Chapter 6

Binomial Theorem and Mathematical Induction

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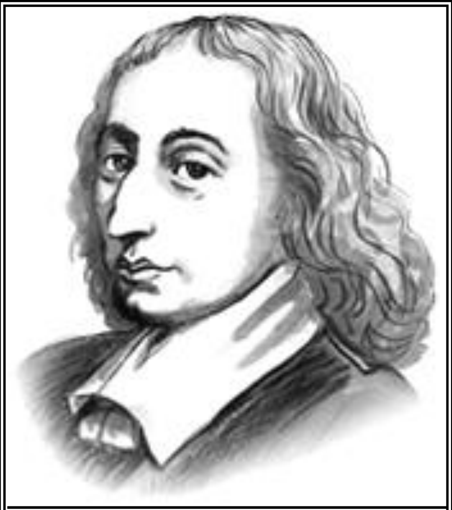

Blaise Pascel

The ancient Indian mathematicians knew the coefficient in the expansion of $(x+y)^n, 0 \le n \le 7$ *. The arrangement of these coefficient was in the form of a diagram called* Meru-Prastara, *provided by Pingla in his book* Chhanda-shastra (200 B.C.). *The term binomial coefficients was first introduced by the German mathematician.* Michael Stipel (1486-1567 A.D.)

The arithmetic triangle popularly known as pascal triangle was constructed by the French mathematician Blaise Pascal (1623-1662 A.D.) *He used the triangle to derive coefficients of a binomial expansion. It was printed in 1665 A.D. The present form of the binomial theorem for integral values of n appeared in Trate du triange arithmetic written by Pascal and published posthumously in 1665 A.D. The* ***generalization of the binomial theorem for negative integral and rational exponents is due to Sir Isaac Newton1*** **(642-1727 A.D)** ***in the same year 1665.***

6.1 Binomial Theorem

6.1.1 Binomial Expression

An algebraic expression consisting of two terms with +*ve* or – *ve* sign between them is called a binomial expression.

For example **:** $(a+b), (2x-3y), \left(\frac{p}{x^2}-\frac{q}{x^4}\right), \left(\frac{1}{x}+\frac{4}{y^3}\right)$ etc.

6.1.2 Binomial Theorem for Positive Integral Index

The rule by which any power of binomial can be expanded is called the binomial theorem.

If n is a positive integer and x, $y \in C$ then

$$(x+y)^n = {}^nC_0 x^{n-0} y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + ... + {}^nC_r x^{n-r} y^r + ... + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

i.e., $(x+y)^n = \sum_{r=0}^{n} {}^nC_r . x^{n-r} . y^r$ .....(i)

Here ${}^nC_0, {}^nC_1, {}^nC_2, ...{}^nC_n$ are called binomial coefficients and ${}^nC_r = \frac{n!}{r!(n-r)!}$ for $0 \le r \le n$.

Important Tips

- *The number of terms in the expansion of* $(x+y)^n$ *are (n + 1).*
- *The expansion contains decreasing power of x and increasing power of y. The sum of the powers of x and y in each term is equal to n.*
- *The binomial coefficients* ${}^nC_0, {}^nC_1, {}^nC_2........$ *equidistant from beginning and end are equal i.e.,* ${}^nC_r = {}^nC_{n-r}$.
- $(x+y)^n =$ *Sum of odd terms + sum of even terms.*

6.1.3 Some Important Expansions

(1) Replacing y by – y in (i), we get,

$$(x-y)^n = {}^nC_0 x^{n-0} . y^0 - {}^nC_1 x^{n-1} . y^1 + {}^nC_2 x^{n-2} . y^2 .... + (-1)^r \, {}^nC_r x^{n-r} . y^r + ... + (-1)^n \, {}^nC_n x^0 . y^n$$

i.e., $(x-y)^n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r x^{n-r} . y^r$ .....(ii)

The terms in the expansion of $(x-y)^n$ are alternatively positive and negative, the last term is positive or negative according as n is even or odd.

(2) Replacing x by 1 and y by x in equation (i) we get,

$$(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 + ... + {}^nC_r x^r + ... + {}^nC_n x^n$$ *i.e.,* $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$

This is expansion of $(1+x)^n$ in ascending power of x.

(3) Replacing x by 1 and y by – x in (i) we get, $(1-x)^n = {}^nC_0x^0 - {}^nC_1x^1 + {}^nC_2x^2 - ...... + (-1)^r\ {}^nC_rx^r + ... + (-1)^n\ {}^nC_nx^n$ *i.e.,* $(1-x)^n = \sum_{r=0}^{n}(-1)^r\ {}^nC_rx^r$

(4) $(x+y)^n + (x-y)^n = 2[{}^nC_0x^ny^0 + {}^nC_2x^{n-2}y^2 + {}^nC_4x^{n-4}y^4 + ...]$ and

$(x+y)^n - (x-y)^n = 2[{}^nC_1x^{n-1}y^1 + {}^nC_3x^{n-3}y^3 + {}^nC_5x^{n-5}y^5 + ...]$

(5) The coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^n$ is ${}^nC_r$.

(6) The coefficient of x^r in the expansion of $(1+x)^n$ is ${}^nC_r$.

Note : ❑ If n is odd, then $(x+y)^n + (x-y)^n$ and $(x+y)^n - (x-y)^n$, both have the same number of terms equal to $\left(\frac{n+1}{2}\right)$.

❑ If n is even, then $(x+y)^n + (x-y)^n$ has $\left(\frac{n}{2}+1\right)$ terms and $(x+y)^n - (x-y)^n$ has $\frac{n}{2}$ terms.

Example: 1 $x^5 + 10x^4a + 40x^3a^2 + 80x^2a^3 + 80xa^4 + 32a^5 =$

(a) $(x+a)^5$ (b) $(3x+a)^5$ (c) $(x+2a)^5$ (d) $(x+2a)^3$

Solution: (c) Conversely $(x+y)^n = {}^nC_0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + ... + {}^nC_nx^0y^n$

$$(x+2a)^5 = {}^5C_0x^5 + {}^5C_1x^4(2a)^1 + {}^5C_2x^3(2a)^2 + {}^5C_3x^2(2a)^3 + {}^5C_4x^1(2a)^4 + {}^5C_5x^0(2a)^5$$
$$= x^5 + 10x^4a + 40x^3a^2 + 80x^2a^3 + 80xa^4 + 32a^5.$$

Example: 2 The value of $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$ will be

(a) – 198 (b) 198 (c) 98 (d) – 99

Solution: (b) We know that, $(x+y)^n + (x-y)^n = 2[x^n + {}^nC_2x^{n-2}y^2 + {}^nC_4x^{n-4}y^4 + ...]$

$(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2[(\sqrt{2})^6 + {}^6C_2(\sqrt{2})^4(1)^2 + {}^6C_4(\sqrt{2})^2(1)^4 + {}^6C_6(\sqrt{2})^0(1)^6] = 2[8 + 15\times4 + 30 + 1] = 198$

Example: 3 The larger of $99^{50} + 100^{50}$ and 101^{50} is **[IIT 1980]**

(a) $99^{50} + 100^{50}$ (b) Both are equal (c) 101^{50} (d) None of these

Solution: (c) We have, $101^{50} = (100+1)^{50} = 100^{50} + 50.100^{49} + \frac{50.49}{2.1}100^{48} +$ .....(i)

and $99^{50} = (100-1)^{50} = 100^{50} - 50.100^{49} + \frac{50.49}{2.1}100^{48} - ....$ ..... (ii)

Subtracting, $101^{50} - 99^{50} = 100^{50} + 2.\frac{50.49.48}{3.2.1}100^{47} + ... > 100^{50}$. Hence $101^{50} > 100^{50} + 99^{50}$.

Example: 4 Sum of odd terms is A and sum of even terms is B in the expansion of $(x+a)^n$, then

(a) $AB = \frac{1}{4}(x-a)^{2n} - (x+a)^{2n}$ (b) $2AB = (x+a)^{2n} - (x-a)^{2n}$

(c) $4AB = (x+a)^{2n} - (x-a)^{2n}$ (d) None of these

Solution: (c) $(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a^1 + {}^nC_2x^{n-2}a^2 + ... + {}^nC_nx^{n-n}.a^n = (x^n + {}^nC_2x^{n-2}a^2 + ..) + ({}^nC_1x^{n-1}a^1 + {}^nC_3x^{n-3}a^3 + ...) = A + B$ .....(i)

Similarly, $(x-a)^n = A - B$ .....(ii)

From (i) and (ii), we get $4AB = (x+a)^{2n} - (x-a)^{2n}$

Trick: Put $n=1$ in $(x+a)^n$. Then, $x+a = A+B$. Comparing both sides $A = x, B = a$.

Option (c) L.H.S. $4AB = 4xa$, R.H.S. $(x+a)^2 - (x-a)^2 = 4ax$. *i.e.,* L.H.S. = R.H.S

6.1.4 General Term

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + ... + {}^nC_rx^{n-r}y^r + ... + {}^nC_nx^0y^n$$

The first term = ${}^nC_0 x^n y^0$

The second term = ${}^nC_1 x^{n-1} y^1$. The third term = ${}^nC_2 x^{n-2} y^2$ and so on

The term ${}^nC_r x^{n-r} y^r$ is the $(r+1)^{th}$ term from beginning in the expansion of $(x+y)^n$.

