CHAPTER

24

Lines and Angles

There are some basic facts related to the lines and angles which are taken for granted without any proof. These facts are also known as axioms or postulates.

Axioms/Postulates

- A point is represented by a dot of which length, breadth and height cannot be measured.

  Example

  In the figure shown above a point has been represented by *P*.

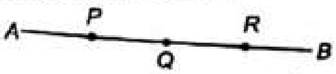
- There occurs atleast two distinct points in the space.
- A line contains infinitely many points and contains atleast two distinct points.
- A plane is considered to be a set of many points and contains atleast three non-collinear points.
- The intersection of two distinct plane is always a straight line.
- An infinite number of lines can pass through a given point.
- Through the given two distinct points, there is one and only one line that contains both the points.

  Example

  In the figure shown above *AB* is a line that passes through two given points as *P* and *Q*.

- Three or more than three points are said to be collinear when there is a line which contains them all.

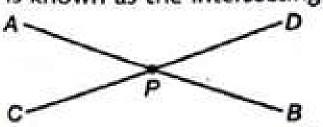
  Example



  In the figure shown above, three points *P*, *Q* and *R* are collinear as the line *AB* contains all of them.

- Two lines can intersect at the most at one point and this common point is known as the intersecting point.

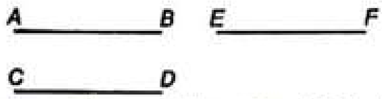
  Example



  In the figure shown above, two lines *AB* and *CD* intersect at a single point *P* that is also known as the intersecting point.

- Two lines which are both parallel to the same line are parallel to each other.

  Example



  In the figure shown above, two lines *AB* and *CD* both are parallel to a third line *EF*. Then, the lines *AB* and *CD* are also parallel to each other. This can be represented as

  $\because \quad AB \parallel EF$ and $CD \parallel EF$; Thus, $AB \parallel CD$

Line Segment

The straight path between two points *P* and *Q* is called a line segment $\overrightarrow{PQ}$. This can be represented as

- *P* and *Q* are called the end points of line segment.
- The line segment has a definite length.
- Distance between *P* and *Q* is called the length of the line segment *PQ*.

Ray

A ray extends indefinitely in one direction. This is exhibited by an arrow *i.e.*, $\overrightarrow{PQ}$

- *P* is called the initial point of the ray.
- The ray has no definite length.
- The ray cannot be drawn but can simply be represented on the plane of a paper.

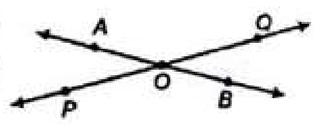
Line

A line segment $\overline{PQ}$ when extended indefinitely in both the directions is called line $\overleftrightarrow{PQ}$.

- A line is a set of infinite points.
- A line has no end points.
- A line has no definite length.
- A line cannot be drawn.

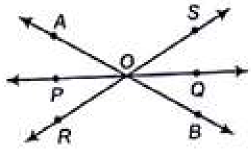
Intersecting Lines

Two lines having a common point are called intersecting lines. This common point is point of intersection *i.e.*, 'O'.



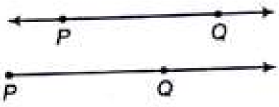
Concurrent Lines

Three or more lines intersecting at the same point are said to be concurrent.



Parallel Lines

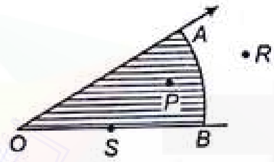
Two lines l and m in a plane are said to be parallel. If they have no common point and we write $l \parallel m$.



Angles

A figure consisting of two rays with end points is called an angle.

- P is a point in the interior of $\angle AOB$.
- S is a point on $\angle AOB$.
- R is a point in the exterior of $\angle AOB$.



An Angle of 360° If a ray OP starting from its original position OP, rotates about O, in the anti-clockwise direction and after making a complete revolution it comes back to its original position we say it has rotated through 360 degree.

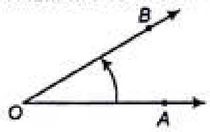
Written as 360°

- $1° = 60$ min, written as $60'$
- $1' = 60$ s, written as $60''$

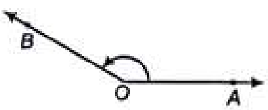
Classification of Angles

Angles can be classified on the basis of their measurement that are as follows.

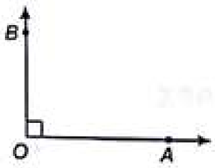
1. **Acute Angles** Measure less than 90°.



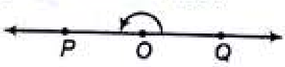
2. **Obtuse Angles** Measure less than 180° but more than 90°.



3. **Right Angles** Measure 90°.

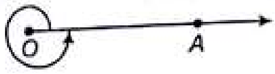


4. **Straight Angles** Measure 180°.



5. **Reflex Angles** Measures more than 180°.

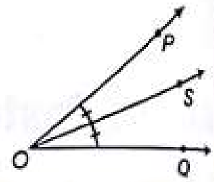
6. **Complete Angle** Measure 360°.



Bisector of an Angle

A ray OS is called the bisector of $\angle POQ$, if $m\angle POS = m\angle QOS$.

- $\angle POS = \angle QOS = \frac{1}{2}\angle POQ$



Complementary Angles

Two angles are said to be complementary, if the sum of their measure is 90°.

- Complementary angles are complement of each other.
- Complement of θ is $(90° - \theta)$.

Supplementary Angles

Two angles are said to be supplementary, if the sum of their measures is 180°.

- Supplementary angles are supplement of each other.
- Supplement of θ is $(180° - \theta)$.

Example 1. The measure of an angle which is 28° more than its complement is

(a) 23° (b) 59°
(c) 77° (d) None of these

Sol. (*b*) Let measure of the required angle be $x°$. Then, measure of its complement $= 90° - x$

$\therefore \quad x - (90° - x) = 28° \Leftrightarrow 2x = 118°; x = 59°$

Hence, the measure of the required angle is 59°.

Example 2. The measure of an angle, which is 32° less than its supplement is

(a) 31° (b) 64° (c) 74° (d) 148°

Sol. (*c*) Let the measure of the required angle be x. Then, measure of its supplement $= (180° - x)$

$\therefore \quad (180° - x) - x = 32° \Leftrightarrow 2x = 148° \Leftrightarrow x = 74°$

Example 3. Two supplementary angles are in the ratio 3 : 2 Then, the measurement of the smaller angle is

(a) 36° (b) 72° (c) 108° (d) 112°

Sol. (*b*) Let the supplementary angles be $3x$ and $2x$ Then, according to the definition of supplementary angle.

$3x + 2x = 180° \Rightarrow 5x = 180° \quad \therefore x = 36°$

$\therefore$ Angles will be $3x = 3 \times 36 = 108°$ and $2x = 2 \times 36 = 72°$. Thus the smaller angle is 72°.

Example 4. The measure of the complement of an angle of $48° 36' 24''$ is

(a) $41°23'36''$ (b) $42°23'36'$ (c) $41°24'36''$ (d) $42°24'36''$

Sol. (*a*) As, $90° = 89° 59' 60''$

$\therefore$ Complement of a angle of $(48° 36' 24'')$

$= \text{Angle of } [90° - 48° 36' 24''] = \text{Angle of } (41° 23' 36'')$

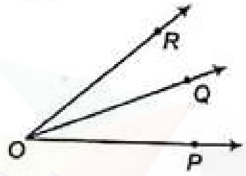
Deg	Min	Sec
89	59	60
− 48	36	24
41	23	36

Some Angle Related Theorems

Adjacent Angles

Two angles are said to be adjacent, if

- They have a common vertex.
- They have a common arm and
- Their non-common arms are on either side of the common arm.



Here, $\angle POQ$ and $\angle ROQ$ are adjacent angles and have

- The same vertex is O,
- a common arm is OQ
- non-common arm OP, OR on either side of OQ.

