Chapter

Numerical Methods

CONTENTS

Simpson

In problems related to science and technology, we generally come across various types of equations, simultaneous equations integrals for which we do not have standard methods for solutions. In such cases we find their approximate solutions through a series of numerical computations, using certain techniques which are generally known as numerical technequies or numerical computation or numerical methods. The numerical methods give approximate value (answer).

As a thumb rule, if we want the final answer to n places of decimal, we take the numbers involved and the intermediate computations results to (n + 2) places of decimal.

Numerical integration is the process of computing the value of a definite integral.

Assignment (Basic and Advance Level)

Answer Sheet of Assignment

Numerical Methods

5.1 Introduction

The limitations of analytical methods have led the engineers and scientists to evolve graphical and numerical methods. The graphical methods, though simple, give results to a low degree of accuracy. Numerical methods can, however, be derived which are more accurate.

5.2 Significant digits and Rounding off of Numbers

(1) **Significant digits :** The significant digits in a number are determined by the following rules :

(i) All non-zero digits in a number are significant.

(ii) All zeros between two non-zero digits are significant.

(iii) If a number having embedded decimal point ends with a non-zero or a sequences of zeros, then all these zeros are significant digits.

(iv) All zeros preceding a non-zero digit are non-significant.

Number	Number of significant digits
3.0450	5
0.0025	2
102.030070	9
35.9200	6
0.0002050	4
20.00	4
2000	1

(2) **Rounding off of numbers :** If a number is to be rounded off to n significant digits, then we follow the following rules :

(i) Discard all digits to the right of the nth digit.

(ii) If the $(n+1)$th digit is greater then 5 or it is 5 followed by a nonzero digit, then nth digit is increased by 1. If the $(n+1)$th digit is less then 5, then digit remains unchanged.

(iii) If the $(n+1)$th digit is 5 and is followed by zero or zeros, then nth digit is increased by 1 if it is odd and it remains unchanged if it is even.

5.3 Error due to Rounding off of Numbers

If a number is rounded off according to the rules, the maximum error due to rounding does not exceed the one half of the place value of the last retained digit in the number.

The difference between a numerical value X and its rounded value X_1 is called round off error is given by $E = X - X_1$.

5.4 Truncation and Error due to Truncation of Numbers

Leaving out the extra digits that are not required in a number without rounding off, is called truncation or chopping off.

The difference between a numerical value X and its truncated value X_1 is called truncation error and is given by $E = X - X_1$.

The maximum error due to truncation of a number cannot exceed the place value of the last retained digit in the number.

Remark **1 :** In truncation the numerical value of a positive number is decreased and that of a negative number is increased.

Remark **2 :** If we round off a large number of positive numbers to the same number of decimal places, then the average error due to rounding off is zero.

Remark **3 :** In case of truncation of a large number of positive numbers to the same number of decimal places the average truncation error is one half of the place value of the last retained digit.

Remark **4 :** If the number is rounded off and truncated to the same number of decimal places, then truncation error is greater than the round off error.

Remark **5 :** Round of error may be positive or negative but truncation error is always positive in case of positive numbers and negative in case of negative numbers.

Number	**Approximated number obtained by**	
	Chopping off	**Rounding off**
0.335217...	0.3352	0.3352
0.666666...	0.6666	0.6667
0.123451...	0.1234	0.1235
0.213450...	0.2134	0.2134
0.213950...	0.2139	0.2140
0.335750...	0.3357	0.3358
0.999999...	0.9999	1.0000
0.555555...	0.5555	0.5556

5.5 Relative and Percentage errors of Numbers

The difference between the exact value of a number X and its approximate value X_1, obtained by rounding off or truncation, is known as absolute error.

The quantity $\frac{X - X_1}{X}$ is called the relative error and is denoted by E_R.

Thus $E_R = \frac{X - X_1}{X} = \frac{\Delta X}{X}$. This is a dimensionless quantity.

The quantity $\frac{\Delta X}{X} \times 100$ is known as percentage error and is denoted by E_p, *i.e.* $E_p = \frac{\Delta X}{X} \times 100$.

Remark 1 : If a number is rounded off to n decimal digits, then $|E_R| < 0.5 \times 10^{-n+1}$

Remark 2 : If a number is truncated to n decimal places, then $|E_R| < 10^{-n+1}$

Example: 1 The number of significant digits in 0.0001 is

(a) 5 (b) 4 (c) 1 (d) None of these

Solution: (c) 0.0001 has only one significant digit 1.

Example: 2 When a number is rounded off to n decimal places, then the magnitude of relative error does not exceed **[DCE 1998]**

(a) 10^{-n} (b) 10^{-n+1} (c) $0.5 \times 10^{-n+1}$ (d) None of these

Solution: (c) When a number is rounded off to n decimal places, then the magnitude of relative error *i.e.* $|E_R|$ does not exceed $0.5 \times 10^{-n+1}$.

Example: 3 When the number 2.089 is rounded off to three significant digits, then the absolute error is

(a) 0.01 (b) –0.01 (c) 0.001 (d) –0.001

Solution: (d) When the number 2.089 is rounded off to three significant digits, it becomes 2.09. Hence, the absolute error that occurs is $2.089 - 2.09 = -0.001$

5.6 Algebraic and Transcendental Equation

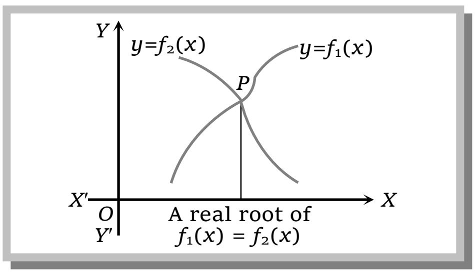
An equation of the form $f(x)=0$, is said to an algebraic or a transcendental equation according as $f(x)$ is a polynomial or a transcendental function respectively.

e.g. $ax^2 + bx + c = 0$, $ax^3 + bx^2 + cx + d = 0$ etc., where $a, b, c, d \in Q$, are algebraic equations whereas $ae^x + b\sin x = 0$; $a\log x + bx = 3$ *etc.* are transcendental equations.

5.7 Location of real Roots of an Equation

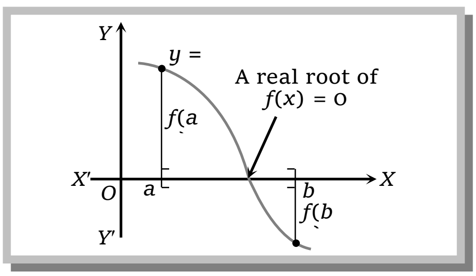
By location of a real root of an equation, we mean finding an approximate value of the root graphically or otherwise.

(1) **Graphical Method :** It is often possible to write $f(x) = 0$ in the form $f_1(x) = f_2(x)$ and then plot the graphs of the functions $y = f_1(x)$ and $y = f_2(x)$.