Let T_{r+1} denote the $(r+1)^{\text{th}}$ term $\therefore$ $T_{r+1} = {}^nC_r x^{n-r} y^r$

This is called general term, because by giving different values to r, we can determine all terms of the expansion.

In the binomial expansion of $(x-y)^n, T_{r+1} = (-1)^r \, {}^nC_r x^{n-r} y^r$

In the binomial expansion of $(1+x)^n, T_{r+1} = {}^nC_r x^r$

In the binomial expansion of $(1-x)^n, T_{r+1} = (-1)^r \, {}^nC_r x^r$

Note : ❑ In the binomial expansion of $(x+y)^n$, the p^{th} term from the end is $(n-p+2)^{th}$ term from beginning.

Important Tips

☞ *In the expansion of* $(x+y)^n, n \in N$

$$\frac{T_{r+1}}{T_r} = \left(\frac{n-r+1}{r}\right)\frac{y}{x}$$

☞ *The coefficient of* x^{n-1} *in the expansion of* $(x-1)(x-2)\dots\dots(x-n) = -\frac{n(n+1)}{2}$

☞ *The coefficient of* x^{n-1} *in the expansion of* $(x+1)(x+2)\dots\dots(x+n) = \frac{n(n+1)}{2}$

Example: 5 If the 4^{th} term in the expansion of $(px + x^{-1})^m$ is 2.5 for all $x \in R$ then

(a) $p = 5/2, m = 3$ (b) $p = \frac{1}{2}, m = 6$ (c) $p = -\frac{1}{2}, m = 6$ (d) None of these

Solution: (b) We have $T_4 = \frac{5}{2} \Rightarrow T_{3+1} = \frac{5}{2} \Rightarrow {}^mC_3 (px)^{m-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2} \Rightarrow {}^mC_3\, p^{m-3} x^{m-6} = \frac{5}{2}$ ....(i)

Clearly, R.H.S. of the above equality is independent of x

$\therefore m-6=0$, $m=6$

Putting $m=6$ in (i) we get ${}^6C_3 p^3 = \frac{5}{2} \Rightarrow p = \frac{1}{2}$. Hence $p = 1/2, m = 6$.

Example: 6 If the second, third and fourth term in the expansion of $(x+a)^n$ are 240, 720 and 1080 respectively, then the value of n is

[Kurukshetra CEE 1991; DCE 1995, 2001]

(a) 15 (b) 20 (c) 10 (d) 5

Solution: (d) It is given that $T_2 = 240, T_3 = 720, T_4 = 1080$

Now, $T_2 = 240 \Rightarrow T_2 = {}^nC_1 x^{n-1} a^1 = 240$ .....(i) and $T_3 = 720 \Rightarrow T_3 = {}^nC_2 x^{n-2} a^2 = 720$ .....(ii)

$T_4 = 1080 \Rightarrow T_4 = {}^nC_3 x^{n-3} a^3 = 1080$ .....(iii)

To eliminate x, $\frac{T_2 . T_4}{T_3^2} = \frac{240 . 1080}{720 . 720} = \frac{1}{2} \Rightarrow \frac{T_2}{T_3} . \frac{T_4}{T_3} = \frac{1}{2}$.

Now $\frac{T_{r+1}}{T_r} = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$. Putting $r=3$ and 2 in above expression, we get $\frac{n-2}{3} . \frac{2}{n-1} = \frac{1}{2} \Rightarrow n = 5$

Example: 7 The 5^{th} term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^3}\right)^9$ is

(a) $63x^3$ (b) $-\frac{252}{x^3}$ (c) $\frac{672}{x^{18}}$ (d) None of these

Solution: (b) 5^{th} term from the end = $(9-5+2)^{th}$ term from the beginning in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^3}\right)^9$ = T_6

$\Rightarrow T_6 = T_{5+1} = {}^9C_5\left(\frac{x^3}{2}\right)^4\left(-\frac{2}{x^3}\right)^5 = -{}^9C_4.2.\frac{1}{x^3} = -\frac{252}{x^3}$.

Example: 8 If $\frac{T_2}{T_3}$ in the expansion of $(a+b)^n$ and $\frac{T_3}{T_4}$ in the expansion of $(a+b)^{n+3}$ are equal, then $n=$

[Rajasthan PET 1987, 96]

(a) 3 (b) 4 (c) 5 (d) 6

Solution: (c) $\because \frac{T_2}{T_3} = \frac{2}{n-2+1}.\frac{b}{a} = \frac{2}{n-1}\left(\frac{b}{a}\right)$ and $\frac{T_3}{T_4} = \frac{3}{n+3-3+1}.\left(\frac{b}{a}\right) = \frac{3}{n+1}\left(\frac{b}{a}\right)$

$\because \frac{T_2}{T_3} = \frac{T_3}{T_4}$ (given) ; $\therefore \frac{2}{n-1}\left(\frac{b}{a}\right) = \frac{3}{n+1}\left(\frac{b}{a}\right) \Rightarrow 2n+2 = 3n-3 \Rightarrow n=5$

6.1.5 Independent Term or Constant Term

Independent term or constant term of a binomial expansion is the term in which exponent of the variable is zero.

Condition : $(n-r)$ [Power of x] + r . [Power of y] = 0, in the expansion of $[x+y]^n$.

Example: 9 The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ will be

[IIT 1965; BIT Ranchi 1993; Karnataka CET 2000; UPSEAT 2001]

(a) $\frac{3}{2}$ (b) $\frac{5}{4}$ (c) $\frac{5}{2}$ (d) None of these

Solution: (b) $(10-r)\left(\frac{1}{2}\right)+r(-2)=0 \Rightarrow r=2 \;\therefore\; T_3 = {}^{10}C_2\left(\frac{1}{3}\right)^{8/2}\left(\frac{3}{2}\right)^2 = \frac{5}{4}$

Example: 10 The term independent of x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is **[EAMCET 1989]**

(a) $C_0^2+2C_1^2+......+(n+1)C_n^2$ (b) $(C_0+C_1+......+C_n)^2$ (c) $C_0^2+C_1^2+......+C_n^2$ (d)

Solution: (c) We know that, $(1+x)^n = {}^nC_0 + {}^nC_1x^1 + {}^nC_2x^2 + ... + {}^nC_nx^n$

$\left(1+\frac{1}{x}\right)^n = {}^nC_0 + {}^nC_1\frac{1}{x^1} + {}^nC_2\frac{1}{x^2} + ... + {}^nC_n\frac{1}{x^n}$

Obviously, the term independent of x will be ${}^nC_0.{}^nC_0 + {}^nC_1{}^nC_1 + ... + {}^nC_n.{}^nC_n = C_0^2+C_1^2+.......+C_n^2$

Trick : Put $n=1$ in the expansion of $(1+x)^1\left(1+\frac{1}{x}\right)^1 = 1+x+\frac{1}{x}+1 = 2+x+\frac{1}{x}$ .....(i)

We want coefficient of x^0. Comparing to equation (i). Then, we get 2 *i.e.,* independent of x.
Option (c) : $C_0^2+C_1^2+.....C_n^2$; Put $n=1$; Then ${}^1C_0^2+{}^1C_1^2 = 1+1 = 2$.

Example: 11 The coefficient of x^{-7} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$ will be **[IIT 1967; Rajasthan PET 1996]**

(a) $\frac{462a^6}{b^5}$ (b) $\frac{462a^5}{b^6}$ (c) $\frac{-462a^5}{b^6}$ (d) $-\frac{462a^6}{b^5}$

Solution: (b) For coefficient of x^{-7}, $(11-r)(1)+(-2).r=-7 \Rightarrow 11-r-2r=-7 \Rightarrow r=6$; $T_7 = {}^{11}C_6(a)^5\left(-\frac{1}{b}\right)^6 = \frac{462a^5}{b^6}$

Example: 12 If the coefficients of second, third and fourth term in the expansion of $(1+x)^{2n}$ are in A.P., then $2n^2-9n+7$ is equal to

[AMU 2001]

(a) –1 (b) 0 (c) 1 (d) 3/2

Solution: (b) $T_2={}^{2n}C_1$, $T_3={}^{2n}C_2, T_4={}^{2n}C_3$ are in A.P. then, $2.{}^{2n}C_2={}^{2n}C_1+{}^{2n}C_3$

$$2.\frac{2n(.2n-1)}{2.1}=\frac{2n}{1}+\frac{2n(2n-1)(2n-2)}{3.2.1}$$

On solving, $2n^2-9n+7=0$

Example: 13 The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$ is **[EAMCET 1996; UPSEAT 2001; Pb. CET 2002]**

(a) 30 (b) 60 (c) 40 (d) None of these

Solution: (b) We have $(1+x^2)^5(1+x)^4 = ({}^5C_0+{}^5C_1x^2+{}^5C_2x^4+.....)\ ({}^4C_0+{}^4C_1x^1+{}^4C_2x^2+.......)$

So coefficient of x^5 in $[(1+x^2)^5(1+x)^4] = {}^5C_2.{}^4C_1+{}^4C_3.{}^5C_1=60$

Example: 14 If A and B are the coefficient of x^n in the expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then**[MP PET 1999**

(a) $A=B$ (b) $A=2B$ (c) $2A=B$ (d) None of these

Solution: (b) $A=$coefficient of x^n in $(1+x)^{2n} = {}^{2n}C_n=\frac{(2n)!}{n!n!}=\frac{2.(2n-1)!}{(n-1)!n!}$ .....(i)

$B=$ coefficient of x^n in $(1+x)^{2n-1}={}^{2n-1}C_n=\frac{(2n-1)!}{n!(n-1)!}$ .....(ii)

By (i) and (ii) we get, $A=2B$

Example: 15 The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$ (c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

Solution: (b) Coefficient of x^n in $(1+x)(1-x)^n$ = Coefficient of x^n in $(1-x)^n$ + coefficient of x^{n-1} in $(1-x)^n$

= Coefficient of x^n in $[{}^nC_n(-x)^n+x.{}^nC_{n-1}(-x)^{n-1}]$ = $(-1)^n\ {}^nC_n+(-1)^{n-1}.{}^nC_1$ = $(-1)^n+(-1)^n.(-n)=(-1)^n[1-n]$.

