Class- X Session- 2022-23

Subject- Mathematics (Basic)

Sample Question Paper - 1

with Solution

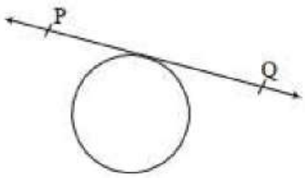
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given circle in Figure, number of tangents parallel to tangent PQ is: **[1]**



a) many b) 1

c) 0 d) 2

2. If the distance between the points (4, p) and (1,0) is 5, then the value of p is **[1]**

a) 0 b) 4 only

c) -4 only d) ± 4

3. The distance between the points (0, 5) and (–5, 0) is **[1]**

a) $5\sqrt{2}$ b) 10

c) 5 d) $2\sqrt{5}$

4. Which of the the following can be the probability of an event? **[1]**

a) $\frac{18}{23}$ b) 1.004

c) – 0.04 d) $\frac{8}{7}$

5. ________ is an algebraic tool for studying geometry. **[1]**

a) Statistics

b) None of these

c) Coordinate Geometry

d) Trigonometry

6. If a pair of linear equation is consistent, then the lines will be **[1]**

a) always intersecting

b) intersecting or coincident

c) always coincident

d) parallel

7. A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal then the ratio of its radius and the slant height of the conical part is **[1]**

a) 4 : 1

b) 1 : 4

c) 1 : 2

d) 2 : 1

8. Two dice are thrown together. The probability of getting the same number on both dice is **[1]**

a) $\frac{1}{6}$

b) $\frac{1}{12}$

c) $\frac{1}{3}$

d) $\frac{1}{2}$

9. If x = 2 is a root of the quadratic equation $3x^2 - px - 2 = 0$, then the value of p is **[1]**

a) 0

b) 3

c) 5

d) 1

10. Which of the following cannot be the probability of an event **[1]**

a) $\frac{7}{6}$

b) 0.3

c) $\frac{1}{3}$

d) 33%

11. The roots of the equation $2x^2 - 6x + 7 = 0$ are **[1]**

a) imaginary

b) real and equal

c) real, unequal and rational

d) real, unequal and irrational

12. $7 \times 11 \times 13 + 13$ is a/an: **[1]**

a) odd number but not composite

b) square number

c) prime number

d) composite number

13. If $\sin\theta = \frac{\sqrt{3}}{2}$ then $(\operatorname{cosec}\theta + \cot\theta) = ?$ **[1]**

a) $\sqrt{2}$

b) $(2 + \sqrt{3})$

c) $2\sqrt{3}$

d) $\sqrt{3}$

14. The angle of elevation and the angle of depression from an object on the ground to an object in the air are related as ________. **[1]**

a) greater than

b) equal

c) less than

d) None of these

15. The distance between the points (a cosθ + b sinθ, 0) and (0, a sinθ - b cosθ) is **[1]**

a) $\sqrt{a^2+b^2}$

b) a + b

c) $a^2 + b^2$

d) $a^2 - b^2$

16. $3 + 2\sqrt{5}$ is a/an: **[1]**

a) natural Number

b) integer

c) irrational number

d) rational number

17. The mean of first n odd natural numbers is $\frac{n^2}{81}$, then n = **[1]**

a) 9

b) 18

c) 81

d) 27

18. **Assertion (A):** If L.C.M. {p, q} = 30 and H.C.F. {p, q} = 5, then $p \cdot q = 150$ **[1]**
Reason (R): L.C.M. of (a, b) × H.C.F. of (a, b) = $a \cdot b$

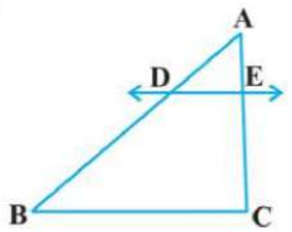
a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. **Assertion (A):** D and E are points on the sides AB and AC respectively of a $\triangle$ABC such that AD = 5.7 cm, DB = 9.5 cm, AE = 4.8 cm and EC = 8 cm then DE is not parallel to BC. **[1]**
Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.



a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. The value of c for which the pair of equations cx – y = 2 and 6x – 2y = 3 will have infinitely many solutions is **[1]**

a) – 3 b) no value

c) 3 d) –12

Section B

21. Solve the pair of linear equations by substitution method: 7x – 15y = 2 and x + 2y = 3 **[2]**

OR

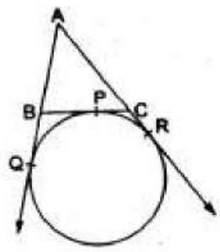
By the graphical method, find whether the pair of equation is consistent or not. If consistent, solve it.
3x + y + 4 = 0
6x – 2y + 4 = 0

22. Find the zeroes of a quadratic polynomial given as $3x^2$ - x - 4 and also verify the relationship between the zeroes and the coefficients. **[2]**

23. Find the ratio in which the line segment joining the points(-3, 10) and (6, -8) is divided by (-1, 6). **[2]**

24. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. **[2]**
    i. What is the probability that the card is the queen?
    ii. If the queen is drawn and put aside, what is the probability that the second card picked up is
        a. an ace?
        b. a queen?