Linear Pair

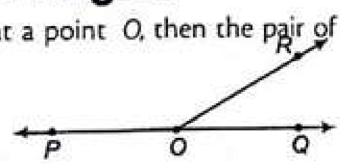
Two angles are said to form a linear pair of angles, if

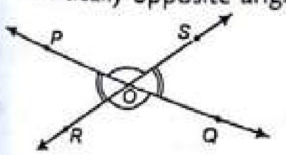
- they are adjacent angles and
- they are supplementary.

> - If a ray stands on a line, then the sum of the adjacent angles, so formed is 180° *i.e.*, $\angle POR + \angle QOR = 180°$.
> - Sum of all the angles around a point is 360°.

Vertically Opposite Angles

If two lines PQ and RS intersects at a point O, then the pair of $\angle POR$ and $\angle QOS$ or pair of $\angle POS$ and $\angle ROQ$ is said to be a pair of vertically opposite angles.



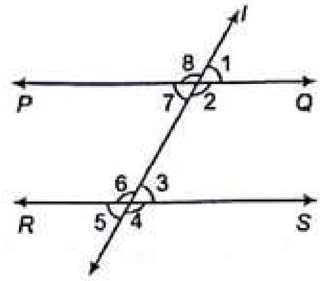


- Vertically opposite angles are always equal *i.e.*, $\angle POS = \angle ROQ$ and $\angle POR = \angle SOQ$.

Angles Made by a Transversal on Parallel Lines

Let PQ and RS be two lines, cut by a transversal *l*. Then,

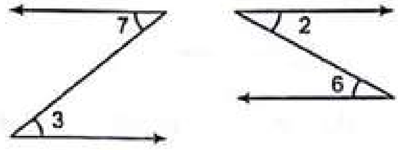
1. Pairs of corresponding angles is always equal.



Here, $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 7$ and $\angle 5$ and $\angle 8$ and $\angle 6$ are all pair of corresponding angles.

Thus, $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 7 = \angle 5$ and $\angle 8 = \angle 6$

2. Pairs of alternate interior angles are alway equal. Here, the pairs of alternate interior angles are.



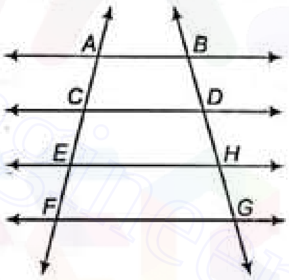
Thus, $\angle 7 = \angle 3$ and $\angle 2 = \angle 6$

3. The sum of pair of consecutive interior angles or allied angles or unjoined angles are always 180°.

   Here, such pairs are : $\angle 2$ and $\angle 3$ and $\angle 7$ and $\angle 6$.

   Thus, $\angle 2 + \angle 3 = \angle 7 + \angle 6 = 180°$.

4.



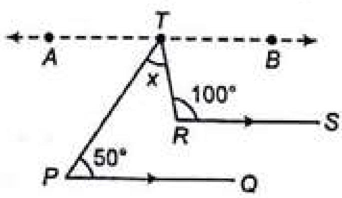
The intercepts cut by parallel lines (more than two) are in proportion $\frac{AC}{EC} = \frac{BD}{DF}$ or $\frac{AC}{EG} = \frac{BD}{FH}$, etc.

Example 5. In the given figure, $PQ \parallel RS$. The value of x is

(a) 50° (b) 80° (c) 75° (d) 65°

Sol. (*a*) Draw $AB \parallel PQ$

$\angle ATP = \angle TPQ = 50°$ and $\angle BTR + \angle TRS = 180°$



$\angle BTR = 80°$

and $\angle ATP + x + \angle BTR = 180°$

$\therefore$ $x = 180° - (50° + 80°) = 50°$

Example 6. In the given figure $AB \parallel CD$. Then, the value of x is

(a) 90° (b) 110° (c) 130° (d) 135°

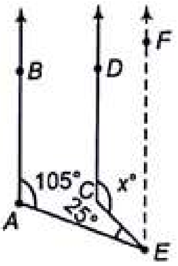
Sol. (*c*) $x + \angle CEF = 180°$,

$\angle CEF = 180° - x$

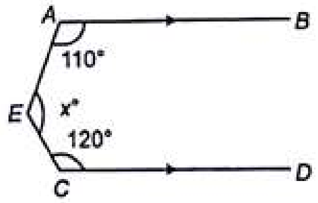
But $\angle BAE + \angle AEF = 180°$

$105° + 25° + (180° - x) = 180°$

$x = 130°$

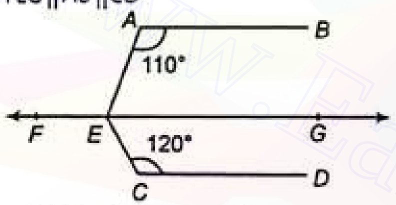


Example 7. In the given figure $AB \parallel CD$ and $\angle BAE = 110°$, $\angle ECD = 120°$ and $\angle AEC = x°$. Then, the value of x is



(a) 70° (b) 105° (c) 120° (d) 130°

Sol. (*d*) Draw $FEG \parallel AB \parallel CD$



$AB \parallel EG$ and AE is the transversal.

$\therefore \quad \angle BAE + \angle AEG = 180° \Rightarrow 110° + \angle AEG = 180°$

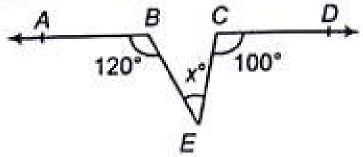
$\therefore \quad \angle AEG = 70°$

Again, $EG \parallel CD$ and EC is the transversal

$\therefore \quad \angle GEC + \angle ECD = 180° \Rightarrow \angle GEC + 120° = 180° \Rightarrow \angle GEC = 60°$

$\therefore \quad x = 70° + 60° = 130°$

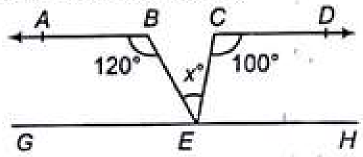
Example 8. In the given figure $AB \parallel CD$, $\angle ABE = 120°$, $\angle DCE = 100°$ and $\angle BEC = x°$ Then, the measurement of x is



(a) 15° (b) 25°
(c) 45° (d) None of these

Sol. (*d*) Through the point E, draw $GEH \parallel AB \parallel CD$.

$AB \parallel GE$ and BE is the transversal.



$\therefore \quad \angle ABE + \angle GEB = 180° \Rightarrow 120° + \angle GEB = 180°$

$\therefore \quad \angle GEB = 60°$

Again, $CD \parallel EH$ and CE is the transversal.

$\therefore \quad \angle DCE + \angle CEH = 180° \Rightarrow 100° + \angle CEH = 180°$

$\Rightarrow \quad \angle CEH = 80°$

Now, $\angle GEB + \angle BEC + \angle CEH = 180°$

$\Rightarrow \quad 60° + x + 80° = 180° \Rightarrow x = 40°$

Exercise

1. How many least number of distinct points determine a unique line?
   (a) One (b) Two (c) Three (d) Infinite
2. Which one of the following determines a plane?
   (a) A line and a point on it (b) Two points
   (c) Three non-collinear points (d) None of these
3. Which of the following statements is false?
   (a) A line segment can be produced to any desired length.
   (b) Through a given point, only one straight line can be drawn.
   (c) Through two given points, it is possible to draw one and only one straight line.
   (d) Two straight lines can intersect in only one point.
4. Number of pairs of vertical angles formed when two lines intersect is/a
   (a) one pair (b) two pairs
   (c) four pairs (d) None of these
5. A set of concurrent lines contain
   (a) an indefinite number of common points
   (b) a common line
   (c) exactly two common points
   (d) a unique common point
6. Two parallel lines are cut by a transversal, then which of the following are true?
   I. Pair of alternate interior angles are congruent.
   II. Pair of corresponding angles are congruent.
   III. Pair of interior angles on the same side of the transversal are supplementary.
   (a) I, II and III are true (b) I and III are true
   (c) I and II are true (d) II and III are true
7. Consider the following statements related to three lines L_1, L_2 and L_3 in the same plane.
   I. If L_2 and L_3 are both parallel to L_1, then they are parallel to each other.
   II. If L_1 and L_3 are both perpendicular to L_1 and they are parallel to each other.
   III. If there is acute angle between L_1 and L_3, then L_2 is parallel to L_3.
   Which of these statements are correct?
   (a) I and II only (b) II and III only
   (c) All of these (d) None of these
8. Three lines intersect each other in pairs. What is the number of angles so formed?
   (a) 3 (b) 6
   (c) 9 (d) 12

