Reflection

Exercise 12(A)

Question 1.

Complete the following table:

Point	Transformation	Image
(5, -7)		(-5, 7)
(4, 2)	Reflection in x-axis	
	Reflection in y-axis	(0, 6)
(6, -6)		(-6, 6)
(4, -8)		(-4, -8)

Solution:

Point	Transformation	Image
(5, -7)	Reflection in origin	(-5, 7)
(4, 2)	Reflection in x-axis	(4, -2)
(0, 6)	Reflection in y-axis	(0, 6)
(6, -6)	Reflection in origin	(-6, 6)
(4, -8)	Reflection in y-axis	(-4, -8)

Question 2.

A point P is its own image under the reflection in a line I. Describe the position of point the P with respect to the line I.

Solution:

Since, the point P is its own image under the reflection in the line I. So, point P is an invariant point.

Hence, the position of point P remains unaltered.

Question 3.

State the co-ordinates of the following points under reflection in x-axis:

(i) (3, 2)

(ii) (-5, 4)

(iii) (0, 0)

Solution:

(i) (3, 2)

The co-ordinate of the given point under reflection in the x-axis is (3, -2).

(ii) (-5, 4)

The co-ordinate of the given point under reflection in the x-axis is (-5, -4). (iii) (0, 0)

The co-ordinate of the given point under reflection in the x-axis is (0, 0).

Question 4.

State the co-ordinates of the following points under reflection in y-axis:

(i) (6, -3)

(ii) (-1, 0)

(iii) (-8, -2)

Solution:

(i) (6, -3)

The co-ordinate of the given point under reflection in the y-axis is (-6, -3). (ii) (-1, 0)

The co-ordinate of the given point under reflection in the y-axis is (1, 0). (iii) (-8, -2)

The co-ordinate of the given point under reflection in the y-axis is (8, -2).

Question 5.

State the co-ordinates of the following points under reflection in origin:

- (i) (-2, -4)
- (ii) (-2, 7)
- (iii) (0, 0)

Solution:

(i) (-2, -4)

The co-ordinate of the given point under reflection in origin is (2, 4).

(ii) (-2, 7)

The co-ordinate of the given point under reflection in origin is (2, -7). (iii) (0, 0)

The co-ordinate of the given point under reflection in origin is (0, 0).

Question 6.

State the co-ordinates of the following points under reflection in the line x = 0:

(i) (-6, 4)

(ii) (0, 5)

(iii) (3, -4)

Solution:

(i) (-6, 4)

The co-ordinate of the given point under reflection in the line x = 0 is (6, 4). (ii) (0, 5)

The co-ordinate of the given point under reflection in the line x = 0 is (0, 5). (iii) (3, -4)

The co-ordinate of the given point under reflection in the line x = 0 is (-3, -4).

Question 7.

State the co-ordinates of the following points under reflection in the line y = 0: (i) (-3, 0) (ii) (8, -5) (iii) (-1, -3) Solution: (i) (-3, 0)

The co-ordinate of the given point under reflection in the line y = 0 is (-3, 0). (ii) (8, -5)

The co-ordinate of the given point under reflection in the line y = 0 is (8, 5). (iii) (-1, -3)

The co-ordinate of the given point under reflection in the line y = 0 is (-1, 3).

Question 8.

A point P is reflected in the x-axis. Co-ordinates of its image are (-4, 5).

(i) Find the co-ordinates of P.

(ii) Find the co-ordinates of the image of P under reflection in the y-axis. **Solution:**

(i) Since, M_x (-4, -5) = (-4, 5)

So, the co-ordinates of P are (-4, -5).

(ii) Co-ordinates of the image of P under reflection in the y-axis (4, -5).

Question 9.

A point P is reflected in the origin. Co-ordinates of its image are (-2, 7).

(i) Find the co-ordinates of P.

(ii) Find the co-ordinates of the image of P under reflection in the x-axis. **Solution:**

(i) Since, $M_{\circ}(2, -7) = (-2, 7)$

So, the co-ordinates of P are (2, -7).

(ii) Co-ordinates of the image of P under reflection in the x-axis (2, 7).

Question 10.

The point (a, b) is first reflected in the origin and then reflected in the y-axis to P'. If P' has co-ordinates (4, 6); evaluate a and b.

Solution:

 $M_{\circ}(a, b) = (-a, -b)$ $M_{v}(-a, -b) = (a, -b)$

Thus, we get the co-ordinates of the point P' as (a, -b). It is given that the co-ordinates of P' are (4, 6).

