CBSE Class 10th Mathematics
Basic Sample Paper - 04

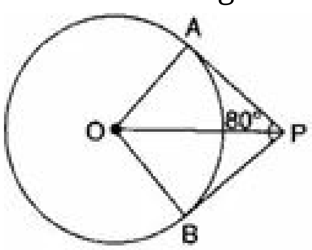
Maximum Marks:
Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. If $x_i's$ are the midpoints of the class intervals of grouped data, $f_i's$ are the corresponding frequencies and $\overline{x}$ is the mean, then $\sum\left(f_i x_i-\overline{x}\right)$ is equal to
   a. 2
   b. 0
   c. – 1
   d. 1
2. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then find $\angle$POA



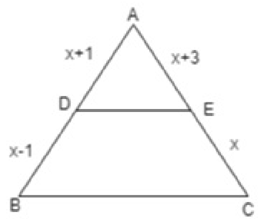
a. 40°
b. 100°
c. 50°
d. 60°

3. If $112 = q \times 6 + r$, then the possible values of r are:
   a. 1, 2, 3, 4
   b. 0, 1, 2, 3
   c. 2, 3, 5
   d. 0, 1, 2, 3, 4, 5
4. If m^2 - 1 is divisible by 8, then 'm' is
   a. an odd integer
   b. a natural number
   c. an even integer
   d. a whole number
5. The product of a rational number and an irrational number is
   a. a rational number or an irrational number
   b. none of these
   c. an irrational number
   d. a rational number
6. The largest power of 'x' in p(x) is the _________ of the polynomial.
   a. zero
   b. root
   c. solution
   d. degree
7. If 'α' and 'β' are the zeroes of the polynomial $x^2 - 6x + 8$, then the value of $\alpha^3 + \beta^3$ is
   a. 76
   b. 72
   c. 74
   d. 80
8. An unbiased die is thrown once. The probability of getting a composite number is
   a. $\frac{2}{5}$
   b. $\frac{1}{3}$

c. $\frac{2}{3}$
d. $\frac{1}{2}$

9. If the point P(2, 1) lies on the line segment joining points A(4, 2) and B(8, 4), then AP is equal to
   a. $AP = \frac{1}{4}AB$
   b. $AP = \frac{1}{2}AB$
   c. $AP = \frac{1}{3}AB$
   d. AP = PB
10. The point on the x-axis which is equidistant from the points (2, – 5) and (– 2, 9) is
    a. (0, – 7)
    b. (– 7, 0)
    c. (0, 7)
    d. (7, 0)
11. Fill in the blanks:

    In the given figure, $DE \parallel BC$. Then the value of x is ________.



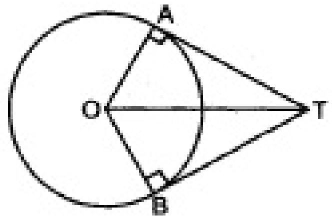
12. Fill in the blanks:
    The distance of point P(3, 4) from the origin is ________.

    OR

    Fill in the blanks:
    The distance of point P(3, 4) from the origin is ________.
13. Fill in the blanks:
    The value of trigonometric function cosec $\frac{\pi}{3} =$ _______.
14. Fill in the blanks:
    If tan θ = $\sqrt{3}$, then sec θ = _______.
15. Fill in the blanks:

    The value of 3sin 30^o - 4$\sin^3 60^o$ is _______.

16. In figure if $\angle$ATO = 40°, find $\angle$AOB.



OR

An observer 1.5 m tall is 28.5 m away from a tower. The angle of elevation of the top of the tower from her eyes is 45°. What is the height of the tower?

17. Which term of the AP 14,11,8,....is -1?

18. Find the radius of a circle whose perimeter and area are numerically equal.

19. The perimeter of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, then what is the length of AB?

20. Why is tossing a coin considered to be a fair way of deciding which team should choose ends in a game of cricket?

Section B

21. Find the zeroes of the quadratic polynomial $9t^2 - 6t + 1$ and verify the relationship between the zeroes and the coefficients.

22. A point P is 25 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

OR

ABC is a right triangle, right-angled at B, such that BC = 6 cm and AB = 8 cm, find the radius of circle.

23. If $\cos\theta + \cos^2\theta = 1$, prove that $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2 = 1$

OR

Prove : (sec A - tan A)^2 (1 + sin A) = 1 - sin A

24. The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute.

25. Out of 1200 students of a school, 125 students participated in Republic Day parade. A student was selected at random at random from the school. Find the probality that the selected student participated in the parade.

26. A box contains 3 blue marbles, 2 white marbles and 4 red marbles. If a marble is taken out at random from the box, What is the probability that it will be a white one? Blue one? Red one?

Section C

27. On dividing a polynomial 3x^3 + 4x^2 + 5x -13 by a polynomial g(x), the quotient and the remainder are (3x + 10) and (16x - 43) respectively. Find g(x).

28. Draw a circle of radius 4.2 cm. Draw a pair of tangents to this circle inclined to each other at an angle of 45°.

OR

Draw a circle of radius 3 cm. Draw a tangent to the circle making an angle of 30° with a line passing through the centre.

29. The interior of a building is in the form of a right circular cylinder of diameter 4.2 m and height 4 m surmounted by a cone of same diameter. The height of the cone is 2.8 m. Find the outer surface area of the building.

30. Prove the identity:

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A- cosec^3 A} = \sin^2 A \cos^2 A$$

OR

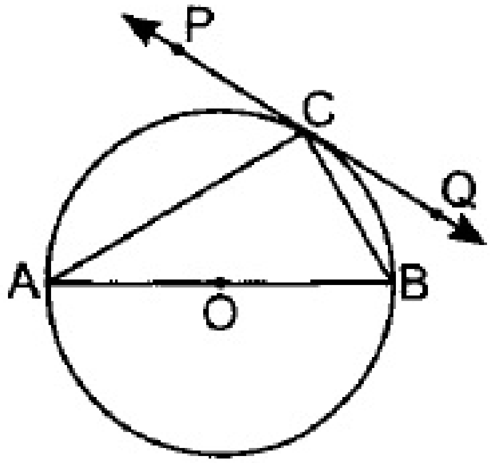
Simplify: $\frac{\sin\theta \sec(90^\circ-\theta) \tan\theta}{\operatorname{cosec}(90^\circ-\theta) \cos\theta \cot(90^\circ-\theta)}$ - $\frac{\tan(90^\circ-\theta)}{\cot\theta}$.

