6th nov,
SUNDAY

CURVES

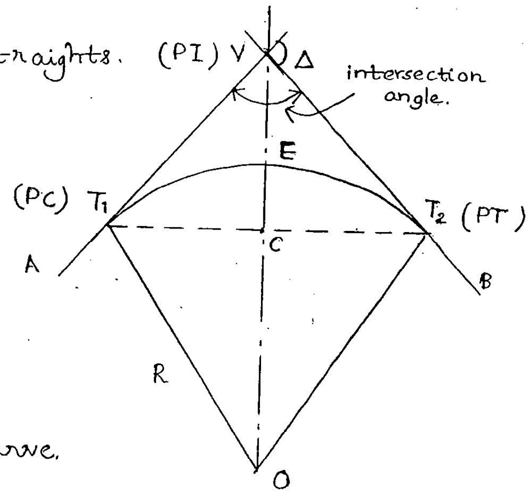
Curves are used to change the direction of highways or railways

Curves are classified as follows:

1. Horizontal Curves
   (i) Simple Curves
   (ii) Compound Curves
   (iii) Reverse curves.
   (iv) Transition curves.
2. Vertical Curves
   (i) Summit curves.
   (ii) Sag curves.

→ Simple Curves

– Arc connecting two straights.



T_1 → point of curve.

V → point of intersection.

T_2 → point of tangency

- Point of Curve is the one where straight line becomes a curve.
- Point of tangency is where a curve becomes a tangent.
- Point of intersection is the intersection of two tangents VT_1 and VT_2

$AT_1 \rightarrow$ back tangent

$BT_2 \rightarrow$ forward tangent.

→ intersection angle.

It is an angle b/w two tangents VT_1 and VT_2.

→ deflection angle (or) deviation angle (Δ)

$$\boxed{\Delta = 180 - \text{intersection angle}}$$

→ normal chord

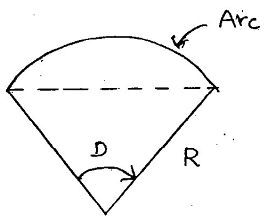
A chord b/w two successive regular stations on a curve. Generally it is equal to '<u>peg interval</u>.'

→ sub chord

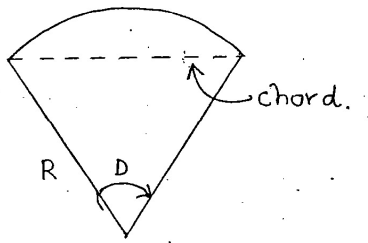
It is any chord shorter than the normal chord

→ Designation of a Curve.

<u>The sharpness of the curve is designated either</u> by its radius or by its degree of curvature.



▪ Arc definition



▪ Chord definition.

* Degree of a Curve:

It is a central angle of the curve, ie, subtended by an arc or chord of fixed length.
Curve is designated by radius in UK, by degree in USA, Canada, India.

For 30 m arc, $30 : 2\pi R = D : 360^\circ$

$$\boxed{R = \frac{1718.9}{D}}$$

For 20 m arc or chord, $20 : 2\pi R = D : 360$

$$\boxed{R = \frac{1145.9}{D}}$$

→ Elements of Simple Curve.

1. Tangent length

$$\boxed{VT_1 = VT_2 = R\tan\left(\frac{\Delta}{2}\right)}$$

2. External distance or Apex distance.

$$VE = R\left(\sec\frac{\Delta}{2} - 1\right)$$

3. Mid ordinate or Versed sine.

$$CE = R\left(1 - \cos\frac{\Delta}{2}\right).$$

4. Length of Long chord.

$$\boxed{T_1 C_2 T_2 = 2R\sin\frac{\Delta}{2}}$$

5. Length of curve

$$\boxed{l = R\Delta} \quad (\Delta \text{ in radians})$$

$$= \frac{\pi R \Delta}{180} \quad (\Delta \text{ in degrees}).$$

$$l = \frac{\pi\Delta}{180}\left(\frac{1718.9}{D}\right) = \frac{30\Delta}{D} \quad (\text{for } 30 \text{ m arc}).$$

Similarly, $l = \frac{20\Delta}{D}$ (For 20 m arc).

→ Methods for setting Simple Curve.

(i) Linear method.