25. A circle is touching the side BC of $\triangle ABC$ at P and touching AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}$(perimeter of ΔABC). **[2]**



OR

Two concentric circles are of radii 6.5 cm and 2.5 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Section C

26. The coach of the cricket team buys 3 bat and 6 balls for Rs.3900. Later, she buys another bat and 3 more balls of the same kind for Rs.1300. Representing this situation algebraically and geometrically. **[3]**

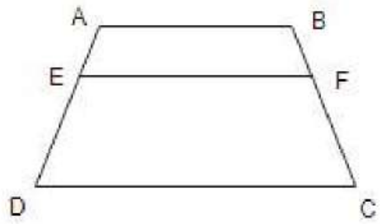
27. In $\triangle ABC$, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$. Find the value of sin A cos C + cos A sin C. **[3]**

28. Prove that $\frac{1}{\sqrt{2}}$ is irrational. **[3]**

OR

If (x–k) is the HCF of $\left(2x^2 - kx - 9\right)$ and $x^2 + x - 12$, find the value of k.

29. ABCD is a trapezium with AB || DC. E and F are two points on non-parallel sides AD and BC respectively, such that EF is parallel to AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$ **[3]**

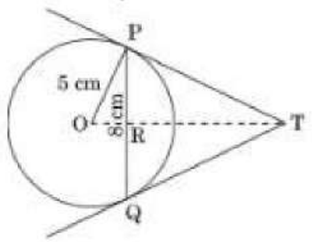


30. An observer 1.5m tall is 30m away from a chimney. The angle of elevation of the top of the chimney from his eye is 60°. Find the height of the chimney. **[3]**

31. From an external point P,two tangents,PA and PB are drawn to a circle with centre O.At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA=10 cm,find the perimeter of the triangle PCD. **[3]**

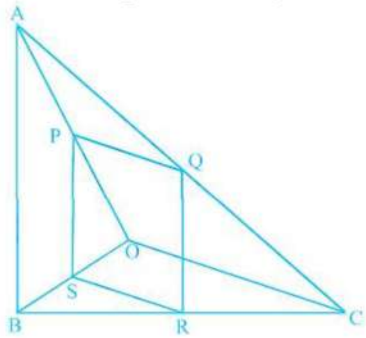
OR

In a given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.



Section D

32. In the figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR. **[5]**



33. A motorboat whose speed is 18 km/h in still water takes 1 hr 30 minutes more to go 36 km upstream than to return downstream to the same spot. Find the speed of the stream. **[5]**

OR

Solve for x: $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

34. The following table gives the distribution of the life time of 400 neon lamps: **[5]**

Lite time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life time of a lamp.

35. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}cm^2$. Find the radius of each circle. **[5]**

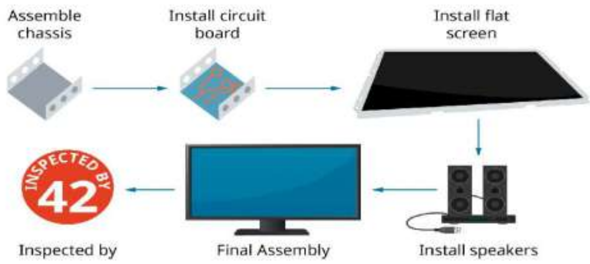
OR

A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year.



Assuming that production increases uniformly by a fixed number every year.

(i) Find the production in the 1st year.

(ii) Find the production in the 5th year.

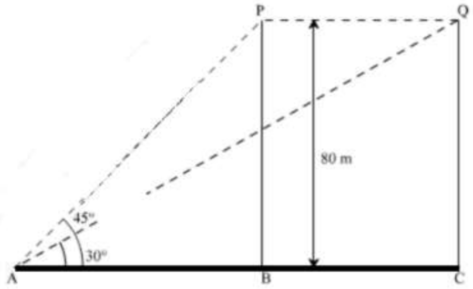
(iii) Find the total production in 7 years.

OR

Find in which year 10000 units are produced?

37. **Read the text carefully and answer the questions:** **[4]**

A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45^{o}. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.



(i) Find the distance between observer and the bottom of the tree?

(ii) Find the speed of the bird?

(iii) Find the distance between second position of bird and observer?

OR

Find the distance between initial position of bird and observer?

38. **Read the text carefully and answer the questions:** **[4]**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.

(i) If the height of a glass was 10 cm, find the apparent capacity of the glass.

(ii) Also, find its actual capacity. (Use $\pi = 3.14$)

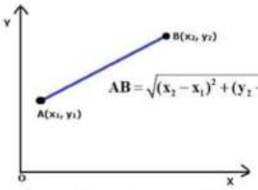
(iii) Find the capacity of the container in liter?

OR

How many glasses he serves if the container is full?

Solution

Section A

1. **(b)** 1
   Explanation: 1
2. **(d)** ±4
   Explanation: Distance between (4, p) and (1, 0) = 5
   $\Rightarrow \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} = 5$
   $\Rightarrow \sqrt{(1-4)^2 + (0-p)^2} = 5$
   $\sqrt{(-3)^2 + (-p)^2} = 5$
   Squaring, both sides
   $(-3)^2 + (-p)^2 = (5)^2 \Rightarrow 9 + p^2 = 25$
   $\Rightarrow p^2 = 25 - 9 = 16$
   $\therefore p = \pm\sqrt{16} = \pm 4$
3. **(a)** $5\sqrt{2}$
   Explanation:
   By using the formula:
   $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
   

   To calculate distance between the points (x_1, y_1) and (x_2, y_2)
   Here we have;
   $x_1 = 0, x_2 = -5$
   $y_2 = 5, y_2 = 0$
   $d^2 = [(-5) - 0]^2 + [0 - 5]^2$
   $d = \sqrt{(-5-0)^2 + (0-5)^2}$
   $d = \sqrt{25 + 25}$
   $d = \sqrt{50} = 5\sqrt{2}$
4. **(a)** $\frac{18}{23}$
   Explanation: Probability should be greater than or equal to 0 and less than or equal to 1.
5. **(c)** Coordinate Geometry
   Explanation: Coordinate Geometry is an algebraic tool for studying geometry.
6. **(b)** intersecting or coincident
   Explanation: If a consistent system has an infinite number of solutions, it is dependent. When you graph; the equations, both equations represent the same line. So for consistent line it has to be parallel or even they intersect at one point. If a system has no solution, it is

said to be inconsistent. The graphs of the lines do not intersect, so the graphs are parallel and there is no solution.

