09

Co-ordinate Geometry

9.01 Introduction

In earlier classes we have studied geometry which is called Euclidean geometry. Now we will study the analytic geometry. Where position of the point is expressed by specific numbers which are called co-ordinates, lines and curves so formed are represented by algebraic equations. **Due to the use of co-ordinates in Analytic geometry, this is called as co-ordinate geometry.**

9.02 Cartesian co–ordinates

Let $X'OX$ and $Y'OY$ be two perpendicular lines in any plane, which intersects each other at point O. These lines are called coordinate axes and O is called origin. $X'OX$ and $Y'OY$ are perpendicular to each other. Thus $X'OX$ and $Y'OY$ are called rectangular axes.

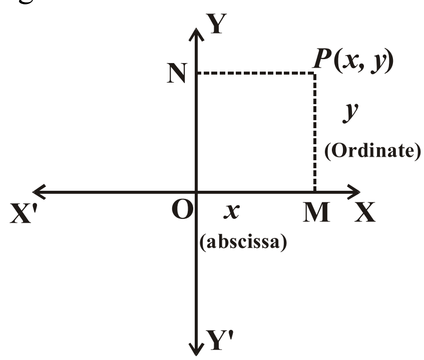


Fig. 9.01

Now to find co-ordinates of P, draw perpendiculars PM and PN from P on x and y axes respectively. The length of the segment OM ($OM=x$) is called the x-coordinate or abscissa of point P. Similarly, the length, of line segment ON is called the y-coordinate or ordinate of point. These coordinates are written in ordered pair (x,y) *i.e.,* while writing the co-ordinates of a point, write co-ordinate first and then y co-ordinate in parentheses.

9.03 Sign of Co–ordinates in quadrants

In figure 9.02, two axes $X'OX$ and $Y'OY$ divide the plane into four equal parts which are called quadrants. XOY, YOX', $X'OY'$ and $Y'OX$ are called respectively I,II,III, and IV quadrants. We always take OX and OY as + ve and OX' and OY' as - ve directions.

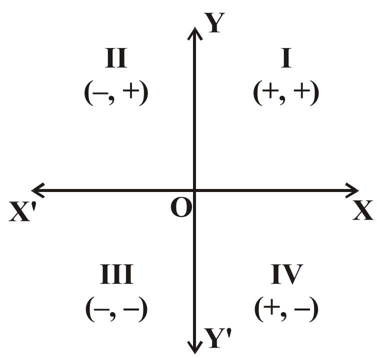


Fig.9.02

If (x,y) be coordinates of any point P in plane, then in

I Quadrant $x > 0, y > 0$; Coordinates(+, +)

II Quadrant $x < 0, y > 0$; Coordinates (–, +)

III Quadrant $x < 0, y < 0$; Coordinates (–, –)

IV Quadrant $x > 0, y < 0$; Coordinates (+, –)

Note:(i) If coordinate of any point P is (x,y) then we can write as P (x,y).

(ii) The abscissa of any point is at a perpendicular distance from y - axis.

(iii) The ordinate of any point is at a perpendicular distance form x-axis.

(iv) The abscissa of any point is positive at R.H.S of y axis and negative at LHS of y axis.

(v) The ordinate of any point is positive above the x-axis and negative below the x-axis.

(vi) If $y = 0$, then point lies on x-axis.

(vii) If $x = 0$, then point lies on y-axis.

(viii) If $x = 0, y = 0$ then point is origin.

9.04 Distance between two points

Let XOX' and YOY' are co-ordinate axes and two points in the plane are $P(x_1, y_1)$ and $Q(x_2, y_2)$ we have to find distance between these two points. From point P and Q draw perpendicular PM and QN on x-axis, respectively and draw perpendicular PR form P to QN.