9. A point P moves such that its distance from two given points A and B are equal. Then, what is the locus of the point P?
   (a) A straight line which is the right bisector of AB
   (b) A circle with centre at B
   (c) A circle with centre at A
   (d) A straight line passing through either A or B

10. If Q lies between P and R and $PR = 10$, $QR = 6$, then PQ^2 is
    (a) 4 (b) 16 (c) 5 (d) 9

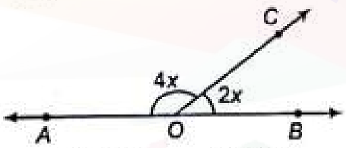
11. The measure of the supplementary angle of 130° is
    (a) 180° (b) 260° (c) 50° (d) 80°

12. The measure of complementary angle of 12° 25′ 40″ is
    (a) 77° 34′ 20″ (b) 77° 36′ 20″
    (c) 77° 24′ 20″ (d) 77° 34′

13. An angle is 14° more than its complement. Then, its measure is
    (a) 166° (b) 86° (c) 76° (d) 52°

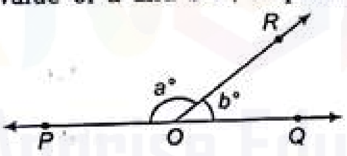
14. The measure of an angle is twice the measure of its supplementary angle. So, its measure is
    (a) 120° (b) 60° (c) 100° (d) 90°

15. In figure $\angle AOC$ and $\angle BOC$ form a linear pair. Then, the value of x



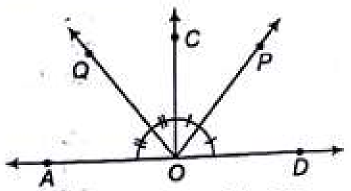
   (a) 15° (b) 40° (c) 25° (d) 30°

16. $\angle POR$ and $\angle QOR$ form a linear pair. If $a - b = 80°$, then the value of a and b is, respectively.



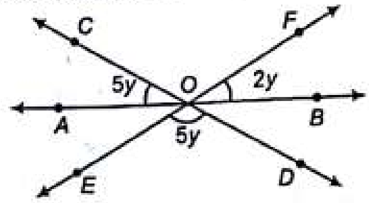
   (a) 95°, 85° (b) 108°, 72°
   (c) 130°, 50° (d) 105°, 75°

17. In figure OP bisects $\angle DOC$ and OQ bisects $\angle AOC$, then $\angle POQ$ is equal to



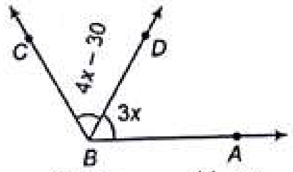
   (a) 90° (b) 75° (c) 105° (d) 80°

18. In figure determine the value of y



   (a) 25° (b) 35° (c) 15° (d) 40°

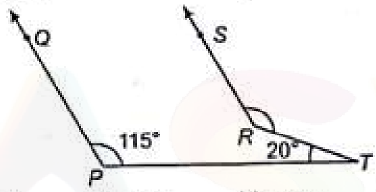
19. In figure, the value of x which would make ABC a line.



   (a) 40° (b) 35° (c) 45° (d) 30°

20. If P and Q are points on the opposite sides of a straight line AB. If O is a point on AB such that $\angle AOP = \angle BOQ$, then when one of the following is correct?
    (a) $\angle AOQ < \angle BOP$
    (b) $\angle AOQ > \angle BOP$
    (c) $\angle AOP = 180° - \angle AOQ$
    (d) $\angle AOP = 90° - \angle AOQ$

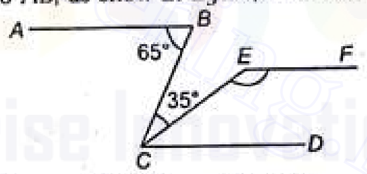
21. In the given figure, if $PQ \parallel RS$, $\angle QPT = 115°$ and $\angle PTR = 20°$, then $\angle SRT$ is equal to



   (a) 155° (b) 150° (c) 135° (d) 95°

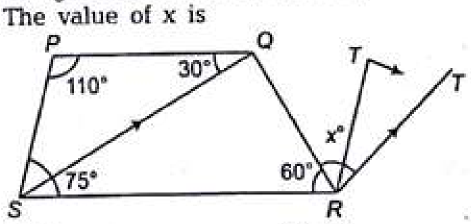
22. AB and CD are two parallel lines. PQ cuts AB and CD at E and F, respectively. EL is the bisector of $\angle FEB$. If $\angle LEB = 35°$; then $\angle CFQ$ will be
    (a) 110° (b) 85° (c) 70° (d) 95°

23. AB and CD are two parallel lines. The points B and C are joined such that $\angle ABC = 65°$. A line CE is drawn making angle of 35° with the line CB, EF is drawn parallel to AB, as show in figure, then $\angle CEF$ is equal to



   (a) 160° (b) 155° (c) 150° (d) 145°

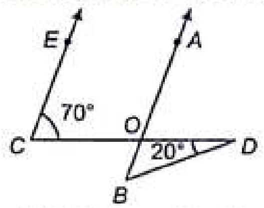
24. In the figure below, RT is drawn parallel to the line SQ. The value of x is



   (a) 85° (b) 45°
   (c) 120° (d) 75°

25. AB is a straight line and O is a point on AB. If one draws a line OC not coinciding with OA or OB, then the $\angle AOC$ and $\angle BOC$ are
    (a) equal
    (b) complementary
    (c) supplementary
    (d) together equal to 130°

26. In the given figure, if $EC \parallel AB$, $\angle ECD = 70°$, $\angle BDO = 20°$, then $\angle OBD$ is equal to



(a) 70° (b) 60° (c) 50° (d) 20°

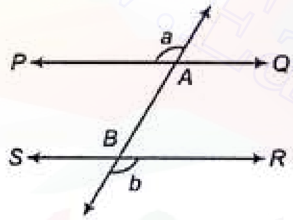
27. Two parallel lines AB and CD are intersected by a transversal line EF at M and N, respectively. The lines MP and NP are the bisectors of the interior angles BMN and DNM on the same side of the transversal. Then, $\angle MPN$ is equal to

(a) 90° (b) 45° (c) 135° (d) 60°

28. AB and CD are parallel straight lines of lengths 5 cm and 4 cm, respectively. AD and BC intersect at a point O such that $AO = 10$ cm, then OD equals

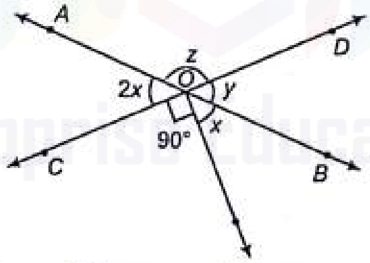
(a) 7 cm (b) 8 cm (c) 5 cm d) 6 cm

29. In the given figure, if $PQ \parallel SR$ the relation between $\angle a$ and $\angle b$ is



(a) $\angle a \neq \angle b$ (b) $\angle a < \angle b$ (c) $\angle a = \angle b$ (d) $\angle a > \angle b$

