Chapter 7

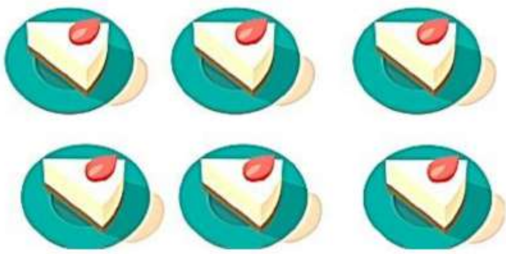
Fractions

Introduction to Fractions

Ethan is celebrating his birthday at home. His mother has baked a cake for his birthday. When his friends came home, he cuts the cake. Now, his mother wants to distribute the cake equally among all his friends.

There are 6 people (including Ethan's mother) at the party.

So, his mother cuts the cake into 6 equal parts.



Can you tell what fraction of the cake does Ethan gets?

Total number of slices of cake = 6

Ethan got (one-sixth) part of the cake. So, Ethan ate one part out of six parts of the cake.

Now, Ethan and his friends had learnt about fractions at school.

So, one of Ethan's friends while eating the cake, cuts his slice of cake into two equal pieces and asked Ethan what fraction of the whole slice was that piece?

So Ethan said that each equal piece is one – half () of one whole slice and the two pieces together will be or 1 whole slice.

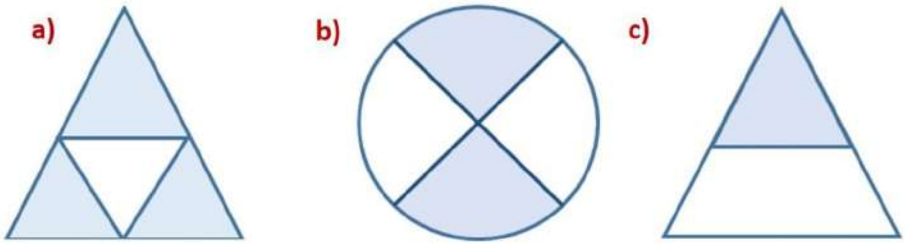
We can say that a fraction is a number representing part of the whole. The whole may be a single object or a group of objects.

A fraction is a number of the form such that $q \neq 0$

In the fraction

- p and q are whole numbers.
- p is called the numerator and q is called the denominator.

Example: Write the fraction representing the shaded portion.

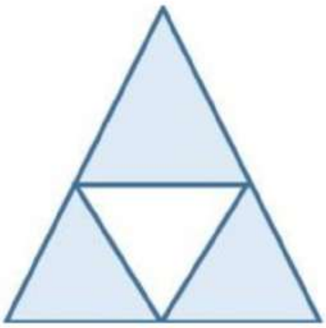


a) The given figure is divided into 4 equal parts.

Number of shaded parts = 3

Total number of equal parts = 4

Fraction representing the shaded portion =



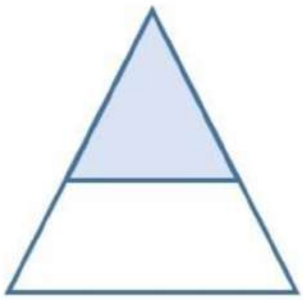
b) The given figure is divided into 4 equal parts.

Number of shaded parts = 1

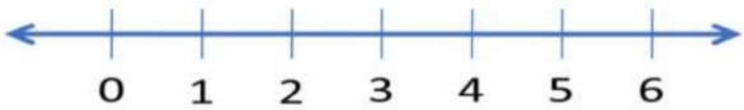
Total number of equal parts = 4

Fraction representing the shaded portion =

c) The given figure is not divided into equal parts. For making fractions the figure should be divided into equal parts.