31. Find the HCF of 180, 252 and 324 by using Euclid's division lemma.

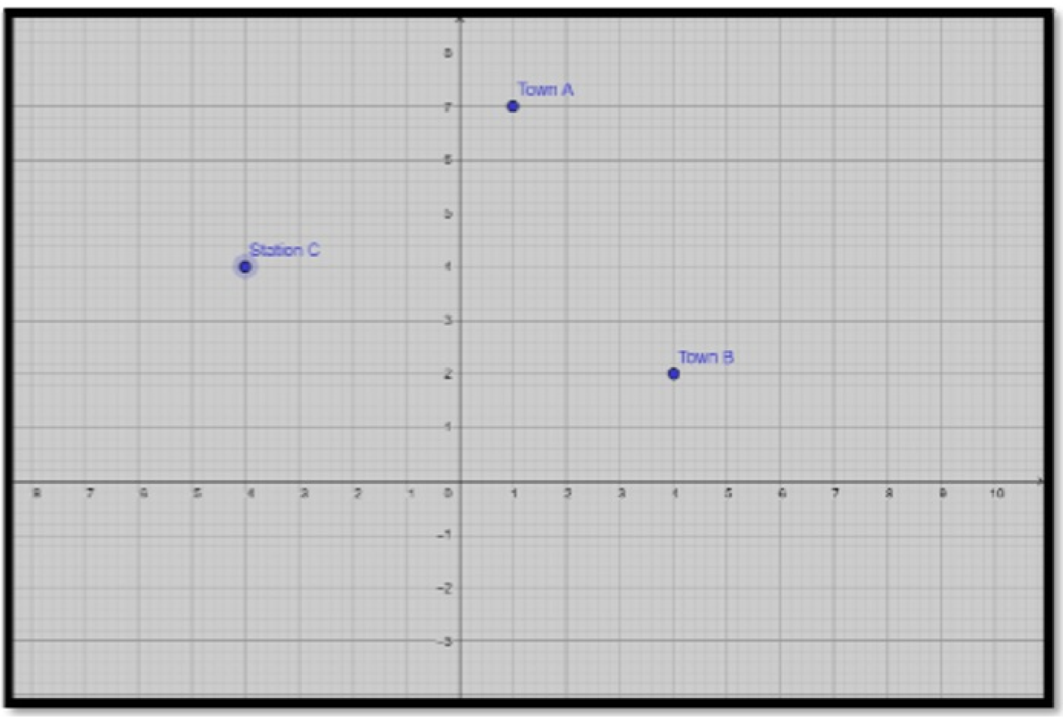
OR

Three bells toll at intervals of 9,12,15 minutes respectively. If they start tolling together, after what time will they next toll together?

32. In figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$.



33. Two friends Seema and Aditya work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation answer the following questions:



i. Who will travel more distance, Seema or Aditya, to reach to their hometown?
ii. Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the point represented by Town A and Town B. Find the coordinates of the point represented by the point D.
iii. Find the area of the triangle formed by joining the points represented by A, B and C.

34. From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to beα and β. If the height of the light house be h metres and the line joining the ships passes through the foot of the light house, show that the distance between the ship is $\frac{h(\tan\alpha+\tan\beta)}{\tan\alpha\tan\beta}$ metres.

Section D

35. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train.

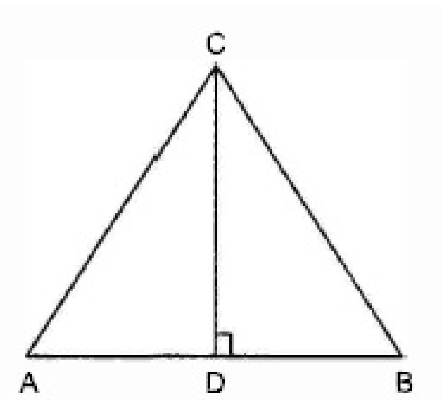
36. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find n and a, if a_n = 4, d = 2 and S_n = -14.

OR

Find the 20th term from the last term of the AP : 3, 8, 13, …., 253.

37. If we add 1 to the numerator of a fraction, it reduces to $\frac{1}{2}$. If we subtract 1 from the denominator, it reduces to $\frac{1}{3}$. Represents This situation algebraically and graphically.

38. In Fig. ∠ACB = 90° and $CD \perp AB$. Prove that $\frac{CB^2}{CA^2} = \frac{BD}{AD}$.



OR

In Fig. $\angle BAC = 90^\circ$, AD is its bisector. If $DE \perp AC$, prove that DE $\times$ (AB + AC) = AB $\times$ AC.

39. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid (Use π = 22/7)

OR

A solid cone of base radius 10 cm is cut into two parts through the mid-point of its height by a plane parallel to its base. Find the ratio of the volumes of the two parts of the cone.

40. If the mean of the following frequency distribution is 91, and sum of frequencies is 150, find the missing frequency x and y :

Classes	0 - 30	30 -60	60 -90	90 -120	120 -150	150 -180
Frequency	12	21	x	52	y	11

Solution

Section A

1. (b) 0

   Explanation:

   If $x_i{}'s$ are the midpoints of the class intervals of grouped data, $f_i{}'s$ are the corresponding frequencies and $\overline{x}$ is the mean, then $\sum(f_ix_i - \overline{x})$ is equal to 0. i.e the difference between the sum of product of frequencies and mid values of corresponding class intervals of the grouped data and the sum of their mean value is equal to zero.

2. (c) 50°

   Explanation:

   Here ∠OAP = 90°
   And $\angle\text{OPA} = \frac{1}{2}\angle\text{BPA}$ [Centre lies on the bisector of the angle between the two tangents]
   $\Rightarrow\angle\text{OPA} = \frac{1}{2} \times 80^\circ = 40^\circ$
   Now, in triangle OPA,
   $\angle\text{OAP} + \angle\text{OPA} + \angle\text{POA} = 180^\circ$
   ⇒90° + 40° + ∠POA = 180°
   ⇒ ∠POA = 50°

3. (d) 0, 1, 2, 3, 4, 5

   Explanation:

   For the relation x = qy + r, $0 \leqslant r < y$

   So, here r lies between $0 \leqslant r < 6$.