7. **(c)** 1 : 2

Explanation: $2\pi r^2 = \pi rl \Rightarrow \frac{r}{l} = \frac{1}{2}$

8. **(a)** $\frac{1}{6}$

Explanation: Here 2 dice are thrown together.

∴ Number of total outcomes = 6 × 6 = 36

Number which should come together are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) = 6 pairs

Therefore, probablity = $\frac{1}{6}$

9. **(c)** 5

Explanation: Given: $p(x) = 3x^2 - px - 2 = 0$

$\therefore\ p(2) = 3(2)^2 - p(2) - 2 = 0$

$\Rightarrow\ 12 - 2p - 2 = 0$

$\Rightarrow\ -2p = -10$

$\Rightarrow\ p = 5$

10. **(a)** $\frac{7}{6}$

Explanation: On actual division $\frac{7}{6}$ comes out to be 1.67 which is greater than 1. The probability can be less than or equal to 1.

11. **(a)** imaginary

Explanation: $D = (-6)^2 - 4 \times 2 \times 7 = (36 - 56) = -20 < 0.$

So, the roots of the given equation are imaginary.

12. **(d)** composite number

Explanation: We have 7 × 11 × 13 + 13 = 13 (77 + 1) = 13 × 78. Since the given number has 2 more factors other than 1 and itself, therefore it is a composite number.

13. **(d)** $\sqrt{3}$

Explanation: Given: $\sin\theta = \frac{\sqrt{3}}{2}$ and $\operatorname{cosec}\theta = \frac{2}{\sqrt{3}}$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow\ \cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\Rightarrow\ \cot^2\theta = \frac{4}{3} - 1$ [Given]

$\Rightarrow\ \cot\theta = \frac{1}{\sqrt{3}}$

$\therefore\ \operatorname{cosec}\theta + \cot\theta = \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= \frac{3}{\sqrt{3}}$

$= \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}}$

$= \sqrt{3}$

14. **(b)** equal

Explanation: The angle of elevation and the angle of depression from an object on the ground to an object in the air are related as equal if the height of objects is the same.

15. **(a)** $\sqrt{a^2 + b^2}$

Explanation: Distance between $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$

$$= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$= \sqrt{\sqrt{(0 - (a\cos\theta + b\sin\theta))^2 + \{(a\sin\theta - b\cos\theta - 0)\}^2}}$$

$$= \sqrt{\{0 - (a\cos\theta + b\sin\theta)\}^2 + \{(a\sin\theta - b\cos\theta - 0)\}^2}$$

$$= \sqrt{\begin{pmatrix} a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + \\ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta \end{pmatrix}}$$

$$= \sqrt{a^2 \times 1 + b^2 \times 1} = \sqrt{a^2 + b^2}$$

$$\left\{ \because \sin^2\theta + \cos^2\theta = 1 \right\}$$

16. **(c)** irrational number

Explanation: Here, 3 is rational and $2\sqrt{5}$ is irrational.

We know that the sum of a rational and an irrational is an irrational number, therefore, $3 + 2\sqrt{5}$ is irrational.

17. **(c)** 81

Explanation: Mean of first n odd natural numbers = $\frac{n^2}{81}$ ...(i)

Also, First odd natural number = 1
n-th odd natural number = (2n - 1)
Sum of first 'n' odd natural numbers

$= (\frac{n}{2}) \times (1 + 2n - 1) = (n \times n) = n^2$

Therefore,

Mean of first n odd natural numbers = $\frac{n^2}{n}$ = n ...(ii)

From (i) and (ii) we get

$\frac{n^2}{81} = n \Rightarrow n = 81$

18. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Both A and R are true and R is the correct explanation of A.
19. **(d)** A is false but R is true.
Explanation: If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side. This is the Converse of the Basic Proportionality theorem.
So, the Reason is correct.
Now, $\frac{AD}{DB} = \frac{5.7}{9.5} = \frac{57}{95} = \frac{3}{5}$
and $\frac{AE}{EC} = \frac{4.8}{8} = \frac{48}{80} = \frac{3}{5}$
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$
By Converse of Basic Proportionality theorem, DE||BC
So, the Assertion is not correct.
20. **(b)** no value
Explanation: Condition for infinitely many solutions
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ...(i)
The given lines are cx - y - 2 = 0 and 6x - 2y - 3 = 0;
Comparing with standard form we get,
$a_1 = c$, $b_1 = -1$, $c_1 = -2$
and $a_2 = 6$, $b_2 = -2$, $c_2 = -3$
From Eq. (i), we have
c/6 = -1/-2
Also c/6 = -2/-3
Solving, we get, c = 3 and c = 4.
Since, c has different values and so it's not possible.
Hence, for no value of c the pair of equations will have infinitely many solutions.