$\therefore$ OM = Abscissa of $P = x_1$

Similarly $ON = x_2$, $PM = y_1$

and $QN = y_2$

According to figure $PR = MN = ON - OM = x_2 - x_1$

and $QR = QN - RN = QN - PM = y_2 - y_1$

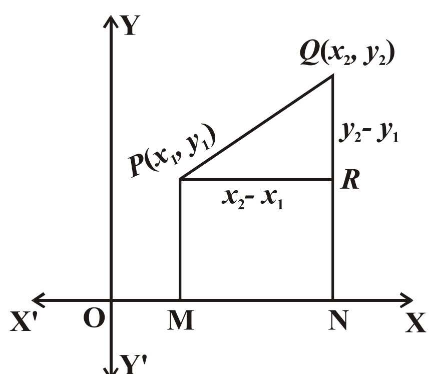


Fig. 9.03

Thus, by Bodhayan formula in right angled ΔPRQ

$$PQ^2 = PR^2 + QR^2$$

or
$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$\therefore$
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(\text{difference of } x - \text{coordinates})^2 + (\text{difference of } y - \text{coordinate})^2}$$

which is a formula to find distance between two points.

Special case : Distance of point P(x,y) from origin O (0, 0)

$$OP = \sqrt{x^2 + y^2}$$

Illustrative Examples

Example 1. Plot the points (2, 4), (–2, 3), (–4, –3) and (5, –2) in the rectangular co-ordinate system.

Solution :

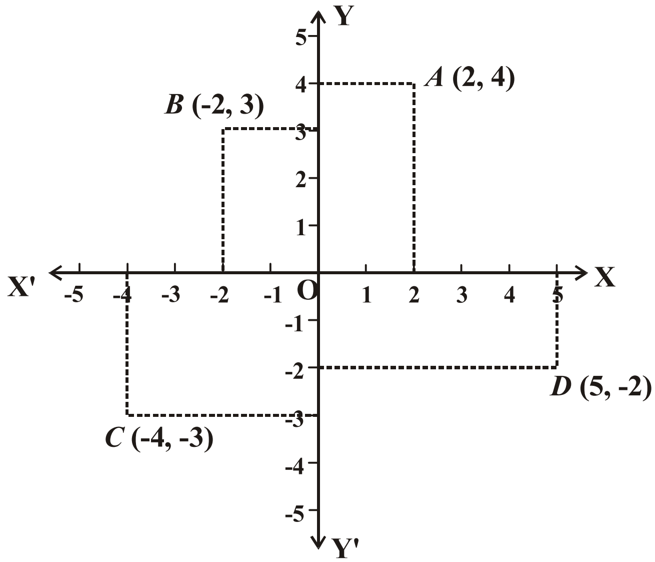


Fig. 9.04

Example 2. If $2a$ is a side of an equilateral triangle, then find the co-ordinates of its vertices.

Solution : According to figure 9.05

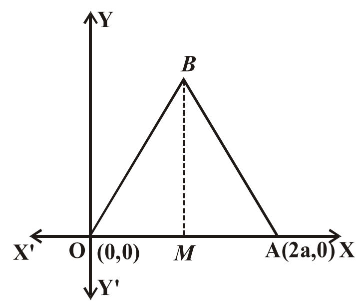


Fig. 9.05

$\because$ OAB is an equilateral triangle of side $2a$

$\therefore$ $OA = AB = OB = 2a$

Draw perpendicular BM from point B to OA

$\therefore$ $OM = MA = a$

In rigtht angle ΔOMB,

$$OB^2 = OM^2 + MB^2$$

or $$(2a)^2 = (a)^2 + MB^2$$

or $$MB^2 = 3a^2$$

$\therefore$ $$MB = \sqrt{3}a$$

Thus, co-ordinates of vertices of an equilateral triangle are O (0, 0), $A(2a,\ 0)$ and $B(a, \sqrt{3}a)$ since $OM = a$ and $MB = \sqrt{3}a$.

Example 3. Find the distance between the points (2,3) and (5,6).

Solution : Let points (2, 3) and (5, 6) are P and Q respectively so distance between them

$$PQ = \sqrt{(5-2)^2 + (6-3)^2}$$

$$= \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2}$$

Example 4. If distance between points $(x, 3)$ and $(5, 7)$ is 5, then find the value of x.