6.1.6 Number of Terms in the Expansion of $(a+b+c)^n$ and $(a+b+c+d)^n$

$(a+b+c)^n$ can be expanded as : $(a+b+c)^n=\{(a+b)+c\}^n$

$=(a+b)^n+{}^nC_1(a+b)^{n-1}(c)^1+{}^nC_2(a+b)^{n-2}(c)^2+.....+{}^nC_n c^n=(n+1)\text{term}+n\,\text{term}+(n-1)\text{term}+...+1\text{term}$

$\therefore$ Total number of terms = $(n+1)+(n)+(n-1)+......+1=\frac{(n+1)(n+2)}{2}$.

Similarly, Number of terms in the expansion of $(a+b+c+d)^n=\frac{(n+1)(n+2)(n+3)}{6}$.

Example: 16 If the number of terms in the expansion of $(x-2y+3z)^n$ is 45, then $n=$

(a) 7 (b) 8 (c) 9 (d) None of these

Solution: (b) Given, total number of terms = $\frac{(n+1)(n+2)}{2}=45 \Rightarrow (n+1)(n+2)=90 \Rightarrow n=8$.

Example: 17 The number of terms in the expansion of $[(x+3y)^2(3x-y)^2]^3$ is **[Rajasthan PET 1986]**

(a) 14 (b) 28 (c) 32 (d) 56

Solution: (b) We have $[(x+3y)(3x-y)]^6$ = $[3x^2+8xy-3y^2]^6$; Number of terms = $\frac{(6+1)(6+2)}{2}=28$

6.1.7 Middle Term

The middle term depends upon the value of n.

(1) **When *n* is even,** then total number of terms in the expansion of $(x+y)^n$ is $n+1$ (odd). So there is only one middle term *i.e.,* $\left(\frac{n}{2}+1\right)^{\text{th}}$ term is the middle term. $T_{\left[\frac{n}{2}+1\right]} = {}^nC_{n/2}x^{n/2}y^{n/2}$

(2) **When *n* is odd**, then total number of terms in the expansion of $(x+y)^n$ is $n+1$ (even). So, there are two middle terms *i.e.,* $\left(\frac{n+1}{2}\right)^{\text{th}}$ and $\left(\frac{n+3}{2}\right)^{\text{th}}$ are two middle terms. $T_{\left(\frac{n+1}{2}\right)} = {}^nC_{\frac{n-1}{2}}x^{\frac{n+1}{2}}y^{\frac{n-1}{2}}$ and $T_{\left(\frac{n+3}{2}\right)} = {}^nC_{\frac{n+1}{2}}x^{\frac{n-1}{2}}y^{\frac{n+1}{2}}$

Note : ❑ When there are two middle terms in the expansion then their binomial coefficients are equal.

❑ Binomial coefficient of middle term is the greatest binomial coefficient.

Example: 18 The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$ is **[BIT Ranchi 1991; Rajasthan PET 2002; Pb. CET 1991]**

(a) ${}^{10}C_4\frac{1}{x}$ (b) ${}^{10}C_5$ (c) ${}^{10}C_5x$ (d) ${}^{10}C_7x^4$

Solution: (b) $\because$ n is even so middle term $T_{\left(\frac{10}{2}+1\right)} = T_6 \Rightarrow T_6 = T_{5+1} = {}^{10}C_5\, x^5.\frac{1}{x^5} = {}^{10}C_5$

Example: 19 The middle term in the expansion of $(1+x)^{2n}$ is **[Pb. CET 1998]**

(a) $\frac{1.3.5......(2n-1)}{n!}x^{2n+1}$ (b) $\frac{2.4.6......2n}{n!}x^{2n+1}$ (c) $\frac{1.3.5......(2n-1)}{n!}x^n$ (d) $\frac{1.3.5......(2n-1)}{n!}x^n.2^n$

Solution: (d) Since $2n$ is even, so middle term $= T_{\frac{2n}{2}+1} = T_{n+1} \Rightarrow T_{n+1} = {}^{2n}C_nx^n = \frac{(2n)!}{n!.n!}x^n = \frac{1.3.5........(2n-1)}{n!}.2^nx^n$.

6.1.8 To Determine a Particular Term in the Expansion

In the expansion of $\left(x^\alpha \pm \frac{1}{x^\beta}\right)^n$, if x^m occurs in T_{r+1}, then r is given by $n\alpha - r(\alpha+\beta) = m \Rightarrow r = \frac{n\alpha - m}{\alpha+\beta}$

Thus in above expansion if constant term which is independent of *x*, occurs in T_{r+1} then r is determined by

$$n\alpha - r(\alpha+\beta) = 0 \Rightarrow r = \frac{n\alpha}{\alpha+\beta}$$

Example: 20 The term independent of *x* in the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ is

(a) 7/12 (b) 7/18 (c) – 7/12 (d) – 7/16

Solution: (b) n =9, $\alpha = 2$, $\beta = 1$. Then $r = \frac{9(2)}{1+2} = 6$. Hence, $T_7 = {}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6 = \frac{9\times8\times7}{3\times2\times1}.\frac{1}{2^3.3^3} = \frac{7}{18}$.

Example: 21 If the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ then ab =

[MP PET 1999; AMU 2001; Pb. CET 2002]

(a) 1 (b) 1/2 (c) 2 (d) 3

Solution: (a) For coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$; n = 11, $\alpha=2,\beta=1$, $m=7$

$r=\frac{11.2-7}{2+1}=\frac{15}{3}=5$

Coefficient of x^7 in $T_6={}^{11}C_5\,a^6.\frac{1}{b^5}$ ......(i)

and for coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$; $n=11,\alpha=1,\beta=2$, $m=-7$; $r=\frac{11.1+7}{3}=6$

Coefficient of x^{-7} in $T_7={}^{11}C_6.a^5.\frac{1}{b^6}$ .....(ii)

From equation (i) and (ii), we get $ab=1$

6.1.9 Greatest Term and Greatest Coefficient

(1) **Greatest term :** If T_r and T_{r+1} be the r^{th} and $(r+1)^{th}$ terms in the expansion of $(1+x)^n$, then

$$\frac{T_{r+1}}{T_r}=\frac{{}^nC_r x^r}{{}^nC_{r-1}x^{r-1}}=\frac{n-r+1}{r}x$$

Let numerically, T_{r+1} be the greatest term in the above expansion. Then $T_{r+1}\geq T_r$ or $\frac{T_{r+1}}{T_r}\geq 1$

$\therefore\ \frac{n-r+1}{r}|x|\geq 1$ or $r\leq\frac{(n+1)}{(1+|x|)}|x|$ ......(i)

Now substituting values of n and x in (i), we get $r\leq m+f$ or $r\leq m$

where m is a positive integer and f is a fraction such that $0<f<1$.

When n is even T_{m+1} is the greatest term, when n is odd T_m and T_{m+1} are the greatest terms and both are equal.

Short cut method : To find the greatest term (numerically) in the expansion of $(1+x)^n$.

(i) Calculate m = $\left|\frac{x(n+1)}{x+1}\right|$

(ii) If m is integer, then T_m and T_{m+1} are equal and both are greatest term.

(iii) If m is not integer, there $T_{[m]+1}$ is the greatest term, where [.] denotes the greatest integral part.

(2) **Greatest coefficient**

(i) If n is even, then greatest coefficient is ${}^nC_{n/2}$

(ii) If n is odd, then greatest coefficient are ${}^nC_{\frac{n+1}{2}}$ and ${}^nC_{\frac{n+3}{2}}$

Important Tips

☞ *For finding the greatest term in the expansion of* $(x+y)^n$. *we rewrite the expansion in this form* $(x+y)^n=x^n\left[1+\frac{y}{x}\right]^n$.