The abscissae of the points of intersection of these two graphs are the real roots of $f(x) = 0$.

(2) **Location Theorem :** Let $y = f(x)$ be a real-valued, continuous function defined on [a, b]. If $f(a)$ and $f(b)$



have opposite signs *i.e.* $f(a).f(b) < 0$, then the equation $f(x)=0$ has at least one real root between *a* and *b*.

5.8 Position of Real Roots

If $f(x)=0$ be a polynomial equation and $x_1, x_2 ....x_k$ are the consecutive real roots of $f(x)=0$, then positive or negative sign of the values of $f(-\infty), f(x)............f(x_k), f(\infty)$ will determine the intervals in which the root of $f(x)=0$ will lie whenever there is a change of sign from $f(x_r)$ to $f(x_{r+1})$ the root lies in the interval $[x_r, x_{r+1}]$.

Example: 4 If all roots of equation $x^3 - 3x + k = 0$ are real, then range of value of *k*

(a) (–2, 2) (b) [–2, 2] (c) Both (d) None of these

Solution: (a) Let $f(x) = x^3 - 3x + k$, then $f(x) = 3x^2 - 3$ and so $f(x) = 0 \Rightarrow x = \pm 1$. The values of $f(x)$ at $x = -\infty, -1, 1, \infty$ are :

x:	$-\infty$	-1	1	∞
$f(x)$:	$-\infty$	$k+2$	$k-2$	∞

If all roots of given equation are real, then $k+2>0$ and $k-2<0 \Rightarrow -2<k<2$. Hence the range of *k* is (–2, 2)

Example: 5 For the smallest positive root of transcendental equation $x - e^{-x} = 0$, interval is **[MP PET 1996]**

(a) (0, 1) (b) (–1, 0) (c) (1, 2) (d) (2, 3)

Solution: (a) Let $f(x) = x - e^{-x} = 0 \Rightarrow xe^x - 1 = 0$ but $f(0) = -ive$ and $f(1) = +ive$. Therefore root lie in (0,1).

Example: 6 The maximum number of real roots of the equation $x^{2n} - 1 = 0$ is **[MP PET 2001]**

(a) 2 (b) 3 (c) *n* (d) 2*n*

Solution: (a) Let $f(x) = x^{2n} - 1$, then $f'(x) = x^{2n-1} = 0 \Rightarrow x = 0$

Sign of $f(x)$ at $x = -\infty, 0, +\infty$ are

x:	$-\infty$	0	$+\infty$
$f(x)$:	$+ive$	$-ive$	$+ive$

This show that there are two real roots of $f(x)=0$ which lie in the interval $(-\infty, 0)$ and $(0, +\infty)$. Hence maximum number of real roots are 2.

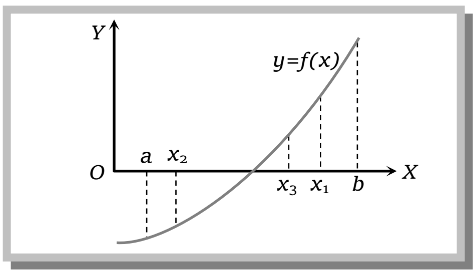
5.9 Solution of Algebraic and Transcendental Equations

There are many numerical methods for solving algebraic and transcendental equations. Some of these methods are given below. After locating root of an equation, we successively approximate it to any desired degree of accuracy.

(1) **Iterative method:** If the equation $f(x) = 0$ can be expressed as $x = g(x)$ (certainly $g(x)$ is non-constant), the value $g(x_0)$ of $g(x)$ at $x = x_0$ is the next approximation to the root α. Let $g(x_0) = x_1$, then $x_2 = g(x_1)$ is a third approximation to α. This process is repeated until a number, whose absolute difference from α is as small as we please, is obtained. This number is the required root of $f(x)=0$, calculated upto a desired accuracy.

Thus, if x_i is an approximation to α, then the next approximation $x_{i+1} = g(x_i)$ ....(i)

The relation (i) is known as Iterative formula or recursion formula and this method of approximating a real root of an equation $f(x)=0$ is called iterative method.



(2) **Successive bisection method :** This method consists in locating the root of the equation $f(x)=0$ between *a* and *b*. If $f(x)$ is continuous between *a* and *b*, and $f(a)$ and $f(b)$ are of opposite

signs *i.e.* $f(a).f(b)<0$, then there is (at least one) root between *a* and *b*. For definiteness, let $f(a)$ be negative and $f(b)$ be positive. Then the first approximation to the root $x_1 = \frac{1}{2}(a+b)$.

Working Rule : (i) Find $f(a)$ by the above formula.

(ii) Let $f(a)$ be negative and $f(b)$ be positive, then take $x_1 = \frac{a+b}{2}$.

(iii) If $f(x_1)=0$, then *c* is the required root or otherwise if $f(x_1)$ is negative then root will be in (x_1, b) and if $f(x_1)$ is positive then root will be in (a, x_1).

(iv) Repeat it until you get the root nearest to the actual root.

Note : ❑ This method of approximation is very slow but it is reliable and can be applied to any type of algebraic or *transcendental equations*.

❑ This method may give a false root if $f(x)$ is discontinuous on [*a*, *b*].

Example: 7 By bisection method, the real root of the equation $x^3 - 9x + 1 = 0$ lying between $x = 2$ and $x = 4$ is nearer to

[MP PET 1997]

(a) 2.2 (b) 2.75 (c) 3.5 (d) 4.0

Solution: (b) Since $f(2) = 2^3 - 9(2) + 1 < 0$ and $f(4) = 4^3 - 9(4) + 1 > 0$

$\therefore$ Root will lie between 2 and 4.

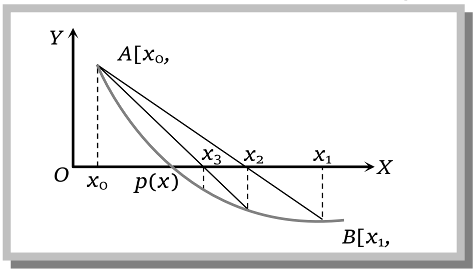
At $x = \frac{2+4}{2} = 3$, $f(3) = 3^3 - 9(3) + 1 > 0$

$\therefore$ Root lie between 2 and 3.

At $x = \frac{2+3}{2} = 2.5$, $f(2.5)$ is *–ive*, $\therefore$ root lies between 2.5 and 3

At $x = \frac{2.5+3}{2} = 2.75$ and $f(2.75) = (2.75)^3 - 9(2.75) + 1 < 0$. $\therefore$ Root is near to 2.75.

(3) **Method of false position or Regula-Falsi method :** This is the oldest method of finding the real root of an equation $f(x) = 0$ and closely resembles the bisection method. Here we choose two points x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are of opposite signs *i.e.* the graph of $y = f(x)$ crosses the *x*-axis between these points. This indicates that a root lies between x_0 and x_1 consequently $f(x_0)f(x_1) < 0$.