On comparing the two points, we get, a = 4 and b = -6

Question 11.

The point P (x, y) is first reflected in the x-axis and reflected in the origin to P'. If P' has co-ordinates (-8, 5); evaluate x and y.

Solution:

 $M_x (x, y) = (x, -y)$ $M_0 (x, -y) = (-x, y)$ Thus, we get the co-ordinates of the point P' as (-x, y). It is given that the co-ordinates of P' are (-8, 5).

On comparing the two points, we get, x = 8 and y = 5

Question 12.

The point A (-3, 2) is reflected in the x-axis to the point A'. Point A' is then reflected in the origin to point A".

(i) Write down the co-ordinates of A".

(ii) Write down a single transformation that maps A onto A".

Solution:

(i) The reflection in x-axis is given by $M_x(x, y) = (x, -y)$.

A' = reflection of A (-3, 2) in the x- axis = (-3, -2).

The reflection in origin is given by $M_{\circ}(x, y) = (-x, -y)$.

A" = reflection of A' (-3, -2) in the origin = (3, 2)

(ii) The reflection in y-axis is given by $M_y(x, y) = (-x, y)$.

The reflection of A (-3, 2) in y-axis is (3, 2).

Thus, the required single transformation is the reflection of A in the y-axis to the point A".

Question 13.

The point A (4, 6) is first reflected in the origin to point A'. Point A' is then reflected in the y-axis to the point A".

(i) Write down the co-ordinates of A".

(ii) Write down a single transformation that maps A onto A".

Solution:

(i) The reflection in origin is given by $M_{\circ}(x, y) = (-x, -y)$.

A' = reflection of A (4, 6) in the origin = (-4, -6)

The reflection in y-axis is given by $M_y(x, y) = (-x, y)$.

A" = reflection of A' (-4, -6) in the y-axis = (4, -6)

(ii) The reflection in x-axis is given by $M_x(x, y) = (x, -y)$.

The reflection of A (4, 6) in x-axis is (4, -6).

Thus, the required single transformation is the reflection of A in the x-axis to the point A".

Question 14.

The triangle ABC, where A is (2, 6), B is (-3, 5) and C is (4, 7), is reflected in the y-axis to triangle A'B'C'. Triangle A'B'C' is then reflected in the origin to triangle A"B"C".

(i) Write down the co-ordinates of A", B" and C".

(ii) Write down a single transformation that maps triangle ABC onto triangle A"B"C". **Solution:**

(i) Reflection in y-axis is given by $M_y(x, y) = (-x, y)$

 \therefore A' = Reflection of A (2, 6) in y-axis = (-2, 6)

Similarly, B' = (3, 5) and C' = (-4, 7)

Reflection in origin is given by $M_{\circ}(x, y) = (-x, -y)$

 \therefore A" = Reflection of A' (-2, 6) in origin = (2, -6)

Similarly, B" = (-3, -5) and C" = (4, -7)

(ii) A single transformation which maps triangle ABC to triangle A"B"C" is reflection in x-axis.

Question 15.

P and Q have co-ordinates (-2, 3) and (5, 4) respectively. Reflect P in the x-axis to P' and Q in the y-axis to Q'. State the co-ordinates of P' and Q'. **Solution:**

Reflection in x-axis is given by $M_x (x, y) = (x, -y)$ P' = Reflection of P(-2, 3) in x-axis = (-2, -3) Reflection in y-axis is given by $M_y (x, y) = (-x, y)$ Q' = Reflection of Q(5, 4) in y-axis = (-5, 4) Thus, the co-ordinates of points P' and Q' are (-2, -3) and (-5, 4) respectively.

Question 16.

On a graph paper, plot the triangle ABC, whose vertices are at points A (3, 1), B (5, 0) and C (7, 4).

On the same diagram, draw the image of the triangle ABC under reflection in the origin O(0, 0).

Solution:

The graph shows triangle ABC and triangle A'B'C' which is obtained when ABC is reflected in the origin.



Question 17.