30. In the given figure, if $\angle COE = 90°$, then the value of x is

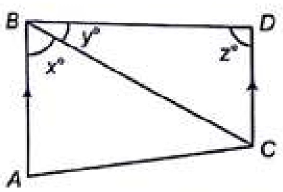


(a) 120° (b) 60° (c) 45° (d) 30°

31. $\angle a$ and $\angle b$ form a linear pair. If $a - 2b = 30°$, then a and b are respectively

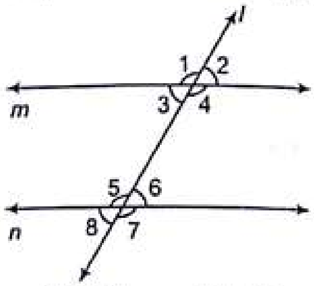
(a) 128°, 52° (b) 120°, 60° (c) 130°, 50° (d) 110°, 70°

32. In figure $AB \parallel CD$, if $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$, then the value of x



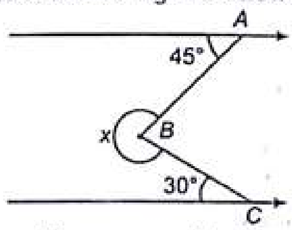
(a) 48° (b) 96°
(c) 36° (d) None of these

33. If $\angle 1 = (5x - 20)°$ and $\angle 7 = (2x + 10)°$, then $\angle 7$ is



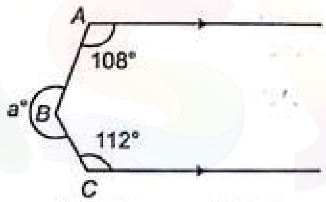
(a) 38° (b) 10° (c) 30° (d) 70°

34. The value of x in the figure below is



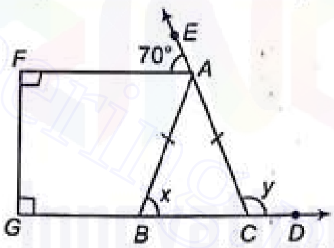
(a) 75° (b) 185° (c) 285° (d) 245°

35. Find the value of 'a'.



(a) 120° (b) 140° (c) 90° (d) 150°

36. In the figure given values of x and y are, respectively



(a) 70°, 110° (b) 110°, 70°
(c) 120°, 60° (d) 70°, 90°

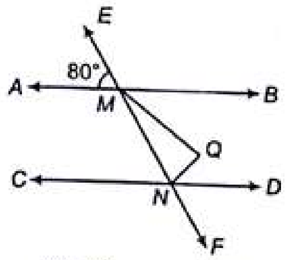
37. If the distances XY, YZ, XZ, XU, YU and ZU among points X, Y, Z, U are respectively 5, 5, 10, 10, 15 and 20, then

(a) X, Z, U are collinear and Y is not collinear with any pair
(b) X, Y, Z are collinear and U is not collinear with any pair
(c) X, Y, U are collinear and Z is not collinear with any pair
(d) U, Y, Z are collinear and X is not collinear with any pair

38. X, Y, Z, U are four points in a straight line. If distance from X to Y is 15, Y to Z is 5, Z to U is 8 and X to U is 2, then the correct sequence of the points will be

(a) $X - Z - Y - U$
(b) $X - Y - Z - U$
(c) $X - Z - U - Y$
(d) $X - U - Z - Y$

39. If AB is parallel to CD, EF intersects them at M and N. The bisectors of $\angle BMN$ and $\angle MND$ meet at Q. If $\angle AME = 80°$, then $\angle MQN$ is

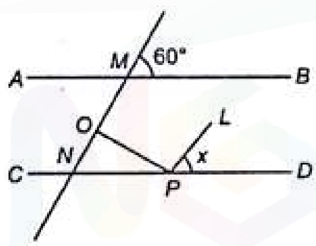


(a) 90° (b) 80° (c) 60° (d) 70°

40. Which of the following pairs of lines can be parallel?
I. Two diameters of a circle.
II. Two tangents to a circle.
III. Two chords of a circle.
IV. A chord of a circle and a tangent to a circle
Select the correct answer using the codes given below.
(a) I, II, and IV (b) I, III and IV
(c) II, III and IV (d) I, II and III

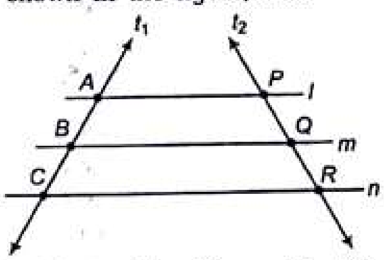
41. If on the number line, three points P, Q, R represents the numbers $\frac{1}{4}, \frac{5}{11}, \frac{9}{17}$, respectively. Then,
(a) Q lies between P and R (b) P lies between Q and R
(c) R lies between Q and P (d) R is to the left of Q

42. In the figure $AB \parallel CD$, $\angle DPL = \frac{1}{2}\angle NPO$, the value of $x°$ is



(a) 30° (b) 40° (c) 15° (d) 25°

43. If l, m, n are three parallel lines and the transversal t_1 and t_2 cut the lines l, m, n at the point A, B, C and P, Q, R as shown in the figure, then

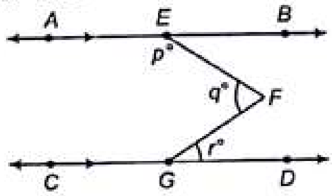


(a) $\frac{BC}{PQ} = \frac{AB}{QR}$ (b) $\frac{AP}{BQ} = \frac{BQ}{CR}$ (c) $\frac{AB}{BC} = \frac{PQ}{QR}$ (d) $\frac{BQ}{AP} = \frac{PQ}{AB}$

44. Let D be the mid-point of a straight line AB and let C be a point different from D such that $AC = BC$. Then,
(a) $\angle CDB$ is acute (b) $\angle CDB > 90°$
(c) $\angle CDB = 90°$ (d) $CA \perp AB$

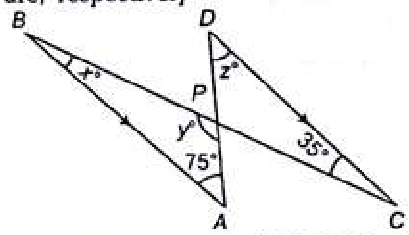
45. Let P, Q, R be three non-collinear points. The number of circles passing through the points P, Q, R is
(a) one (b) two
(c) three (d) None of these

46. In the given figure, $AB \parallel CD$, then which of the following is true?



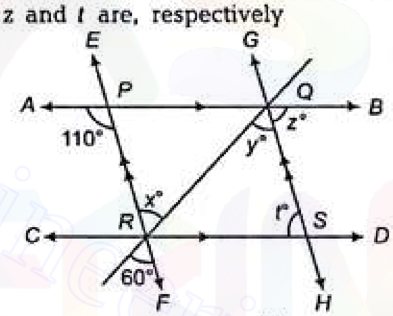
(a) $P + q - r = 180°$ (b) $p + q + r = 180°$
(c) $p - q + r = 180°$ (d) $p + q - 2r = 180°$

47. In the given figure $AB \parallel CD$, then the values of x, y and z are, respectively



(a) 75°, 35°, 80° (b) 70°, 35°, 60°
(c) 35°, 70°, 75° (d) 70°, 35°, 80°

48. In the given figure $AB \parallel CD$ and $EF \parallel GH$. The values of x, y, z and t are, respectively

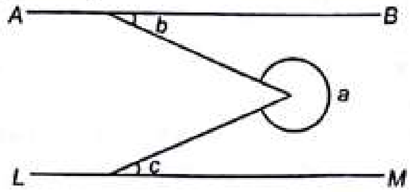


(a) 60, 75, 75, 60 (b) 50, 75, 75, 65
(c) 60, 70, 60, 70 (d) 60, 60, 70, 70

49. AB is a straight line. C is a point whose perpendicular distance from AB is 3 cm. What are the number of points which are at a perpendicular distance of 1 cm from AB and at a distance 4 cm from C? **(CDS 2011 I)**
(a) 1 (b) 2 (c) 3 (d) 4