Equation of the chord joining the points $A[x_0, f(x_0)]$ and $B[x_1, f(x_1)]$ is

$$y - f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) \qquad \text{........(i)}$$

The method consists in replacing the curve *AB* by means of the chord *AB* and taking the point of intersection of the chord with the *x*-axis as an approximation to the root. So the abscissa of the point where the chord cuts the x-axis $(y = 0)$ is given by $x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_2)} f(x_0)$

.........(ii)

which is an approximation to the root.

If now $f(x_0)$ and $f(x_2)$ are of opposite signs, then the root lies between x_0 and x_2. So replacing x_1 by x_2 in (ii), we obtain the next approximation x_3. (The root could as well lie between x_1 and x_2 and we would obtain x_3 accordingly). This procedure is repeated till the root is found to desired accuracy. The iteration process based on (i) is known as the *method of false position*.

Working rule

(i) Calculate $f(x_0)$ and $f(x_1)$, if these are of opposite sign then the root lies between x_0 and x_1.

(ii) Calculate x_2 by the above formula.

(iii) Now if $f(x_2) = 0$, then x_2 is the required root.

(iv) If $f(x_2)$ is negative, then the root lies in (x_2, x_1).

(v) If $f(x_2)$ is positive, then the root lies in (x_0, x_2).

(vi) Repeat it until you get the root nearest to the real root.

Note :
- This method is also known as the method of false position.
- The method may give a false root or may not converge if either a and b are not sufficiently close to each other or $f(x)$ is discontinuous on $[a, b]$.
- Geometrically speaking, in this method, part of the curve between the points $P(a, f(a))$ and $Q\ (b, f(b))$ is replaced by the secant PQ and the point of intersection of this secant with x-axis gives an approximate value of the root.
- It converges more rapidly than bisection.

Example: 8 The root of the equation $x^3 + x - 3 = 0$ lies in interval (1, 2) after second iteration by false position method, it will be in

[MP PET 2003]

(a) (1.178, 2.00) (b) (1.25, 1.75) (c) (1.125, 1.375) (d) (1.875, 2.00)

Solution: (a) $f(x) = x^3 + x - 3$

$f(1) = -1$ and $f(2) = 7$

Therefore, root lie in (1, 2).

Now, take $x_0 = 1$, $x_1 = 2$

$\Rightarrow x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} . f(x_0) \Rightarrow x_2 = 1 - \frac{2 - 1}{+7 - (-1)} . (-1) = 1.125$ and so $f(x_2) = -0.451$

Hence, roots lie in (1.125, 2)

$\Rightarrow x_3 = 1.125 - \frac{2 - 1.125}{7 - (-0.451)}(-0.451) = 1.178$. So required root lie in (1.178, 2)

(4) **Newton-Raphson method :** Let x_0 be an approximate root of the equation $f(x) = 0$. If $x_1 = x_0 + h$ be the exact root, then $f(x_1) = 0$

$\therefore$ Expanding $f(x_0 + h)$ by Taylor's series

$$f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + ... = 0$$

Since h is small, neglecting h^2 and higher powers of h, we get

$$f(x_0) + hf'(x_0) = 0$$

or $$h = -\frac{f(x_0)}{f'(x_0)} \quad ....(i)$$

$\therefore$ A closer approximation to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly, starting with x_1, a still better approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general, $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad ....(ii)$$

Which is known as the Newton-Raphson formula or Newton's iteration formula.

***Working rule*:** (i) Find $|f(a)|$ and $|f(b)|$. If $|f(a)| < |f(b)|$, then let $a = x_0$, otherwise $b = x_0$.

(ii) Calculate $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

(iii) x_1 is the required root if $f(x_1) = 0$.

(iv) To find nearest to the real root, repeat it.

Note : ❑ Geometrically speaking, in Newton-Raphson method, the part of the graph of the function $y = f(x)$ between the point $P(a, f(a))$ and the x-axis is replaced by a tangent to the curve at the point at each step in the approximation process.

❑ This method is very useful for approximating isolated roots.

❑ The Newton-Raphson method fails if $f'(x)$ is difficult to compute or vanishes in a neighbourhood of the desired root. In such cases, the Regula-Falsi method should be used.

❑ The Newton-Raphson method is widely used since in a neighbourhood of the desired root, it converges more rapidly than the bisection method or the Regula-Falsi method.

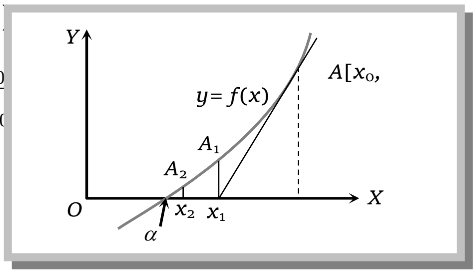
❑ If the starting value a is not close enough to the desired root, the method may give a false root or may not converge.

- If $f(x_0)/f'(x_0)$ is not sufficiently small, this method does not work. Also if it work, it works faster.

Geometrical Interpretation

Let x_0 be a point near the root α of the equation $f(x)=0$. Then the equation of the tangent at $A_0[x_0, f(x_0)]$ is $y-f(x_0)=f'(x_0)(x-x_0$

It cuts the x-axis at $x_1 = x_0 - \frac{f(x_0}{f'(x_0}$



Which is a first approximation to the root α. If A_1 is the point corresponding to x_1 on the curve, then the tangent at A_1 will cut the x-axis of x_2 which is nearer to α and is, therefore, a second approximation to the root. Repeating this process, we approach to the root α quite rapidly. Hence the method consists in replacing the part of the curve between the point A_0 and the x-axis by means of the tangent to the curve at A_0.

Example: 9 The value of the root nearest to the 2, after first iteration of the equation $x^4 - x - 10 = 0$ by Newton-Raphson method is

[MP PET 2003]

(a) 2.321 (b) 2.125 (c) 1.983 (d) 1.871

Solution: (d) Let $f(x)=x^4-x-10$, then $f(1)=-10$ and $f(2)=4$

Thus roots lie in (1, 2). Also $|f(2)|<|f(1)|$, So take $x_0=2$

Also $f'(x)=4x^3-1$

$f'(2)=4(8)-1=31$

$\therefore$ By Newton's rule, the first iteration,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{4}{31} = 1.871$$

Example: 10 For finding real roots of the equation $x^2 - x = 2$ by Newton-Raphson method, choose $x_0 = 1$, then the value of x_2 is

[MP PET 2000, 02]

(a) –1 (b) 3 (c) $\frac{11}{5}$ (d) None of these

Solution: (c) Let $f(x)=x^2-x-2$. Given $x_0=1$.