Find the image of point (4, -6) under the following operations: (i) $M_x . M_y$ (ii) $M_y . M_x$ (iii) $M_0 . M_x$ (iv) $M_x . M_0$

(v) M_{o} . M_{v} (vi) M_{v} . M_{o}

Write down a single transformation equivalent to each operation given above. State whether:

(a) $M_{o} \cdot M_{x} = M_{x} \cdot M_{o}$ (b) $M_{y} \cdot M_{o} = M_{o} \cdot M_{y}$ Solution:

(i) $M_x \cdot M_y(4, -6) = M_x(-4, -6) = (-4, 6)$ Single transformation equivalent to $M_x \cdot M_y$ is M_0 . (ii) $M_y \cdot M_x(4, -6) = M_y(4, 6) = (-4, 6)$ Single transformation equivalent to $M_y \cdot M_x$ is M_0 . (iii) $M_0 \cdot M_x(4, -6) = M_0(4, 6) = (-4, -6)$ Single transformation equivalent to $M_0 \cdot M_x$ is M_y . (iv) $M_x \cdot M_0(4, -6) = M_x(-4, 6) = (-4, -6)$ Single transformation equivalent to $M_x \cdot M_0$ is M_y . (v) $M_0 \cdot M_y(4, -6) = M_0(-4, -6) = (4, 6)$ Single transformation equivalent to $M_0 \cdot M_y$ is M_x . (vi) $M_y \cdot M_0(4, -6) = M_y(-4, 6) = (4, 6)$ Single transformation equivalent to $M_x \cdot M_0$ is M_x . From (iii) and (iv), it is clear that $M_0 \cdot M_x = M_x \cdot M_0$.

Question 18.

Point A (4, -1) is reflected as A' in the y-axis. Point B on reflection in the x-axis is mapped as B' (-2, 5). Write down the co-ordinates of A' and B. **Solution:**

Reflection in y-axis is given by $M_y(x, y) = (-x, y)$ A' = Reflection of A(4, -1) in y-axis = (-4, -1) Reflection in x-axis is given by $M_x(x, y) = (x, -y)$ B' = Reflection of B in x-axis = (-2, 5) Thus, B = (-2, -5)

Question 19.

The point (-5, 0) on reflection in a line is mapped as (5, 0) and the point (-2, -6) on reflection in the same line is mapped as (2, -6).

(a) Name the line of reflection.

(b) Write down the co-ordinates of the image of (5, -8) in the line obtained in (a). **Solution:**

(a) We know that reflection in the line x = 0 is the reflection in the y-axis.

It is given that:

Point (-5, 0) on reflection in a line is mapped as (5, 0).

Point (-2, -6) on reflection in the same line is mapped as (2, -6).

Hence, the line of reflection is x = 0.

(b) It is known that $M_y(x, y) = (-x, y)$

Co-ordinates of the image of (5, -8) in the line x = 0 are (-5, -8).

Exercise 9(B)

Question 1.

Attempt this question on graph paper.

(a) Plot A (3, 2) and B (5, 4) on graph paper. Take 2 cm = 1 unit on both the axes.

(b) Reflect A and B in the x-axis to A' and B' respectively. Plot these points also on the same graph paper.

(c) Write down:

(i) the geometrical name of the figure ABB'A';

(ii) the measure of angle ABB';

(iii) the image of A" of A, when A is reflected in the origin.

(iv) the single transformation that maps A' to A".

Solution:



(C)

(i) From graph, it is clear that ABB'A' is an isosceles trapezium.

(ii) The measure of angle ABB' is 45°.

(iii) A" = (-3, -2)

(iv) Single transformation that maps A' to A" is the reflection in y-axis.

Question 2.

Points (3, 0) and (-1, 0) are invariant points under reflection in the line L₁; points (0, -3)

and (0, 1) are invariant points on reflection in line L₂.

(i) Name or write equations for the lines L_1 and L_2 .

(ii) Write down the images of the points P (3, 4) and Q (-5, -2) on reflection in line L₁. Name the images as P' and Q' respectively.

(iii) Write down the images of P and Q on reflection in L_2 . Name the images as P" and Q" respectively.

(iv) State or describe a single transformation that maps P' onto P".

Solution:

(i) We know that every point in a line is invariant under the reflection in the same line. Since points (3, 0) and (-1, 0) lie on the x-axis.

So, (3, 0) and (-1, 0) are invariant under reflection in x-axis.

Hence, the equation of line L_1 is y = 0.

Similarly, (0, -3) and (0, 1) are invariant under reflection in y-axis.

Hence, the equation of line L_2 is x = 0.

(ii) P' = Image of P (3, 4) in L₁ = (3, -4)

Q' = Image of Q (-5, -2) in $L_1 = (-5, 2)$

(iii) $P'' = Image of P(3, 4) in L_2 = (-3, 4)$

Q" = Image of Q (-5, -2) in $L_2 = (5, -2)$

(iv) Single transformation that maps P' onto P" is reflection in origin.

Question 3.

