# 4. Time Series

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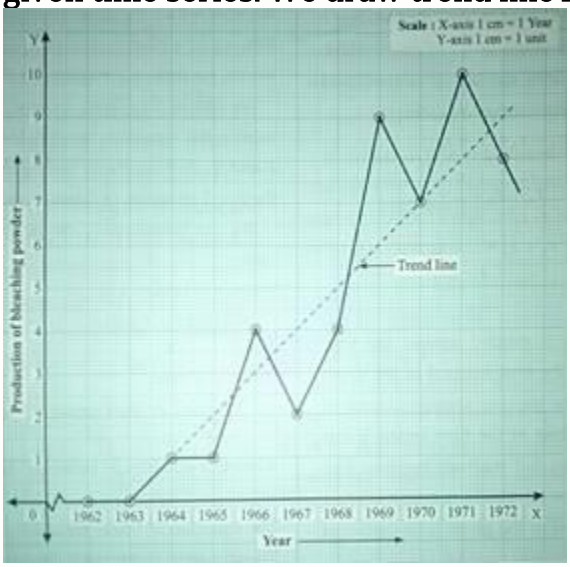
**Exercise 4.1**

**1. The following data gives the production of bleaching Powder (in '000 tonnes) for the year 1962 to 1972.**

| Year | 1962 | 1963 | 1964 | 1965 | 1966 | |
|---|---|---|---|---|---|---|
| Production | 0 | 0 | 1 | 1 | 4 | |
| Year | 1967 | 1968 | 1969 | 1970 | 1971 | 1972 |
| Production | 2 | 4 | 9 | 7 | 10 | 8 |

**Fit a trend line by graphical method to the above data.**

**Solution:**
**Taking year on x-axis and production on y-axis, we plot the point for Production corresponding to the year. Joining the point by straight Line we get the graph for the given time series. We draw trend line As shown in the figure 4.1**




**2. Use the method of least square to fit a trend to The data in problem 1 and above. Also obtain the trend Value for the year 1975.**

**Solution:**
Here, n = 11. Hence we transform year t to 'u' taking Origin at 1967.
We construct the following table for calculation of $\sum u, \sum u^2, \sum x_t, \sum u x_t$

| Year t | Production $x_t$ ('000 tones) | U = t – '67 | $u^2$ | $ux_t$ |
|---|---|---|---|---|
| 1962 | 0 | -5 | 25 | 0 |
| 1963 | 0 | -4 | 16 | 0 |
| 1964 | 1 | -3 | 9 | -3 |
| 1965 | 1 | -2 | 4 | -2 |
| 1966 | 4 | -1 | 1 | -4 |
| 1967 | 2 | 0 | 0 | 0 |
| 1968 | 4 | 1 | 1 | 4 |
| 1969 | 9 | 2 | 4 | 18 |
| 1970 | 7 | 3 | 9 | 21 |
| 1971 | 10 | 4 | 16 | 40 |
| 1972 | 8 | 5 | 25 | 40 |
| Total | $\sum x_t = 46$ | $\sum u = 0$ | $\sum u^2 = 110$ | 123<br>-9<br>$\sum ux_t = 114$ |

The Equation on the trend line is

→ $x_t = a' + b'u.$

The Two normal equation are

$\sum x_t = na' + b'\sum u \quad ...(1)$

$\sum ux_t = a'\sum u + b'\sum u^2 ...(2)$

Putting the values

$\sum x_t = 46, n = 11,$

→ $\sum u = 0, \sum u^2 = 110.$

$\sum ux_t = 114$ *in the equation, we get*

46 = 11a' + b' (0) ... (3)

114 = a' (0) + b' (110) ... (4)

From equation (3),

$a' = \frac{46}{11} = 4.182$

From Equation (4),

$b' = \frac{114}{110} = 1.036$

We get the equation of trend line,

$x_t = 4.182 + 1.036u,$

Where u = t – 1967.

Trend value for the year 1975:

For t = 1975, u = 1975 – 1967 = 8

Putting u = 8 in

$x_t = 4.182 + 1.036u, we\ get$

$x_{1975} = 4.182, +1.036(8)$

$= 4.182 + 8.288$

$\therefore\ x_{1975} = 12.47$

Hence,

The trend value for the year 1975 is 12.47 (in'000 tonnes)

**3. Obtain the trends line for the above data using 5 yearly moving average.**

**Solution:**

**We construct the following table to obtain 5 yearly**

**Moving averages for the data in problem 1:**

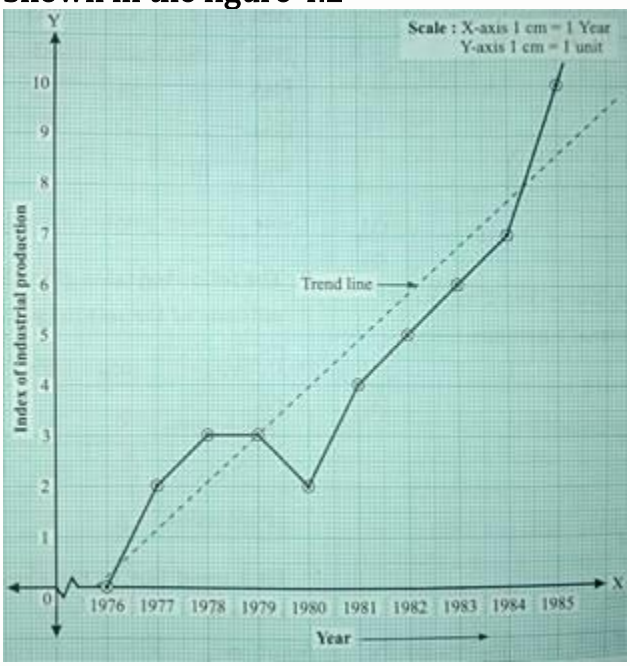
| Year<br>t | Production<br>(in '000 tonnes)<br>xt | 5 Yearly<br>Moving<br>total | 5 Yearly<br>Moving average<br>trend value |
|---|---|---|---|
| 1962 | 0 | - | - |
| 1963 | 0 | - | - |
| 1964 | 1 | 6 | 1.2 |
| 1965 | 1 | 8 | 1.6 |
| 1966 | 4 | 12 | 2.4 |
| 1967 | 2 | 20 | 4.0 |
| 1968 | 4 | 26 | 5.2 |
| 1969 | 9 | 32 | 6.4 |
| 1970 | 7 | 38 | 7.6 |
| 1971 | 10 | - | - |
| 1972 | 8 | - | - |

**4. The following table shows the index of industrial production For the period from 1976 to 1985, using the year 1976 as the base Year:**

| **Year** | 1976 | 1977 | 1978 | 1979 | 1980 |
|---|---|---|---|---|---|
| **Index** | 0 | 2 | 3 | 3 | 2 |
| **Year** | 1981 | 1982 | 1983 | 1984 | 19885 |
| **Index** | 4 | 5 | 6 | 7 | 10 |

**Fit a trend line to above data by graphical method.**

**Solution:**
**Taking year on X axis and index on y axis, we, plot**
**The points for indices corresponding to the year.**
**We get the graph time series. We draw trend line as**
**Shown in the figure 4.2**




**5. Fit a trends line to the data in problem 4 above by The method of least squares. Also, obtain the trend Value for the index of industrial production for the Year 1987.**

**Solution:**
Here, n = 10 we transform t to u by taking
U = 2(t - 1980.5)
We construct the following table for calculation:

| Year t | Index of Industrial Production xt | U = 2(t – 1980.5) | $u^2$ | $ux_t$ |
|---|---|---|---|---|
| 1976 | 0 | -9 | 81 | 0 |
| 1977 | 2 | -7 | 49 | -14 |
| 1978 | 3 | -5 | 25 | -15 |
| 1979 | 3 | -3 | 9 | -9 |
| 1980 | 2 | -1 | 1 | -2 |
| 1981 | 4 | 1 | 1 | 4 |
| 1982 | 5 | 3 | 9 | 15 |

| | | | | |
|---|---|---|---|---|
| 1983 | 6 | 5 | 25 | 30 |
| 1984 | 7 | 7 | 49 | 49 |
| 1985 | 10 | 9 | 81 | 90 |
| Total | $\sum x_t = 42$ | $\sum u = 0$ | $\sum u^2 = 330$ | 188<br>-40<br>$\sum ux_t = 148$ |

The equation of trend line,

$x_t = a' + b'u$

The normal equation are,

$\sum x_t = na'b'\sum u$ .... (1)

$\sum ux_t = a'\sum u + b'\sum u^2$ ... (2)

$Here, n = 10, \sum x_t = 42, \sum u$

$\rightarrow = 0, \sum u^2 = 330,$

$\sum ux_t = 148$

Putting these value in the normal equation, we get

42 = 10a' + b' (0) ... (3)

148 = a' (0) + b' (330) ... (4)

From the equation (3), we get

$a' = \frac{42}{10} = 4.2$

From the equation (4), we get

$b' = \frac{148}{330} = 0.4485$

Putting a' = 4.2 and b' = 0.4485 in xt = a' + b' u, we get the

Equation of trend line as

Xt = 4.2 + 0.4485u, where u = 2 (t – 1980.5)

Trend value for the year 1987:

For t = 1987, u = 2(1987 – 1980.5)

= 2(6.5) = 13

Putting u = 13 in xt = 4.2 + 0.4485u, we get

$x_{1987} = 4.2 + 0.4485 \times 13$

$x_{1987} = 4.2 + 5.8305$

$x_{1987} = 10.0305$

Hence, the trend value for the index of industrial product-

Tion for the year 1987 is 10.0305.

**6. Obtain the trend values for the data in problem 4 Using 4- yearly centered moving average.**

**Solution:**

We construct the following table to obtain 4 – yearly
Moving average for the data in problem 4:

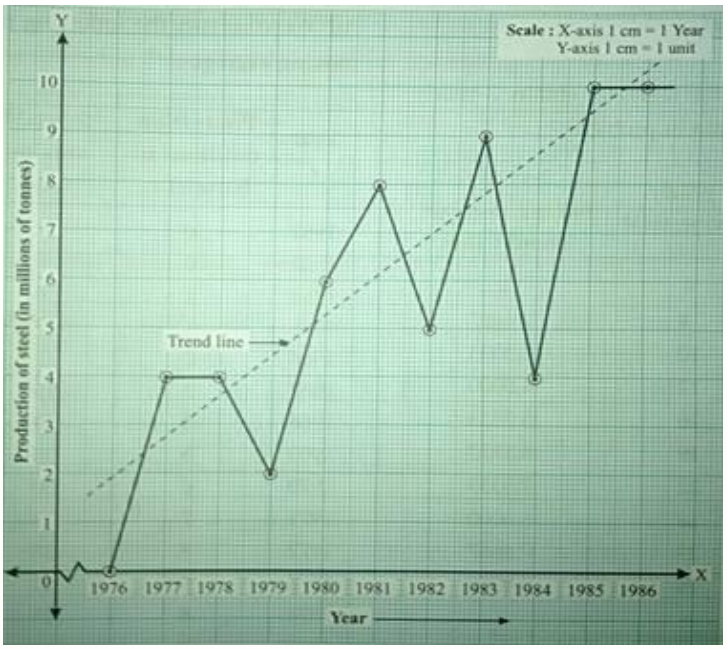
| Year t | Index xt | 4- yearly moving total | 4- yearly moving averages | 2 unit Moving total | 4 yearly centered moving averages (trend value) |
|---|---|---|---|---|---|
| 1976 | 0 | - | - | - | - |
| 1977 | 2 | 8 | 2.0 | - | - |
| 1978 | 3 | 10 | 2.5 | 4.5 | 2.25 |
| 1979 | 3 | 12 | 3.0 | 5.5 | 2.75 |
| 1980 | 2 | 14 | 3.5 | 6.5 | 3.25 |
| 1981 | 4 | 17 | 4.25 | 7.75 | 3.875 |
| 1982 | 5 | 22 | 5.5 | 9.75 | 4.875 |
| 1983 | 6 | 28 | 7.0 | 12.5 | 6.25 |
| 1984 | 7 | - | - | - | - |
| 1985 | 10 | | | - | - |

**7. The following table given the production of steel (in Millions of tonnes) for the year 1976 to 1986.**

| **Year** | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 |
|---|---|---|---|---|---|---|
| **Production** | 0 | 4 | 4 | 2 | 6 | 8 |
| **Year** | 1982 | 1983 | 1984 | 1985 | 1986 | |
| **Production** | 5 | 9 | 4 | 10 | 10 | |

**Fit a trend line to the above data by the graphical method.**

**Solution:**
Taking year on x – axis and production on y – axis, we plot
The points for production corresponding to year, Joining
These points by straight lines, we get the graph of the given
Time series, we draw trend line as shown in the figure 4.3.




**8. Fit a trend line to the data in problem 7 by the method Of least squares. Also, obtain the trend value For the year 1990.**

**Solution:**

Here, n = 11, We transform year - t to u by taking u = t – 1981.