Fractions on Number Line

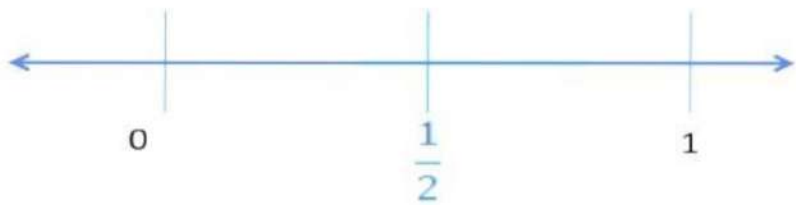


Here, it is a number line of whole numbers and we know how to plot 0, 1, 2, 3..... on a number line.

Let us try to draw a number line and try to mark $\frac{1}{2}$ on the same.

Now, $0 < \frac{1}{2}$ and $\frac{1}{2} < 1$.

So it will lie between 0 and 1.



Example: Mark the following on the number line:

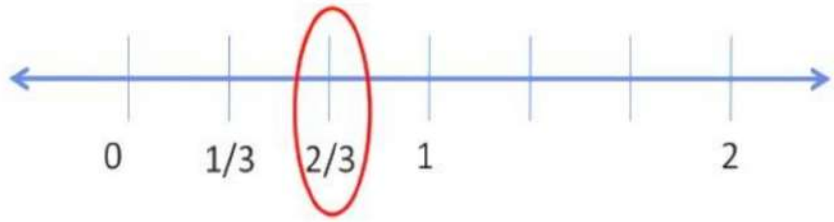
i) $\frac{2}{3}$

ii) $\frac{3}{2}$

Sol.

i) Now to plot $\frac{2}{3}$ we know it lies between 0 and 1.

To plot this we will divide the gap between 0 and 1 into three equal gaps.



ii) Now to $\frac{3}{2}$ plot we know it lies between 1 and 2.

To plot this we will divide the gap between 1 and 2 into two equal gaps.

Types of Fraction

Fractions are mainly of three types:

- Proper fraction
- Improper fraction
- Mixed Fraction

Proper fraction:

Let us see some of the fraction less than 1 like $\frac{1}{2}, \frac{1}{7}, \frac{7}{10}, \ldots\ldots$

Here, the numerator is always less than denominator such fractions are termed as a proper fraction.

Improper fraction:

Let us see some of the fraction greater than 1 like $\frac{3}{2}, \frac{10}{7}, \frac{11}{10}, \ldots\ldots$

Here, the numerator is always greater than denominator such fractions are termed as an improper fraction.

Mixed fraction:

Fractions having whole numbers along with proper fraction.

Proper Fraction	Improper Fraction	Mixed Fraction
• The numerator of a fraction is smaller than the denominator	• The numerator of a fraction is greater than the denominator.	• Fractions having whole numbers along with proper fraction
• $\frac{1}{9}, \frac{6}{11}, \frac{10}{30}$	• $\frac{11}{7}, \frac{34}{27}, \frac{101}{100}$	• $5\frac{6}{7}, 3\frac{4}{11}, 8\frac{2}{13}$

Example: Convert the following into mixed fraction:

i) $\frac{17}{4}$

ii) $\frac{11}{3}$

Sol.

i) $\frac{17}{4}$

$$\begin{array}{r} 4 \\ 4\overline{)17} \\ -16 \\ \hline 1 \\ \hline \end{array}$$

i.e. 4 whole and $\frac{1}{4}$ more, or $4\frac{1}{4}$

ii) $\frac{11}{3}$

$$\begin{array}{r} 3 \\ 3\overline{)11} \\ -9 \\ \hline 2 \\ \hline \end{array}$$

i.e. 3 whole and $\frac{2}{3}$ more, or $3\frac{2}{3}$

Like Fraction

- Fractions with same denominators are called like fractions
- $\frac{5}{7}, \frac{23}{7}, \frac{44}{7}$

Unlike Fraction

- Fractions with different denominators are called unlike fractions
- $\frac{3}{7}, \frac{16}{15}, \frac{100}{9}$

Equivalent Fractions and Simplest Form of Fraction

Equivalent fractions

If we multiply or divide numerator and denominator of a fraction by same non-zero integer, then we will get another equivalent fraction.

Thus, a fraction can be written in several equivalent forms.