By Newton-Raphson method, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f'(x)=2x-1=2(1)-1=1$ and $f(1)=-2$. Now $x_1 = 1 - \frac{-2}{1} = 3$

$f(x_1)=f(3)=9-3-2=4$

$f'(x_1)=f'(3)=2(3)-1=5$; $x_2 = 3 - \frac{4}{5} = \frac{11}{5}$

Example: 11 The value of $\sqrt{12}$ correct to 3 decimal places by Newton-Raphson method is given by **[DCE 1999, 2000]**

(a) 3.463 (b) 3.462 (c) 3.467 (d) None of these

Solution: (a) Let $x=\sqrt{12} \Rightarrow x^2-12=0 \Rightarrow f(x)=x^2-12,\ f'(x)=2x$

$\therefore f(3)=-3$ *i.e. –ive* and $f(4)=+4$ *i.e. +ive*

Hence roots lie between 3 and 4. $\because |f(3)|<|f(4)|$, $\therefore x_0=3$

First iteration, $x_1=x_0-\frac{f(x_0)}{f'(x_0)}=3-\frac{-3}{6}=3.5$

Now second iteration, $f(3.5)=0.25,\ f'(3.5)=7$, $x_2=3.5-\frac{0.25}{7}=3.463$

5.10 Numerical Integration

It is the process of computing the value of a definite integral when we are given a set of numerical values of the integrand $f(x)$ corresponding to some values of the independent variable x.

If $I=\int_a^b y.dx$. Then I represents the area of the region R under the curve $y=f(x)$ between the ordinates $x=a, x=b$ and the x-axis.

(1) **Trapezoidal rule**

Let $y=f(x)$ be a function defined on [a, b] which is divided into n equal sub-intervals each of width h so that $b-a=nh$.

Let the values of $f(x)$ for $(n+1)$ equidistant arguments $x_0=a, x_1=x_0+h,\ x_2=x_0+2h, ..., x_n=x_0+nh=b$ be $y_0, y_1, y_2, ...y_n$ respectively.

Then $$\int_a^b f(x)dx=\int_{x_0}^{x_0+nh} y\,dx=h\left[\frac{1}{2}(y_0+y_n)+(y_1+y_2+......+y_{n-1})\right]=\frac{h}{2}[(y_0+y_n)+2(y_1+y_2+......+y_{n-1})]$$

This rule is known as **Trapezoidal rule .**

The geometrical significance of this rule is that the curve $y=f(x)$ is replaced by n straight lines joining the points (x_0,y_0) and (x_1,y_1); (x_1,y_1) and (x_2,y_2); (x_{n-1},y_{n-1}) and (x_n,y_n). The area bounded by the curve $y=f(x)$. The ordinate $x=x_0$ and $x=x_n$ and the x-axis, is then approximately equivalent to the sum of the areas of the n trapeziums obtained.

Example: 12 If $e^0=1, e^1=2.72, e^2=7.39, e^3=20.09, e^4=54.60$, then by Trapezoidal rule $\int_0^4 e^x dx=$ **[MP PET 1995, 2001]**

(a) 53.87 (b) 53.60 (c) 58.00 (d) None of these

Solution: (c) $h=\frac{b-a}{n}=\frac{4-0}{4}=1$

x :	0	1	2	3	4
y :	1	2.72	7.39	20.09	54.60

By Trapezoidal rule, $\int_0^4 f(x)dx=\int_0^4 e^x\,dx=\frac{h}{2}[(y_0+y_4)+2(y_1+y_2+y_3)]$

$$=\frac{1}{2}[(1+54.6)+2(2.72+7.39+20.09)]=\frac{1}{2}[55.6+60.4]=58.00$$

Example: 13 By trapezoidal rule the value of the integral $\int_1^5 x^2\,dx$ on dividing the interval into four equal parts is **[MP PET**

(a) 42 (b) 41.3 (c) 41 (d) 40

Solution: (a) $h=\frac{5-1}{4}=1$

x :	1	2	3	4	5
y :	1	4	9	16	25

$\int_1^5 x^2\,dx=\frac{h}{2}[(y_0+y_4)+2(y_1+y_2+y_3)]=\frac{1}{2}[(1+25)+2(4+9+16)]=\frac{1}{2}[26+58]=\frac{84}{2}=42$

Example: 14 If for $n=4$, the approximate value of integral $\int_1^9 x^2\,dx$ by Trapezoidal rule is $2\left[\frac{1}{2}(1+9^2)+\alpha^2+\beta^2+7^2\right]$, then

[MP PET 2002]

(a) $\alpha=1,\ \beta=3$ (b) $\alpha=2,\ \beta=4$ (c) $\alpha=3,\ \beta=5$ (d) $\alpha=4,\ \beta=6$

Solution: (c) $h=\frac{b-a}{n}=\frac{9-1}{4}=2$

$x_0=1$, $x_1=x_0+nh=1+1.2=3$, $x_2=x_0+2.2=5,\quad x_3=x_0+3.2=7,\ \ x_4=x_0+4.2=9$

$y_0=1,\ y_1=9,\ y_2=25,\ y_3=49,\ y_4=81$

By trapezoidal rule,

$\int_a^b f(x)dx=\frac{h}{2}[(y_0+y_4)+2(y_1+y_2+y_3)]\ =\frac{2}{2}[(1+81)+2(9+25+49)]=2\left[\frac{1}{2}(1+9^2)+(3^2+5^2+7^2)\right]$

Obvious from above equation, $\alpha=3,\ \beta=5$.

(2) **Simpson's one third rule :** Let $y=f(x)$ be a function defined on [*a, b*] which is divided into *n* (an even number) equal parts each of width *h* so that $b-a=nh$.

Suppose the function $y=f(x)$ attains values $y_0,y_1,y_2,..........y_n$ at $n+1$ equidistant points $x_0=a,\ x_1=x_0+h,\ x_2=x_0+2h,..........,\ x_n=x_0+nh=b$ respectively. Then

$$\int_a^b f(x)dx=\int_{x_0}^{x_0+nh} ydx=\frac{h}{3}[(y_0+y_n)+4(y_1+y_3+y_5+....+y_{n-1})+2(y_2+y_4+...+y_{n-2})]$$

= (one-third of the distance between two consecutive ordinates)

× [(sum of the extreme ordinates)+4(sum of odd ordinates)+2(sum of even ordinates)]

This formula is known as Simpson's one-third rule. Its geometric significance is that we replace the graph of the given function by $\frac{n}{2}$ arcs of second degree polynomials, or parabolas with vertical axes. It is to note here that the interval [*a, b*] is divided into an even number of subinterval of equal width.

Simpson's rule yield more accurate results than the trapezoidal rule. Small size of interval gives more accuracy.