   Hence, r = 0, 1, 2, 3, 4, 5.

4. (a) an odd integer

Explanation:

Since m^2 is off for some non-negative integer k,

we have $m^2 = 2k + 1$

$\Rightarrow 2k = m^2 - 1$

= $(m - 1)(m + 1)$

Here, 2 divides left-hand side,

it must be divide one of the factors on the right-hand side.

Suppose 2 divides (m - 1),

then (m - 1) is even,

i.e., m is odd.

5. (a) a rational number or an irrational number
   Explanation:

   The product of a rational number and an irrational number can be either a rational number or an irrational number.

   e.g $\sqrt{5} \times \sqrt{2}$ = $\sqrt{10}$ which irrational

   but $\sqrt{8} \times \sqrt{2}$ = $\sqrt{16}$ = 4 which is a rational number

   Thus, product of two irrational numbers can be either rational or irrational

   similarly, product of rational and irrational numbers can be either rational or irrational

   $5 \times \sqrt{2}$ = $5\sqrt{2}$ which is irrational.

   but $4 \times \sqrt{4}$ = 4 $2\times$ = 8 which is rational.

6. (d) degree
   Explanation:

A degree in a polynomial function is the greatest exponent of that equation. The degree of the constant is zero,

7. (b) 72

   Explanation:

   Here $a = 1, b = -6, c = 8$ Since
   $\alpha^3 + \beta^3 = (\alpha + \beta)\left[\alpha^2 + \beta^2 - \alpha\beta\right] = (\alpha + \beta)\left[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta\right]$

   = $(\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]$

   = $\left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3 \times \frac{c}{a}\right]$

   = $\left(\frac{-b}{a}\right)\left[\frac{b^2}{a^2} - \frac{3c}{a}\right]$

   = $\left(\frac{-b}{a}\right)\left[\frac{b^2 - 3ac}{a^2}\right]$

   = $\frac{-b^3 + 3abc}{a^3}$

   Putting the values of a, b and c, we get = $\frac{-(-6)^3 + 3 \times 1 \times (-6) \times 8}{(1)^3}$ = $\frac{216 - 144}{1}$ = 72

8. (b) $\frac{1}{3}$

   Explanation:

   Number of composite numbers on a dice = {4, 6} = 2
   Number of possible outcomes = 2
   Number of Total outcomes = 6
   $\therefore$ Required Probability = $\frac{2}{6} = \frac{1}{3}$

9. (b) $AP = \frac{1}{2}AB$

   Explanation:

   AP = $\sqrt{(2-4)^2 + (1-2)^2}$
   = $\sqrt{4+1}$ = $\sqrt{5}$ = units

AB = $\sqrt{(8-4)^2+(4-2)^2}$

= $\sqrt{16+4}$ = $\sqrt{20}$ = $2\sqrt{5}$ units

Here AB = 2 $\times$AP

$\therefore$ AP = $\frac{1}{2}$ AB

10. (b) (– 7, 0)

    Explanation:

    Let A$(2,-5)$ and B$(-2,9)$.

    Since point is on $x-$axis C$(x,0)$.

    $\therefore$AC^2 = BC^2

    $\Rightarrow(2-x)^2+(-5-0)^2=(-2-x)^2+(9-0)^2$

    $\Rightarrow4+x^2-4x+25=4+x^2+4x+81$

    $\Rightarrow-8x=56$

    $\Rightarrow x=-7$

    Therefore, the point on $x-$axis is $(-7,0)$.

11. x = 3

12. 5 units

    OR

    5 units

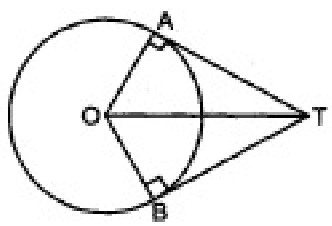
13. $\frac{2}{\sqrt{3}}$

14. 2

15. $\frac{3(1-\sqrt{3})}{2}$

16. According to the question,

    $\angle$ATO = 40°



In $\triangle$OAT, $\angle$OAT = 90°

$\angle$AOT = 50° [Angle sum property]

Now $\angle$BTO = 40° as OT bisects $\angle$ATB

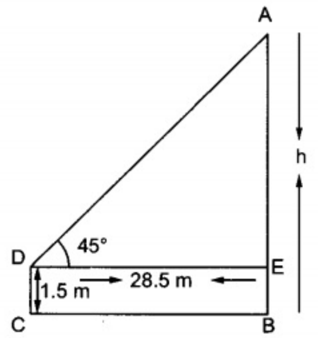
Similarly, $\angle$BOT= 50°

$\angle$AOB = $\angle AOT + \angle$BOT = 50° + 50° = 100°

OR

Let AB be the tower of height h and CD be the observer of height 1.5 m at a distance of 28.5 m from the tower AB.

In $\triangle AED,$ we have



$\tan 45^\circ = \frac{AE}{DE}$

$\Rightarrow 1 = \frac{AE}{28.5}$

$\Rightarrow AE = 28.5\text{m}$

$\therefore$ h = AE + BE

= AE + DC

= (28.5 + 1.5) m = 30m

Hence, the height of the tower is 30 m.

17. Here, $a = 14,\ d = 11 - 14 = -3$

Let a_n = -1 $\Rightarrow$ a + (n - 1)d = -1

$\Rightarrow 14 + (n-1)(-3) = -1$

$\Rightarrow (n-1)(-3) = -1 - 14$

$\Rightarrow$ n - 1 = $\frac{-15}{-3}$ = 5 $\Rightarrow$ n = 6

18. Assume radius of a circle = r

Then, area of a circle $= \pi r^2$

and Perimeter of a circle $= 2\pi r$

According to question we have,

Perimeter of a circle = area of a circle

$\Rightarrow \pi r^2 = 2\pi r$

$\Rightarrow$ r = 2 units

19. According to the question,we have to find the length of AB.

Given that $\triangle \mathrm{ABC} \sim \Delta \mathrm{LMN}$

So, ratio of corresponding sides of the two triangles is equal to the ratio of their perimeters.