The general form for Equivalent fraction can be written as:

If $\frac{p}{q}$ is any fraction and $\frac{r}{s}$ be its equivalent fraction then,

$$\frac{p}{q} = \frac{r}{s}$$

such that ps = rq where p, q, r, and s are whole numbers such that q and s are non-zero non zero whole numbers.

For example,

1. Find any 3 equivalent fractions of $\frac{1}{2}$.

Sol. If we multiply both the numerator and denominator with the same non zero number, we will get its equivalent fraction.

If we multiply given fraction by:

- 2

$$\frac{1}{2}=\frac{1x2}{2x2}=\frac{2}{4}$$

- 3

$$\frac{1}{2}=\frac{1x3}{2x3}=\frac{3}{6}$$

- 4

$$\frac{1}{2}=\frac{1x4}{2x4}=\frac{4}{8}$$

Simplest forms of the fraction

A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.

The shortest way to find the equivalent fraction in the simplest form is to find the HCF of the numerator and denominator, and then divide both of them by the HCF

Comparing Fractions

Fractions are of 3 types of proper fractions, improper fractions, and mixed fractions.

- Comparison of like fractions

Like fractions are the fractions that have the same denominators.

Example: $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{6}{2}$ ... .

In like fractions, the one with the greater numerator is greater.

Example: Which is greater?

i) $\frac{3}{7}$ or $\frac{9}{7}$

Here, we see that denominators of both the fractions are 7 and hence they are like fractions And on checking numerators, 9>3

So, $\frac{3}{7} < \frac{9}{7}$

$\frac{9}{7}$ is the greater fraction.

ii) $\frac{5}{8}$ or $\frac{13}{8}$

Here, we see that denominators of both the fractions are 8 and hence they are like fractions And on checking numerators, $13 > 5$.

So, $\frac{5}{8} < \frac{13}{8}$

$\frac{13}{8}$ is the greater fraction.

- Comparison of unlike fractions

Unlike fractions are the fractions which have different denominators.

Example: $\frac{1}{5}, \frac{3}{2}, \frac{5}{9}, \frac{6}{7}$ ... .

<u>Unlike fraction with the same numerator</u>

In unlike fractions with the same numerator, the one with greater denominator is smaller.

Example: Which is greater?

i) $\frac{1}{7}$ or $\frac{1}{9}$

Here, we see that numerators of both the fractions are 1 and now checking on denominators, 9>7

So, $\frac{1}{7} > \frac{1}{9}$

$\frac{1}{7}$ is the greater fraction.

Unlike fractions with different numerator

When we have unlike fractions with different numerator then we find the equivalent fractions of given fractions such that the denominators become the same.

So, the one with the greater numerator is greater.

Example: Which is greater?

i) $\frac{1}{7}$ or $\frac{2}{5}$

Here, we see that numerators of both the fractions are different.

$$\frac{1}{7} = \frac{1x5}{7x5} = \frac{5}{35}$$

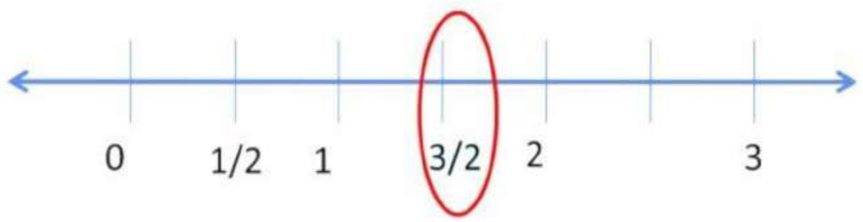
$$\frac{2}{5} = \frac{2x7}{5x7} = \frac{14}{35}$$

Now, we see that 14>5

So, $\frac{14}{35} > \frac{5}{35}$

$\frac{2}{5} > \frac{1}{7}$

$\frac{2}{5}$ is the greater fraction.



Addition and Subtraction of Fractions

1) Addition and Subtraction of two like fractions

When we add or subtract like fractions, we add or subtract their numerators and the denominator remains the same.