Greatest term in (x + y)ⁿ $= x^n$. *Greatest term in* $\left(1+\frac{y}{x}\right)^n$

Example: 22 The largest term in the expansion of $(3+2x)^{50}$, where $x=\frac{1}{5}$ is

(a) 5^{th} (b) 8^{th} (c) 7^{th} (d) 6^{th}

Solution: (c,d) $(3+2x)^{50}=3^{50}\left[1+\frac{2x}{3}\right]^{50}$, Now greatest term in $\left(1+\frac{2x}{3}\right)^{50}$

$$r=\left|\frac{x(n+1)}{1+x}\right|=\left|\frac{\frac{2x}{3}(50+1)}{\frac{2x}{3}+1}\right|=\frac{\frac{2.\frac{1}{5}}{3}(51)}{\frac{2}{15}+1}=6 \text{ (an integer)}$$

$\therefore$ T_r and $T_{[r]+1}=T_6$ and $T_{[6]+1}=T_6$ and T_7 are numerically greatest terms

Example: 23 The greatest coefficient in the expansion of $(1+x)^{2n+2}$ is

(a) $\frac{(2n)!}{n!^2}$ (b) $\frac{(2n+2)!}{[(n+1)!]^2}$ (c) $\frac{(2n+2)!}{n!(n+1)!}$ (d) $\frac{(2n)!}{n!.(n+1)!}$

Solution: (b) $\because$ n is even so greatest coefficient in $(1+x)^{2n+2}$ is = ${}^{2n+2}C_{n+1}=\frac{(2n+2)!}{[(n+1)!]^2}$

Example: 24 The interval in which x must lie so that the greatest term in the expansion of $(1+x)^{2n}$ has the greatest coefficient is

(a) $\left(\frac{n-1}{n},\frac{n}{n-1}\right)$ (b) $\left(\frac{n}{n+1},\frac{n+1}{n}\right)$ (c) $\left(\frac{n}{n+2},\frac{n+2}{n}\right)$ (d) None of these

Solution: (b) Here the greatest coefficient is ${}^{2n}C_n$

$\therefore$ ${}^{2n}C_nx^n>{}^{2n}C_{n+1}x^{n-1}\Rightarrow x>\frac{n}{n+1}$ and ${}^{2n}C_nx^n>{}^{2n}C_{n-1}x^{n+1}\Rightarrow x<\frac{n+1}{n}$. Hence the result is (b)

6.1.10 Properties of Binomial Coefficients

In the binomial expansion of $(1+x)^n$, $(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+.....+{}^nC_rx^r+....+{}^nC_nx^n$.

where ${}^nC_0,{}^nC_1,{}^nC_2,......,{}^nC_n$ are the coefficients of various powers of x and called binomial coefficients, and they are written as $C_0,C_1,C_2,.....C_n$.

Hence, $(1+x)^n=C_0+C_1x+C_2x^2+.....+C_rx^r+.....+C_nx^n$ .....(i)

(1) The sum of binomial coefficients in the expansion of $(1+x)^n$ is 2^n.

Putting $x=1$ in (i), we get $2^n=C_0+C_1+C_2+.....+C_n$ .....(ii)

(2) Sum of binomial coefficients with alternate signs : Putting $x=-1$ in (i)

We get, $0=C_0-C_1+C_2-C_3+......$ .....(iii)

(3) Sum of the coefficients of the odd terms in the expansion of $(1+x)^n$ is equal to sum of the coefficients of even terms and each is equal to 2^{n-1}.

From (iii), we have $C_0+C_2+C_4+.....=C_1+C_3+C_5+........$ ......(iv)

i.e., sum of coefficients of even and odd terms are equal.

From (ii) and (iv), $C_0+C_2+C_4+.....=C_1+C_3+C_5+.....=2^{n-1}$ ......(v)

(4) ${}^nC_r=\frac{n}{r}{}^{n-1}C_{r-1}=\frac{n}{r}.\frac{n-1}{r-1}{}^{n-2}C_{r-2}$ and so on.

(5) Sum of product of coefficients : Replacing x by $\frac{1}{x}$ in (i) we get $\left(1+\frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \dots \frac{C_n}{x^n} + \dots$ (vi)

Multiplying (i) by (vi), we get $\frac{(1+x)^{2n}}{x^n} = (C_0 + C_1 x + C_2 x^2 + \dots\dots)\left(C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \dots\dots\right)$

Now comparing coefficient of x^r on both sides. We get, ${}^{2n}C_{n+r} = C_0 C_r + C_1 C_{r+1} + \dots\dots C_{n-r}.C_n$ .....(vii)

(6) Sum of squares of coefficients : Putting $r=0$ in (vii), we get ${}^{2n}C_n = C_0^2 + C_1^2 + \dots\dots C_n^2$

(7) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Example: 25 The value of $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \dots\dots$ is equal to **[Karnataka CET 2000]**

(a) $\frac{2^n - 1}{n+1}$ (b) $n.2^n$ (c) $\frac{2^n}{n}$ (d) $\frac{2^n + 1}{n+1}$

Solution: (a) We have $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \dots\dots = \frac{n}{2.1} + \frac{n(n-1)(n-2)}{4.3.2.1} + \frac{n(n-1)(n-2)(n-3)(n-4)}{6.5.4.3.2.1} + \dots\dots$

$= \frac{1}{n+1}\left[\frac{(n+1)n}{2!} + \frac{(n+1)(n)(n-1)(n-2)}{4!} + \dots\dots\right] = \frac{1}{n+1}[2^{(n+1)-1} - 1] = \frac{2^n - 1}{n+1}$

Trick: For $n=1$, $= \frac{C_1}{2} = \frac{{}^1C_1}{2} = \frac{1}{2}$

Which is given by option (a) $\frac{2^n - 1}{n+1} = \frac{2^1 - 1}{1+1} = \frac{1}{2}$.

Example: 26 The value of $C_0 + 3C_1 + 5C_2 + \dots\dots + (2n+1)C_n$ is equal to

(a) 2^n (b) $2^n + n.2^{n-1}$ (c) $2^n(n+1)$ (d) None of these

Solution: (c) We have $C_0 + 3C_1 + 5C_2 + \dots\dots + (2n+1)C_n = \sum_{r=0}^{n}(2r+1)C_r = \sum_{r=0}^{n}(2r+1)\,{}^nC_r = \sum_{r=0}^{n} 2r\ {}^nC_r + \sum_{r=0}^{n} {}^nC_r$

$= 2.\sum_{r=1}^{n} r.\frac{n}{r}.{}^{n-1}C_{r-1} + \sum_{r=0}^{n} {}^nC_r = 2n\sum_{r=1}^{n} {}^{n-1}C_{r-1} + \sum_{r=0}^{n} {}^nC_r = 2n[(1+1)^{n-1}] + [1+1]^n = 2n.2^{n-1} + 2^n = 2^n.[n+1]$.

Trick: Put $n=1$ in given expansion ${}^1C_0 + 3.{}^1C_1 = 1 + 3 = 4$.

Which is given by option (c) $2^n.(n+1) = 2^1(1+1) = 4$.

Example: 27 If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$. Then $\frac{t_n}{S_n}$ is equal to

(a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n - 1$ (c) $n-1$ (d) $\frac{n}{2}$

Solution: (d) Take $n = 2m$, then, $S_n = \frac{1}{{}^{2m}C_0} + \frac{1}{{}^{2m}C_1} + \dots\dots + \frac{1}{{}^{2m}C_{2m}} = 2\left[\frac{1}{{}^{2m}C_0} + \frac{1}{{}^{2m}C_1} + \dots\dots + \frac{1}{{}^{2m}C_{m-1}}\right] + \frac{1}{{}^{2m}C_m}$

$t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{2m} \frac{r}{{}^{2m}C_r} = \frac{1}{{}^{2m}C_1} + \frac{2}{{}^{2m}C_2} + \dots\dots + \frac{2m}{{}^{2m}C_{2m}}$

$$t_n = \left(\frac{1}{{}^{2m}C_1} + \frac{2m-1}{{}^{2m}C_{2m-1}}\right) + \left(\frac{2}{{}^{2m}C_2} + \frac{2m-2}{{}^{2m}C_{2m-2}}\right) + ... + \left(\frac{m-1}{{}^{2m}C_{m-1}} + \frac{m+1}{{}^{2m}C_{m+1}}\right) + \frac{m}{{}^{2m}C_m} + \frac{2m}{{}^{2m}C_{2m}}$$

$$= 2m\left[\frac{1}{{}^{2m}C_1} + \frac{1}{{}^{2m}C_2} + ...\right] + \frac{m}{{}^{2m}C_m} + 2m = 2m\left[\frac{1}{{}^{2m}C_0} + \frac{1}{{}^{2m}C_1} + ... + \frac{1}{{}^{2m}C_{m-1}}\right] + \frac{m}{{}^{2m}C_m} = m\left[S_n - \frac{1}{{}^{2m}C_m}\right] + \frac{m}{{}^{2m}C_m} = mSn$$

$$t_n = mS_n \Rightarrow \frac{t_n}{S_n} = m = \frac{n}{2}$$

Example: 28 If $(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$. Then $a_0 + a_2 + a_4 + ... + a_{2n} =$

[MNR 1992; DCE 1996; AMU 1998; Rajasthan PET 1999; Karnataka CET 1999; UPSEAT 1999]

(a) $\frac{3^n + 1}{2}$ (b) $\frac{3^n - 1}{2}$ (c) $\frac{1 - 3^n}{2}$ (d) $3^n + \frac{1}{2}$

Solution: (a) $(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$

Putting $x = 1$, we get $(1 - 1 + 1)^n = a_0 + a_1 + a_2 + ... + a_{2n}$; $1 = a_0 + a_1 + a_2 + ... + a_{2n}$ .....(i)

Again putting $x = -1$, we get $3^n = a_0 - a_1 + a_2 - ...... + a_{2n}$ ......(ii)