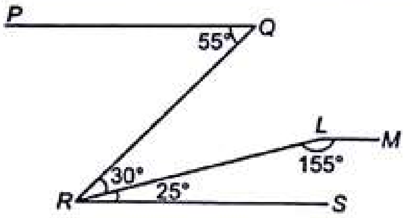
50. The line segments AB and CD intersect at O. OF is the internal bisector of obtuse angle BOC and OE is the internal bisector of acute angle AOC. If $\angle BOC = 130°$, what is the measure of $\angle FOE$? **(CDS 2010 II)**
(a) 90° (b) 110° (c) 115° (d) 120°

51. In the figure given below. AB is parallel to LM. What is the angle a equal to? **(CDS 2010 I)**



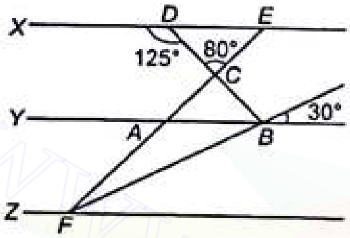
(a) $\pi + b + c$ (b) $2\pi - b + c$
(c) $2\pi - b - c$ (d) $2\pi + b - c$

52. In the figure given below, PQ is parallel to RS. What is the angle between the lines PQ and LM? **(CDS 2010 II)**



(a) 175° (b) 177° (c) 179° (d) 180°

53. Three straight lines X, Y and Z are parallel and the angles are as shown in the figure below. What is $\angle AFB$ equal to? **(CDS 2010 I)**

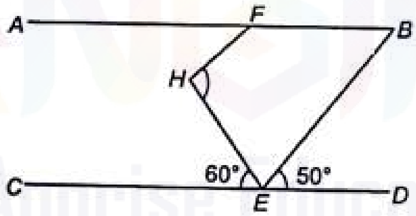


(a) 20° (b) 15° (c) 30° (d) 10°

54. The length of a line segment AB is 2 units. It is divided into two parts at the point C such that $AC^2 = AB \times CB$. What is the length of CB?

(a) $3+\sqrt{5}$ units (b) $3-\sqrt{5}$ units
(c) $2-\sqrt{5}$ units (d) $\sqrt{3}$ units

55. In the figure, AB is parallel to CD and BE is parallel to FH. What is $\angle FHE$ equal to? **(CDS 2009 II)**



(a) 110° (b) 120° (c) 125° (d) 130°

56. Let AB and AC be two rays intersecting at A. Let D, E be the points lying on AB, AC, respectively and P be the point such that P divides the line DE such that $PD : PE = AD : AE$. What is the locus of the point P? **(CDS 2009 I)**

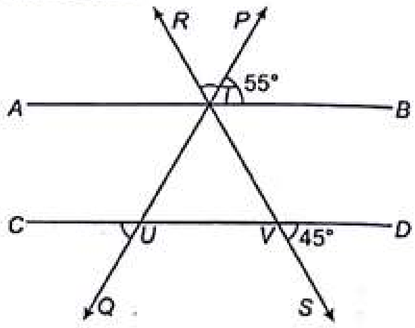
(a) The angle bisector of angle A
(b) The angle trisector of angle A
(c) The perpendicular bisector of angle A
(d) None of the above

57. Assertion (A) Two distinct lines cannot have more than one point in common.

Reason (R) Any number of lines can be drawn through one point. **(CDS 2008 I)**

(a) A and R are correct and R is correct explanation of A.
(b) A and R are correct but R is not correct explanation of A.
(c) A is correct but R is wrong.
(d) A is wrong but R is correct.

58. In the given figure, if $AB \| CD$, $\angle PTB = 55°$ and $\angle DVS = 45°$, then what is the sum of the measures of $\angle CUQ$ and $\angle RTP$? **(CDS 2008 II)**

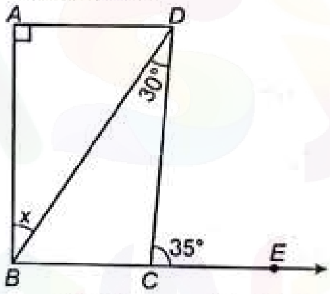


(a) 180° (b) 135° (c) 110° (d) 100°

59. The angle between the legs of a compass is 60° and each leg is 10 cm long. How far apart are the points on which the legs of the compass rest?

(a) 5 cm (b) 10 cm (c) $5\sqrt{3}$ cm (d) $10\sqrt{3}$ cm

60. Given that, $AD \| BE$, $\angle DCE = 85°$, $\angle BDC = 30°$, then what is the value of x? **(CDS 2008 I)**



(a) 30° (b) 35° (c) 45° (d) 55°

61. LM is a straight line and O is a point on LM. Line ON is drawn not coinciding with OL or OM. If $\angle MON$ is one-third of $\angle LON$, what is $\angle MON$ equal to?

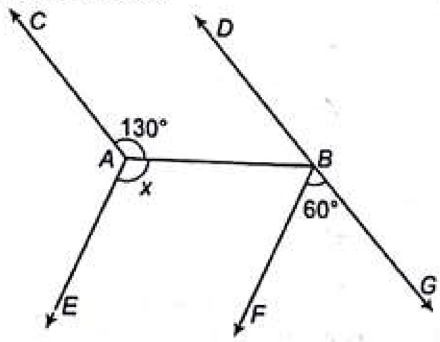
(a) 45° (b) 60° (c) 75° (d) 80°

62. Consider the following statements
Two lines intersected by a transversal are parallel, if
I. the pairs of corresponding angles are equal.
II. the interior angles on the same side of the transversal are supplementary.

Which of the statements given above is/are correct?

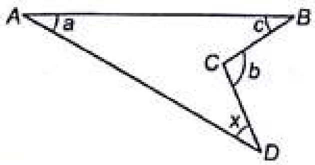
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

63. In the figure given below, $AC \| BD$ and $AE \| BF$. What is the value of $\angle x$? **(CDS 2007 II)**



(a) 130° (b) 110° (c) 70° (d) 50°

64. What is the value of x in the figure given below? **(CDS 2007 II)**



(a) $b-a-c$
(b) $b-a+c$
(c) $b+a-c$
(d) $\pi-(a+b+c)$

65. Consider the following statements

If two parallel lines are intersected by a transversal, then

I. each pair of corresponding angles are equal.

II. each pair of alternate are unequal.

Which of the statements given above is/are correct?

(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

Answers

1. (b)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (b)
11. (c)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (c)
21. (c)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (d)
31. (c)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (a)	**40.** (c)
41. (a)	**42.** (c)	**43.** (c)	**44.** (c)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (d)	**49.** (c)	**50.** (a)
51. (c)	**52.** (d)	**53.** (b)	**54.** (b)	**55.** (a)	**56.** (a)	**57.** (b)	**58.** (b)	**59.** (b)	**60.** (b)
61. (a)	**62.** (c)	**63.** (b)	**64.** (a)	**65.** (a)					

Hints and Solutions

1. In order to determine a unique line, atleast two distinct points are required.

2. Three non-collinear points are sufficient enough to determine a plane.

3. Since, an infinite number of straight lines can be drawn to pass through a given point. Hence, (b) is false statement.

4. When two lines intersect, then four angles are formed. Thus, there are 2 pairs of vertical angles.

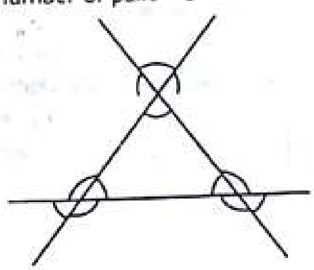
5. A set of concurrent lines contain a unique common point, through which all the lines pass.