(ii) Angular method

Q. Calculate the necessary data for setting a simple curve for the following data

a) Chainage of PI = 55 + 60 (55 chains, 60 links).

b) Radius of curve = 300 m.

c) Deflection angle = 30°

d) Peg interval = 20 m. (length of chain)

Length of tangent $= R\tan\frac{\Delta}{2}$

$= 300 \tan 15 = \underline{80.38\ m}$

Length of curve $= \frac{\pi R \Delta}{180} = \frac{\pi \times 300 \times 30}{180} = 157.07\ m$

Chainage of point of intersection = 55 + 60

$= 55 \times 20 + 60 \times 0.2$

$= 1112\ m$

Chainage of T_1 = 1112 − 80.38 = 1031.62 m

Chainage of T_2 = 1031.62 + length of a curve

= 1031.62 + 157.07 = 1188.69 m

First subchord length, C_1 = 1040 − 1031.62

$= \underline{\underline{8.38\ m}}$

Last subchord length, $C_L = 1188.69 - 1180$

$= 8.69$ m.

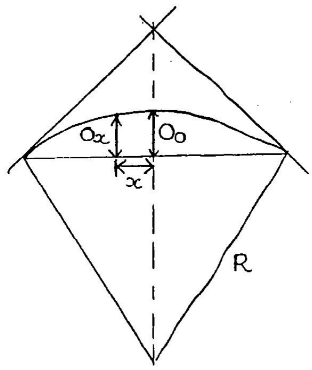
No: of normal chords $= \dfrac{1180 - 1040}{2} = \underline{\underline{7 \text{ no.s}}}$

→ Linear method

* Offset from Long chord.

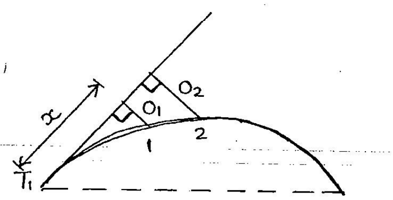
$$O_0 = R - \sqrt{R^2 - (L/2)^2}$$

$$O_x = O_0 - \frac{x^2}{2R}$$



* Perpendicular offset from tangent.

$$O_1 = \frac{x^2}{2R}$$



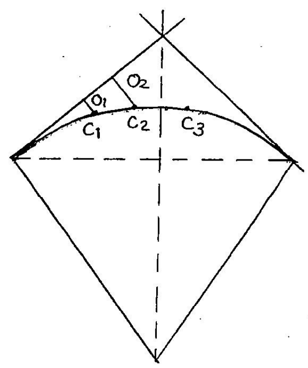
* By offsets from the chords Produced.

$$O_1 = \frac{C_1^{\,2}}{2R}.$$

$$O_2 = \frac{C_2}{2R}(C_1 + C_2).$$

$$O_3 = \frac{C_3}{2R}(C_2 + C_3).$$

$$O_n = \frac{C_n}{2R}(C_{n-1} + C_n).$$



$C_1 = 8.38$ m

$C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = 20$ m.

$C_9 = 8.69$ m

→ Angular method.

* Rankine's method of Deflection Angles (or) Method of tangential Angles

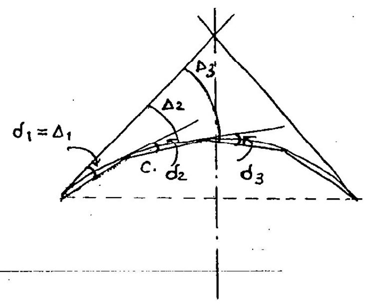
Tangential angle, $\delta = 1718.9 \times \frac{C}{R}$ minutes

$\delta_1 = 1718.9 \times \frac{8.38}{300} = \underline{\underline{48.015}}$ minutes

$\delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = \delta_8 = 1718.9 \times \frac{20}{300}$

$= \underline{\underline{114.59'}}$

$\delta_9 = 1718.9 \times \frac{8.69}{300} = \underline{\underline{49.79'}}$



Deflection angle, $\Delta_1 = \delta_1$

$\Delta_2 = \Delta_1 + \delta_2$

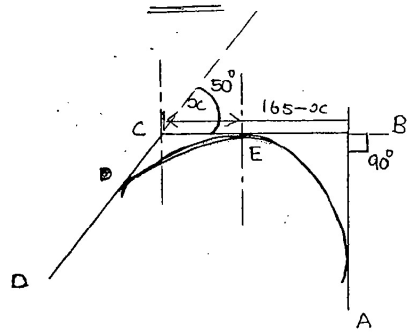
$\Delta_3 = \Delta_2 + \delta_3$

P-58.