$\therefore \quad \frac{\text{Perimeter of } \Delta \mathrm{ABC}}{\text{Perimeter of } \Delta \mathrm{LMN}} = \frac{\mathrm{AB}}{\mathrm{LM}} = \frac{\mathrm{BC}}{\mathrm{MN}} = \frac{\mathrm{AC}}{\mathrm{LN}}$

Let AB = x cm

$\Rightarrow \quad \frac{60}{48} = \frac{x}{8}$

$\Rightarrow x = \frac{60}{48} \times 8$

= 10 cm

$\therefore$ AB = 10 cm

20. We know that a coin has only two choices-head or tail and both are equally likely events i.e. the probability of occurrence of both is same. Hence, a coin is a fair option to decide which team will choose ends in the game.

Section B

21. The given polynomial is

p(t) = $9t^2$ - 6t + 1

For zeroes of p(t)

$9t^2$ - 6t + 1 = 0

$\Rightarrow 9t^2$ - 3t - 3t+ 1 = 0

$\Rightarrow$3t(3t - 1) -1(3t - 1)= 0

$\Rightarrow$ (3t - 1) (3t - 1)= 0

$\Rightarrow t = \frac{1}{3}, \frac{1}{3}$

$\therefore$ zeroes are $\frac{1}{3}, \frac{1}{3}$

Now, a = 9, b = -6 , c = 1

$\frac{-b}{a} = \frac{-(-6)}{9} = \frac{2}{3}$ ...... (1)

Also, sum of the zeroes = $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ .... (2)

From (1) and (2)

Sum of zeroes = $\frac{-b}{a}$

Also, $\frac{c}{a}=\frac{1}{9}$ .... (3)

and product of the zeroes = $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$....... (4)

From (3) and (4)

Product of zeroes = $\frac{c}{a}$

22. OP = 25cm.

Let TP be the tangent, so that TP = 24cm

Join OT where OT is radius.

Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

$\therefore OT\perp PT$

In $\triangle OTP$,

By Pythagoras theorem, $OT^2 + TP^2 = OP^2$

$OT^2 + 24^2 = 25^2$

$OT^2 = 625 - 576$

$OT^2 = 49$

OT = 7

The radius of the circle will be 7cm.

OR

Let ABC be the right angled triangle such that ∠B = 90° , BC = 6 cm, AB = 8 cm. Let O be the centre and *r* be the radius of the in circle.

AB, BC and CA are tangents to the circle at P, Nand M.

∴ OP = ON = OM = r (radius of the circle)

Area of the $\triangle$ABC = $\frac{1}{2}\times 6\times 8$= 24 cm^2

By Pythagoras theorem,

$CA^2 = AB^2 + BC^2$

$\Rightarrow CA^2 = 8^2 + 6^2$

$\Rightarrow CA^2 = 100$

$\Rightarrow$ CA = 10 cm

Area of the $\triangle$ABC = Area $\triangle$OAB + Area $\triangle$OBC + Area $\triangle$OCA

24 = $\frac{1}{2}r \times \mathrm{AB} + \frac{1}{2}r \times \mathrm{BC} + \frac{1}{2}r \times \mathrm{CA}$

24 = $\frac{1}{2}r$(AB + BC + CA)

$\Rightarrow r = \frac{2\times 24}{(\mathrm{AB}+\mathrm{BC}+\mathrm{CA})}$

$\Rightarrow r = \frac{48}{8+6+10}$

$\Rightarrow r = \frac{48}{24}$

$\Rightarrow$ r = 2 cm

23. We have, $\cos\theta + \cos^2\theta = 1$

$\Rightarrow \cos\theta = 1 - \cos^2\theta$

$\Rightarrow \cos\theta = \sin^2\theta \left(\because \sin^2\theta + \cos^2\theta = 1\right)$......(1)

Now, $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta$ - 2

$= \left[\left(\sin^4\theta\right)^3 + 3\sin^4\theta\cdot\sin^2\theta\left(\sin^4\theta + \sin^2\theta\right) + \left(\sin^2\theta\right)^3\right] + 2\left(\sin^2\theta\right)^2 + 2\sin^2\theta$

2

Using (a + b)^3 = a^3 + b^3 + 3ab (a + b)

Also, from (1) $\sin^2\theta = \cos\theta$

$= \left(\sin^4\theta + \sin^2\theta\right)^3 + 2(\cos\theta)^2 + 2\sin^2\theta$ - 2

$= \left(\left(\sin^2\theta\right)^2 + \sin^2\theta\right)^3 + 2\left(\sin^2\theta + \cos^2\theta\right)$ - 2

$= \left(\cos^2\theta + \sin^2\theta\right)^3 + 2 - 2 \left(\because \sin^2\theta + \cos^2\theta = 1\right)$

= (1)^3 + 0

= 1 = R.H.S

therefore, $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2 = 1$

Hence proved.

OR

L.H.S = (secA - tanA)^2(1 + sinA)

= (sec^{2}A + tan^{2}A - 2 secA tanA) (1 + sinA)

$= \left[\frac{1}{cos^2A} + \frac{sin^2A}{cos^2A} - 2\frac{sinA}{cos^2A}\right]$(1 + sinA)

$= \left[\frac{1+sin^2A-2sinA}{cos^2A}\right]$(1 + sin A)

$= \left[\frac{(1-sinA)^2}{1-sin^2A}\right]$ (1 + sinA)

$= \frac{(1-sinA)(1-sinA)(1+sinA)}{(1-sinA)(1+sinA)}$

= 1 - sin A = R.H.S

24. Clearly, minute hand of a clock describes a circle of radius equal to its length i.e. 14 cm. Since the minute hand rotates through 6° in one minute. Therefore, area swept by the minute hand in one minute is the area of a sector of angle 6° in a circle of radius 14 cm. Hence, required area A is given by

$A = \frac{\theta}{360} \times \pi r^2$

$\Rightarrow \quad A = \left\{\frac{6}{360} \times \frac{22}{7} \times (14)^2\right\} \text{cm}^2$

$= \left\{\frac{1}{60} \times \frac{22}{7} \times 14 \times 14\right\} \text{cm}^2 = \frac{154}{15} \text{cm}^2$

= 10.26 cm^2

25. Let K be the event to get selected the student who participated in the parade.

Total students = 1200

Total no of outcomes = 1200

No of students participated in parade = 125

Outcomes favouring K = 125

$\text{P(K)} = \frac{125}{1200} = \frac{5}{48}$

26. Total no. of possible outcomes = 3+2+4 = 9

No. of favourable outcomes for white marbles = 2

$Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

Required probability

P(white marbles) = $\frac{2}{9}$

No. of favourable outcomes for blue marbles = 3

Required probability

P(Blue marbles) = $\frac{3}{9} = \frac{1}{3}$

No. of favourable outcomes for red marbles = 4

Required probability

P(red marbles) = $\frac{4}{9}$

Section C

27. Dividend = $3x^3 + 4x^2 + 5x - 13$

Divisor = g(x)

Quotient = (3x + 10)

Remainder = (16x - 43)

Dividend = Divisor $\times$ Quotient + Remainder

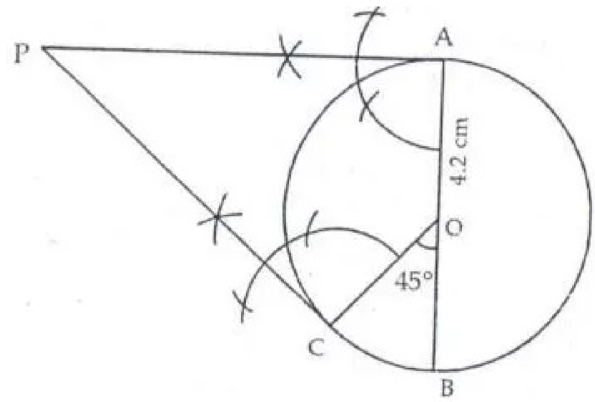
$3x^3 + 4x^2 + 5x - 13 = (3x + 10)g(x) + (16x - 43)$

$g(x)(3x + 10) = (3x^3 + 4x^2 + 5x - 13) - (16x - 43)$

$\Rightarrow g(x) = \frac{3x^3+4x^2-11x+30}{3x+10}$

$$
\begin{array}{r|l}
 & x^2 - 2x + 3 \\
\hline
3x + 10 & 3x^3 + 4x^2 - 11x + 30 \\
 & 3x^3 + 10x^2 \\
 & - \quad - \\
\hline
 & -6x^2 - 11x \\
 & -6x^2 - 20x \\
 & + \quad + \\
\hline
 & 9x + 30 \\
 & 9x + 30 \\
 & - \quad - \\
\hline
 & 0 \\
\hline
\end{array}
$$

Hence, g(x) = $x^2 - 2x + 3$

28. 

Steps of construction:

1. Draw a circle with centre O and radius 4.2 cm
2. Draw diameter AB
3. With OB as a base, draw BOC = 45°

4. At A, draw a line perpendicular to OA.
5. At C, draw a line perpendicular to OC.

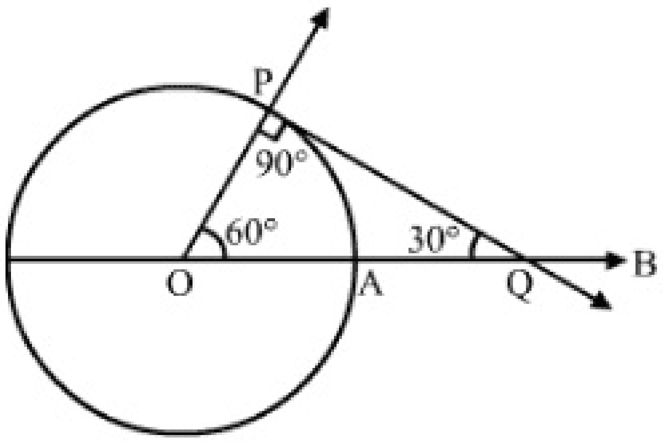
These lines intersect each other at P.
Thus, PA and PC are the required tangents.

OR

Steps of construction:

1. Draw a circle with center O and radius 3 cm.
2. Draw radius OA and produce it to B.
3. Make ∠AOP = 60°.
4. Draw PQ perpendicular to OP, meeting OB at Q.
5. **T**hen, PQ is the desired tangent, such that ∠OQP = 30°.

Construction:



29. According to the question,we are given that,

 height of the cone = 2.8 m
Diameter of cylinder = 4.2 m

radius of the cylinder = 2.1 m
radius of the cylinder = radius of the cone = 2.1 m

Height of the cylinder=4 m
Therefore, Slant height of the cone $l = \sqrt{(2.8)^2 + (2.1)^2}$
$= \sqrt{7.84 + 4.41}$

$= \sqrt{(2.8)^2 + (2.1)^2}$

= 3.5 m

outer surface area of the building

= Curved surface area of cylinder + Curved surface area of cone

$= 2\pi rh + \pi rl$

$= 2 \times \left(\frac{22}{7}\right) \times 2.1 \times 4 + \left(\frac{22}{7}\right) \times 2.1 \times 3.5$

= 44×0.3×4+22×0.3×3.5

= 44×1.2+6.6×3.5

= 52.8+23.1

= 75.90 m^2

30. We have,

$$\text{LHS} = \frac{\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)}{\left(\frac{1}{\cos^3 A}-\frac{1}{\sin^3 A}\right)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{\cos^2 A+\sin^2 A}{\sin A\cos A}\right)(\sin A-\cos A)}{\left(\frac{\sin^3 A-\cos^3 A}{\sin^3 A\cos^3 A}\right)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{1}{\sin A\cos A}\right)(\sin A-\cos A)\sin^3 A\cos^3 A}{(\sin^3 A-\cos^3 A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{(\sin A\cos A+1)(\sin A-\cos A)\sin^2 A\cos^2 A}{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)}$$ [∵ $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$]

$$\Rightarrow \quad \text{LHS} = \frac{(\sin A\cos A+1)\sin^2 A\cos^2 A}{(1+\sin A\cos A)}$$ = $\sin^2 A \cos^2 A$ = RHS

OR

$sec(90^\circ - \theta) = cosec\theta,$

$tan(90^\circ - \theta) = cot\theta,$

$cot(90^\circ - \theta) = tan\theta,$

$cosec(90^\circ - \theta) = sec\theta$

Hence,

$\frac{\sin\theta\sec(90^\circ-\theta)\tan\theta}{cosec(90^\circ-\theta)\cos\theta\cot(90^\circ-\theta)}$ - $\frac{\tan(90^\circ-\theta)}{\cot\theta}$

= $\frac{\sin\theta cosec\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} - \frac{\cot\theta}{\cot\theta}$

= $\frac{\sin\theta\times\frac{1}{\sin\theta}\times\tan\theta}{\frac{1}{\cos\theta}\times\cos\theta\tan\theta} - 1$

$= \frac{tan\theta}{tan\theta} - 1$

= 1 - 1

= 0

31. Given numbers are 180, 252 and 324.

$324 > 252 > 180$

On applying Euclid's Division lemma for 324 and 252, we get

$324 = (252 \times 1) + 72$

Here, remainder = $72 \neq 0$

So, again applying Euclid's Division lemma with new dividend 252 and new divisor 72, we get

$252 = (72 \times 3) + 36$

Here, remainder = $36 \neq 0$

So, again applying Euclid's Division lemma with new dividend 72 and new divisor 36, we get

$72 = (36 \times 2) + 0$

Here, remainder = 0 and divisor is 36

So, HCF of 324 and 252 is 36

Now, applying Euclid's Division lemma for 180 and 36, we get

$180 = (36 \times 5) + 0$

Here, remainder = 0

So HCF of 180 and 36 is 36.

Hence, HCF of 180,252 and 324 is 36.

OR

We have to find the smallest number which is divisible by 9,12 and 15,that is the time of next together toll.

The smallest number divisible by 9,12,15 = LCM(9, 12, 15)

Now the factorisation of 9,12 and 15 are

$9 = 3 \times 3 = 3^2$

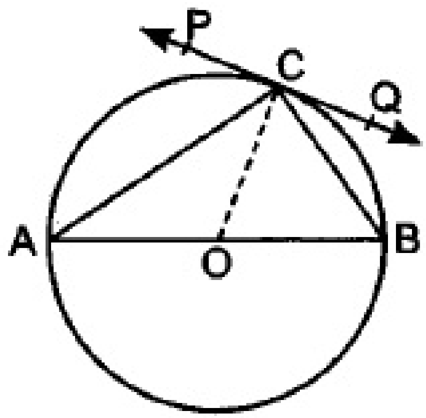
$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

$\therefore$ LCM (9, 12, 15) = $3^2 \times 2^2 \times 5 = 180$

Hence all the three bells will toll together after 180 minutes or 3 hours.

32. Given,



In $\triangle$AOC,

$AO = CO$[radius of circle]

$\therefore \angle OCA = \angle CAB = 30^\circ$

$\text{OC} \perp \text{PQ},$

$\Rightarrow \angle\text{OCP} = 90^\circ$

$\Rightarrow \angle PCA + \angle OCA = 90^\circ$

$\Rightarrow \angle\text{PCA} + 30^\circ = 90^\circ$

$\Rightarrow \angle\text{PCA} = 60^\circ$

33. i. A(1, 7), B(4, 2), C(-4, 4)

Distance travelled by Seema, AC = $\sqrt{[-4-1]^2 + [4-7]^2} = \sqrt{34}$ units

Distance travelled by Aditya, BC = $\sqrt{[-4-4]^2 + [4-2]^2} = \sqrt{68}$ units

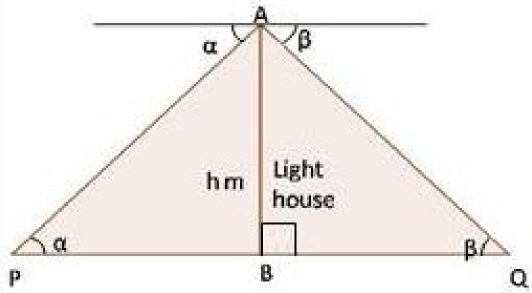
$\therefore$ Aditya travels more distance

ii. By using mid-point formula,

Coordinates of D are $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

iii. $\text{ar}(\Delta\text{ABC}) = \frac{1}{2}$ [1(2 - 4) + 4(4 - 7) - 4(7 - 2)]

= 17 sq. units

34. 

Height of light house AB = h metres

In ΔABP

$\tan\alpha = \frac{AB}{PB}$

$\Rightarrow \tan\alpha = \frac{h}{PB}$

$\Rightarrow PB = \frac{h}{\tan\alpha}$ .... (i)

In ΔABQ

$\tan\beta = \frac{AB}{BQ}$

$\Rightarrow \tan\beta = \frac{h}{BQ}$

$\Rightarrow BQ = \frac{h}{\tan\beta}$ .... (ii)

$\therefore$ Distance between the ships $= PQ$

$= PB + BQ$

$= \frac{h}{\tan\alpha} + \frac{h}{\tan\beta}$

$= \frac{h\tan\beta + h\tan\alpha}{\tan\alpha\tan\beta}$

$= \frac{h(\tan\beta+\tan\alpha)}{\tan\alpha+\tan\beta}\ m$

Section D

35. Let the usual speed of train be x km/hr

$\frac{300}{x} - \frac{300}{x+5} = 2$

300 (x+5 - x) = 2x(x+5)

150(5) = x^2 + 5x

750 = x^2 + 5x

or, $x^2 + 5x - 750 = 0$

or, $x^2 + 30x - 25x - 750 = 0$

or, (x + 30) (x - 25) = 0

or, x = - 30 or x = 25

Since, speed cannot be negative.

$\therefore \quad x \neq -30, x = 25\text{km/hr}$

$\therefore$ Speed of train = 25 km/hr

36. Given,

Common difference(d) = 2

and, n^{th} term (a_n) = 4

a + (n - 1)d = 4

$\Rightarrow$a + (n - 1)(2) = 4

$\Rightarrow$a + 2n - 2 = 4

$\Rightarrow$a + 2n= 6

and, S_n = -14

$\Rightarrow \frac{n}{2}[a + a_n]$ = -14

$\Rightarrow \quad \frac{n}{2}[a + 4] = -14$

$\Rightarrow \quad \frac{n}{2}[6 - 2n + 4] = -14$ [using a=6-2n]

$\Rightarrow \quad \frac{n}{2}[10 - 2n] = -14$

$\Rightarrow$5n - n^2 = -14

$\Rightarrow$$n^2$ - 5n - 14 = 0

$\Rightarrow$$n^2$ - 7n + 2n - 14 = 0

$\Rightarrow$n(n - 7) + 2(n - 7) = 0

If, n - 7 = 0

n = 7

or n + 2 = 0

n = -2(Rejected as n cannot be negative)

Put value of n in (i),

a + 2$\times$7= 6

$\Rightarrow$a = 6 -14

$\Rightarrow$a = -8

OR

The given AP is 3, 8, 13, ..., 253

Here, a = 3

d = 8 - 3 = 5

l = 253

Let the number of terms of the AP be n.