6. All the three statements are true regarding the condition.

7. Only I and II statements are true in context of the given condition.

8. We know, when two lines intersect each other it makes 4 angles.

$\therefore$ The total number of pairs $=3$



$\therefore$ Total number of angles $=3\times4=12$

9. The locus of P is a straight line which is the right bisector of AB.

10. Here, $PQ+QR=PR \Rightarrow PQ=PR-QR=10-6=4$

$PQ^2=4^2=16$

11. Supplementary angle of $130°=180°-130°=50°$

12. Complementary angle of $12°\,25'\,40''=90°-12°\,25'\,40''$

$=89°\,59'\,60''-12°\,25'\,40''$

$=(89-12)°+(59'-25')+(60-40)''$

$=77°+34'+20''=77°\,34'\,20''$

13. Let angle be x, and its complement is $90°-x$.

Here, $x=(90°-x)+14$

$2x=104° \Rightarrow x=\dfrac{104°}{2}=52°$

14. Let angle be x, and its supplementary $=180°-x$

So, $x=2(180°-x)$

$3x=360° \Rightarrow x=120°$

15. Here, $\angle AOC+\angle BOC=180°$

$4x+2x=180°$

$6x=180° \Rightarrow x=30°$

16. Here, $a+b=180°$ and $a-b=80° \Rightarrow 2a=260° \Rightarrow a=130°$

$b=180°-130°=50°$

So, $a=130°, b=50°$

17. $\angle AOC+\angle BOC=180°$

$\dfrac{1}{2}\angle AOC+\dfrac{1}{2}\angle BOC=90°$

$\angle QOC+\angle COP=90° \Rightarrow \angle QOP=90°$

18. Here, as OA, OB are opposite rays.

$\angle AOC+\angle COF+\angle FOB=180°$

$5y+5y+2y=180°$ [vertically opposite angles]

$12y=180° \Rightarrow y=\dfrac{180°}{12°}=15°$

19. Here, if ABC is a straight line,

then $\angle ABD+\angle DBC=180°$

$4x-30°+3x=180° \Rightarrow 7x=180°+30°$

$7x=210° \Rightarrow x=30°$

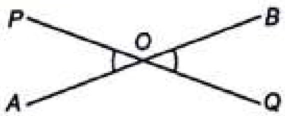
20. Here, as given $\angle AOP = \angle BOQ$

POQ is a straight line.

So, $\angle AOP + \angle AOQ = 180°$

$\angle AOP = 180° - \angle AOQ$

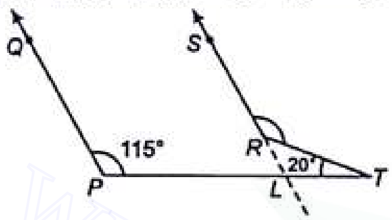
21. Here, $\angle QPL + \angle PLR = 180°$



$\angle PLR = 180° - \angle QPL = 180° - 115° = 65°$

$\angle RLP = \angle LRT + \angle RTL$

$\angle LRT = \angle RLP - \angle RTL = 65° - 20° = 45°$



$\angle SRT + \angle LRT = 180°$

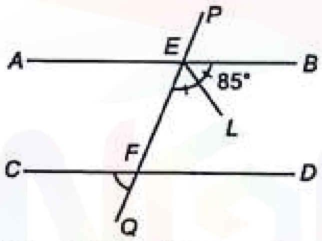
$\angle SRT = 180° - \angle LRT$

$\angle SRT = 180° - 45° = 135°$

$\angle SRT = 135°$

22. $\angle LEB = 35°$

$\angle FEB = 2 \times \angle LEB = 70°$



$\angle AEB = \angle AEF + \angle BEF = 180°$

$\Rightarrow \quad \angle AEF = 180° - 70 = 110°$

$\angle CFQ = \angle AEQ = 110°$ (alternate angles)

23. $\angle ABC = \angle BCD$ as $AB \parallel CD$

$\angle BCD = 65°$

$\angle ECD = 65° - \angle BCE = 65° - 35° = 30°$

$\angle CEF + \angle ECD = 180°$

$\angle CEF = 180° - 30° = 150°$

24. Here, $\angle PSQ = 180° - (110 + 30)° = 40°$

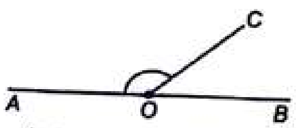
$\angle QSR = 75° - 40° = 35°$

$\angle QSR + \angle SRT = 180°$ $(\because SQ \parallel RT)$

$35° + 60° + x = 180°$

$x = 180° - 95° \; x = 85°$

25. As, AOB are collinear, so



$\angle AOC + \angle BOC = 180°$

Hence, are supplementary.

26. $\angle AOD = \angle ECO \Rightarrow \angle AOD = 70°$

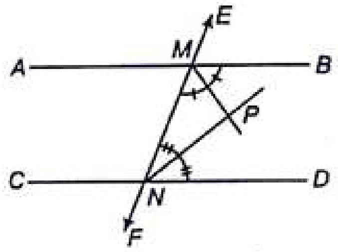
So, $\angle BOD = 110°$

Hence, in ΔBOD

$\angle OBD + \angle BOD + \angle ODB = 180°$

$\angle OBD = 180° - (110 + 20)° \Rightarrow \angle OBD = 50°$

27. As, $\angle BMN + \angle DNM = 180°$

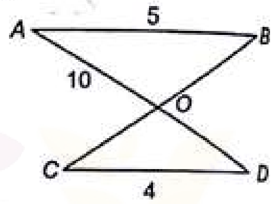


$\angle PMN + \angle PNM = 90°$

$\angle MPN = 180° - (\angle PMN + \angle PNM)$

$\angle MPN = 180° - (90°)$

28. Here, as $AB \parallel CD$



So, $\dfrac{AB}{CD} = \dfrac{AO}{OD}$

$\dfrac{5}{4} = \dfrac{10}{OD} \Rightarrow OD = \dfrac{4 \times 10}{5} = 8$ cm

29. $\angle a = \angle QAB$ (Vertically opposite angles)

$\angle QAB = \angle b$ (corresponding angles)

$\Rightarrow \quad \angle a = \angle b$

30. Here, $\angle BOD = \angle AOC$

$\Rightarrow \quad 2x = y$

New, $\angle COE + \angle EOB + \angle BOD = 180°$

$90° + x + 2x = 180°$

$3x = 90° \Rightarrow x = 30°$

31. Here, $a + b = 180°$ ...(i)

and $a - 2b = 30°$ given...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$3b = 150° \Rightarrow b = 50°$

Put in Eq. (i)

$a = 180° - b = 180° - 50° \Rightarrow a = 130°$

32. As, $AB \parallel CD$ and BD cuts them

$\angle ABD = \angle BDC$, so $\angle BDC = x$

So, $x + y + z = 180°$

In ΔBCD

$\dfrac{4}{3}y + y + z = 180° \Rightarrow \dfrac{7y}{3} + z = 180°$

$\dfrac{7}{3}\left(\dfrac{3}{8}z\right) + z = 180° \Rightarrow \dfrac{7}{8}z + z = 180°$