(i) Point P (a, b) is reflected in the x-axis to P' (5, -2). Write down the values of a and b.

(ii) P" is the image of P when reflected in the y-axis. Write down the co-ordinates of P".

(iii) Name a single transformation that maps P' to P".

Solution:

(i) We know $M_x(x, y) = (x, -y)$

P'(5, -2) = reflection of P(a, b) in x-axis.

Thus, the co-ordinates of P are (5, 2).

Hence, a = 5 and b = 2.

(ii) P'' = image of P(5, 2) reflected in y-axis = (-5, 2)

(iii) Single transformation that maps P' to P" is the reflection in origin.

Question 4.

The point (-2, 0) on reflection in a line is mapped to (2, 0) and the point (5, -6) on reflection in the same line is mapped to (-5, -6).

(i) State the name of the mirror line and write its equation.

(ii) State the co-ordinates of the image of (-8, -5) in the mirror line.

Solution:

(i) We know reflection of a point (x, y) in y-axis is (-x, y).

Hence, the point (-2, 0) when reflected in y-axis is mapped to (2, 0).

Thus, the mirror line is the y-axis and its equation is x = 0.

(ii) Co-ordinates of the image of (-8, -5) in the mirror line (i.e., y-axis) are (8, -5).

Question 5.

The points P (4, 1) and Q (-2, 4) are reflected in line y = 3. Find the co-ordinates of P',

the image of P and Q', the image of Q. **Solution:**

The line y = 3 is a line parallel to x-axis and at a distance of 3 units from it.

Mark points P (4, 1) and Q (-2, 4).

From P, draw a straight line perpendicular to line CD and produce. On this line mark a point P' which is at the same distance above CD as P is below it.

The co-ordinates of P' are (4, 5).

Similarly, from Q, draw a line perpendicular to CD and mark point Q' which is at the same distance below CD as Q is above it.

The co-ordinates of Q' are (-2, 2).



Question 6.

A point P (-2, 3) is reflected in line x = 2 to point P'. Find the coordinates of P'. **Solution:**

The line x = 2 is a line parallel to y-axis and at a distance of 2 units from it. Mark point P (-2, 3).

From P, draw a straight line perpendicular to line CD and produce. On this line mark a point P' which is at the same distance to the right of CD as P is to the left of it.



Question 7.

A point P (a, b) is reflected in the x-axis to P' (2, -3). Write down the values of a and b. P" is the image of P, reflected in the y-axis. Write down the co-ordinates of P". Find the co-ordinates of P"', when P is reflected in the line, parallel to y-axis, such that x = 4. **Solution:**



A point P (a, b) is reflected in the x-axis to P' (2, -3). We know $M_x (x, y) = (x, -y)$ Thus, co-ordinates of P are (2, 3). Hence, a = 2 and b = 3. P" = Image of P reflected in the y-axis = (-2, 3) P" = Reflection of P in the line (x = 4) = (6, 3)

Question 8.

Points A and B have co-ordinates (3, 4) and (0, 2) respectively. Find the image:

(a) A' of A under reflection in the x-axis.

(b) B' of B under reflection in the line AA'.

(c) A" of A under reflection in the y-axis.

(d) B" of B under reflection in the line AA".

Solution:



(d) B" = Image of B under reflection in the line AA" = (0, 6)

Question 9.

(i) Plot the points A (3, 5) and B (-2, -4). Use 1 cm = 1 unit on both the axes.

(ii) A' is the image of A when reflected in the x-axis. Write down the co-ordinates of A' and plot it on the graph paper.

(iii) B' is the image of B when reflected in the y-axis, followed by reflection in the origin. Write down the co-ordinates of B' and plot it on the graph paper.

(iv) Write down the geometrical name of the figure AA'BB'.

(v) Name the invariant points under reflection in the x-axis. **Solution:**



(i) The points A (3, 5) and B (-2, -4) can be plotted on a graph as shown.

(ii) A' = Image of A when reflected in the x-axis = (3, -5)

(iii) C = Image of B when reflected in the y-axis = (2, -4)

B' = Image when C is reflected in the origin = (-2, 4)

(iv) Isosceles trapezium

(v) Any point that remains unaltered under a given transformation is called an invariant. Thus, the required two points are (3, 0) and (-2, 0).

Question 10.

The point P (5, 3) was reflected in the origin to get the image P'.

(a) Write down the co-ordinates of P'.

(b) If M is the foot if the perpendicular from P to the x-axis, find the co-ordinates of M.