01. Length of middle ordinate, $O_0 = R - \sqrt{R^2 - (L/2)^2}$

$= 180 - \sqrt{180^2 - 30^2}$

$= \underline{\underline{2.52 \text{ m}}}$



02. $R \tan 45 = 165 - x$

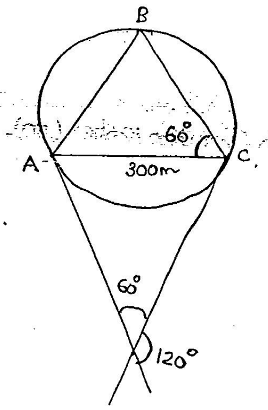
$R \tan \frac{50}{2} = x.$

$R = \underline{\underline{112.56}}$ m

03. $O_0 = 50 - \sqrt{50^2 - 30^2}$

$= \underline{\underline{10\ m}}$

07. $\Delta = 120^\circ$



$300 = R \tan\left(\frac{\Delta}{2}\right)$

$R = \frac{300}{\tan 60} = \underline{\underline{173.2\ m}}$

08. $L = 2R \sin\frac{\Delta}{2}$

$= 2 \times 600 \times \sin 30 = \underline{\underline{600\ m}}$

Mid ordinate $= R\left(1 - \cos\frac{\Delta}{2}\right).$

$= 600\,(1 - \cos 30) = \underline{\underline{80.38\ m}}$

13.
Q9. $2R \sin\frac{\Delta}{2} = R \tan\frac{\Delta}{2}$

$2 \sin\frac{\Delta}{2} = \tan\frac{\Delta}{2}.$

$\Delta = \underline{\underline{120^\circ}}$

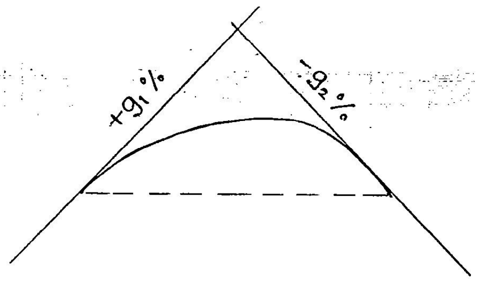
→ Vertical Curves

– It is used to connect two different grade lines of railways or highways to smooth out the changes in vertical motion. It contributes safety, comfort and appearance.

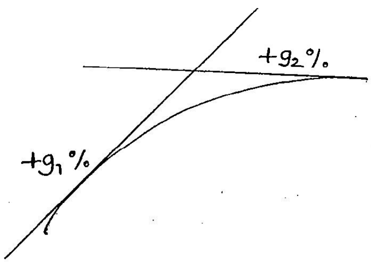
– <u>Parabola</u> or <u>circular arc</u> is the best suited vertical curve because the rate of change of grade is uniform.

* Summit Curves

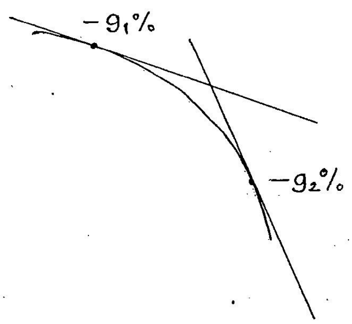
(i) $+G_1\%$ followed by $-G_2\%$



(ii) $+G_1\%$ followed by $+G_2\%$

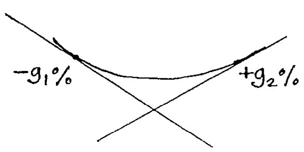


(iii) $-G_1\%$ followed by $-G_2\%$

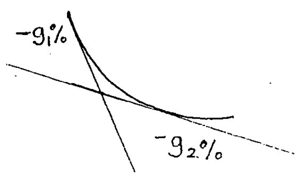


* Sag Curves

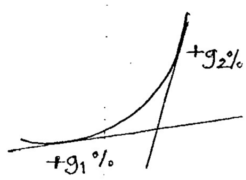
(i) $-g_1\%$, $+g_2\%$



(ii) $-g_1\%$, $-g_2\%$



(iii) $+g_1\%$, $+g_2\%$



→ Length of Vertical Curve.

$$\boxed{L = \frac{g_1 - g_2}{r}}$$

r → rate of change of grade.