Adding (i) and (ii), we get, $3^n + 1 = 2[a_0 + a_2 + a_4 + ... + a_{2n}]$

$$\frac{3^n + 1}{2} = a_0 + a_2 + a_4 + ... + a_{2n}$$

Example: 29 If $(1 + x)^n = \sum_{r=0}^{n} C_r x^r$, then $\left(1 + \frac{C_1}{C_0}\right)\left(1 + \frac{C_2}{C_1}\right)......\left(1 + \frac{C_n}{C_{n-1}}\right) =$

(a) $\frac{n^{n-1}}{(n-1)!}$ (b) $\frac{(n+1)^{n-1}}{(n-1)!}$ (c) $\frac{(n+1)^n}{n!}$ (d) $\frac{(n+1)^{n+1}}{n!}$

Solution: (c) We have $\left(1 + \frac{C_1}{C_0}\right)\left(1 + \frac{C_2}{C_1}\right)........\left(1 + \frac{C_n}{C_{n-1}}\right) = \left(1 + \frac{n}{1}\right)\left(1 + \frac{n(n-1)/2!}{n}\right)......\left(1 + \frac{1}{n}\right)$

$$= \left(\frac{1+n}{1}\right)\left(\frac{1+n}{2}\right)\left(\frac{1+n}{3}\right).......\left(\frac{1+n}{n}\right) = \frac{(n+1)^n}{n!}$$

Trick : Put $n = 1, 2, 3......., S_1 = 1 + \frac{{}^1C_1}{{}^1C_0} = 2, S_2 = \left(1 + \frac{{}^2C_1}{{}^2C_1}\right)\left(1 + \frac{{}^2C_2}{{}^2C_1}\right) = \frac{9}{2}$

Which is given by option (c) $n = 1$, $\frac{(1+1)^1}{1!} = 2$; For $n = 2$, $\frac{(2+1)^2}{2!} = \frac{9}{2}$

Example: 30 In the expansion of $(1 + x)^5$, the sum of the coefficient of the terms is **[Rajasthan PET 1992, 97; Kurukshetra CEE**

(a) 80 (b) 16 (c) 32 (d) 64

Solution: (c) Putting $x = 1$ in $(1 + x)^5$, the required sum of coefficient = $(1 + 1)^5 = 2^5 = 32$

Example: 31 If the sum of coefficient in the expansion of $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$ vanishes, then the value of α is **[IIT 1991; Pb. CET 1988]**

(a) 2 (b) –1 (c) 1 (d) – 2

Solution: (c) The sum of coefficient of polynomial $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$ is obtained by putting $x = 1$ in $(\alpha^2 x^2 - 2\alpha x + 1)^{51}$. Therefore by hypothesis $(\alpha^2 - 2\alpha + 1)^{51} = 0 \Rightarrow \alpha = 1$

Example: 32 If C_r stands for ${}^nC_r$, the sum of given series $\frac{2(n/2)!(n/2)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 - ...... + (-1)^n (n+1) C_n^2]$ where n is an even positive integer, is

(a) 0 (b) $(-1)^{n/2}(n+1)$ (c) $(-1)^n (n+2)$ (d) $(-1)^{n/2}(n+2)$

Solution: (d) We have $C_0^2 - 2C_1^2 + 3C_2^2 - .... + (-1)^n (n+1) C_n^2 = [C_0^2 - C_1^2 + C_2^2 - .... + (-1)^n C_n^2] - [C_1^2 - 2C_2^2 + 3C_3^2 ...... + (-1)^n n.C_n^2]$

$$= (-1)^{n/2}.{}^nC_{n/2} - (-1)^{n/2-1}.\frac{1}{2}n.{}^nC_{n/2} = (-1)^{n/2}\left[1 + \frac{n}{2}\right]{}^nC_{n/2}$$

Therefore the value of given expression $= \frac{2.\frac{n}{2}!\frac{n}{2}!}{n!}\left[(-1)^{n/2}.\left(1+\frac{n}{2}\right)\frac{n!}{\frac{n}{2}!\frac{n}{2}!}\right] = (-1)^{n/2}(n+2)$

Example: 33 If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then the value of $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ will be

[MP PET 1996; Rajasthan PET 1997; DCE 1995; IIT 1971; AMU 1995; EAMCET 2001]

(a) $(n+2)2^{n-1}$ (b) $(n+1)2^n$ (c) $(n+1)2^{n-1}$ (d) $(n+2)2^n$

Solution: (a) **Trick:** Put $n=1$ the expansion is equivalent to ${}^1C_0 + 2.{}^1C_1 = 1+2 = 3$.

Which is given by option (a) = $(n+2)2^{n-1} = (1+2)2^0 = 3$

(1) **Use of Differentiation :** This method applied only when the numericals occur as the product of binomial coefficients.

Solution process : (i) If last term of the series leaving the plus or minus sign be *m*, then divide *m* by *n* if *q* be the quotient and *r* be the remainder. *i.e.,* $m = nq + r$

Then replace *x* by x^q in the given series and multiplying both sides of expansion by x^r.

(ii) After process (i), differentiate both sides, w.r.t. *x* and put $x=1$ or -1 or *i* or – *i* etc. according to given series.

(iii) If product of two numericals (or square of numericals) or three numericals (or cube of numerical) then differentiate twice or thrice.

Example: 34 $C_1 + 2C_2 + 3C_3 + ...{}^nC_n =$ **[Rajasthan PET 1995; MP PET 2002]**

(a) 2^n (b) $n.2^n$ (c) $n.2^{n-1}$ (d) $n.2^{n+1}$

Solution: (c) We know that, $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ ......(i)

Differentiating both sides w.r.t. *x*, we get $n(1+x)^{n-1} = 0 + C_1 + 2.C_2x + 3C_3x^2 + ... + nC_nx^{n-1}$

Putting $x=1$, we get, $n.2^{n-1} = C_1 + 2C_2 + 3C_3 + ... + nC_n$.

Example: 35 If *n* is an integer greater than 1, then $a - {}^nC_1(a-1) + {}^nC_2(a-2) - .... + (-1)^n(a-n) =$

(a) a (b) 0 (c) a^2 (d) 2^n

Solution: (b) We have $a[C_0 - C_1 + C_2......] + [C_1 - 2C_2 + 3C_3.....] = a[C_0 - C_1 + C_2......] - [-C_1 + 2C_2 - 3C_3 + ...]$

We know that $(1-x)^n = C_0 - C_1x + C_2x^2...... + (-1)^nC_nx^n$; Put $x=1$, $0 = C_0 - C_1 + C_2 - ....$

Then differentiating both sides w.r.t. to x, we get $n(1-x)^{n-1} = 0 - C_1 + 2C_2x - 3C_3x^2 + ...$

Put $x=1$, $0 = -C_1 + 2C_2 - 3C_3 + ...$ $= a[0] - [0] = 0$.

(2) **Use of Integration :** This method is applied only when the numericals occur as the denominator of the binomial coefficients.

Solution process : If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then we integrate both sides between the suitable limits which gives the required series.

(i) If the sum contains $C_0, C_1, C_2, ... C_n$ with all positive signs, then integrate between limit 0 to 1.

(ii) If the sum contains alternate signs (*i.e.* +, –) then integrate between limit – 1 to 0.

(iii) If the sum contains odd coefficients *i.e.,* (C_0, C_2, C_4.....) then integrate between –1 to 1.

(iv) If the sum contains even coefficients (*i.e.,* $C_1, C_3, C_5.....$) then subtracting (ii) from (i) and then dividing by 2.

(v) If in denominator of binomial coefficients is product of two numericals then integrate two times, first taking limit between 0 to *x* and second time take suitable limits.

Example: 36 $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}+.....+\frac{C_n}{n+1}=$ **[Rajasthan PET 1996]**

(a) $\frac{2^n}{n+1}$ (b) $\frac{2^n-1}{n+1}$ (c) $\frac{2^{n+1}-1}{n+1}$ (d) None of these

Solution: (c) Consider the expansion $(1+x)^n=C_0+C_1x+C_2x^2+....+C_nx^n$ .....(i)

Integrating both sides of (i) within limits 0 to 1 we get,

$\int_0^1(1+x)^n dx=\int_0^1 C_0+\int_0^1 C_1x+\int_0^1 C_2x^2+.......+\int_0^1 C_nx^n dx$

$\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1=C_0[x]_0^1+C_1\left[\frac{x^2}{2}\right]_0^1+........+C_n\left[\frac{x^{n+1}}{n+1}\right]_0^1$

$\frac{2^{n+1}}{n+1}-\frac{1}{n+1}=C_0[1]+C_1\frac{1}{2}+C_2\frac{1}{3}+.......+C_n.\frac{1}{n+1}$; $C_0+\frac{C_1}{2}+\frac{C_2}{3}+....+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$.

Example: 37 $2C_0+\frac{2^2}{2}C_1+\frac{2^3}{3}C_2+......+\frac{2^{11}}{11}C_{10}=$ **[MP PET 1999; EAMCET 1992]**

(a) $\frac{3^{11}-1}{11}$ (b) $\frac{2^{11}-1}{11}$ (c) $\frac{11^3-1}{11}$ (d) $\frac{11^2-1}{11}$

Solution: (a) It is clear that it is a expansion of $(1+x)^{10}=C_0+C_1x+C_2x^2+.....+C_{10}x^{10}$

Integrating w.r.t. x both sides between the limit 0 to 2.