Example: 15 By Simpson's rule the value of the interval $\int_1^6 x\,dx$ on dividing, the interval into four equal parts is **[MP PET 2**

(a) 16 (b) 16.5 (c) 17 (d) 17.5

Solution: (d) $h=\frac{6-1}{4}=\frac{5}{4}=1.25$

$x_0=a=1,\ x_1=x_0+h=1+1.25=2.25$

$x_2=x_0+2h=1+2(1.25)=3.50$

$x_3 = x_0 + 3h = 1 + 3(1.25) = 4.75$

$x_4 = x_0 + 4h = 1 + 4(1.25) = 6.0$

By Simpson's rule, $\int_a^b f(x)dx = \int_1^6 xdx = \frac{1.25}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)]$

$$= \frac{1.25}{3}[(1+6) + 4(2.25 + 4.75) + 2(3.5)] = \frac{1.25}{3}[7 + 28 + 7] = 17.5$$

Example: 16 Considering six sub-interval, the value of $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's rule is

(a) 1.3562 (b) 1.3662 (c) 1.3456 (d) 1.2662

Solution: (b) $h = \frac{6-0}{6} = 1$

$x_0 = 0, \ x_1 = 0 + 1.1 = 1$

$x_2 = 0 + 2.1 = 2, \ x_3 = 3, \ x_4 = 4, \ x_5 = 5, \ x_6 = 6$

and $y_0 = \frac{1}{1+0} = 1, \ y_1 = \frac{1}{2}, \ y_2 = \frac{1}{5}, \ y_3 = \frac{1}{10}, \ y_4 = \frac{1}{17}, \ y_5 = \frac{1}{26}, \ y_6 = \frac{1}{37}$

By Simpson's rule ,

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)] = \frac{1}{3}\left[\left(1 + \frac{1}{37}\right) + 4\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{26}\right) + 2\left(\frac{1}{5} + \frac{1}{17}\right)\right] = 1.3662$$

Example: 17 By Simpson's rule taking $n = 4$, the value of the integral $\int_0^1 \frac{1}{1+x^2}dx$ is equal to **[Kurukshetra CEE 1998; DCE 2**

(a) 0.785 (b) 0.788 (c) 0.781 (d) None of these

Solution: (a) $h = \frac{1-0}{4} = 0.25$

$x_0 = 0 \quad x_1 = 0 + 0.25 = 0.25$

$x_2 = 0 + 2(0.25) = 0.50, \ x_3 = 0.75, \ x_4 = 1$

and $y_0 = \frac{1}{1+x_0^2} = \frac{1}{1+0} = 1$, $y_1 = \frac{1}{1+(0.25)^2} = 0.941$, $y_2 = \frac{1}{1+(0.50)^2} = 0.8$, $y_3 = 0.64$ and $y_4 = 0.5$

$$\int_0^1 \frac{dx}{1+x^2} = \frac{0.25}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)]$$

$$= \frac{0.25}{3}[(1 + 0.5) + 4(0.941 + 0.64) + 2(0.8)] = 0.785 \ .$$

Assignment

Significant Digit and Rounding off Numbers, Fundamental concepts

Basic Level

1. The number 3.14150 rounded to 3 decimals is [MP PET 2000]

   (a) 3.14 (b) 3.141 (c) 3.142 (d) None of these

2. The number of significant digits in 0.003050 is

   (a) 7 (b) 6 (c) 4 (d) None of these

3. The number of significant digits in 20.035 is

   (a) 3 (b) 5 (c) 4 (d) None of these

4. The number of significant digits in 20340 is

   (a) 4 (b) 5 (c) 3 (d) None of these

5. The number 0.0008857 when rounded off to three significant digits yields

   (a) 0.001 (b) 0.000886 (c) 0.000885 (d) None of these

6. The number 3.68451 when rounded off to three decimal places becomes

   (a) 3.68 (b) 3.684 (c) 3.685 (d) None of these

7. The number of significant digits in the number 0.00452000 is

   (a) 3 (b) 5 (c) 8 (d) None of these

8. When a number is approximated to n decimal places by chopping off the extra digits, then the absolute value of the relative error does not exceed

   (a) 10^{-n} (b) 10^{-n+1} (c) $0.5 \times 10^{-n+1}$ (d) None of these

9. When the number 6.878652 is rounded off to five significant figures, then the round off error is

   (a) – 0.000048 (b) –0.00048 (c) 0.000048 (d) 0.00048

10. The number 0.0009845 when rounded off to three significant digits yields [DCE 1998]

(a) 0.001 (b) 0.000987 (c) 0.000985 (d) None of these

11. A decimal number is chopped off to four decimal places, then the absolute value of the relative error is not greater than [DCE 1996]

(a) 10^{-2} (b) 10^{-3} (c) 10^{-4} (d) None of these

12. If e_1 and e_2 are absolute errors in two numbers n_1 and n_2 respectively due to rounding or truncation, then $\left|\frac{e_1}{n_1}+\frac{e_2}{n_2}\right|$

(a) Is equal to e_1+e_2 (b) Is less then e_1+e_2

(c) Is less then or equal to e_1+e_2 (d) Is greater then or equal to e_1+e_2

13. In general the ratio of the truncation error to that of round off error is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None of these

14. The equation $e^{-2x}-\sin x+1=0$ is of the form

(a) Algebraic (b) Linear (c) Quadratic (d) Transcendental

15. The root of the equation $x^3-6x+1=0$ lies in the interval

(a) (2, 3) (b) (3, 4) (c) (3, 5) (d) (4, 6)

16. The root of the equation $x^3-3x-5=0$ in the interval (1, 2) is

(a) 1.13 (b) 1.98

(c) 1.54 (d) No root lies in the interval (1, 2)

17. The equation $f(x)=0$ has repeated root $a\in(x_1,x_2)$, if

(a) $f'(a)<0$ (b) $f'(a)>0$ (c) $f'(a)=0$ (d) None of these

18. The root of the equation $2x-\log_{10}x=7$ lies between

(a) 3 and 3.5 (b) 2 and 3 (c) 3.5 and 4 (d) None of these

19. For the equation $f(x)=0$, if $f(a)<0,\ f(b)>0,\ f(c)>0$ and $b>c$ then we will discard the value of the function $f(x)$ at the point

(a) *a* (b) *b* (c) *c* (d) Anyone out of *a, b, c*

20. The positive root of the equation $e^x+x-3=0$ lies in the interval

(a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (2, 4)

21. The positive root of the equation $x^3-2x-5=0$ lies in the interval

(a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (3, 4)

22. One real root of the equation x^3-5x+1 = 0 must lie in the interval

(a) (0, 1) (b) (1, 2) (c) (–1, 0) (d) (–2, 0)

23. The number of positive roots of the equation $x^3-3x+5=0$ is [MP PET 1998]

(a) 1 (b) 2 (c) 3 (d) None of these

24. Let $f(x)=0$ be an equation and x_1, x_2 be two real numbers such that $f(x_1)f(x_2)<0$, then **[MP PET 1989, 1997]**