Section B

21. **Step 1:** By substitution method, we pick either of the equations and write one variable in terms of the other.
7x – 15y = 2 ...(1)
and x + 2y = 3 ...(2)
Let us consider the Equation (2):
x + 2y = 3 and write it as x = 3 – 2y ...(3)
Step 2: Now substitute the value of x in Equation (1)
We get 7(3 – 2y) – 15y = 2
i.e., 21 – 14y – 15y = 2
i.e., – 29y = –19
Therefore $y = \frac{19}{29}$
Step 3: Substituting this value of y in Equation (3), we get

$x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$

Therefore, the solution is $x = \frac{49}{29}, y = \frac{19}{29}$

OR

Given pair of equations is

3x + y + 4 = 0 …(i)

and 6x -2y + 4 = 0…(ii)

comparing with ax + by + c = 0

Here, $a_1 = 3$, $b_1 = 1$, $c_1 = 4$;

And $a_2 = 6$, $b_2 = -2$, $c_2 = 4$;

$a_1 / a_2 = 1/2$

$b_1 / b_2 = -1/2$

$c_1 / c_2 = 1$

since $a_1/a_2 \neq b_1/b_2$

so system of equations is consistent with a unique solution.

We have, $3x + y + 4 = 0$

$y = -4 - 3x$

When x = 0, then y = - 4

When x = - 1, then y = - 1

When x = - 2, then y = 2

x	0	-1	-2
y	-4	-1	2
Points	B	C	A

and $6x - 2y + 4 = 0$

$6x + 4 = 2y$

$y = 3x + 2$

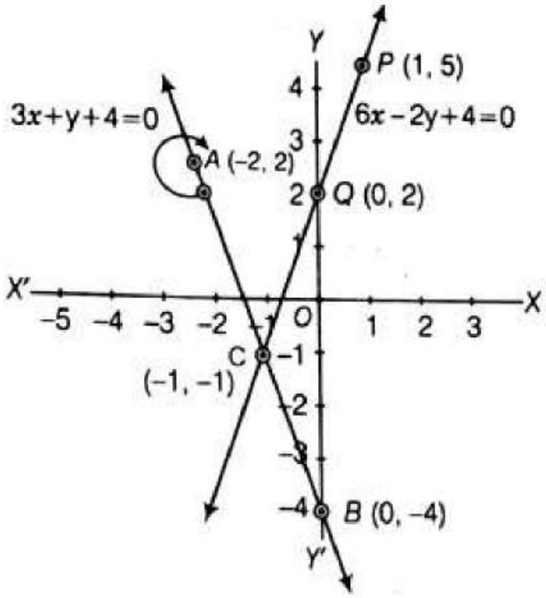
When x = 0, then y = 2

When x = - 1,then y = - 1

When x = 1,then y = 5

x	-1	0	1
y	-1	2	5
Points	C	Q	P

Plotting the points B(0, - 4) and A(- 2,2),we get the straight line AB. Plotting the points Q(0,2) and P(1,5) we get the straight line PQ. The lines AB and PQ intersect at C (-1, -1).



So, (-1, -1) is the required solution.

22. The quadratic equation is given as: $3x^2 - x - 4$
(Now we will factorize 1 in such a way that the product of factors is equal to -12 and the sum is equal to 1)
$= 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$
The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$,
when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1

Sum of zeroes $= \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-\text{ coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= \frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence verified.

23. Let (-1, 6) divides line segment joining the points (-3, 10) and(6, -8) in k:1.
Using Section formula, we get
$-1 = \frac{(-3) \times 1 + 6 \times k}{k+1} 4 \Rightarrow -k - 1 = (-3 + 6k)$

$\Rightarrow -7k = -2 \Rightarrow k = \frac{2}{7}$

Therefore, the ratio is $\frac{2}{7} : 1$ which is equivalent to 2:7.

Therefore, (-1, 6) divides line segment joining the points (-3, 10) and (6, -8) in 2:7.

24. Total number of favourable outcomes = 5

$\textit{Proaibility of the event} = \frac{\textit{Number of favourble outcomes}}{\textit{Total number of possible outcomes}}$

i. (i) There is only one queen.
$\therefore$ Favourable outcome = 1

Hence, P (the queen) = $\frac{1}{5}$

ii. In this situation, total number of favourable outcomes = 4

a. Favourable outcome = 1

Hence, P (an ace) = $\frac{1}{4}$

b. There is no card as queen.

$\therefore$ Favourable outcome = 0

Hence, P (the queen) = $\frac{0}{4}$ = 0

25. We know that the lengths of tangents drawn from an external point to a circle are equal.

AQ = AR, ...(i) [tangents from A]

BP = BQ ...(ii) [tangents from B]

CP = CR ... (iii) [tangents from C]

Perimeter of $\triangle ABC$

= AB + BC + AC

= AB + BP + CP + AC

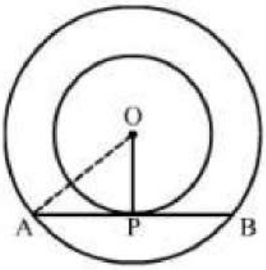
= AB + BQ + CR + AC [using (ii) and (iii)]

= AQ + AR

= 2AQ [using (i)

$\therefore \quad AQ = \frac{1}{2}(\text{ perimeter of } \triangle ABC)$

OR



We know that the radius and tangent are perpendicular at their point of contact

In right Triangle AOP

$AO^2 = OP^2 + PA^2$

$\Rightarrow (6.5)^2 = (2.5)^2 + PA^2$

$\Rightarrow PA^2 = 36$

$\Rightarrow$ PA = 6cm

Since, the perpendicular drawn from the center bisects the chord.