Solution : Let $P(x, 3)$ and $Q(5, 7)$ are given points then according to question.

$$PQ = 5$$

$$\sqrt{(x-5)^2+(3-7)^2} = 5$$

Squaring both sides,

$$(x-5)^2+(-4)^2 = 25$$

or $\quad x^2-10x+25+16 = 25$

or $\quad x^2-10x+16 = 0$

or $\quad (x-2)(x-8) = 0$

$\therefore \quad x = 2, 8$

Example 5. Prove that points $(-2, -1)$, $(-1, 1)$, $(5, -2)$ and $(4, -4)$ taken in order, are vertices of a rectangle.

Solution : Let given points are $P(-2, -1)$, $Q(-1, 1)$, $R(5, -2)$ and $S(4, -4)$

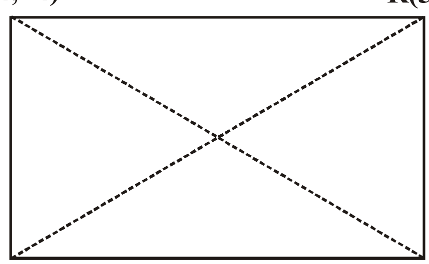


Fig. 9.06

$$PQ = \sqrt{[-2-(-1)]^2+[-1-1]^2} = \sqrt{(-1)^2+(-2)^2} = \sqrt{5}$$

$$QR = \sqrt{[5-(-1)]^2+[-2-1]^2} = \sqrt{(6)^2+(-3)^2} = \sqrt{45}$$

$$RS = \sqrt{[4-5]^2+[-4-(-2)]^2} = \sqrt{(-1)^2+(-2)^2} = \sqrt{5}$$

$$SP = \sqrt{[4-(-2)]^2+[-4-(-1)]^2} = \sqrt{(6)^2+(-3)^2} = \sqrt{45}$$

$\therefore \quad PQ = RS$ and $QR = SP$

So opposite sides are equal

Again, diagonal $PR = \sqrt{[5-(-2)]^2+[-2-(-1)]^2} = \sqrt{(7)^2+(-1)^2} = \sqrt{50}$

$$QS = \sqrt{[4-(-1)]^2+[-4-1)]^2} = \sqrt{(5)^2+(-5)^2} = \sqrt{50}$$

Thus, diagonals are equal. So given points are vertices of rectangle $PQRS$.

Example 6. If points (x, y) lies at equal distance form points $(a+b, b-a)$ and $(a-b, a+b)$ then prove that $bx = ay$.

Solution : Let $P(x, y)$, $Q(a+b, b-a)$ and $R(a-b, a+b)$ are given points, so according to question