(a) At least one root of the equation lies in the interval (x_1, x_2)

(b) No root of the equation lies in the interval (x_1, x_2)

(c) Either no root or more than one root of the equation lies the interval (x_1, x_2)

(d) None of these

25. Let $f(x)=0$ be an equation let x_1, x_2 be two real numbers such that $f(x_1)f(x_2)>0$, then

(a) At least one root of the equation lies in (x_1, x_2)

(b) No root of the equation lies in (x_1, x_2)

(c) Either no root or an even number of roots lie in (x_1, x_2)

(d) None of these

26. If for $f(x)=0$, $f(a)<0$ and $f(b)>0$, then one root of $f(x)=0$ is

(a) Between *a* and *b* (b) One of from *a* and *b*

(c) Less than *a* and greater than *b* (d) None of these

27. If $f(a)f(b)<0$, then an approximate value of a real root of $f(x)=0$ lying between *a* and *b* is given by

(a) $\frac{af(b)-bf(a)}{b-a}$ (b) $\frac{bf(a)-af(b)}{b-a}$

(c) $\frac{af(b)-bf(a)}{f(b)-f(a)}$ (d) None of these

Successive bisection method

Basic Level

28. One root of $x^3-x-4=0$ lies in (1, 2). In bisection method, after first iteration the root lies in the interval

(a) (1, 1.5) (b) (1.5, 2.0) (c) (1.25, 1.75) (d) (1.75, 2)

29. A root of the equation $x^3-x-1=0$ lies between 1 and 2. Its approximate value as obtained by applying bisection method 3 times is

[MP PET 1993]

(a) 1.375 (b) 1.625 (c) 1.125 (d) 1.25

30. A root of the equation $x^3-x-4=0$ lies between 1 and 2. Its approximate value, as obtained by applying bisection method 3 times, is

(a) 1.375 (b) 1.750 (c) 1.975 (d) 1.875

31. Performing 3 iterations of bisection method, the smallest positive approximate root of equation $x^3 - 5x + 1 = 0$ is [MP PET 1996]

(a) 0.25 (b) 0.125 (c) 0.50 (d) 0.1875

32. A root of the equation $x^3 - 3x - 5 = 0$ lies between 2 and 2.5. Its approximate value, by applying bisection method 3 times is

(a) 2.0625 (b) 2.3125 (c) 2.3725 (d) 2.4225

33. If for the function $f(x) = 0$, $f(a) < 0$ and $f(b) > 0$, then the value of x in first iteration is

(a) $\frac{a+b}{2}$ (b) $\frac{b-a}{2}$ (c) $\frac{2a-b}{2}$ (d) $\frac{2b-a}{2}$

34. Using successive bisection method, a root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2, at the end of first interaction, it lies between [DCE 1996]

(a) 1.62 and 1.75 (b) 1.5 and 1.75 (c) 1.75 and 1.87 (d) None of these

35. The nearest real root of the equation $xe^x - 2 = 0$ correct to two decimal places, is

(a) 1.08 (b) 0.92 (c) 0.85 (d) 0.80

Regula-Falsi method

Basic Level

36. By the false position method, the root of the equation $x^3 - 9x + 1 = 0$ lies in interval (2, 4) after first iteration. It is

(a) 3 (b) 2.5 (c) 3.57 (d) 2.47

37. The formula [where $f(x_{n<1})$ and $f(x_n)$ have opposite sign at each step $n \geq 1$] of method of False position of successive approximation to find the approximate value of a root of the equation $f(x) = 0$ is [MP PET 1995, 97]

(a) $x_{n+1} = x_n - \frac{f(x_n) - f(x_{n-1})}{f(x_n)}(x_n - x_{n-1})$ (b) $x_{n+1} = x_n - \frac{f(x_n)}{f(x_n) - f(x_{n-1})}(x_n - x_{n-1})$

(c) $x_{n+1} = x_n + \frac{f(x_n) + f(x_{n-1})}{f(x_n)}(x_n - x_{n-1})$ (d) $x_{n+1} = x_n + \frac{f(x_n)}{f(x_n) + f(x_{n-1})}(x_n - x_{n-1})$

38. By false positioning, the second approximation of a root of equation $f(x) = 0$ is (where x_0, x_1 are initial and first approximations respectively) [MP PET 1996; DCE 2001]

(a) $x_0 - \frac{f(x_0)}{f(x_1) - f(x_0)}$ (b) $\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$ (c) $\frac{x_0 f(x_0) - x_1 f(x_1)}{f(x_1) - f(x_0)}$ (d) $x_0 - \frac{f(x_0)}{f(x_1) - f(x_0)}$

39. A root of the equation $x^3 - 18 = 0$ lies between 2 and 3. The value of the root by the method of false position is

(a) 2.526 (b) 2.536 (c) 2.546 (d) 2.556

40. The equation $x^3 - 3x + 4 = 0$ has only one real root. What is its first approximate value as obtained by the method of false position in (–3, –2) **[MP PET 1999]**

(a) –2.125 (b) 2.125 (c) –2.812 (d) 2.812

41. A root of equation $x^3 + 2x - 5 = 0$ lies between 1 and 1.5. its value as obtained by applying the method of false position only once is

[MP PET 1993]

(a) $\frac{4}{3}$ (b) $\frac{35}{27}$ (c) $\frac{23}{25}$ (d) $\frac{5}{4}$

Newton-Raphson method

Basic Level

42. If successive approximations are given by $x_1, x_2, x_3, ... x_n, x_{n+1}$, then Newton-Raphson formula is given as **[MP PET 1993, 95]**

(a) $x_{n+1} = x_n + \frac{f(x_{n+1})}{f'(x)}$ (b) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

(c) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ (d) $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$

43. Newton-Raphson method is applicable only when

(a) $f(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$ (b) $f'(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$

(c) $f''(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$ (d) None of these

44. Newton-Raphson processes has a

(a) Linear convergence (b) Quadratic convergence (c) Cubic convergence (d) None of these

45. The condition for convergence of the Newton-Raphson method to a root α is **[MP PET 2001]**

(a) $\frac{1}{2}\frac{f'(\alpha)}{f''(\alpha)} < 1$ (b) $\frac{f'(\alpha)}{f''(\alpha)} < 1$

(c) $\frac{1}{2}\frac{f'(\alpha)}{f''(\alpha)} > 1$ (d) None of these

46. The real root of the equation $x^3 - x - 5 = 0$ lying between –1 and 2 after first iteration by Newton-Raphson method is

(a) 1.909 (b) 1.904 (c) 1.921 (d) 1.940

47. A root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2. Its value as obtained by using Newton-Raphson method is

(a) 1.775 (b) 1.850 (c) 1.875 (d) 1.950

48. The value of x_0 (the initial value of *x*) to get the solution in interval (0.5, 0.75) of the equation $x^3 - 5x + 3 = 0$ by Newton-Raphson method, is