$\left[\frac{(1+x)^{11}}{11}\right]_0^2=C_0[x]_0^2+C_1\left[\frac{x^2}{2}\right]_0^2+C_2\left[\frac{x^3}{3}\right]_0^2+......+C_{10}\left[\frac{x^{11}}{11}\right]_0^2 \Rightarrow \frac{3^{11}-1}{11}=2C_0+\frac{2^2}{2}.C_1+\frac{2^3}{2}.C_2+......+\frac{2^{11}}{11}C_{10}$.

Example: 38 The sum to $(n+1)$ terms of the following series $\frac{C_0}{2}-\frac{C_1}{3}+\frac{C_2}{4}-\frac{C_3}{5}+.....$ is

(a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$ (c) $\frac{1}{n(n+1)}$ (d) None of these

Solution: (d) $(1-x)^n=C_0-C_1x+C_2x^2-C_3x^3+.......$

$\Rightarrow x(1-x)^n=C_0x-C_1x^2+C_2x^3-C_3x^4+..... \Rightarrow \int_0^1 x(1-x)^n dx=C_0\left[\frac{x^2}{2}\right]_0^1-C_1\left[\frac{x^3}{3}\right]_0^1+C_2\left[\frac{x^4}{4}\right]_0^1-.......$ (i)

The integral on L.H.S. of (i) = $\int_1^0(1-t)t^n(-dt)$ by putting $1-x=t$, $\Rightarrow \int_0^1(t^n-t^{n+1})dt=\frac{1}{n+1}-\frac{1}{n+2}$

Whereas the integral on the R.H.S. of (i)

$=C_0\left[\frac{1}{2}\right]-C_1\left[\frac{1}{3}\right]+\frac{C_2}{4}-.......=\frac{C_0}{2}-\frac{C_1}{3}+\frac{C_2}{4}-.......$ to $(n+1)$ terms $=\frac{1}{n+1}-\frac{1}{n+2}=\frac{1}{(n+1)(n+2)}$

Trick : Put $n=1$ in given series $=\frac{{}^1C_0}{2}-\frac{{}^1C_1}{3}=\frac{1}{6}$. Which is given by option (d).

6.1.11 An Important Theorem

If $(\sqrt{A}+B)^n=I+f$ where I and n are positive integers, n being odd and $0\le f<1$ then $(I+f).f=K^n$ where $A-B^2=K>0$ and $\sqrt{A}-B<1$.

Note : ❑ If n is even integer then $(\sqrt{A}+B)^n+(\sqrt{A}-B)^n=I+f+f'$

Hence L.H.S. and I are integers.

$\therefore f+f'$ is also integer; $\Rightarrow f+f'=1$; $\therefore f'=(1-f)$

Hence $(I+f)(1-f)=(I+f)f' = (\sqrt{A}+B)^n(\sqrt{A}-B)^n = (A-B^2)^n = K^n$.

Example: 39 Let $R=(5\sqrt{5}+11)^{2n+1}$ and $f=R-[R]$ where [.] denotes the greatest integer function. The value of R.f is **[IIT 19**

(a) 4^{2n+1} (b) 4^{2n} (c) 4^{2n-1} (d) 4^{-2n}

Solution: (a) Since $f=R-[R]$, $R=f+[R]$

$[5\sqrt{5}+11]^{2n+1}=f+[R]$, where $[R]$ is integer

Now let $f'=[5\sqrt{5}-11]^{2n+1}, 0<f'<1$

$f+[R]-f' = [5\sqrt{5}+11]^{2n+1}-[5\sqrt{5}-11]^{2n+1} = 2\left[{}^{2n+1}C_1(5\sqrt{5})^{2n}(11)^1+{}^{2n+1}C_3(5\sqrt{5})^{2n-2}(11)^3+.......\right]$

$= 2.(\text{Integer}) = 2K\ (K\in N)$ = Even integer

Hence $f-f'$ = even integer – [R], but $-1<f-f'<1$. Therefore, $f-f'=0 \quad \therefore f=f'$

Hence R.f = $R.f'=(5\sqrt{5}+11)^{2n+1}(5\sqrt{5}-11)^{2n+1}=4^{2n+1}$.

6.1.12 Multinomial Theorem (For positive integral index)

If n is positive integer and $a_1,a_2,a_3,....a_n\in C$ then $(a_1+a_2+a_3+...+a_m)^n=\sum\frac{n!}{n_1!n_2!n_3!...n_m!}a_1^{n_1}a_2^{n_2}...a_m^{n_m}$

Where $n_1,n_2,n_3,.....n_m$ are all non-negative integers subject to the condition, $n_1+n_2+n_3+.....n_m=n$.

(1) The coefficient of $a_1^{n_1}.a_2^{n_2}.....a_m^{n_m}$ in the expansion of $(a_1+a_2+a_3+....a_m)^n$ is $\frac{n!}{n_1!n_2!n_3!....n_m!}$

(2) The greatest coefficient in the expansion of $(a_1+a_2+a_3+....a_m)^n$ is $\frac{n!}{(q!)^{m-r}[(q+1)!]^r}$

Where q is the quotient and r is the remainder when n is divided by m.

(3) If n is $+ve$ integer and $a_1,a_2,.....a_m\in C$, $a_1^{n_1}.a_2^{n_2}.........a_m^{n_m}$ then coefficient of x^r in the expansion of $(a_1+a_2x+.....a_mx^{m-1})^n$ is $\sum\frac{n!}{n_1!n_2!n_3!.....n_m!}$

Where $n_1,n_2.....n_m$ are all non-negative integers subject to the condition: $n_1+n_2+.....n_m=n$ and $n_2+2n_3+3n_4+....+(m-1)n_m=r$.

(4) The number of distinct or dissimilar terms in the multinomial expansion $(a_1+a_2+a_3+....a_m)^n$ is ${}^{n+m-1}C_{m-1}$.

Example: 40 The coefficient of x^5 in the expansion of $(x^2-x-2)^5$ is

(a) – 83 (b) – 82 (c) – 81 (d) 0

Solution: (c) Coefficient of x^5 in the expansion of $(x^2-x-2)^5$ is $\sum\frac{5!}{n_1!.n_2!n_3!}(1)^{n_1}(-1)^{n_2}(-2)^{n_3}$.

where $n_1+n_2+n_3=5$ and $n_2+2n_3=5$. The possible value of n_1,n_2 and n_3 are shown in margin

n_1	n_2	n_3
1	3	1
2	1	2
0	5	0

$\therefore$ The coefficient of x^5 = $\frac{5!}{1!3!1!}(1)^1(-1)^3(-2)^1+\frac{5!}{2!1!2!}(1)^2(-1)^1(-2)^2+\frac{5!}{0!5!0!}(1)^0(-1)^5(-2)^0 = 40-120-1=-81$

Example: 41 Find the coefficient of $a^3b^4c^5$ in the expansion of $(bc+ca+ab)^6$

(a) 0 (b) 60 (c) – 60 (d) None of these

Solution: (b) In this case, $a^3b^4c^5=(ab)^x(bc)^y(ca)^z=a^{x+z}.b^{x+y}.c^{y+z}$

$z+x=3$, $x+y=4, y+z=5$; $2(x+y+z)=12$; $x+y+z=6$. Then $x=1, y=3, z=2$

Therefore the coefficient of $a^3b^4c^5$ in the expansion of $(bc+ca+ab)^6$ = $\frac{6!}{1!3!2!}=60$.

6.1.13 Binomial Theorem for any Index

Statement : $(1+x)^n=1+nx+\frac{n(n-1)x^2}{2!}+\frac{n(n-1)(n-2)}{3!}x^3+....+\frac{n(n-1)......(n-r+1)}{r!}x^r+...\text{terms up to }\infty$

When n is a negative integer or a fraction, where $-1<x<1$, otherwise expansion will not be possible.

If $x<1$, the terms of the above expansion go on decreasing and if x be very small a stage may be reached when we may neglect the terms containing higher power of x in the expansion, then $(1+x)^n=1+nx$.

Important Tips

- *Expansion is valid only when* $-1<x<1$.
- ${}^nC_r$ *can not be used because it is defined only for natural number, so* ${}^nC_r$ *will be written as* $\frac{(n)(n-1)......(n-r+1)}{r!}$
- *The number of terms in the series is infinite.*
- *If first term is not 1, then make first term unity in the following way,* $(x+y)^n=x^n\left[1+\frac{y}{x}\right]^n$, *if* $\left|\frac{y}{x}\right|<1$.

General term : $T_{r+1}=\frac{n(n-1)(n-2)......(n-r+1)}{r!}x^r$

Some important expansions:

(i) $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+.......+\frac{n(n-1)(n-2)......(n-r+1)}{r!}x^r+......$

(ii) $(1-x)^n=1-nx+\frac{n(n-1)}{2!}x^2-.......+\frac{n(n-1)(n-2).....(n-r+1)}{r!}(-x)^r+.......$

(iii) $(1-x)^{-n}=1+nx+\frac{n(n+1)}{2!}x^2+\frac{n(n+1)(n+2)}{3!}x^3+.....+\frac{n(n+1)......(n+r-1)}{r!}x^r+.....$

(iv) $(1+x)^{-n}=1-nx+\frac{n(n+1)}{2!}x^2-\frac{n(n+1)(n+2)}{3!}x^3+.....+\frac{n(n+1)......(n+r-1)}{r!}(-x)^r+......$

(a) **Replace *n* by 1 in (iii) :** $(1-x)^{-1}=1+x+x^2+.....+x^r+......\infty$, General term, $T_{r+1}=x^r$

(b) **Replace *n* by 1 in (iv) :** $(1+x)^{-1}=1-x+x^2-x^3+.....+(-x)^r+......\infty$, General term, $T_{r+1}=(-x)^r$.