$$PQ = PR$$

or $\quad PQ^2 = PR^2$

or $$[x-(a+b)]^2+[y-(b-a)]^2=[x-(a-b)]^2+[y-(a+b)]^2$$

or $$x^2-2(a+b)x+(a+b)^2+y^2-2(b-a)y+(b-a)^2$$

$$=x^2-2(a-b)x+(a-b)^2+y^2-2(a+b)y+(a+b)^2$$

or $$-2(a+b)x-2(b-a)y=-2(a-b)x-2(a+b)y$$

or $$ax+bx+by-ay=ax-bx-ay-by$$

or $$2bx=2ay \Rightarrow bx=ay$$

Exercise 9.1

1. Find the co-ordinates of points *P, Q, R* and *S* from given figure.

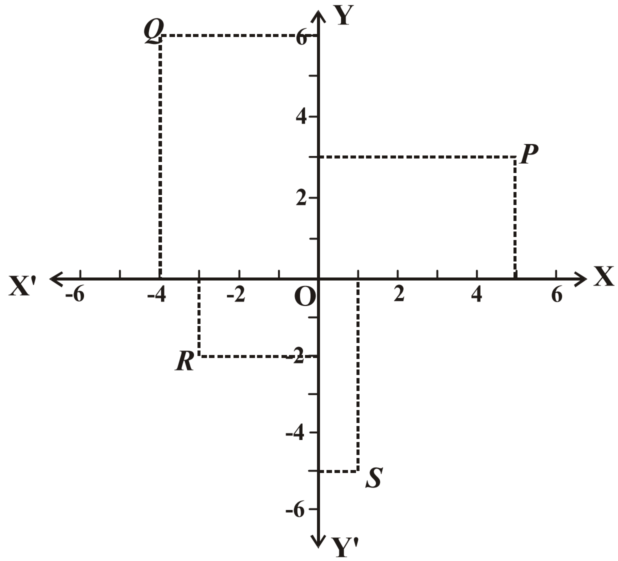


Fig. 9.07

2. Plot the points of the following co-ordinates.
   (1, 2), (–1, 3), (–2, –4), (3, –2), (2, 0), (0, 3)
3. By taking rectangular coordinate axis plot the points *O* (0,0), *P* (3, 0) and *R*(0, 4). If *OPQR* is rectangle then find coordinates of *Q*.
4. Plot the points (–1, 0), (1, 0), (1, 1), (0, 2), (–1, 1). Which figure is obtained, by joining them serially?
5. Draw quadrilateral, if its vertices are following :
   (i) (1,1), (2, 4), (8, 4) and (10, 1)
   (ii) (–2, –2), (–4, 2), (–6, –2) and (–4, –6)
   Also, mention type of obtained quadrilateral.
6. Find the distance between the following points :
   (i) (–6, 7) and (–1, –5)
   (ii) (–1, –1) and (8, –2)
   (iii) $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$
7. Prove that the points (2, –2), (–2, 1) and (5, 2) are vertics of a right angled triangle.
8. Prove that points (1, –2), (3, 0), (1, 2) and (–1, 0) are vertices of a square.
9. Prove that points (*a, a*), (*–a, –a*) and $(-\sqrt{3}a, \sqrt{3}a)$ are vertices of a equilateral triangle.

10. Prove that points (1, 1), (–2, 7) and (3, –3) are collinear.
11. Find that point on x-axis which is equidistant form points (–2, –5) and (2, –3).
12. Find that point on y-axis which is equidistant form points (–5, –2) and (3, 2).
13. If points (3, K) and (K, 5) are equidistant from a point (0, 2), then find the value of K.
14. If co-ordinates of P and Q are $(a\cos\theta, b\sin\theta)$ and $(-a\sin\theta, b\cos\theta)$ respectively, then show that

    $OP^2 + OQ^2 = a^2 + b^2$, where O is origin.

15. If (0, 0) and $(3, \sqrt{3})$ are two vertices of an equilateral triangle then find third vertex.

9.05 Internal and external division of distance between two points

Let A and B are two points in plane. If point P lies in the middle of line AB then this type of division is called internal division.If point P is not in the middle of A and B, but it lies either left of A or right of B then such division is called external division.

(i) Internal division :

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points in plane and point $P(x, y)$ divides line segment AB in the ratio $m_1 : m_2$, internally. AL, PM and BN are perpendicular drawn form A, P and B on x-axis, respectively. Draw perpendicular AQ and PR form A to PM and from P to BN. Then

$OL = x_1,\ OM = x,\ ON = x_2$

$AL = y_1,\ PM = y$ and $BN = y_2$

$\therefore \quad AQ = LM = OM - OL = x - x_1$

$PR = MN = ON - OM = x_2 - x$

$PQ = PM - QM = PM - AL = y - y_1$

$BR = BN - RN = BN - PM = y_2 - y$

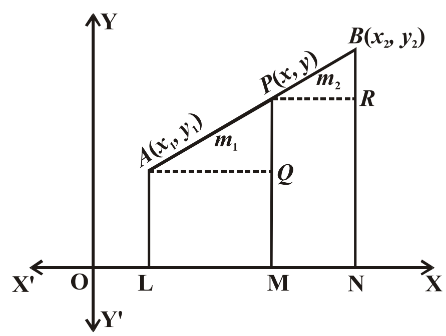


Fig. 9.8

In Fig. 9.8, ΔAQP and ΔPRB are similar triangles.