PA = PB = 6cm

Now, AB = AP = PB = 6 + 6 = 12cm

Hence, the length of the chord of the larger circle is 12 cm.

Section C

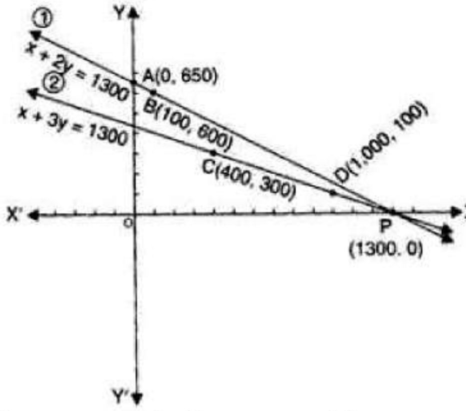
26. Let us denote the cost of 1 bat by Rs. x and one ball by Rs. y.

Then the algebraic representation is given by the following Equations:

3x + 6y = 3900 and x + 3y = 1300

$\Rightarrow$ x + 2y = 1300 ...(1)

And x + 3y = 1300 ...(2)



To represents these equations graphically, We find two solutions for each equation, These solution are given below:

For equation (1)x + 2y = 1300

$\Rightarrow$ 2y = 1300 - x $\Rightarrow y = \dfrac{1300-x}{2}$

Table 1 of solutions

x	0	100
y	650	600

For equation (2) x + 3y = 1300

$\Rightarrow$ 3y = 1300 - x $\Rightarrow y = \dfrac{1300-x}{3}$

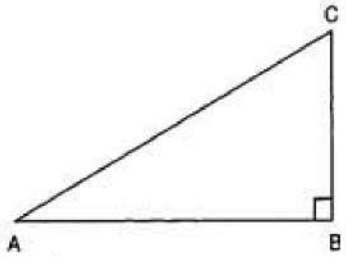
Table 2 of solutions

x	400	1000
y	300	100

We plots the points A(0,650) and B (100,600) corresponding to the solutions in table 1 on the graph paper to get the line AB representing the equation (1) and the points C(400, 300) and D(1000, 100) corresponding to the solution in table 2 on the same graph paper to get the line CD representing the equation (2), as shown in figure.

We observe in the figure that the two lines representing the two equations are intersecting at the point P(13000)

27.



we have,

$\tan A = \dfrac{1}{\sqrt{3}} = tan 30^\circ$

$\therefore A = 30^\circ$

In $\triangle$ABC, we have

$\boldsymbol{\angle A + \angle B + \angle C = 180^\circ}$

$\Rightarrow 30^\circ + 90^\circ + \angle C = 180^\circ$

$\Rightarrow 120^\circ + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$

So,

sinA . cosC + cos A . sin C

$= \sin 30^\circ . \cos 60^\circ + \cos 30^\circ . \sin 60^\circ$

$= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = 1$

28. We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers *a* and *b* ($b \neq 0$)
Such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$..........(1)

R.H.S of (1) is rational but we know that is $\sqrt{2}$ irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ can not be rational.

Hence, it is irrational.

OR

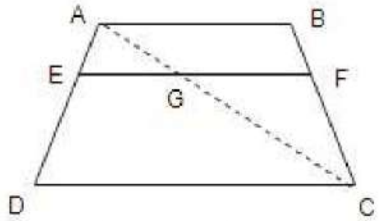
Let p(x) $= 2x^2 - kx - 9$ and $q(x) = x^2 + x - 12$
Since (x-k) is the HCF of both p(x) and q(x),
therefore (x-k) divides both p(x) and q(x) exactly.
$\Rightarrow x - k$ is a factor of both $p(x)$ and $q(x)$
$\therefore$ By factor theorem p(k) = 0
and also q(k) = 0
Now p(k) = 0

$\Rightarrow 2k^2 - kk - 9 = 0 \Rightarrow k^2 - 9 = 0$ Using Identity $a^2 - b^2 = (a + b)(a - b)$
$\Rightarrow (k - 3)(k + 3) = 0 \Rightarrow k = 3, -3$ ....(i)
Again q(k) = 0
$\Rightarrow k^2 + k - 12 = 0 \Rightarrow k^2 + 4k - 3k - 12 = 0$
$\Rightarrow k(k + 4) - 3(k + 4) \Rightarrow (k + 4)(k - 3) = 0$
$\Rightarrow k = -4, 3$ ....(ii)
Hence from (i) and (ii), k = 3

29. Given, In trapezium ABCD,
$AB || DC$ and $EF || DC$

To prove $\frac{AE}{ED} = \frac{BF}{FC}$

Construction: Join AC to intersect EF at G.