$r = 0.06\%$ per 20 m for 1st class railways (150-300 kmph)

$r = 0.03\%$ per 20 m for IInd class railways (< 150 kmph)

Q. If two grades of +1.2% and −0.9% meet to form a vertical curve, rate of change of grade is 0.1% per 30m the length of vertical curve is ___ ?

$$L = \frac{g_1 - g_2}{r} = \frac{1.2 - (-0.9)}{0.1/30} = \underline{\underline{630 \text{ m}}}$$

→ Transition Curves.

A curve of varying radius and varying curvature introduced b/w the tangent length and a circular curve or b/w two branches of compound curve or a reverse curve is known as 'transition or easement curve'

* Advantages

(i) It allows a gradual transition of curvature from tangent to the circular curve or from circular curve to the tangent.

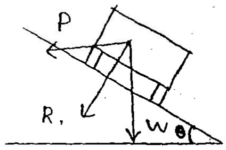
(ii) The radius of curvature increases or decreases gradually.

(iii) It is provided for the gradual change in super elevation in a convenient manner.

(iv) It eliminates danger of derailments, overturning, slide slipping of vehicles and discomfort to the passengers.

* Super elevation (or) cant

$$\tan\theta = \frac{P}{W} = \frac{v^2}{Rg} = \frac{B\,h}{h\,B}$$



$$\therefore \boxed{h = \frac{Bv^2}{Rg}} \Rightarrow \text{Roads.}$$

$$\boxed{h = \frac{Gv^2}{Rg}} \Rightarrow \text{Railways.}$$

* Centrifugal ratio $= \frac{P}{W} = \frac{1}{4}$ (for roads)

$= \frac{1}{8}$ (for railways).

* Hands off Velocity (Design Speed).

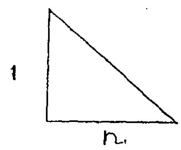
Speed of vehicle in the absence of frictional force

$$\tan(\theta + \cancel{\phi}^{\,0}) = \frac{v^2}{Rg}$$

$$\Rightarrow \boxed{V = \sqrt{\tan\theta * Rg}}$$

→ Length of transition curve

(1) By rate of superelevation.



$$\boxed{L = nh} \qquad \left\{\frac{1}{n} = \frac{h}{L}\right\}$$

$$= \frac{nBv^2}{Rg} \Rightarrow \text{roads}$$

$$= n\frac{Gv^2}{Rg} \Rightarrow \text{railways} \quad ; V \text{ in m/sec}$$

(ii) By time rate.

Let the time rate of application be x cm/sec.

Let superelevation be h cm

Let speed be v m/s.

$$L = v * t = v * \frac{h}{x}$$

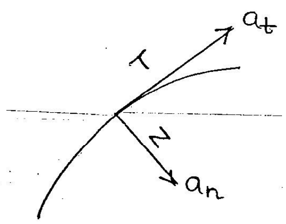
$$L = \frac{Bv^3}{Rgx} \quad ; \text{ for highways}$$

$$= \frac{Gv^3}{Rgx} \quad ; \text{ for railways.}$$

(iii) By rate of change of radial acceleration.

- It is the most efficient method

Radial acceleration $\} \; a_n = \frac{v^2}{R}$.



Rate of change of radial acceleration, $\alpha = \frac{a_n}{t} = \frac{v^2}{Rt}$.

$a_t \rightarrow$ tangential accl.
$a_n \rightarrow$ radial accl.

Length of transition curve, $L = vt$.

$$= \frac{v^3}{R\alpha}$$

$$\boxed{L = \frac{v^3}{R\alpha}} \quad ; \; v \text{ in m/s}$$

NOTE:

⊙ α = ~~0.0~~ 0.3 m/sec²/sec experienced by Mr. Shortt for comfort to the passengers.

$\therefore$ length of transition curve, $L = \frac{V^3}{14R}$; V in kmph.