$$\therefore \quad \frac{AP}{BP} = \frac{AQ}{PR} = \frac{PQ}{BR}$$

$$\text{or} \quad \frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y_1}$$

$$\text{Now} \quad \frac{m_1}{m_2} = \frac{x - x_1}{x_2 - x}$$

$$\text{or} \quad m_1 x_2 - m_1 x = m_2 x - m_2 x_1$$

$$\text{or} \quad (m_1 + m_2)x = m_1 x_2 + m_2 x_1$$

$$\therefore \quad x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\text{Again} \quad \frac{m_1}{m_2} = \frac{y - y_1}{y_2 - y}$$

or $\qquad m_1 y_2 - m_1 y = m_2 y - m_2 y_1$

or $\qquad (m_1 + m_2) y = m_1 y_2 + m_2 y_1$

$\therefore \qquad y = \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

Thus, required coordinate of $P\left(\dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

(ii) External division :

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are points lie in plane. Point P externally divides line segment AB it the ratio $m_1 : m_2$. AL, BN and PM are perpendiculars drawn form A, B and P respectively. AQ and BR are perpendicular from point A and B on PM and BN respectively. Then $OL = x_1$, $ON = x_2$, $OM = x$, $AL = y_1$, $BN = y_2$ and $PM = y$

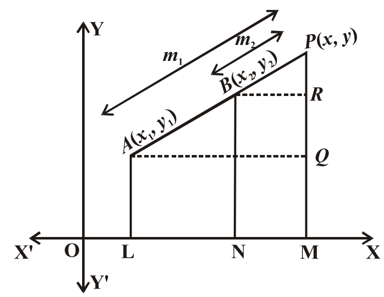


Fig. 9.9

$\therefore \quad AQ = LM = OM - OL = x - x_1$

$\qquad BR = NM = OM - ON = x - x_2$

$\qquad PQ = PM - QM = PM - AL = y - y_1$

and $\quad PR = PM - RM = PM - BN = y - y_2$

In fig. 9.09, ΔAPQ and ? BPR are smilar triangles

$\therefore \qquad \dfrac{AP}{BP} = \dfrac{AQ}{BR} = \dfrac{PQ}{PR}$

or $\qquad \dfrac{m_1}{m_2} = \dfrac{x - x_1}{x - x_2} = \dfrac{y - y_1}{y - y_2}$

Now $\qquad \dfrac{m_1}{m_2} = \dfrac{x - x_1}{x - x_2}$

or $\qquad m_1 x - m_1 x_2 = m_2 x - m_2 x_1$

or $\qquad (m_1 - m_2) x = m_1 x_2 - m_2 x_1$

$\therefore \qquad x = \dfrac{m_1 x_2 - m_2 x_1}{m_1 - m_2}$

Again $\qquad \dfrac{m_1}{m_2} = \dfrac{y - y_1}{y - y_2}$

or $\qquad m_1 y - m_1 y_2 = m_2 y - m_2 y_1$

or $\qquad (m_1 - m_2) y = m_1 y_2 - m_2 y_1$

$\therefore \qquad y = \dfrac{m_1 y_2 - m_2 y_1}{m_1 - m_2}$

Thus, required coordinates of P

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

Special case : If point P lie in the mid of line segment AB *i.e.,* P divides AB in the ratio 1 : 1, then co-ordinates of P are $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

Note :

(i) Internal division formula is replaced by external division formula by putting –ve sign of m_1 or m_2.

(ii) In external division, if $|m_1| > |m_2|$ then division point is in right of B and if $|m_1| < |m_2|$ then, division point is in left of A.

(iii) If point $P(x_1, y_1)$ divides line segment AB in ratio λ : 1 then co-ordinates of P are $\left(\frac{x_1 + \lambda x_2}{1+\lambda}, \frac{y_1 + \lambda y_2}{1+\lambda}\right)$.

So co-ordinates of any point of the line joining the points (x_1, y_1) and (x_2, y_2) can be expressed as above.

Illustrative Examples

Example 1. Find the co-ordinates of that point which divides the line joining the points (–2, 1) and (5, 4) internally in the ratio2 : 3.