Term, nth term = l

$\Rightarrow$ 3 + (n - 1)5 = 253 $\because a_n = a + (n-1)d$

$\Rightarrow$ (n - 1)5 = 253 - 3

$\Rightarrow$ (n - 1)5 = 250

$\Rightarrow n - 1 = \frac{250}{5}$

$\Rightarrow$ n - 1 = 50

$\Rightarrow$ n = 50 + 1

$\Rightarrow$ n = 51

So, there are 51 terms in the given AP.

Now, 20th term from the last term

= (51 - 20 + 1)th term from the beginning

= 32th term from the beginning

= 3 + (32 - 1)5 $\because a_n = a + (n-1)d$

= 3 + 155

= 158

Hence, the 20th term from the last term of the given AP is 158.

Aliter. Let us write the given AP in the reverse order.

Then the AP becomes 253, 248, 243, ..., 3

Here, a = 253

d = 248 - 253 = -5

Therefore, required term

= 20th term of the AP

= 253 + (20 - 1) (-5) $\because a_n = a + (n-1)d$

= 253 - 95

= 158

Hence, the 20th term from the last term of the given AP is 158.

37. Let, the numerator and the denominator of the fraction be x and y respectively. Then the algebraical representation is given by the following equations:

$\frac{x+1}{y} = \frac{1}{2}$

$\Rightarrow$ 2(x + 1) = y

$\Rightarrow$ 2(x + 1) = y

$\Rightarrow$ 2x - y = -2 ...(1)

and $\frac{x}{y-1} = \frac{1}{3}$ $\Rightarrow$ 3x = y - 1

$\Rightarrow$ 3x - y = -1 ...(2)

To represent these equation graphically, we find two solutions for each equation.

These solution are given below:

For equation (1) 2x - y = -2

Table 1 of solutions

x	0	-1
y	2	0

For equation (2),

3x - y = -1

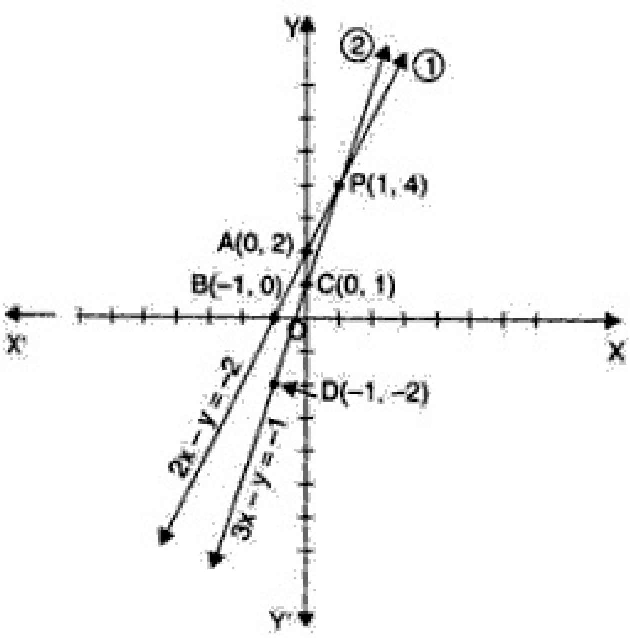
$\Rightarrow$ y = 3x + 1

Table 2 of solutions

x	0	-1
y	1	-2

We plot the points A(0, 2) and B(-1, 0) corresponding to the solutions in table 1 on a Graph paper to get the line AB representing.

The equation (1) and the points C(0, 1) and D(-1, 2) corresponding to the solutions in Table 2 on the same graph paper to get the line CD representing the equation (2) as shown in the figure given below



We observe in the figure that the two lines representing the two equations are intersecting at the point P(1, 4).

38. We have to prove that $\frac{CB^2}{CA^2} = \frac{BD}{AD}$.

Now, so far as the given figure is concerned,in triangles ACD and ABC, we have

$\angle$ ADC = $\angle$ ACB [Each equal to 90°]

and, $\angle$ DAC = $\angle$ BAC [Common]

Therefore, by AA-criterion of similarity, we obtain

$\Delta ACD \sim \Delta ABC$

$\Rightarrow \quad \frac{AC}{AB} = \frac{AD}{AC}$

$\Rightarrow \quad AC^2 = AB \times AD$ ....(i)

Now , in Δ's BCD and BAC, we have,

$\angle$BDC = $\angle$BCA

and, $\angle$DBC = $\angle$ABC

Therefore, by AA-criterion of similarity, we obtain,

$BCD \sim BAC$

$\Rightarrow \quad \frac{BC}{BD} = \frac{BA}{BC}$

$\Rightarrow \quad BC^2 = AB \times BD$ ....(ii)

Dividing (ii) by(i), we get

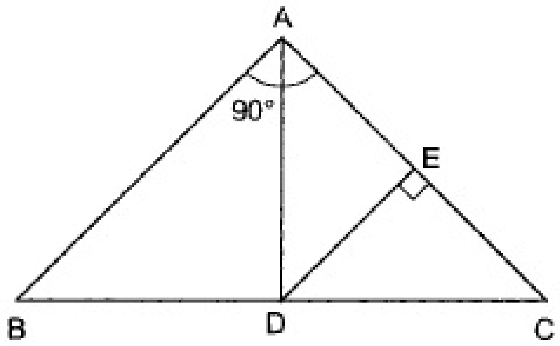
$\frac{BC^2}{AC^2} = \frac{AB \times BD}{AB \times AD} \Rightarrow \frac{BC^2}{AC^2} = \frac{BD}{AD}. Hence, proved.$

OR

To prove the given result,we will use the following theorm.

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle

Since AD is the bisector of ∠A of ΔABC.