Proof Since, $AB||DC$ and $EF||DC$

$EF||AB$ [since, lines parallel to the same line are also parallel to each other]...... (i)

In $\triangle ADC$, $EG||DC$ [$\because$ $EF||DC$]

By using basic proportionality theorem,

$$\frac{AE}{ED} = \frac{AG}{GC} \text{ ....(ii)}$$

In $\triangle ABC$, $GF||AB$ [$\because$ $EF||AB$ from (i)]

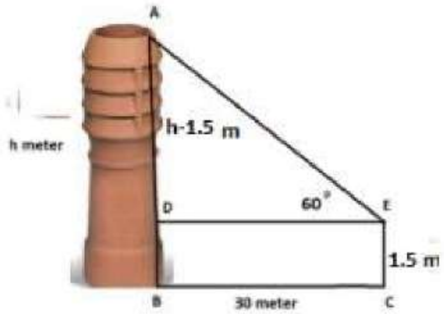
By using basic proportionality theorem ,

$$\frac{CG}{AG} = \frac{CF}{BF} \text{ or } \frac{AG}{GC} = \frac{BF}{CF}$$ [On taking reciprocal of the terms]............ (iii)

From Equations (ii) and (iii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence Proved.

30. 

Let AB be the chimney and CE is the observer.

Let h be the height of the chimney.

Height of the observer CE = 1.5 m

So AD= AB - BD = AB - CE = h - 1.5

and DE = BC = 30 m

Now in right ΔADE we have,

$$\frac{\text{AD}}{\text{DE}} = \tan 60^\circ$$

$$\frac{\text{h} - 1.5}{30} = \sqrt{3}$$

$$\text{h - 1.5} = 30\sqrt{3}$$

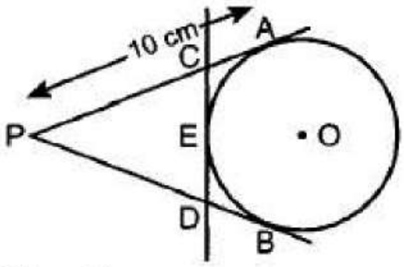
$$\text{h} = 1.5 + 30\sqrt{3}$$

$= 1.5 + 30 \times 1.732$

$= 1.5 + 51.96 = 53.46$ meter

Therefore the height of the chimney = 53.46 meter.

31. Given,



PA = 10 cm.

PA = PB [If P is external point] ...(i)[From an external point tangents drawn to a circle are equal in length]

If C is external point, then CA = CE

If D is external point, then

DB = DE ...(ii)

Perimeter of triangle △PCD

= PC + CD + PD

= PC + CE + ED + PD

= PC + CA + DB + PD

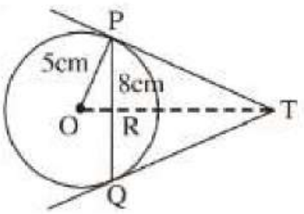
= PA + PB

= PA + PA

=2 PA

= 2 × 10 = 20cm [From (i)]

OR



Let TR be x cm and TP be y cm

OT is perpendicular bisector of PQ

So PR = 4 cm ($PR = \frac{PQ}{2} = \frac{8}{2}$)

In △OPR, $OP^2 = PR^2 + OR^2$

$5^2 = 4^2 + OR^2$

$OR = \sqrt{25 - 16}$

$\therefore$ OR = 3 cm

In △PRT, $PR^2 + RT^2 = PT^2$

$y^2 = x^2 + 4^2$ .....(1)

In △OPT, $OP^2 + PT^2 = OT^2$

$(x + 3)^2 = 5^2 + y^2$ (OT = OR + RT = 3 + x)

$\therefore$ $(x + 3)^2 = 5^2 + x^2 + 16$ [using (1)]

Solving, we get $x = \frac{16}{3}$ cm

From (1), $y^2 = \frac{256}{9} + 16 = \frac{400}{9}$

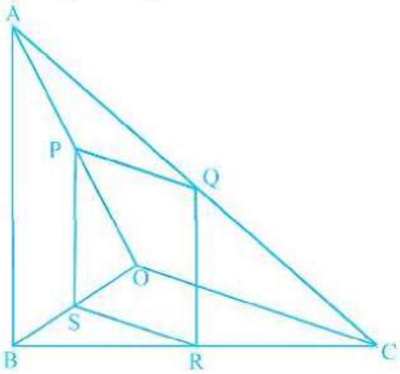
So, $y = \frac{20}{3}$ cm = 6.667 cm

Section D

32. It is given that PQRS is a parallelogram,

So, PQ || SR and PS || QR.

Also, AB || PS.



To prove OC || SR

In $\triangle$OPS and OAB,

PS || AB

$\angle POS = \angle AOB$ [common angle]

$\angle OSP = \angle OBA$ [corresponding angles]

$\therefore$ OPS $\sim$ $\triangle$OAB [by AAA similarity criteria]

Then,

$\frac{PS}{AB} = \frac{OS}{OB}$ …(i) [by basic proportionality theorem]

In $\triangle$CQR and ΔCAB,

QR || PS || AB

$\angle QCR = \angle ACB$ [common angle]

$\angle CRQ = \angle CBA$ [corresponding angles]

$\therefore$ $\triangle CQR \sim \triangle CAB$

Then, by basic proportionality theorem

$= \frac{QR}{AB} = \frac{CR}{CB}$

$\Rightarrow \frac{PC}{AB} = \frac{CR}{CB}$ .....(ii)

[PS $\cong$ QR Since, PQRS is a parallelogram,]

From Equation (i) and (ii),

$\frac{OS}{OB} = \frac{CR}{CB}$

or $\frac{OB}{OS} = \frac{CB}{CR}$

On subtracting from both sides, we get,

$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$

$$\Rightarrow \frac{OB - OS}{OS} = \frac{(CB - CR)}{CR}$$

$$\Rightarrow \frac{BS}{OS} = \frac{BR}{CR}$$

By converse of basic proportionality theorem, SR || OC

Hence proved.