$\dfrac{15z}{8} = 180° \Rightarrow z = 96°$

So, $y = \dfrac{3}{8} \times 96° = 36°$ and $x = \dfrac{4}{3} \times 36° = 48°$

33. Here, $\angle 1 = \angle 7$ (corresponding angles)

So, $5x - 20° = 2x + 10°$

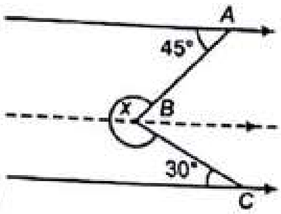
$3x = 30° \Rightarrow x = 10°$

Hence, $\angle 7 = 2x + 10° = 2(10°) + 10° = 30°$

34. Here, $\angle ABC = 45^\circ + 30^\circ$

$x = 360^\circ - \angle ABC = 360^\circ - 75^\circ$

$x = 285^\circ$



35. Here, $\angle A + \angle C + a = 360^\circ$

$108^\circ + 112^\circ + a = 360^\circ$

$a = 360^\circ - (220^\circ)$

$\Rightarrow \quad a = 140^\circ$

36. Here, $\angle ABC = \angle ACB = x$

So, $x + y = 180^\circ$ and

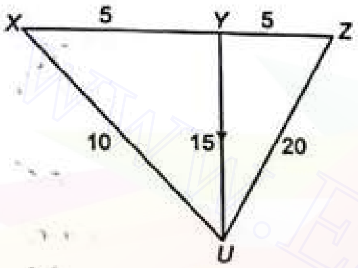
$\angle EAF = \angle ACB = 70^\circ$ (corresponding angles)

$x = 70^\circ$

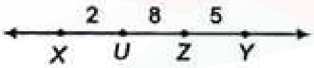
$y = 180^\circ - 70^\circ \Rightarrow y = 110^\circ$

37. Here, $XY = 5$, $YZ = 5$, $XZ = 10$

So, only if X, Y, Z are collinear and U is not collinear with any pair.



38. Here, $XY = 15$, $YZ = 5$, $ZU = 8$ and $XU = 2$



39. $\angle BMN + \angle DNM = 180^\circ$

$\frac{1}{2}\angle BMN + \frac{1}{2}\angle DNM = 90^\circ$

$\angle QMN + \angle MNQ = 90^\circ$

$\angle MQN = 180^\circ - (\angle QMN + \angle MNQ) \Rightarrow \angle MQN = 90^\circ$

40. Clearly, two diameters of a circle can't be parallel. but it is coincide.

41. Q lies between P and R.

42. $\angle ONP = 60^\circ$

$\angle OPN = 90^\circ - \angle ONP$

$= 90^\circ - 60^\circ \Rightarrow \angle OPN = 30^\circ$

$\angle DPL = \frac{1}{2}\angle NPO = \frac{1}{2}(30)^\circ$

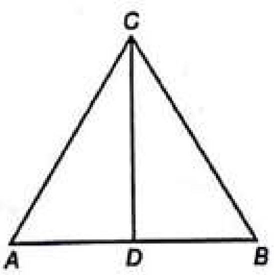
$\Rightarrow \quad \angle DPL = 15^\circ \Rightarrow x = 15^\circ$

43. Here, $\frac{AB}{BC} = \frac{PQ}{QR}$. If three lines are parallel and two transversal cuts the lines, then the intercepts made by the lines are proportional.

44. As, C is equidistant from A and B.

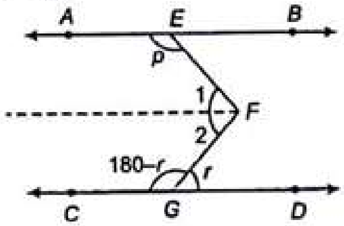
So, $\angle CDA = \angle CDB$

$\Rightarrow \quad \angle CDB = 90^\circ$



45. There is one and only one circle passing through three non-collinear given points.

46. Here, $\angle 1 + p = 180^\circ$



$\angle 2 + 180^\circ - r = 180^\circ$

Adding

$\angle 1 + \angle 2 + p + 180^\circ - r = 360^\circ$

$p + q - r = 180^\circ \quad (\because \angle 1 + \angle 2 = q)$

47. $x = 35^\circ$ (acute angle)

$z^\circ = 75^\circ$ (acute angle)

$x + y + 75^\circ = 180^\circ \quad (\because \Delta BPA)$

$y = 180^\circ - (75^\circ + 35^\circ)\ y = 70^\circ$

48. $x^\circ = 60^\circ$ (vertically opposite)

$x = y$ (acute angles)

$\Rightarrow \quad y = 60^\circ$

$\angle PRS = 110^\circ$

$\angle QRS + x^\circ = 110^\circ$

$\angle QRS = 110^\circ - 60^\circ = 50^\circ$

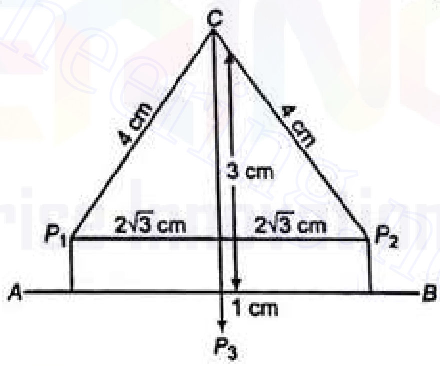
$\therefore \quad t = 180^\circ - (y + \angle QRS) = 180^\circ - (60^\circ + 50^\circ)$

$t = 70^\circ$

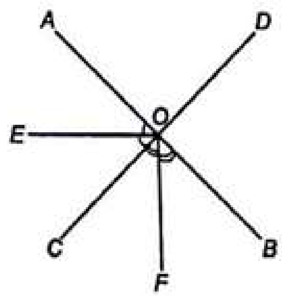
Also, $t = z =$ (acute angles)

$\therefore \quad z = 70^\circ$

49. $\therefore$ Required number of points $= 3 \quad (\because P_1, P_2$ and $P_3)$



50. Given, $\angle BOC = 130^\circ$



$\because \quad \angle BOC + \angle AOC = 180^\circ$ (linear pair)

$\Rightarrow \quad 130^\circ + \angle AOC = 180^\circ \Rightarrow \angle AOC = 50^\circ$

Now, $\angle BOC = 130^\circ$

$\Rightarrow \quad \angle BOF + \angle FOC = 130^\circ$

$$\angle FOC + \angle FOC = 130^\circ \quad (\because OF \text{ is bisector of } \angle BOC)$$

$\Rightarrow$ $\angle FOC = 65^\circ$ and $\angle AOC = 50^\circ$

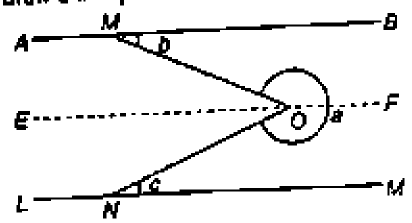
$\Rightarrow$ $\angle AOE + \angle EOC = 50^\circ \Rightarrow \angle EOC + \angle EOC = 50^\circ$

$\Rightarrow$ $\angle EOC = 25^\circ$

$\therefore$ $\angle EOF = \angle EOC + \angle FOC$

$\therefore$ $= 65^\circ + 25^\circ = 90^\circ$

51. Let we draw a line parallel to AB which is EF.



$\therefore$ $\angle EOP = \angle OPB = b$ (alternate angle)

and $\angle EON = \angle ONM$ (alternate angle)

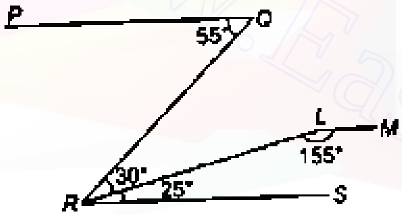
$\angle EON = c$

$\therefore$ $\angle PON = b + c$

$\because$ $\angle PON + a = 2\pi$

$\therefore$ $a = 2\pi - \angle PON = 2\pi - b - c$ ...(i)

52. Since, $PQ \parallel RS$



$\therefore$ $\angle PQR = \angle QRS$ (alternate angle)

and $\angle SRL + \angle RLM = 180^\circ$

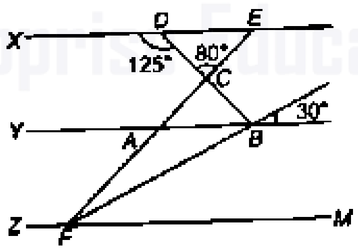
$\Rightarrow$ $RS \parallel LM$ ...(ii)