(a) 0.5 (b) 0.75 (c) 0.625 (d) None of these

49. If a and $a + h$ are two consecutive approximate roots of the equation $f(x) = 0$ as obtained by Newtons method, then h is equal to [MP PET 1999]

(a) $f(a)/f'(a)$ (b) $f'(a)/f(a)$ (c) $-f'(a)/f(a)$ (d) $-f(a)/f'(a)$

50. The Newton-Raphson method converges fast if $f'(\alpha)$ is (α is the exact value of the root) [DCE 1998]

(a) Small (b) Large (c) 0 (d) None of these

Advance Level

51. If one root of the equation $f(x) = 0$ is near to x_0 then the first approximation of this root as calculated by Newton-Raphson method is the abscissa of the point where the following straight line intersects the x-axis [MP PET 1998]

(a) Normal to the curve $y = f(x)$ at the point $(x_0, f(x_0))$

(b) Tangent to the curve $y = f(x)$ at the point $(x_0, f(x_0))$

(c) The straight line through the point $(x_0, f(x_0))$ having the gradient $\frac{1}{f'(x_0)}$

(d) The ordinate through the point $(x_0, f(x_0))$

52. A root of the equation $x^3 - 3x - 5 = 0$ lies between 2 and 2.5. Its value as obtained by using Newton-Raphson method, is

(a) 2.25 (b) 2.33 (c) 2.35 (d) 2.45

53. After second iteration of Newton-Raphson method, the positive root of equation $x^2 = 3$ is (taking initial approximation $\frac{3}{2}$) [MP PET 1996]

(a) $\frac{3}{2}$ (b) $\frac{7}{4}$ (c) $\frac{97}{56}$ (d) $\frac{347}{200}$

54. If one root of the equation $x^3 + x^2 - 1 = 0$ is near to 1.0, then by Newton-Raphson method the first calculated approximate value of this root is [MP PET 1998]

(a) 0.9 (b) 0.6 (c) 1.2 (d) 0.8

55. The approximate value of a root of the equation $x^3 - 3x - 5 = 0$ at the end of the second iteration by taking the initial value of the roots as 2, and by using Newton-Raphson method, is [AI CBSE 1990]

(a) 2.2806 (b) 2.2701 (c) 2.3333 (d) None of these

56. Newton-Raphson method is used to calculate $\sqrt[3]{65}$ by solving $x^3 = 65$. If $x_0 = 4$ is taken as initial approximation then the first approximation x_1 is [AMU 1999]

(a) 65/16 (b) 131/32 (c) 191/48 (d) 193/48

57. Starting with $x_0 = 1$, the next approximation x_1 to $2^{1/3}$ obtained by Newton's method is [DCE 1997]

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{5}{6}$

Trapezoidal rule

Basic Level

58. Approximate value of $\int_{x_0}^{x_0+nh} y\,dx$ by Trapezoidal rule, is **[MP PET 1993, 97]**

[Where $y(x_i)=y_i,\ \ x_{i+1}-x_i=h, i=0,1,2,.....n$]

(a) $\frac{h}{2}[y_0+y_n+2(y_1+y_2+y_3+.....+y_{n-1})]$

(b) $\frac{h}{3}[y_0+y_n+4(y_1+y_3+y_5+.....+y_{n-1})+2(y_2+y_4+....+y_{n-2})]$

(c) $\frac{h}{4}[y_0+y_n+2(y_1+y_3+y_5+.....+y_{n-1})+4(y_2+y_4+....+y_{n-2})]$

(d) $\frac{h}{2}[y_0+y_2+y_4+.....+y_n)+2(y_1+y_3+y_5+....+y_{n-1})]$

59. Trapezoidal rule for evaluation of $\int_a^b f(x)\,dx$ requires the interval (*a*, *b*) to be divided into **[DCE 1994; MP PET 1996]**

(a) 2*n* sub-intervals of equal width (b) 2*n* + 1 sub-intervals of equal width

(c) Any number of sub-intervals of equal width (d) 3*n* sub-intervals of equal width

60. The value of *f*(*x*) is given only at $x=0,\frac{1}{3},\frac{2}{3},1$. Which of the following can be used to evaluate $\int_0^1 f(x)\,dx$ approximately **[MP PET 1999]**

(a) Trapezoidal rule (b) Simpson rule

(c) Trapezoidal as well as Simpson rule (d) None of these

61. A river is 80 *metre* wide. Its depth *d metre* and corresponding distance *x metre* from one bank is given below in table

x :	0	10	20	30	40	50	60	70	80
y :	0	4	7	9	12	15	14	8	3

Then the approximate area of cross-section of river by Trapezoidal rule, is **[MP PET 1994]**

(a) 710 *sq.m* (b) 730 *sq.m* (c) 705 *sq.m* (d) 750 *sq.m*

62. A curve passes through the points given by the following table

x :	1	2	3	4	5
y :	10	50	70	80	100

By Trapezoidal rule, the area bounded by the curve, the *x*-axis and the lines *x* = 1, *x* = 5, is

(a) 310 (b) 255 (c) 305 (d) 275

63. From the following table, using Trapezoidal rule, the area bounded by the curve, the *x*-axis and the lines *x* = 7.47, *x* = 7.52, is

x :	7.47	7.48	7.49	7.50	7.51	7.52

$f(x)$:	1.93	1.95	1.98	2.01	2.03	2.06

(a) 0.0996 (b) 0.0896 (c) 0.1096 (d) 0.0776

64. Let $f(0) = 1$, $f(1) = 2.72$, then the trapezoidal rule gives approximate value of $\int_0^1 f(x)\,dx$ **[MP PET 1999; DCE 2001]**

(a) 3.72 (b) 1.86 (c) 1.72 (d) 0.86

65. By Trapezoidal rule, the value of $\int_0^1 x^3\,dx$ considering five sub-intervals, is

(a) 0.21 (b) 0.23 (c) 0.24 (d) 0.26

Advance Level

66. The approximate value of $\int_1^9 x^2 dx$ by using Trapezoidal rule with 4 equal intervals is **[EAMCET 2002]**

(a) 243 (b) 248 (c) 242.8 (d) 242.5

67. Taking $n = 4$, by trapezoidal rule, the value of $\int_0^2 \frac{dx}{1+x}$ is **[DCE 1999, 2000]**

(a) 1.1125 (b) 1.1176 (c) 1.118 (d) None of these

68. With the help of trapezoidal rule for numerical integration and the following table

x :	0	0.25	0.50	0.75	1
$f(x)$:	0	0.0625	0.2500	0.5625	1

The value of $\int_0^1 f(x)\,dx$ is **[MP PET 1996]**

(a) 0.35342 (b) 0.34375 (c) 0.34457 (d) 0.33334

69. If for $n = 3$, the integral $\int_1^{10} x^3 dx$ is approximately evaluated by Trapezoidal rule $\int_1^{10} x^3 dx = 3\left[\frac{1+10^3}{2} + \alpha + 7^3\right]$, then $\alpha =$