33. Given:

Speed of the boat in still water= 18km/hr

Let speed of stream be x km/hr

Speed of boat in upstream = (18 - x)km/hr

Speed of boat in downstream = (18 + x)km/hr

We know that,

$$Speed = \frac{\text{Distance}}{\text{Time taken}}$$

Time taken to travel 36 km in upstream $= \frac{36}{18 - x} hr$

Time taken to travel 36 km in upstream $= \frac{36}{18 + x} hr$

According to question,

Time taken for upstream - time taken for downstream = 1hr 30 minutes

$$\Rightarrow \frac{36}{18 - x} - \frac{36}{18 + x} = 1\frac{30}{60}$$

$$\Rightarrow \frac{36(18 + x) - 36(18 - x)}{(18 - x)(18 + x)} = \frac{3}{2}$$

$\Rightarrow$ $2[36(18 + x - 18 + x] = 3[(18)^2 - (x)^2]$ ...[$\because$ $(a - b)(a + b) = (a^2 - b^2)$]

$\Rightarrow$ $2[36(2x)] = 3[324 - x^2]$

$\Rightarrow$ $144x = 972 - 3x^2$

$\Rightarrow$ $3x^2 + 144x - 972 = 0$

$\Rightarrow$ $x^2 + 48x - 324 = 0$

Comparing eq. with $ax^2 + bx + c = 0$

Here a = 1, b = 48 and c = -324

We know that,

$D = b^2 - 4ac$

$= (48)^2 - 4(1)(-324)$

$= 2304 + 1296$

$= 3600$

So, the roots to the equation are

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-48 \pm \sqrt{3600}}{2 \times 1}$$

$$x = \frac{-48 \pm \sqrt{36 \times 100}}{2}$$

$$x = \frac{-48 \pm \sqrt{6 \times 6 \times 10 \times 10}}{2}$$

$$x = \frac{-48 \pm 60}{2}$$

x = -24 ± 30

⇒ x = -24 + 30 or x = -24 - 30

⇒ x = 6 or x = -54

Since, x is the speed, so it can't be negative

So, x = 6 km /hr

OR

We have given,

$$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$$

Let $\frac{2x}{(x-5)}$ be y

$\therefore \quad y^2 + 5y - 24 = 0$

Now factorise,

$y^2 + 8y - 3y - 24 = 0$

$y(y + 8) - 3(y + 8) = 0$

$(y + 8)(y - 3) = 0$

$y = 3, \ -8$

Putting y=3

$$\frac{2x}{x-5} = 3$$

2x = 3x - 15

x = 15

Putting y = -8

$$\frac{2x}{x-5} = -8$$

2x = -8x + 40

10x = 40

x = 4

Hence, x is 15 , 4

34.

Life time	Number of lamps (f_i)	Cumulative frequency
1500-2000	14	14
2000-2500	56	14 + 56 = 70
2500-300	60	70 + 60 = 130
3000-3500	86	130 + 86 = 216
3500-4000	74	216 + 74 = 290

Life time	Number of lamps (f_i)	Cumulative frequency
4000-4500	62	290 + 62 = 352
4500-5000	48	352 + 48 = 400
	400	

N = 400

Now we may observe that cumulative frequency just greater than $\frac{n}{2}$ (ie., $\frac{400}{2}$ = 200) is 216

Median class = 3000 - 3500

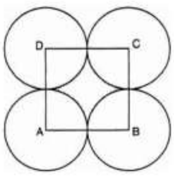
$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Here,
l = Lower limit of median class
F = Cumulative frequency of class prior to median class.
f = Frequency of median class.
h = Class size.
Lower limit (l) of median class = 3000
Frequency (f) of median class 86
Cumulative frequency (cf) of class preceding median class = 130
Class size (h) = 500

$$\text{Median} = 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

= 3406.98

35. 

Let r cm be the radius of each circle.

Area of square - Area of 4 sectors = $\frac{24}{7}\text{cm}^2$

$$(\text{side})^2 - 4\left[\frac{\theta}{360}\pi r^2\right] = \frac{24}{7}\text{cm}^2$$

$$\text{or, } (2r)^2 - 4\left(\frac{90^\circ}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$$

or, $(2r)^2 - 4\left(\frac{1}{4°} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$

or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

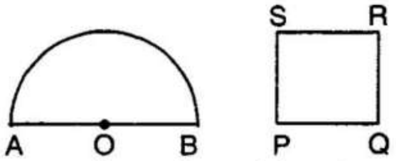
or, $6r^2 = 24$

or, $r^2 = 4$

or, $r = \pm 2$

or, Radius of each circle is 2 cm (r cannot be negative)

OR



Let radius of semicircular region be r units.

Perimeter = $2r + \pi r$

Let side of square be x units

Perimeter = 4x units.