(c) If N is the foot if the perpendicular from P' to the x-axis, find the co-ordinates of N.

(d) Name the figure PMP'N.

(e) Find the area of the figure PMP'N.

Solution:



Question 11.

The point P (3, 4) is reflected to P' in the x-axis; and O' is the image of O (the origin) when reflected in the line PP'. Write:

(i) the co-ordinates of P' and O'.(ii) the length of the segments PP' and OO'.

(iii) the perimeter of the quadrilateral POP'O'.

(iv) the geometrical name of the figure POP'O'. **Solution:**



(ii) PP' = 8 units and OO' = 6 units.

(iii) From the graph it is clear that all sides of the quadrilateral POP'O' are equal. In right Δ PO'Q,

PO' = $\sqrt{(4)^2 + (3)^2} = 5$ units

So, perimeter of quadrilateral POP'O' = 4 PO' = 4 × 5 units = 20 units

(iv) Quadrilateral POP'O' is a rhombus.

Question 12.

A (1, 1), B (5, 1), C (4, 2) and D (2, 2) are vertices of a quadrilateral. Name the quadrilateral ABCD. A, B, C, and D are reflected in the origin on to A', B', C' and D' respectively. Locate A', B', C' and D' on the graph sheet and write their co-ordinates. Are D, A, A' and D' collinear? **Solution:**



Quadrilateral ABCD is an isosceles trapezium. Co-ordinates of A', B', C' and D' are A'(-1, -1), B'(-5, -1), C'(-4, -2) and D'(-2, -2) respectively.

It is clear from the graph that D, A, A' and D' are collinear.

Question 13.

P and Q have co-ordinates (0, 5) and (-2, 4).

(a) P is invariant when reflected in an axis. Name the axis.

(b) Find the image of Q on reflection in the axis found in (i).

(c) (0, k) on reflection in the origin is invariant. Write the value of k.

(d) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin followed by reflection in x-axis.

Solution:

(a) Any point that remains unaltered under a given transformation is called an invariant. It is given that P(0, 5) is invariant when reflected in an axis. Clearly, when P is reflected in the y-axis then it will remain invariant. Thus, the required axis is the y-axis.

(b) The co-ordinates of the image of Q (-2, 4) when reflected in y-axis is (2, 4).

(c) (0, k) on reflection in the origin is invariant. We know the reflection of origin in origin is invariant. Thus, k = 0.

(d) Co-ordinates of image of Q (-2, 4) when reflected in origin = (2, -4)

Co-ordinates of image of (2, -4) when reflected in x-axis = (2, 4)Thus, the co-ordinates of the point are (2, 4).

Question 14.

The points P (1, 2), Q (3, 4) and R (6, 1) are the vertices of PQR.

(a) Write down the co-ordinates of P', Q' and R', if P'Q'R' is the image of PQR, when reflected in the origin.

(b) Write down the co-ordinates of P", Q" and R", if P"Q"R" is the image of PQR, when reflected in the x-axis.

(c) Mention the special name of the quadrilateral QRR"Q" and find its area. **Solution:**



(a) The co-ordinates of P', Q' and R' are (-1, -2), (-3, -4) and (-6, -1) respectively. (b) The co-ordinates of P", Q" and R" are (1, -2), (3, -4) and (6, -1) respectively. (c) The quadrilateral QRR"Q" is an isosceles trapezium. Area of QRR"Q" = $\frac{1}{2}$ (RR"+ QQ") × Height = $\frac{1}{2}$ (2 + 8) × 3 = 15 sq units

Question 15.

(i) The point P (2, -4) is reflected about the line x = 0 to get the image Q. Find the coordinates of Q.

(ii) The point Q is reflected about the line y = 0 to get the image R. Find the co-ordinates



Question 16.

A' and B' are images of A (-3, 5) and B (-5, 3) respectively on reflection in y-axis. Find: (a) the co-ordinates of A' and B'.

- (b) Assign special name of quadrilateral AA'B'B.
- (c) Are AB' and BA' equal in length? **Solution:**



(a) The co-ordinates of A' and B' are (3, 5) and (5, 3).

- (b) Quadrilateral AA'B'B is an isosceles trapezium.
- (c) Yes, AB' and BA' are equal in length.

Question 17.

Using a graph paper, plot the point A (6, 4) and B (0, 4).

- (a) Reflect A and B in the origin to get the image A' and B'.
- (b) Write the co-ordinates of A' and B'.
- (c) Sate the geometrical name for the figure ABA'B'.
- (d) Find its perimeter.

Solution:

