bcan solve that question” are independent, find the probability that only one of them solves the question.

Q14. A problem in mathematics is given to three students whose chance of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem will be solved?

Q15. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases they are likely to contradict each other in stating the same fact.

Q16. There are two identical boxes containing respectively 4 white and 3 red balls, 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. If the ball drawn is white, what is the probability that it is from the first box?

Q17. A can hit a target 4 times in 5 shots, B can hit the target 3 times in 4 shots and C 2 times in 3 shots.

They fire a volley. What is the probability that atleast two shots hit?

Q18. A man takes a step forward with probability 0.4 and backward with probability 0.6 find the probability that at the end of eleven steps he is one step away from the starting point.

Q19. A box contains 13 bulbs, out of which 5 are defective. 3 bulbs are randomly drawn, one by one without replacement, from the box. Find the probability distribution of the number of defective bulbs.

Q20. A coin is biased so that the head is 3 times as likely to occurs as a tail if the coin is tossed twice, find the probability distribution for the number of tails.

Q21. There are three coins, one is a two headed coin, another is a biased coin that comes up head 75% of the time and third is an unsbiased coin. One of the three coins is chosen at random and tossed. If it shows a head. What is the probability that it was a two headed-coin9

Q22. Two cards are darwn from a pack of 52 cards at random and kept out. Then one card is drawn from the remaining 50 cards. Find the probability that it is an ace.

Q23. A random variable xhas the following distribution:

X −2	−1	0	1	2	3
$P(X)$ 0.1	K	0.2	$2K$	0.3	K

Find (i) the value of K, (ii) $p(X \leq 1)$, (iii) $P(X \geq 0)$.

Q24. Two dice are thrown. Find the probability that the number appeared have a sum 8 if it is known that the second dice always exhibits 4.

Q25. A machine operates of all of its three components function. The probability that the first component fails during the year is 0.14, the probability that the second component fails is 0.10 and the probability that the third component fails is 0.05. What is the probability that the machine will fail during the year?

Long Answer Type Questions (6 Marks)

Q26. A company has two plants to manufacture bicylces. The first plant manufactures 60% of the bicycles and the second plant 40%. 80% of the bicyles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be of standard quality. Find the probability that it comes from the second plant.

Q27. In a bolt factory, Machnies A, Band C and manufacture 60%, 25% and 15% of the total output respectively. Of the total output 1 %, 2% and 1 % are defective bolts respectively manufactured by machines A, Band C. A bolt is drawn at random form the total production and is found to be defective. From which machine, the defective bolt is most likely to have been manufactured.

Q28. By examining the chest x-ray, the probability that T.B. is detected when a person is actually suffering from it is 0.99. The probability that the doctor diagnosis incorrectly that a perosn has T.B. on the basis of x-ray is 0.001. In a certain city. 1 in 1000 persons suffers from T.B. a person is sleeted at random and is diagnosed to have T.B. what is the chance that he actually has T.B.?

Q29. A man is known to speak truth 3 out of 4 times. He throws a pair of dice and reports that sum of numbers appeared is six. Find the probability that it is actually six.

Q30. Supose one of the three men A, B, C will be appointed as a vice chanceller of a university. The respective probabilities of their appointment are 0.5,0.3,0. The probabilities that research facilities will be enhanced by these people if they are appointed are 0.3, 0.7, and 0.8 respectively. If the research facilities are enhanced. What is the probability that it was due to the appointment of C ?

Q31. A pair of dice is rolled twice. Let x denote the number of times, a total of 9 is obtained. Find the mean and variance of the random variable X.

Q32. A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that in the bag all balls are white?

Q33. A letter is known to have come either from TATA NAGAR or KOLKATA. On the envelope, only two consecutive letters TA are visible. What is the probability that letter has come from KOLKATA.

Q34. Two cards from a pack of cards are lost. From the remaining cards, a card is drawn and is found to spade. Find the probability of the missing cards to be spades.

Q35. Bag A contains 3 red and 4 black balls and bag B contains 4 red and 5 black balls. Two balls are transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red in colour. Find the probability that the transferred ball were both black.