1) $\frac{5}{9} + \frac{2}{9}$

The two fractions are like fractions, so we add their numerators and keep the denominator the same.

$$\frac{5}{9} + \frac{2}{9} = \frac{5+2}{9} = \frac{7}{9}$$

2) $\frac{14}{25} - \frac{11}{25}$

Here, the given fractions are like fractions. So, we subtract their numerators and keep the denominator the same.

$$\frac{14}{25} - \frac{11}{25} = \frac{14-11}{25} = \frac{3}{25}$$

2) Addition and Subtraction of two unlike fractions

When we add or subtract unlike fractions we follow the following steps:

1. **Find the LCM of their denominators.**
2. **Convert the given fractions into like fractions.**
3. **Then add or subtract the like fractions.**

1) $\frac{5}{8} + \frac{11}{24}$

The given fractions are unlike fractions, so we first find LCM of their denominators.

LCM of 8 and 24 = 2 × 2 × 2 × 3 = 24

Now, we convert the fractions into like fractions.

(Changing the denominator of fractions to 24)

2	8, 24
2	4, 12
2	2, 6
3	1, 3
	1, 1

$\frac{5x3}{8x3} = \frac{15}{24}$ and $\frac{11}{24}$

$\frac{15}{24} + \frac{11}{24} = \frac{15 + 11}{24} = \frac{26}{24}$

2) $\frac{11}{25} - \frac{5}{27}$

As the given fractions are unlike fractions, we find the LCM of their denominator.

LCM of 15 and 27 = 3 × 3 × 3 × 5 = 135

Next, we convert the fractions into like fractions

(Fractions with the same denominator)

3	15, 27
3	5, 9
3	5, 3
5	5, 1
	1, 1

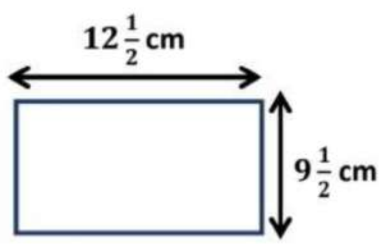
$\frac{11x9}{15x9} = \frac{99}{135}$ and $\frac{11x5}{27x5} = \frac{55}{135}$

$\frac{99}{135} - \frac{55}{135} = \frac{99-55}{135} = \frac{44}{135}$

Example: A rectangular sheet of paper $12\frac{1}{2}$ is cm long and $9\frac{1}{2}$ cm wide. Find its perimeter.

Length of the rectangular sheet = $12\frac{1}{2}$ cm.

$$12\frac{1}{2} = \frac{12x2+1}{2} = \frac{24+1}{2} = \frac{25}{2}$$



Breadth of the rectangular sheet = $9\frac{1}{2}$ cm

$a\frac{b}{c}$

Perimeter of a rectangle = $2(l + b)$

Perimeter of rectangular sheet of paper

$$= 2\left(\frac{25}{2} + \frac{19}{2}\right) = 2\left(\frac{25 + 19}{2}\right) = 2\left(\frac{44}{2}\right) = 44 \text{ cm}$$

Perimeter is the distance around a closed figure.

Example: Michael finished coloring a picture in $\frac{7}{12}$ hour. Vaibhav finished colouring the same picture in $\frac{3}{4}$ hour. Who worked longer?

By what fraction was it longer?

Time taken by Michael to colour the picture = $\frac{7}{12}$ hour

Time taken by Vaibhav to colour the same picture = $\frac{3}{4}$ hour

The two fractions are unlike, so we first convert them to like fractions (fractions having the same denominator).

2	12, 4
2	6, 2
3	3, 1
	1, 1

$\frac{7}{12}, \frac{3}{4}$

LCM of 12 and 4 = $2 \times 2 \times 3 = 12$

$\frac{7}{12}$ and $\frac{3x3}{4x3} = \frac{9}{12}$

On comparing the two fractions we get, $\frac{9}{12} > \frac{7}{12}$

Therefore, Vaibhav worked longer by

$\frac{9}{12}$ - $\frac{7}{12} = \frac{(9-7)}{12} = \frac{2}{12} = \frac{1}{6}$ hour