A.T.Q, $4x = 2r + \pi r \Rightarrow x = \frac{2r + \pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$$\Rightarrow \left(\frac{2r + \pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$$

$$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$$

$$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$$

$$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$$

$$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$$

$$\Rightarrow r^2(\pi - 2)^2 = 64$$

$$\Rightarrow r = \sqrt{\frac{64}{(\pi - 2)^2}}$$

$$\Rightarrow r = \frac{8}{\pi - 2} = \frac{8}{\frac{22}{7} - 2} = 7 \text{ cm}$$

Perimeter of semicircle $= 2 \times 7 + \frac{22}{7} \times 7 = 36$ cm

Perimeter of square = 36 cm

Side of square $= \frac{36}{4} = 9$ cm

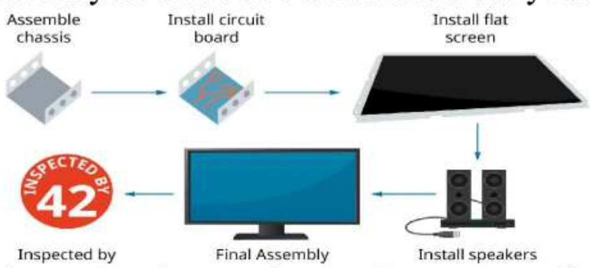
Area of square $= 9 \times 9 = 81 \text{ cm}^2$

Area of semicircle $= \frac{\pi r^2}{2} = \frac{22}{2 \times 7} \times 7 \times 7 = 77 \text{ cm}^2$

Section E

36. **Read the text carefully and answer the questions:**

Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year.



Assuming that production increases uniformly by a fixed number every year.

(i) Let production in a 1st year be 'a' unit and increase in production (every year) be 'd' units.

Increase in production is constant, therefore unit produced every year forms an AP.

Now, $a_3 = 6000$

$a + 2d = 6000 \Rightarrow a = 6000 - 2d$ ...(i)

and $a_7 = 7000 \Rightarrow a + 6d = 7000$

$\Rightarrow (6000 - 2d) + 6d = 7000 \Rightarrow 4d = 1000$ [using eq. (i)]

$\Rightarrow d = 250$

When d = 250, eq. (i) becomes

$a = 6000 - 2(250) = 5500$

$\therefore$ Production in 1st year = 5500

(ii) Production in fifth year

$a_5 = a + 4d = 5500 + 4(250) = 6500$

(iii) Total production in 7 years $= \frac{7}{2}(5500 + 7000) = 43750$

OR

$a_n = 1000$ units

$a_n = 1000$

$\Rightarrow 10000 = a + (n - 1)d$

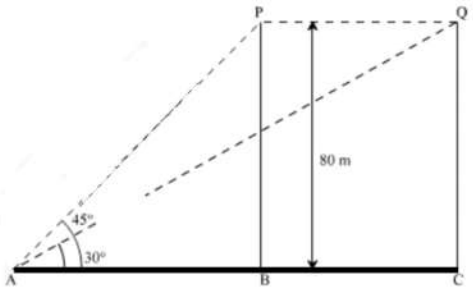
$\Rightarrow 1000 = 5500 + 250n - 250$

$\Rightarrow 10000 - 5500 + 250 = 250n$

$\Rightarrow$ $4750 = 250n$

$\Rightarrow$ $n = \frac{4750}{250} = 19$

37. **Read the text carefully and answer the questions:**
A bird is sitting on the top of a tree, which is 80m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.



(i) Given height of tree = 80m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree
In △ABP

$\tan 45^{o} = \frac{BP}{AB}$

$\Rightarrow$ $1 = \frac{80}{AB}$

$\Rightarrow$ AB = 80 m

(ii) The speed of the bird
In △AQC

$\tan 30^{o} = \frac{QC}{AC}$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{80}{AC}$

$\Rightarrow$ $AC = 80\sqrt{3}$ m

AC - AB = BC

$\Rightarrow$ $BC = 80\sqrt{3} - 80 = 80(\sqrt{3} - 1)$m

Speed of bird = $\frac{\text{Distance}}{\text{Time}}$

$\Rightarrow$ $\frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3} - 1)$

$\Rightarrow$ Speed of the bird = 29.28 m/sec

(iii)The distance between second position of bird and observer.
In △AQC

$\sin 30^{o} = \frac{QC}{AQ}$

$$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$$

$\Rightarrow$ AQ = 160 m

OR

The distance between initial position of bird and observer.

In $\triangle$ABP

$$\sin 45^{o} = \frac{BP}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{80}{AP}$$

$$\Rightarrow AP = 80\sqrt{2}m$$

38. **Read the text carefully and answer the questions:**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.

(i) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The apparent capacity of the glass = Volume of cylinder

$= \pi r^2 h$

$$= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10$$

$$= 3.14 \times \frac{25}{4} \times 10 = 196.25 \text{ cm}^3$$

(ii) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The actual capacity of glass = Apparent capacity of glass - Volume of hemispherical part of the glass

The volume of hemispherical part $= \frac{2}{3}\pi r^2 h = \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 = 32.71 \text{ cm}^3$

Actual capacity of glass = 196.25 - 32.71 = 163.54 cm^3

(iii) We have, inner diameter of the glass, d = 5 cm, height of the glass = 10 cm

Volume of container = V = $\pi r^2 h$

$\Rightarrow$ V = 3.14 × 20 × 20 × 50 = 62800 cm^3

$\Rightarrow$ V = 62.8 litre

OR

We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

$$\text{Number of glasses} = \frac{\text{Volume of container}}{\text{Actual volume of one glass}}$$

$$\Rightarrow \text{Number of glasses} = \frac{20 \times 3.14 \times 20 \times 50}{3.14 \times \frac{2\pi}{4} \times 10 - \frac{2}{3} \times 3.14 \times \frac{125}{8}}$$

$$\Rightarrow \text{Number of glasses} = \frac{20000}{\frac{250}{4} - \frac{125}{12}} = \frac{20000 \times 12}{750 - 125} = \frac{240000}{625} = 384$$

$\Rightarrow$ Number of glasses = 384