Answers

1. $\frac{7}{8}$ 2. $\frac{2}{3}$ 3. $(0.3)^3$

4. $\frac{2}{7}$ 5. 0.65 6. $\frac{1}{2}$

7. $\frac{2}{21}$ 8. $\frac{1}{6}$ 9. Yes

10. $\frac{2}{7}$ 11. $\frac{2}{663}$ 12. $\frac{71}{91}$

13. $\frac{9}{28}$ 14. $\frac{3}{4}$ 15. 42%

16. $\frac{40}{61}$ 17. $\frac{5}{6}$ 18. $462 \times (.24)^5 = 0.3678$

19.

X	0	1	2	3
P(X)	$\frac{84}{429}$	$\frac{210}{429}$	$\frac{120}{429}$	$\frac{15}{429}$

20.

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

21. $\frac{4}{9}$ 22. $\frac{1}{13}$ 23. (i) K = $\frac{1}{10}$ (ii) 0.6 (iii) 0.8

24. $\frac{1}{6}$ 25. .2467 26. $\frac{3}{7}$

27. Machine A. 28. $\frac{110}{221}$ 29. $\frac{15}{46}$

30. $\frac{4}{13}$ 31. $\frac{2}{9}, \frac{16}{9}$ 32. $\frac{3}{5}$

33. $\frac{2}{5}$ 34. $\frac{22}{425}$ 35. $\frac{4}{17}$

Hints

12. Require probability = 1 – P (balls are of same colour)

= $1-\left[\frac{2}{14}\times\frac{1}{13}+\frac{3}{14}\times\frac{2}{13}+\frac{5}{14}\times\frac{4}{13}+\frac{4}{14}\times\frac{3}{13}\right]$

= $\frac{71}{91}$

18. Let X denote the number of steps taken forward

∴ Required probability = P(x= 5) + P(x= 6)

= ${}^{11}C_5(.4)^5(.6)^6 + {}^{11}C_6(.6)^5(.4)^6$

22. Events A_1= drawn cards are both aces

A_2= drawn cards are both non aces

A3 = drawn cards are both one ace and one non ace

B_3= Ace is drawn from 50 cards

$\therefore$ P(B) = $P(A_1)P(B/A_1)+P(A_2)P(B/A_2)+P(A_3)P(B/A_3)$

= $\left(\frac{4}{52}\times\frac{3}{51}\times\frac{2}{50}\right)+\left(\frac{48}{52}\times\frac{47}{51}\times\frac{4}{50}\right)+\left(\frac{4}{52}\times\frac{48}{51}\times 2\times\frac{3}{50}\right)$

= $\frac{1}{13}$

32. Let event A_1= the bag contains 2 white and 2 non white balls

A_2= the bag contains 3 white and 1 non white balls

A_3= the bag contains 4 white balls

B= two white balls are drawn from the bag.

Required probability = $P(A_3/B)$

= $\dfrac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1}=\frac{3}{5}$ [Use Baye's thon]

33. Let event A_1= letter has come from TATA NAGAR

Let event A_2= letter has come from KOLKATA

B= two consecutive letters visible are TA.

$P(A_1)$ = $\frac{1}{2}$ $P(A_2)$ = $\frac{1}{2}$

$P(B/A_1)$ = $\frac{1}{4}$ $P(B/A_2)$ = $\frac{1}{6}$

Required prob. = $P(A_2/B)$

= $\dfrac{\frac{1}{2}\times\frac{1}{6}}{\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{6}}=\frac{2}{5}$

35. Let event A_1= balls transfered are both red

A_2= balls transfered are one red and one black

A_3= balls transfered are both back

B = Red ball is drown form bag B after transfer

$P(A_1) = \frac{3}{7} \times \frac{2}{6}$ $P(A_2) = \frac{3}{7} \times \frac{4}{6} \times 2$ $P(A_3) = \frac{4}{7} \times \frac{3}{6}$

$P(B/A_2) = \frac{6}{11}$ $P(B/A_2) = \frac{5}{11}$ $P(B/A_3) = \frac{4}{11}$

Unit 12

Linear Programming

Teaching-Learning Points

● Linear programming is a technique to find optimal value (maximum or minimum) of a linear function of a number of variables (say xand y) subject to a number of linear inequalities in the variables involved and the variables take non negative values only.

● A problem which seeks to maximise or minimise a linear function is called optimisation problem or linear programming problem.

● Linear function of the form z= ax+ bywhere a, bare constants, which is to be maximized or minimized is called objective function.