(c) **Replace *n* by 2 in (iii) :** $(1-x)^{-2}=1+2x+3x^2+.....+(r+1)x^r+.....\infty$, General term, $T_{r+1}=(r+1)x^r$.

(d) **Replace *n* by 2 in (iv) :** $(1+x)^{-2}=1-2x+3x^2-4x^3+......+(r+1)(-x)^r+.....\infty$

General term, $T_{r+1}=(r+1)(-x)^r$.

(e) **Replace *n* by 3 in (iii) :** $(1-x)^{-3}=1+3x+6x^2+10x^3+.....+\frac{(r+1)(r+2)}{2!}x^r+..........\infty$

General term, $T_{r+1}=(r+1)(r+2)/2!.x^r$

(f) **Replace *n* by 3 in (iv) :** $(1+x)^{-3}=1-3x+6x^2-10x^3+.....+\frac{(r+1)(r+2)}{2!}(-x)^r+.....\infty$

General term, $T_{r+1}=\frac{(r+1)(r+2)}{2!}(-x)^r$

Example: 42 To expand $(1+2x)^{-1/2}$ as an infinite series, the range of *x* should be **[AMU 2002]**

(a) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (b) $\left(-\frac{1}{2},\frac{1}{2}\right)$ (c) $[-2,2]$ (d) (– 2, 2)

Solution: (b) $(1+2x)^{-1/2}$ can be expanded if $|2x|<1$ *i.e.,* if $|x|<\frac{1}{2}$ *i.e.,* if $-\frac{1}{2}<x<\frac{1}{2}$ *i.e.,* if $x\in\left(-\frac{1}{2},\frac{1}{2}\right)$.

Example: 43 If the value of *x* is so small that x^2 and higher power can be neglected, then $\frac{\sqrt{1+x}+\sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$ is equal to

[Roorkee 1962]

(a) $1+\frac{5}{6}x$ (b) $1-\frac{5}{6}x$ (c) $1+\frac{2}{3}x$ (d) $1-\frac{2}{3}x$

Solution: (b) Given expression can be written as $=\frac{(1+x)^{1/2}+(1-x)^{2/3}}{1+x+(1+x)^{1/2}}=$

$$\frac{\left(1+\frac{1}{2}x+\left(-\frac{1}{8}\right)x^2+.....\right)+\left(1-\frac{2}{3}x-\frac{1}{9}x^2-....\right)}{1+x+\left[1+\frac{1}{2}x-\frac{1}{8}x^2+.....\right]}$$

$$=\frac{1-\frac{1}{12}x-\frac{1}{144}x^2+.....}{1+\frac{3}{4}x-\frac{1}{16}x^2+.....}=1-\frac{5}{6}x+.....=1-\frac{5}{6}x$$, when $x^2,x^3....$ are neglected.

Example: 44 If $(1+ax)^n=1+8x+24x^2+.....$ then the value of *a* and *n* is

(a) 2, 4 (b) 2, 3 (c) 3, 6 (d) 1, 2

Solution: (a) We know that $(1+x)^n=1+\frac{nx}{1!}+\frac{n(n-1)x^2}{2!}+.....$

$$(1+ax)^n=1+\frac{n(ax)}{1!}+\frac{n(n-1)(ax)^2}{2!}+...... \Rightarrow 1+8x+24x^2+......=1+\frac{n(ax)}{1!}+\frac{n(n-1)(ax)^2}{2!}+....$$

Comparing coefficients of both sides we get, $na=8$, and $\frac{n(n-1)a^2}{2!}=24$ on solving, $a=2$, b = 4.

Example: 45 Coefficient of x^r in the expansion of $(1-2x)^{-1/2}$ **[Kurukshetra CEE 2001]**

(a) $\frac{(2r)!}{(r!)^2}$ (b) $\frac{(2r)!}{2^r.(r!)^2}$ (c) $\frac{(2r)!}{(r!)^2.2^{2r}}$ (d) $\frac{(2r)!}{2^r.(r+1)!(r-1)!}$

Solution: (b) Coefficient of

$$x^r=\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)....\left(-\frac{1}{2}-r+1\right)}{r!}(-2)^r=\frac{1.3.5...(2r-1).(-1)^r.(-1)^r.2^r}{2^r\,r!}=\frac{1.3.5...(2r-1)}{r!}=\frac{(2r)!}{r!r!2^r}$$

Example: 46 The coefficient of x^{25} in $(1+x+x^2+x^3+x^4)^{-1}$ is

(a) 25 (b) – 25 (c) 1 (d) – 1

Solution: (c) Coefficient of x^{25} in $(1+x+x^2+x^3+x^4)^{-1}$

= Coefficient of x^{25} in $\left[\frac{1(1-x^5)}{1-x}\right]^{-1}$ = Coefficient of x^{25} in $(1-x^5)^{-1}.(1-x)$

= Coefficient of x^{25} in $[(1-x^5)^{-1} - x(1-x^5)^{-1}] = [1+(x^5)^1+(x^5)^2+......] - x[1+(x^5)^1+(x^5)^2]+......]$

= Coefficient of x^{25} in $[1+x^5+x^{10}+x^{15}+.....]$ – Coefficient of x^{24} in $[1+x^5+x^{10}+x^{15}+.....]$ $=1-0=1$.

Example: 47 $1-\frac{1}{8}+\frac{1}{8}.\frac{3}{16}-.........=$ **[EAMCET 1990]**

(a) $\frac{2}{5}$ (b) $\frac{\sqrt{2}}{5}$ (c) $\frac{2}{\sqrt{5}}$ (d) None of these

Solution: (c) We know that $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+......\infty$

Here $nx = -\frac{1}{8}, \frac{n(n-1)}{2}x^2 = \frac{3}{8.16} \Rightarrow x = \frac{1}{4}, n = -\frac{1}{2} \Rightarrow 1-\frac{1}{8}+\frac{1}{8}.\frac{3}{16}-...... = \left(1+\frac{1}{4}\right)^{-1/2} = \frac{2}{\sqrt{5}}$.

Example: 48 If x is so small that its two and higher power can be neglected and $(1-2x)^{-1/2}(1-4x)^{-5/2} = 1+kx$ then k =

[Rajasthan PET 1993]

(a) 1 (b) – 2 (c) 10 (d) 11

Solution: (d) $(1-2x)^{-1/2}(1-4x)^{-5/2} = 1+kx$

$$\left[1+\frac{(-1/2)(-2x)}{1!}+\frac{(-1/2)(-3/2)(-2x)^2}{2!}+......\right]\left[1+\frac{(-5/2)(-4x)}{1!}+\frac{(-5/2)(-7/2)(-4x)^2}{2!}+......\right] = 1+kx$$

Higher power can be neglected. Then $\left[1+\frac{x}{1!}\right]\left[1+\frac{10x}{1!}\right] = 1+kx$; $1+10x+x = 1+kx$; $k = 11$

Example: 49 The cube root of $1+3x+6x^2+10x^3+.....$ is

(a) $1-x+x^2-x^3+.....\infty$ (b) $1+x^3+x^6+x^9+......$ (c) $1+x+x^2+x^3+....$ (d) None of these

Solution: (c) We have $(1+3x+6x^2+10x^3+.....)^{1/3} = [(1-x)^{-3}]^{1/3}$; $[\because (1-x)^{-3} = 1+3x+6x^2+...\infty]$ $\Rightarrow$ $(1-x)^{-1} = 1+x+x^2+...\infty$

Example: 50 The coefficient of x^n in the expansion of $\left(\frac{1}{1-x}\right)\left(\frac{1}{3-x}\right)$ is

(a) $\frac{3^{n+1}-1}{2.3^{n+1}}$ (b) $\frac{3^{n+1}-1}{3^{n+1}}$ (c) $2\left(\frac{3^{n+1}-1}{3^{n+1}}\right)$ (d) None of these

Solution: (a) $\frac{1}{(1-x)(3-x)} = (1-x)^{-1}(3-x)^{-1} = 3^{-1}(1-x)^{-1}\left(1-\frac{x}{3}\right)^{-1} = \frac{1}{3}[1+x+x^2+.....x^n]\left[1+\frac{x}{3}+\frac{x^2}{3^2}+.....+\frac{x^{n-1}}{3^{n-1}}+\frac{x^n}{3^n}\right]$

Coefficient of $x^n = \frac{1}{3^{n+1}}+\frac{1}{3^n}+\frac{1}{3^{n-1}}+.....(n+1)\text{ terms} = \frac{1}{3^{n+1}}\frac{[3^{n+1}-1]}{3-1} = \frac{3^{n+1}-1}{2.3^{n+1}}$.