From relations (i) and (ii), we get

$PQ \parallel LM$

$\therefore$ Angle between the lines PQ and LM is 180°

53. $\angle CDE = 180^\circ - 125^\circ = 55^\circ$



In ΔDCE

$\angle CED = 180^\circ - 55^\circ - 80^\circ = 45^\circ$

and $\angle ABF = 30^\circ$ (vertically opposite)

Also, $\angle ABF = \angle BFM = 30^\circ$ (alternate angle)

and $\angle DEF = \angle EFM$ (alternate angle)

$\angle EFM = 45^\circ$

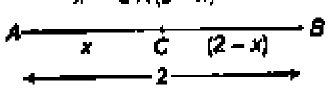
$\Rightarrow$ $\angle EFB + \angle BFM = 45^\circ$

$\Rightarrow$ $\angle EFB = 45^\circ - 30^\circ$

$\Rightarrow$ $\angle AFB = 15^\circ$

54. Given, $AC^2 = AB \times CB$

$\Rightarrow$ $x^2 = 2 \times (2 - x)$



$x^2 = 4 - 2x$

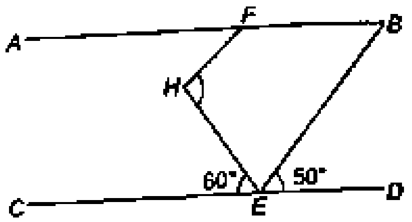
$\Rightarrow$ $x^2 + 2x - 4 = 0$

$\Rightarrow$ $x = \dfrac{-2 \pm \sqrt{4 + 16}}{2 \times 1} \Rightarrow x = -1 \pm \sqrt{5}$

Now, $BC = 2 - (-1 \pm \sqrt{5})$

$= 3 - \sqrt{5}$ (neglect $3 + \sqrt{5} \because 3 + \sqrt{5} > 2$)

55. $\angle HEB = 180^\circ - 60^\circ - 50^\circ = 70^\circ$



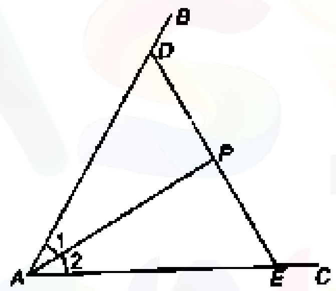
Since, $HF \parallel BE$ and HE is transversal line.

$\therefore$ $\angle FHE + \angle HEB = 180^\circ$ (interior angle)

$\Rightarrow$ $\angle FHE + 70^\circ = 180^\circ$

$\Rightarrow$ $\angle FHE = 110^\circ$

56. Since, $\dfrac{PD}{PE} = \dfrac{AD}{AE} = \dfrac{AP}{AP}$



ΔDAP and ΔAPE are similar. So, $\angle 1 = \angle 2$

AP is bisector of $\angle A$.

Hence, the locus of P is the triangle bisector of angle A.

57. It is true that two distinct lines intersect only one point.

(R) It is true that, from a one point we can draw any number of lines. But (R) is not a correct explanation of (A).

58. Since, $\angle PTB = 55^\circ$

Then, $\angle TUV = 55^\circ$ (corresponding angle)

Also, $\angle PTB = \angle UTA = 55^\circ$ (vertically opposite angle)

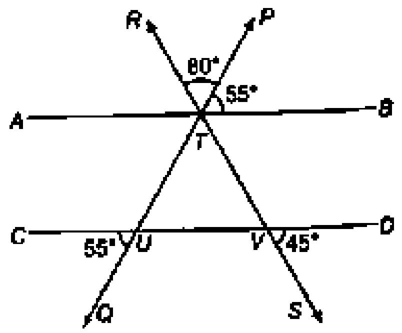
Also given, $\angle DVS = 45^\circ$

Then, $\angle UVT = 45^\circ$ (vertically opposite angle)

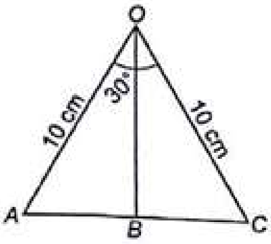
In ΔUTV $\angle T = 180^\circ - (55^\circ + 45^\circ) = 80^\circ$

$\Rightarrow$ $\angle T = \angle PTR = 80^\circ$ (vertically opposite angle)

$\therefore$ $\angle CUQ + \angle RTP = 55^\circ + 80^\circ = 135^\circ$



59. In ΔAOB



$$\sin 30° = \frac{AB}{OA}$$

$\Rightarrow$ $\frac{1}{2} = \frac{AB}{10}$

$\Rightarrow$ $AB = 5$ cm

Now, $AC = 2\,AB = 2 \times 5 = 10$ cm

60. $\because$ $AD \| BE$

$\therefore$ $\angle ADC = \angle DCE$ (alternate angle)

$\Rightarrow$ $\angle ADB + 30° = 85°$

$\Rightarrow$ $\angle ADB = 55°$

and $\angle BAD = 90°$ (given)

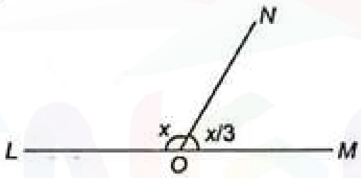
Now in ΔABD

$$\angle ABD + \angle ADB + \angle BAD = 180°$$

$\Rightarrow$ $x + 55° + 90° = 180°$

$\Rightarrow$ $x = 180° - 145° = 35°$

61. Given that,



$$\angle MON = \frac{1}{3}\angle LON$$

Let $\angle LON = x$, then, $\angle MON = \frac{x}{3}$

We know that,

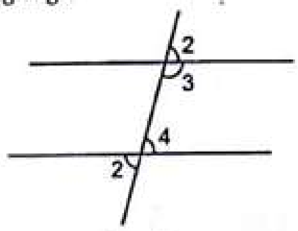
$\angle LON + \angle MON = 180°$ (linear pair)

$\Rightarrow$ $x + \frac{x}{3} = 180°$

$\Rightarrow$ $x = \frac{180° \times 3}{4} = 135°$

Thus, $\angle MON = \frac{x}{3} = \frac{135°}{3} = 45°$

62. Corresponding angle



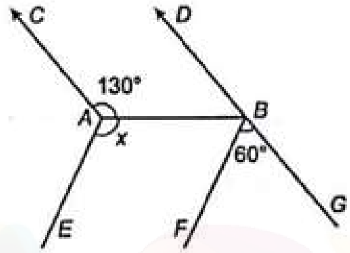
$$\angle 1 = \angle 2$$

and interior angles

$$\angle 3 + \angle 9 = 180°$$

i.e., supplementry.

63. Since, $AC \| BD$



$\therefore$ $\angle DBA = 180° - 130° = 50°$

DBG is straight line.

$\therefore$ $\angle DBA + \angle ABF + \angle FBG = 180°$

$\Rightarrow$ $50° + \angle ABF + 60° = 180°$

$\Rightarrow$ $\angle ABF = 70°$

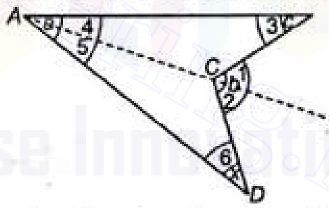
Since $AE \| BF$

$\therefore$ $x = 180° - \angle ABF$

$= 180° - 70° = 110°$

64. $\angle 1 = \angle 3 + \angle 4$, $\angle 2 = \angle 5 + \angle 6$

(exterior angle is equal to sum of two opposite interior angles)



$\Rightarrow$ $\angle 1 + \angle 2 = \angle 3 + \angle 4 + \angle 5 + \angle 6$

$\Rightarrow$ $b = c + a + x$

$\therefore$ $x = b - c - a$

65. If two parallel lines are intersected by a transversal, then each pair of corresponding angles and of alternate angles are equal. Therefore, both statement are correct.