Sequence and Series

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A particular order in which related things follow each other. Called sequence. The sequence having specified patterns is called progression. The real sequence is that sequence whose range is a subset of the real numbers. A series is defined as the expression denoting the sum of the terms of the sequence. The sum is obtained after adding the terms of the sequence. If $a_1, a_2\ a_3, ---, a_n$ is a sequence having n terms, then the sum of the series is given by:

$$\sum_{K=1}^{n} a_k = a_1 + a_2 + a_3 + ----+ a_n$$

Arithmetic Progression (A.P.)

A sequence is said to be in arithmetic progression if the difference between its consecutive terms is a constant. The difference between the consecutive terms of an A.P. is called common difference and nth term of the sequence is called general term. If $a_1, a_2\ a_3, ---, a_n$ be n terms of the sequence in A.P., then nth term of the sequence is given by $a_n = a + (n-1)\text{d}$, where 'a' is the first term of the sequence, 'd' is the common difference and 'n' is the number of terms in the sequence. For example 10th term of the sequence $3, 5, 7, 9, ---$ is given by:

$$a_{10} = a + 9d \qquad \Rightarrow \qquad a_{10} = 3 + 9 \times 2 = 21$$

Sum of n terms of the A.P.

If $a_1, a_2\ a_3, ---, a_n$ be n terms of the sequence in A.P., then the sum of n- terms of the sequence is given by $S_n = \frac{n}{2}[2a + (n-1)d]$.

For example the sum of first 10 terms of the sequence 3, 5, 7, 9, --- is given by:

$$S_{10} = \frac{10}{2}[2 \times 3 + 9 \times 2] \Rightarrow S_{10} = 120$$

If S is the sum of the first n terms of an AP, then its n^{th} term is given by $a_n = S_n - S_{n-1}$

Geometric Progression (G.P.)

A sequence is said to be in G.P., if the ratio between its consecutive terms is constant. The sequence $a_1, a_2\ a_3, ---, a_n$ is said to be in G.P. If the ratio of its consecutive terms is a constant, the constant term is called common ratio of the G.P. and is denoted by r. For example any sequence of the form 2, 4, 8, 16, --- is a G.P. Here the common ratio of any two consecutive terms is 2.

If 'r' is the common ratio, then the nth term of the sequence is given by $a_n = ar^{n-1}$

The sum of n terms of a G.P. is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ if } r > 1 \text{ and } S_n = \frac{a(1 - r^n)}{1 - r} \text{ if } r < 1$$

Sum to infinity of a G.P. is given by $S_\infty = \frac{a}{1-r}$

Harmonic Progression (H.P.)

A sequence is said to be in H.P. If the reciprocal of its consecutive terms are in A.P. It has got wide application in the field of geometry and theory of sound. These progressions are generally solved by inverting the terms and using the property of arithmetic progression.

Three numbers a, b, c are said to be in H.P. if, $\frac{1}{a}, \frac{1}{b}\ and\ \frac{1}{c}$ are in A.P.

Some Useful Results

(i) Sum of first n natural numbers ie. $1 + 2 + 3 + ...n = \frac{(n+1)\text{n}}{2}$

(ii) Sum of the squares of first n natural numbers ie. $1^2 + 2^2 + 3^2 + ...n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) Sum of the cubes of first n natural numbers

ie. $1^3 + 2^3 + 3^3 + ...n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Arithmetic Mean

If two numbers a and b are in an A.P., then their Arithmetic Mean $(A.M.) = \frac{a+b}{2}$

For any three numbers a, b and c of an A.P, their Arithmetic Mean $(A.M.) = b = \frac{a+c}{2}$

Geometric Mean

For any two terms a and b their geometric mean $= \sqrt{ab}$

For any n terms $a_1, a_2\ a_3, ---, a_n$ of a G.P., their geometric mean is given by G.M. $= \sqrt[n]{a_1 a_2 a_3 .....a_n}$

Note: For any two numbers a and b, $\frac{a+b}{2} \geq \sqrt{ab}$

➢ **Example:**

Find the 101th term of the A.P.

$-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$

(a) 230 (b) $\frac{250}{2}$

(c) 240 (d) 245

(e) None of these

Answer (d)

Explanation: Here,

$a = -5,\ d = 0 + \frac{5}{2} = \frac{5}{2},\ n = 101$

$\therefore a_n = a + (n-1)d$

$\Rightarrow \quad a_{101} = -5 + (101-1) \times \frac{5}{2} = 245$