Solution : Let required point is (x, y), then by formula

$$x = \frac{2\times5 + 3\times(-2)}{2+3} = \frac{10-6}{5} = \frac{4}{5}$$

and $$y = \frac{2\times4 + 3\times1}{2+3} = \frac{8+3}{5} = \frac{11}{5}$$

Therefore, co-ordinates of required point are $\left(\frac{4}{5}, \frac{11}{5}\right)$

Example 2. Find the co-oridinates of that point which externally divides the line joining the points (–4, 4) and (7, 2) in the ratio 4 : 7

Solution : Let co-ordinates of requiredpoint is (x, y), then

$$x = \frac{4\times7 - 7\times(-4)}{4-7} = \frac{28+28}{-3} = -\frac{56}{3} = -18\frac{2}{3}$$

and $$y = \frac{4\times2 - 7\times4}{4-7} = \frac{8-28}{-3} = \frac{20}{3} = 6\frac{2}{3}$$

Thus, coordinates of required point are $\left(-18\frac{2}{3}, 6\frac{2}{3}\right)$

Example 3. In which ratio x–axis divides the line joining the points $A(3, -5)$ and $B(-4, 7)$?

Solution : Ordinate of each point on x–axis is zero.So, let point $P(x, 0)$ internally divides the given line segment in the ratio $m_1 : m_2$.

$$\therefore \qquad 0 = \frac{m_1 \times 7 + m_2 \times (-5)}{m_1 + m_2}$$

$$\text{or} \qquad 7m_1 - 5m_2 = 0$$

$$\text{or} \qquad \frac{m_1}{m_2} = \frac{5}{7}$$

Therefore, x-axis internally divides the line joiniung the given points in the ratio 5: 7.

Example 4. In which ratio, point (–2,3) divides the line joining the points (–3, 5) and (4, –9).

Solution : Let point (–2, 3) inetrnally divides the line joining the given points in the ratio λ : 1. So by internal division formula

$$-2 = \frac{\lambda \times 4 + 1 \times (-3)}{\lambda + 1}$$

$$\text{or} \qquad -2 = \frac{4\lambda - 3}{\lambda + 1}$$

$$\text{or} \qquad -2\lambda - 2 = 4\lambda - 3$$

$$\text{or} \qquad 6\lambda = 1 \Rightarrow \lambda = \frac{1}{6}$$

Thus, required ratio is 1 : 6

Note : By ordinate same ratio will be obtained.

Example 5. If point P (–1, 2) divides the line joining the points A(2, 5) and B in the ratio 3: 4 internally, then find co-ordinates of B.

Solution : Let co-ordinates of B are (x_1, y_1) and given that $AP : BP = 3 : 4$

By internal division formula

$$-1 = \frac{3 \times x_1 + 4 \times 2}{3 + 4} = \frac{3x_1 + 8}{7}$$

$$\text{or} \qquad -7 = 3x_1 + 8 \Rightarrow x_1 = -\frac{15}{3} = -5$$

$$\text{and} \qquad 2 = \frac{3 \times y_1 + 4 \times 5}{3 + 4} = \frac{3y_1 + 20}{7}$$

$$\text{or} \qquad 14 = 3y_1 + 20$$

$$\Rightarrow \qquad y_1 = -\frac{6}{3} = -2$$

Therefore, co-ordinates of B are (–5, –2)

Example 6. Find in which ratio line $x + y = 4$ divides the line joining the points (–1, 1) and (5, 7)?

Solution : Let given line divides line joining the points A(–1, 1) and B(5, 7) in the ratio λ : 1. So co-ordinates of P will be