● Linear inequalities in the variables of a linear programming problem are called constraints.

● The common region determined by all the constraints including non negative constraints x, y ≥0 of a linear programming problem is called the feasible region. The region other than feasible region is called infeasible region.

● The points within and on the boundary of the feasible region represent feasible solutions. Any point out the feasible region is called infeasible solution.

● Any point in the feasible region that gives optimal value (maximum or minimum) of the objective function is called an optimal solution.

● Optimal value of an objective function must occurs at a corner point of the feasible region.

● A feasible region is said to be bounded if it can be enclosed within a circle. Otherwise, it is called unbounded.

● If a feasible region is bounded, then the objective function has both a maximum and a minimum value and each of these occurs at a corner point of the feasible region.

● If the feasible region is unbouned, then a maximum or minimum value of a function may not exist.

However if it exists, if must occur at corner point of the feasible region.

● After evaluating the objective function z = ax + by at each corner point, let M and m respetively denote the largest and smallest values of these points and if

(i) Feasible region is bounded, then M and m are the maximum and minimum values of the function.

(ii) Feasible region is unbounded then M is the maximum value if z, provided the open half plane determined by ax + by> M has no point in common with the feasible region, otherwise zhas no maximum value. Similiarly mis the minimum value of z, provided the open half plane determined by ax + by< mhas no point in common with the feasible region, otherwise zhas no minimum value.

● If the obtimal solution of the objective function occures at two vertices (corners) of the feasible region, then every point on the line segment joining these points will give the same optimal value.

Questions for Practice

Long Answer type Questions Carrying 6 Marks each

1. A house wife wishes to mix two types of food xand yin such a way that the vitamin contents at the mixture contains at least 8 units of vitamin A and 11 units of vitamin B. Food xcosts `60 per kg and food ycost `80 per kg. Foodxcontains 3 units per kg of vitamin A and 5 units per kg of vitamin B while food y contains 4 units per kg of vitamin A and 2 units per kg of vitamin B. Formulate is as a linear programming problem to minimize the cost of the mixture and solve it graphically.

2. A dealer has `15000 only for a purchase of rice and wheat. A bag of rice costs `1500 and bag of wheat costs `1200. He has a storage capacity of ten bags only and the dealer gets a profit of `110 and `80 per bag of rice and wheat respectively. Formulate it as a linear programming problem to get the maximum profit and solve it graphically.

3. In a small scale industry a manufacture produces two types of book cases. The first type of book cose requires 3 hours on machine A and 2 hours on machine B for completion. whereas the second type of book case requires 3 hours on machine A and 3 hours on machine B. Machine A can run at most for 18 hours and machine B can run atmost for 14 hours per day. He earns a profit of `30 on each book case of first type and `40 one each book case of 2nd type. How many book cases of each type should he make everyday so as to have a maximum profit? Solve it as a linear programming problem.

4. A machine producing either product A or B can produce A by using 2 units of chemicals and unit of a compound and can produce B by using unit of chemicals and 2 units of compound. If only 800 units of chemicals and 1000 units of the compound are available and the profits per unit of A and B are respectively `30 and `20, then find the number of units of A and B to be produced so as to maximize the total profit. Find the maximum profit also, by solving it as a linear programming problem.

5. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from godowns to the shops are given below in the table:

Transportaion cost per quintal (in Rs)

From/To	A	B
D	6	4

E	3	2
F	2.50	3

How should the supplier be transported in order that the transportation cost is minimum? Also find the minimum cost by solving it as a linear programming problem.

6. A manufactures makes two types of toys A and B. Three machines are needed for this purpose and the time required (in minutes) for each toy on the machines is given below :

Types of toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is `7.50 and that on each toy of type B as `5, find the number of toys of each type to be manufactured in a day to get maximum profit. Solve it as a linear programming problem. Also find maximum profit.

Answers

1. Minimum cast = `160, at all points on the line segment joining $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$.
2. Ten bags of rice only and maximum profit = `1100.
3. First type = 4, second type = 2, and maximum profit = `200.
4. Maximum profit = `14000 for A = 200 units and B = 400 units.
5. From A : 10, 50, 40 units and from B : 50,0,0 units minimum cost = `10.
6. Type A toys = 15, type B toys = 30, maximum profit = `262.50.