$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$ [by above theorm]

$\Rightarrow \quad \frac{AB}{AC} + 1 = \frac{BD}{DC} + 1$ [Adding 1 on both sides]

$\Rightarrow \quad \frac{AB+AC}{AC} = \frac{BD+DC}{DC}$

$\Rightarrow \quad \frac{AB+AC}{AC} = \frac{BC}{DC}$ ... (i)

In Δ's CDE and CBA, we have

∠DCE = ∠BCA = ∠C [Common]

∠BAC = ∠DEC [Each equal to 90°]

So, by AA-criterion of similarity, we have

ΔCDE ~ ΔCBA

$\Rightarrow \quad \frac{CD}{CB} = \frac{DE}{BA}$

$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{DC}$ ...(ii)

From (i) and (ii), we obtain

$\frac{AB+AC}{AC} = \frac{AB}{DE} \Rightarrow DE \times (AB + AC) = AB \times AC$

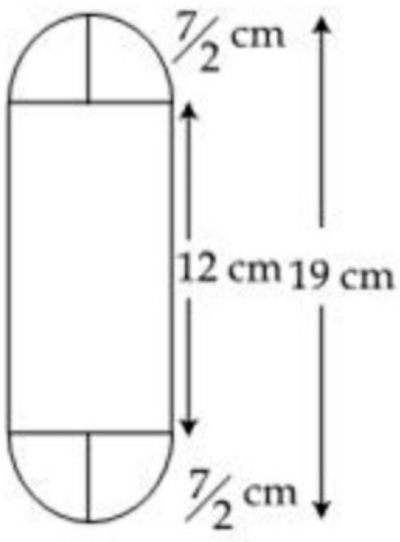
39. Diameter of the cylinder = 7 cm

Therefore radius of the cylinder = $\frac{7}{2} cm$

Total height of the solid = 19 cm

Therefore, Height of the cylinder portion = 19 - 7 = 12 cm

Also, radius of hemisphere = $\frac{7}{2}$ cm



Let V be the volume and S be the surface area of the solid. Then,

V = Volume of the cylinder + Volume of two hemispheres

$\Rightarrow \quad V=\left\{\pi r^2 h+2\left(\frac{2}{3} \pi r^3\right)\right\} \mathrm{cm}^3$

$\Rightarrow \quad V=\pi r^2\left(h+\frac{4 r}{3}\right) \mathrm{cm}^3$

$\Rightarrow \quad V=\left\{\frac{22}{7} \times\left(\frac{7}{2}\right)^2 \times\left(12+\frac{4}{3} \times \frac{7}{2}\right)\right\} \mathrm{cm}^3$

$=\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{50}{3} \mathrm{cm}^3=641.66 \mathrm{cm}^3$

and,

S = Curved surface area of cylinder + Surface area of two hemispheres

$\Rightarrow \quad S=\left(2 \pi r h+2 \times 2 \pi r^2\right) \mathrm{cm}^2$

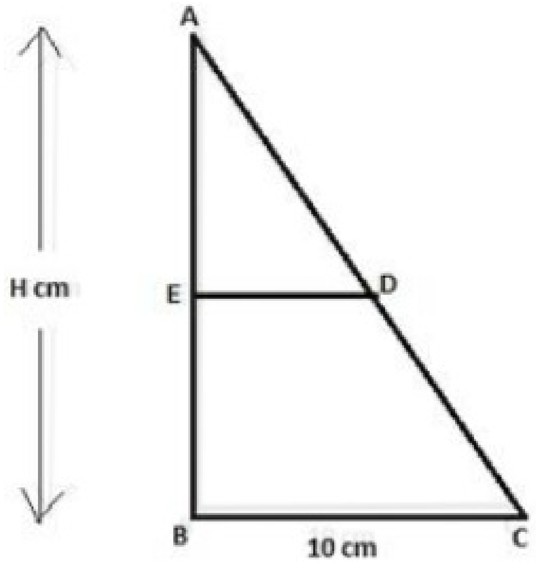
$\Rightarrow \quad S=2 \pi r(h+2 r) \mathrm{cm}^2$

$\Rightarrow \quad S=2 \times \frac{22}{7} \times \frac{7}{2} \times\left(12+2 \times \frac{7}{2}\right) \mathrm{cm}^2$

$=\left(2 \times \frac{22}{7} \times \frac{7}{2} \times 19\right) \mathrm{cm}^2$

= 418 cm^2

OR



Given that AE = EB = $\frac{H}{2}$, where H is the height of the cone ABC.

Consider the ΔAED and ΔABC,

$\angle AED = \angle ABC = 90^\circ$

$\angle EAD = \angle BAC$ ...(common angle)

So, $\Delta \mathrm{AED} \sim \Delta \mathrm{ABC}$ ...(AA criterion for similarity)

$\Rightarrow \frac{AE}{AB} = \frac{ED}{BC} = \frac{r}{10}$, where r is the radius of the cone,

when a plane parallel to its base cuts the heights of its mid-point.

$\Rightarrow \frac{\frac{H}{2}}{H} = \frac{r}{10}$, where H is the height of the cone

$\Rightarrow \frac{1}{2} = \frac{r}{10}$

$\Rightarrow$ r = 5 cm

Volume of the frustum of the cone = Volume of the cone ABC - Volume of the upper part of the cone AED

Here, R = 10 cm

$\Rightarrow$ Volume of the frustum of the cone $= \frac{1}{3}\pi(100)\mathrm{H} - \frac{1}{3}\pi(25)\frac{\mathrm{H}}{2} = \frac{175\pi\mathrm{H}}{6}$

Volume of the cone $AED = \frac{1}{3}\pi(25)\frac{H}{2} = \frac{25\pi H}{6}$

Ratio of their volumes $= \frac{\frac{25\pi\mathrm{H}}{6}}{\frac{175\pi\mathrm{H}}{6}} = \frac{1}{7}$

Hence, the ratio is 1:7.

40. Given that the mean of the frequency distribution is 91, and the sum of frequencies is 150.

Class	$\mathbf{x_i}$		

	(Class marks)	f_i	f_iu_i
0-30	15	12	180
30-60	45	21	945
60-90	75	x	75x
90-120	105	52	5460
120-150	135	y	135y
150-180	165	11	1815

96 + x + y = 150.........(i)

x + y = 54

$\therefore \quad \overline{x} = \frac{\Sigma f_1 x_1}{\Sigma f_i}$

$\Rightarrow \quad 91 = \frac{8400+75x+135y}{150}$

$\Rightarrow$ 13650 = 8,400 + 75x + 135y

$\Rightarrow$ 75x + 135 y = 5250

$\Rightarrow$ 5x + 9y = 350..........(ii)

by solving eqns. (i) and (ii) we get x = 34 and y = 20