$$\left(\frac{5\lambda-1}{\lambda+1}, \frac{7\lambda+1}{\lambda+1}\right)$$

But point P lies on line $x + y = 4$

$\therefore$ $$\frac{5\lambda-1}{\lambda+1} + \frac{7\lambda+1}{\lambda+1} = 4$$

or $$5\lambda - 1 + 7\lambda + 1 = 4\lambda + 4$$

or $$8\lambda = 4$$

$\Rightarrow$ $$\lambda = \frac{1}{2}$$

or $$\lambda : 1 = 1 : 2$$

Exercise 9.2

1. Find the coordinates of the point which divides the line segment joining the points (3,5) and (7,9) in the ratio 2 : 3 internally.
2. Find the coordinates of the point which divides the line segment joining the points (5, –2) and $\left(-1\frac{1}{2}, 4\right)$ in the ratio 7 : 9 externally.
3. Prove that origin O divides the line joining the points $A(1, -3)$ and $B(-3, 9)$ in the ratio 1 : 3 internally. Find the coordinartes of the points which devides the line AB externally in the ratio 1 : 3.
4. Find the mid point of line joining the points (22, 20) and (0, 16).
5. In which ratio, x-axis divides the line segment which joins points (5, 3) and (–3, –2)?
6. In which ratio, y-axis divides the line segment which joins points (2, –3) and (5, 6)?
7. In which ratio, point (11, 15) divides the line segment which joins (15, 5) and (9, 20)?
8. If point P (3, 5) divides line segment which joins $A(-2, 3)$ and B (x, y) in the ratio 4 : 7 intermally, then find the co-ordinates of B.
9. Find the co-ordinates of point which trisects the line joining point (11, 9) and (1, 2).
10. Find the co-ordinates of point which quartersects the line joining point (–4, 0) and (0, 6).
11. Find the ratio in which line $3x + y = 9$ divides the line segment which joins points (1, 3) and (2, 7)
12. Find the ratio where point $(-3, p)$, divides intermally the line segment which joins points (–5, –4) and (–2, 3). Also find p.

Miscellaneous Exercise-9

Objective Question [1 to 10]

1. Distance of point (3, 4) form y-axis will be :

   (a) 1 (b) 4 (c) 2 (d) 3
2. Distance of point (5, –2) from x-axis will be :

   (a) 5 (b) 2 (c) 3 (d) 4
3. Distance between points (0, 3) and (–2, 0) will be :

   (a) $\sqrt{14}$ (b) $\sqrt{15}$ (c) $\sqrt{13}$ (d) $\sqrt{5}$

4. Triangle having vertices (–2, 1), (2, –2) and (5, 2) is :
(a) Right triangle (b) Equilateral (c) Isoceles (d) None of these

5. Quadrilateral having vertices (–1, 1), (0, –3), (5, 2) and (4, 6) will be :
(a) Square (b) Rectangle (c) Rhombus (d) Parallelogram

6. Point equidistance from (0, 0), (2, 0) and (0, 2) is :
(a) (1, 2) (b) (2, 1) (c) (2, 2) (d) (1, 1)

7. *P* divides the line segment which joins points (5, 0) and (0, 4) in the ratio of 2 : 3 internally. Co-ordinates of *P* are :
(a) $\left(3, \frac{8}{5}\right)$ (b) $\left(1, \frac{4}{5}\right)$ (c) $\left(\frac{5}{2}, \frac{3}{4}\right)$ (d) $\left(2, \frac{12}{5}\right)$

8. If points (1, 2), (–1, *x*) and (2, 3) are collinear, then *x* will be :
(a) 2 (b) 0 (c) −1 (d) 1

9. If distance between point (3, *a*) and (4, 1) is $\sqrt{10}$, then *a* will be :
(a) 3, −1 (b) 2, −2 (c) 4, −2 (d) 5, −3

10. If point (x, y) is at equidistant from (2, 1) and (1, –2), then the true statement is :
(a) $x+3y=0$ (b) $3x+y=0$ (c) $x+2y=0$ (d) $2y+3x=0$

11. Find the type of quadrilateral, if its vertices are (1, 4), (–5, 4), (–5, –3) and (1,–3).

12. Which shape will be formed on joining (–2, 0), (2, 0), (2, 2), (0, 4), (–2, 2) in the given order?

13. Find the ratio in which point (3, 4) divides the line segment which joins points (1, 2) and (6, 7).

14. Opposite vetrices of any square are (5, –4) and (–3, 2), then find the length of diagonal.

15. If co-ordinate of one end and mid point of a line segment are (4, 0) and (4, 1) respectively, then find the co-ordinates of other end of line segment.

16. Find the distance between of point (1, 2) from mid point of line segment which joints the points (6, 8) and (2, 4).

17. If in any plane, there are four points *P*(2, –1), *Q*(3, 4), *R*(–2, 3) and *S*(–3, –2), then prove that *PQRS* is not a square but a rhombus.