We contract the following table for calculation:

| Year t | Production xt | U= t - 1981 | $u^2$ | $ux_t$ |
|---|---|---|---|---|
| 1976 | 0 | -5 | 25 | 0 |
| 1977 | 4 | -4 | 16 | -16 |
| 1978 | 4 | -3 | 9 | -12 |
| 1979 | 2 | -2 | 4 | -04 |
| 1980 | 6 | -1 | 1 | -06 |
| 1981 | 8 | 0 | 0 | 0 |
| 1982 | 5 | 1 | 1 | 5 |
| 1983 | 9 | 2 | 4 | 18 |
| 1984 | 4 | 3 | 9 | 12 |
| 1985 | 10 | 4 | 16 | 40 |
| 1986 | 10 | 5 | 25 | 50 |

| | | | | |
|---|---|---|---|---|
| Total | ∑ xt = 62 | ∑ u = 0 | ∑ u2 = 110 | 125<br>-38<br>∑ ux1 = 87 |

The equation of trend line

$x_t = a' + b'u$

The normal equation are

$\sum x_t = na' + b'\sum u \ldots (1)$

$\sum ux_t = a'\sum u + b'\sum u^2 \quad \ldots (2)$

Here, n =11,

$\sum x_t = 62, \sum u = 0, \sum u^2 = 110.$

$\sum ux_t = 87$

Putting these values in normal equation, we get

62 = 11a' + b' (0) … (3)

87 = a' (0) + b' (110) … (4)

From equation (4), we get

$a' = \frac{62}{11} = 5.6364$

From equation (4), we get

$b' = \frac{87}{110} = 0.7909$

Putting a' = 5.6364 and b' = 0.7909 in the equation

$x_t = a' + b'u, we\ get\ the\ equation\ of\ trend\ line\ as$

$x_t = 5.6364 + 0.7909u$

Trend for the year 1990:

For t = 1990, u = 1990 – 19981 = 9

Putting u = 9 in

$x_t = 5.6364 + 0.7909u, we\ get$

$x_{1990} = 5.6364 + 07909 \times 9$

$x_{1990} = 5.6364 + 7.1181$

$x_{1990} = 12.7545$

Hence, trend value for the year 1990 is 12.7545.

**9. Obtain the trend value for the above data using 3 – yearly Moving averages.**

**Solution:**

We contract the following table to obtain 3 – yearly Moving averages

For the data in problem 7:

| Year<br>t | Production<br>xt | 3- yearly<br>Moving total | 3- yearly<br>Moving averages trend value |
|---|---|---|---|
| 1976 | 0 | - | - |

| | | | |
|---|---|---|---|
| 1977 | 4 | 8 | 2.667 |
| 1978 | 4 | 10 | 3.3333 |
| 1979 | 2 | 12 | 4.0000 |
| 1980 | 6 | 16 | 5.3333 |
| 1981 | 8 | 19 | 6.3333 |
| 1982 | 5 | 22 | 7.3333 |
| 1983 | 9 | 18 | 6.0000 |
| 1984 | 4 | 23 | 7.6667 |
| 1985 | 10 | 24 | 8.0000 |
| 1986 | 10 | - | - |

**10. The following table shows the production of gasoline In USA for the year 1962 to 1976:**

| Year | Production (million barrels) | Year | Production (million barrels) |
|---|---|---|---|
| 1962 | 0 | 1970 | 6 |
| 1963 | 0 | 1971 | 7 |
| 1964 | 1 | 1972 | 8 |
| 1965 | 1 | 1973 | 9 |
| 1966 | 2 | 1974 | 8 |
| 1967 | 3 | 1975 | 9 |
| 1968 | 4 | 1976 | 10 |
| 1969 | 5 | | |

**i) Obtain the trend value using 5 – yearly moving averages for the above data.**

**ii) Plot the original time series and the trend values obtained in i) on the same graph.**

**Solution:**

i) We contract the following table to obtain 5 – yearly
Moving averages:

| Year | Production (million barrels) | 5 – yearly Moving total | 5 – yearly Moving Average trend value |
|---|---|---|---|
| 1962 | 0 | - | - |
| 1963 | 0 | - | - |

| | | | |
|---|---|---|---|
| 1964 | 1 | 4 | 0.8 |
| 1965 | 1 | 7 | 1.4 |
| 1966 | 2 | 11 | 2.2 |
| 1967 | 3 | 15 | 3.0 |
| 1968 | 4 | 20 | 4.0 |
| 1969 | 5 | 25 | 5.0 |
| 1970 | 6 | 30 | 6.0 |
| 1971 | 7 | 35 | 7.0 |
| 1972 | 8 | 38 | 7.6 |
| 1973 | 9 | 41 | 8.2 |
| 1974 | 8 | 44 | 8.8 |
| 1975 | 9 | - | - |
| 1976 | 10 | - | - |

**ii) Taking year on x – axis and production trend on y – axis , we plot the plot the points for production corresponding to years to get the graph of time series and plot the point for trend values corresponding to years to get the graph of trend as shown in the figure 4.4**