[MP PET 2000]

(a) 3^3 (b) 4^3 (c) 5^3 (d) 6^3

70. By trapezoidal rule, the value of $\int_1^2 \frac{1}{x}\,dx$, (using five ordinates) is nearly **[DCE 1994]**

(a) 0.216 (b) 0.697 (c) 0.921 (d) None of these

Simpson's one third rule

Basic Level

71. The value of $\int_{x_0}^{x_0+nh} y\,dx$, *n* is even number, by Simpson's one-third rule is [MP PET 1995]

(a) $\frac{h}{3}[(y_0+y_n)+2(y_1+y_3+.....+y_{n-1})+4(y_2+y_4+......+y_{n-2})]$ (b) $\frac{h}{3}[(y_0+y_n)+4(y_1+y_3+.....+y_{n-1})+2(y_2+y_4+......+y_{n-2})]$

(c) $\frac{h}{3}[(y_0+y_n)-2(y_1+y_3+.....+y_{n-1})+4(y_2+y_4+......+y_{n-2})]$ (d) None of these

72. Simpson's one-third rule for evaluation $\int_a^b f(x)dx$ requires the interval [*a*, *b*] to be divided into [DCE 1999]

(a) An even number of sub-intervals of equal width (b) Any number of sub-intervals

(c) Any number of sub-intervals of equal width (d) An odd number of sub-intervals of equal width

73. Simpson rule for evaluation of $\int_a^b f(x)dx$ requires the interval (*a*, *b*) to be divided into [Haryana CEE 1993; DCE 1994]

(a) 3*n* intervals (b) 2*n* + 1 intervals (c) 2*n* intervals (d) Any number of intervals

74. To calculate approximate value of π by Simpson's rule, the approximate formula is [MP PET 2000]

(a) $\int_0^1 \left(\frac{1}{1+x^2}\right)dx$, $n=16$ (b) $\int_0^1 \left(\frac{1}{1+x^2}\right)dx$, $n=9$ (c) $\int_0^1 \left(\frac{1}{1+x}\right)dx$, $n=11$ (d) $\int_0^1 \left(\frac{1}{1+x}\right)dx$, $n=9$

75. In Simpson's one-third rule, the curve $y = f(x)$ is assumed to be a [MP PET 2001]

(a) Circle (b) Parabola (c) Hyperbola (d) None of these

76. A river is 80 feet wide. The depth *d* (in feet) of the river at a distance of *x* feet from one bank is given by the following table

x:	0	10	20	30	40	50	60	70	80
y:	0	4	7	9	12	15	14	8	3

By Simpson's rule, the area of the cross-section of the river is

(a) 705 sq. feet (b) 690 sq. feet (c) 710 sq. feet (d) 715 sq. feet

77. A curve passes through the points given by the following table

x:	1	1.5	2	2.5	3	3.5	4
y:	2	2.4	2.7	2.8	3	2.6	2.1

By Simpson's rule, the area bounded by the curve, the *x*-axis and the lines *x* = 1, *x* = 4, is

(a) 7.583 (b) 6.783

(c) 7.783 (d) 7.275

78. Using Simpson's $\frac{1}{3}$ rule, the value of $\int_1^3 f(x)dx$ for the following data, is

x:	1	1.5	2	2.5	3
f(*x*) :	2.1	2.4	2.2	2.8	3

[MP PET 1993]

(a) 55.5 (b) 11.1 (c) 5.05 (d) 4.975

79. By the application of Simpson's one-third rule for numerical integration, with two subintervals, the value of $\int_0^1 \frac{dx}{1+x}$ is [MP PET 1996]

(a) $\frac{17}{24}$ (b) $\frac{17}{36}$ (c) $\frac{25}{35}$ (d) $\frac{17}{25}$

80. By Simpson's rule, the value of $\int_{-3}^{3} x^4 dx$ by taking 6 sub-intervals, is

(a) 98 (b) 96 (c) 100 (d) 99

81. If $\int_a^b f(x)dx$ is numerically integrated by Simpson's rule, then in any pair of consecutive sub-intervals by which of the following curves, the curve $y = f(x)$ is approximated [MP PET 1998]

(a) Straight line (b) Parabola (c) Circle (d) Ellipse

82. If by Simpson's rule $\int_0^1 \frac{1}{1+x^2}dx = \frac{1}{12}[3.1 + 4(a+b)]$ when the interval [0, 1] is divided into 4 sub-intervals and *a* and *b* are the values of $\frac{1}{1+x^2}$ at two of its division points, then the values of *a* and *b* are the following [MP PET 1998]

(a) $a = \frac{1}{1.0625}, b = \frac{1}{1.25}$ (b) $a = \frac{1}{1.0625}, b = \frac{1}{1.5625}$ (c) $a = \frac{1}{1.25}, b = 1$ (d) $a = \frac{1}{1.5625}, b = \frac{1}{1.25}$

83. If $e^0 = 1, e^1 = 2.72, e^2 = 7.39, e^3 = 20.09$ and $e^4 = 54.60$, then by Simpson's rule, the value of $\int_0^4 e^x dx$ is

[MP PET 1994, 95, 2001, 02]

(a) 5.387 (b) 53.87 (c) 52.78 (d) 53.17

84. If (2, 6) is divided into four intervals of equal region, then the approximate value of $\int_2^6 \frac{1}{x^2 - x}dx$ using Simpson's rule, is

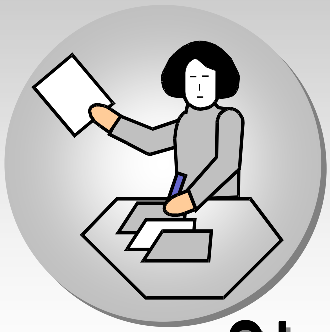
[EAMCET 2002]

(a) 0.3222 (b) 0.2333 (c) 0.5222 (d) 0.2555

85. If $h = 1$ in Simpson's rule, the value of $\int_1^5 \frac{dx}{x}$ is

(a) 1.62 (b) 1.43 (c) 1.48 (d) 1.56

* * *



Answer Sheet

Numerical Methods ***Assignment (Basic and Advance Level)***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
c	c	b	a	b	c	d	b	a	c	b	c	b	d	a	d	c	c	b	a
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	a	d	a	c	a	c	b	a	d	d	b	a	d	c	d	b	b	a	a
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	c	b	b	c	a	c	b	d	b	a	b	c	d	a	d	b	a	c	a
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	b	a	b	d	b	a	b	b	b	b	a	c	a	b	c	c	c	c	a
81	**82**	**83**	**84**	**85**															
b	b	b	c	a															