18. Prove that mid point (*C*) of hypotaneous in a right angled triangle *AOB* is situated at equal distance form vertics *O*, *A* and *B* of triangle.

19. Find the length of median of a triangle whose vertices are (1,–1), (0, 4) and (–5, 3).

20. Prove that mid point of a line segment which joins points (5, 7) and (3, 9) is the same as mid point of line segment which joins points (5,7) and (3, 9).

21. If mid points of sides of a triangle is (1, 2), (0, –1) and (2, –1), then find its vertices.

Important Points

1. Formula of distance between two points $P(x_1, y_1)$ and $Q\ (x_2, y_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or

$$PQ = \sqrt{(\text{difference of abscissas})^2 + (\text{difference of ordinates})^2}$$

2. The coordinates of the point P(x,y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

and

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

3. The coordinates of the point P (x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m_1 : m_2$ are

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2}$$

and

$$y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}$$

4. The mid point of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

or

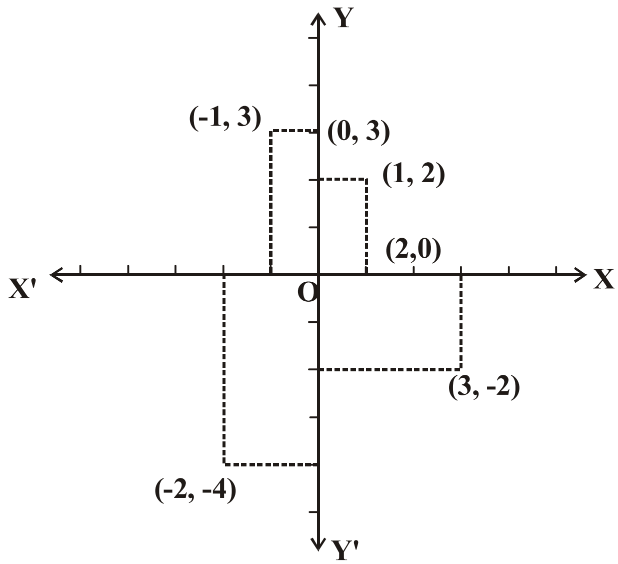
$$\left(\frac{\text{Sum of } x \text{ co-ordinates}}{2}, \frac{\text{Sum of } y \text{ co-ordinates}}{2}\right)$$

Answer Sheet

Exercise 9.1

1. $P(5,3), Q(-4,6), R(-3,-2), S(1,-5)$

2.



3. (3, 4) 4. Pentagon 5. (i) Trapezium (ii) Rhombus

6. (i) 13 (ii) $\sqrt{82}$ (iii) $a(t_2 - t_1)\sqrt{(t_2 + t_1)^2 + 4}$ 11. (–2, 0) 12. (0, –2)

13. 1 15. $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$

Exercise 9.2

1. $\left(\frac{23}{5}, \frac{33}{5}\right)$ 2. $\left(27\frac{3}{4}, -23\right)$ 3. (3, –9) 4. (11, 18)

5. 3 : 2 6. 2 :5 external division 7. 2 :1 8. $\left(\frac{47}{4}, \frac{17}{2}\right)$

9. $\left(\frac{13}{3}, \frac{13}{3}\right), \left(\frac{23}{3}, \frac{20}{3}\right)$ 10. $\left(-3, \frac{3}{2}\right), (-2, 3), \left(-1, \frac{9}{2}\right)$ 11. 3 : 4 12. $2:1, p = \frac{2}{3}$

Miscellaneous Exercise-9

1. (d) 2. (b) 3. (c) 4. (a) 5. (d) 6. (d) 7. (a)
8. (b) 9. (c) 10. (a) 11. Rectangle 12. Pentagon 13. 2 :3. 14. 10

15. (4, 2) 16. 5 19. $\frac{\sqrt{130}}{2}, \frac{\sqrt{130}}{2}, \sqrt{13}$ 21. (1, –4), (3, 2), (–1, 2)