Trick: Put $n = 1, 2, 3......$ and find the coefficients of $x, x^2, x^3......$ and comparing with the given option as :

Coefficient of x^2 is $= \frac{1}{3^3}+\frac{1}{3^2}+\frac{1}{3^1} = \frac{1}{3^3}\frac{[3^3-1]}{3-1} = \frac{13}{27}$; Which is given by option (a) $\frac{3^{n+1}-1}{2.(3^{n+1})} = \frac{3^3-1}{2.3^3} = \frac{13}{27}$.

6.1.14 Three / Four Consecutive terms or Coefficients

(1) **If consecutive coefficients are given:** In this case divide consecutive coefficients pair wise. We get equations and then solve them.

(2) **If consecutive terms are given :** In this case divide consecutive terms pair wise *i.e.* if four consecutive terms be $T_r, T_{r+1}, T_{r+2}, T_{r+3}$ then find $\frac{T_r}{T_{r+1}}, \frac{T_{r+1}}{T_{r+2}}, \frac{T_{r+2}}{T_{r+3}} \Rightarrow \lambda_1, \lambda_2, \lambda_3$ (say) then divide λ_1 by λ_2 and λ_2 by λ_3 and solve.

Example: 51 If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} =$

[IIT 1975]

(a) $\frac{a_2}{a_2+a_3}$ (b) $\frac{1}{2}\frac{a_2}{a_2+a_3}$ (c) $\frac{2a_2}{a_2+a_3}$ (d) $\frac{2a_3}{a_2+a_3}$

Solution: (c) Let a_1, a_2, a_3, a_4 be respectively the coefficients of $(r+1)^{th}, (r+2)^{th}, (r+3)^{th}, (r+4)^{th}$ terms in the expansion of $(1+x)^n$. Then $a_1 = {}^nC_r, a_2 = {}^nC_{r+1}, a_3 = {}^nC_{r+2}, a_4 = {}^nC_{r+3}$.

Now, $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} + \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}} = \frac{{}^nC_r}{{}^{n+1}C_{r+1}} + \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} = \frac{{}^nC_r}{\frac{n+1}{r+1}\,{}^nC_r} + \frac{{}^nC_{r+2}}{\frac{n+1}{r+3}\,{}^nC_{r+2}}$

$$= \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2(r+2)}{n+1} = 2.\frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} = 2.\frac{{}^nC_{r+1}}{{}^nC_{r+1} + {}^nC_{r+2}} = \frac{2a_2}{a_2+a_3}$$

6.1.15 Some Important Points

(1) **Pascal's Triangle :**

1	$(x+y)^0$
1 1	$(x+y)^1$
1 2 1	$(x+y)^2$
1 3 3 1	$(x+y)^3$
1 4 6 4 1	$(x+y)^4$
1 5 10 10 5 1	$(x+y)^5$

Pascal's triangle gives the direct binomial coefficients.

Example : $(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

(2) **Method for finding terms free from radical or rational terms in the expansion of $(a^{1/p} + b^{1/q})^N \forall a, b \in$ prime numbers :** Find the general term $T_{r+1} = {}^NC_r (a^{1/p})^{N-r} (b^{1/q})^r = {}^NC_r\, a^{\frac{N-r}{p}} . b^{\frac{r}{q}}$

Putting the values of $0 \le r \le N$, when indices of *a* and *b* are integers.

Note : ❑ Number of irrational terms = Total terms – Number of rational terms.

Example: 52 The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is **[AIEEE 2003]**

(a) 32 (b) 33 (c) 34 (d) 35

Solution: (b) $T_{r+1} = {}^{256}C_r . 3^{\frac{256-r}{2}} . 5^{\frac{r}{8}}$

First term = ${}^{256}C_0\; 3^{128} 5^0 =$ integer and after eight terms, *i.e.*, 9th term = ${}^{256}C_8\, 3^{124}.5^1 =$ integer

Continuing like this, we get an A.P., $1^{st}, 9^{th}....... 257^{th}$; $T_n = a + (n-1)d \Rightarrow 257 = 1 + (n-1)8 \Rightarrow n = 33$

Example: 53 The number of irrational terms in the expansion of $\left(\sqrt[8]{5}+\sqrt[6]{2}\right)^{100}$ is

(a) 97 (b) 98 (c) 96 (d) 99

Solution: (a) $T_{r+1} = {}^{100}C_r\, 5^{\frac{100-r}{8}}.2^{\frac{r}{6}}$

As 2 and 5 are co-prime. T_{r+1} will be rational if $100-r$ is multiple of 8 and r is multiple of 6 also $0 \le r \le 100$

$\therefore r = 0, 6, 12 .... 96$; $\therefore 100 - r = 4, 10, 16 ..... 100$ ......(i)

But $100-r$ is to be multiple of 8.

So, $100-r$ = 0, 8, 16, 24,......96 .....(ii)

Common terms in (i) and (ii) are 16, 40, 64, 88.

$\therefore$ r = 84, 60, 36, 12 give rational terms $\therefore$ The number of irrational terms = 101 – 4 = 97.

***

6.2 Mathematical Induction

6.2.1 First Principle of Mathematical Induction

The proof of proposition by mathematical induction consists of the following three steps :

Step I : (Verification step) **:** Actual verification of the proposition for the starting value "i"

Step II : (Induction step) **:** Assuming the proposition to be true for "k", $k \geq i$ and proving that it is true for the value $(k + 1)$ which is next higher integer.

Step III : (Generalization step) **:** To combine the above two steps

Let $p(n)$ be a statement involving the natural number n such that

(i) $p(1)$ is true *i.e.* $p(n)$ is true for $n = 1$.

(ii) $p(m + 1)$ is true, whenever $p(m)$ is true *i.e.* $p(m)$ is true $\Rightarrow p(m + 1)$ is true.

Then $p(n)$ is true for all natural numbers n.

6.2.2 Second Principle of Mathematical Induction

The proof of proposition by mathematical induction consists of following steps :

Step I : (Verification step) : Actual verification of the proposition for the starting value i and $(i + 1)$.

Step II : (Induction step) : Assuming the proposition to be true for $k - 1$ and k and then proving that it is true for the value $k + 1$; $k \geq i + 1$.

Step III : (Generalization step) : Combining the above two steps.

Let $p(n)$ be a statement involving the natural number n such that

(i) $p(1)$ is true *i.e.* $p(n)$ is true for $n = 1$ and

(ii) $p(m + 1)$ is true, whenever $p(n)$ is true for all n, where $i \leq n \leq m$

Then $p(n)$ is true for all natural numbers.

For $a \neq b$, The expression $a^n - b^n$ is divisible by

(a) $a + b$ if n is even. (b) $a - b$ is n if odd or even.

6.2.3 Some Formulae based on Principle of Induction

For any natural number n

(i) $\sum n = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$ (ii)

$$\sum n^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(iii) $\sum n^3 = 1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum n\right)^2$

Example: 1 The smallest positive integer n, for which $n! < \left(\frac{n+1}{2}\right)^n$ hold is

(a) 1 (b) 2 (c) 3 (d) 4

Solution: (b) Let $P(n)$: $n! < \left(\frac{n+1}{2}\right)^n$

Step I : For n = 2 $\Rightarrow 2! < \left(\frac{2+1}{2}\right)^2 \Rightarrow 2 < \frac{9}{4} \Rightarrow 2 < 2.25$ which is true. Therefore, $P(2)$ is true.

Step II : Assume that $P(k)$ is true, then $p(k)$: $k! < \left(\frac{k+1}{2}\right)^k$

Step III : For $n = k + 1$,

$$P(k+1):(k+1)! < \left(\frac{k+2}{2}\right)^{k+1} \Rightarrow k! < \left(\frac{k+1}{2}\right)^k \Rightarrow (k+1)k! < \frac{(k+1)^{k+1}}{2^k}$$

$$\Rightarrow (k+1)! < \frac{(k+1)^{k+1}}{2^k} \quad .....(i) \qquad \text{and} \qquad \frac{(k+1)^{k+1}}{2^k} < \left(\frac{k+2}{2}\right)^{k+1} \quad .....(ii)$$

$$\Rightarrow \left(\frac{k+2}{k+1}\right)^{k+1} > 2 \Rightarrow \left[1+\frac{1}{k+1}\right]^{k+1} > 2 \Rightarrow 1+(k+1)\frac{1}{k+1} + {}^{k+1}C_2\left(\frac{1}{k+1}\right)^2 + ... > 2$$

$$\Rightarrow 1+1+{}^{k+1}C_2\left(\frac{1}{k+1}\right)^2 + ... > 2$$

Which is true, hence (ii) is true.

From (i) and (ii), $(k+1)! < \frac{(k+1)^{k+1}}{2^k} < \left(\frac{k+2}{2}\right)^{k+1} \Rightarrow (k+1)! < \left(\frac{k+2}{2}\right)^{k+1}$

Hence $P(k+1)$ is true. Hence by the principle of mathematical induction $P(n)$ is true for all $n \in N$

Trick : By check option

(a) For n = 1, $1! < \left(\frac{1+1}{2}\right)^1 \Rightarrow 1 < 1$ which is wrong (b) For n = 2, $2! < \left(\frac{3}{2}\right)^2 \Rightarrow 2 < \frac{9}{4}$ which is correct

(c) For n = 3, $3! < \left(\frac{3+1}{2}\right)^3 \Rightarrow 6 < 8$ which is correct

(d) For n = 4, $4! < \left(\frac{4+1}{2}\right)^4 \Rightarrow 24 < \left(\frac{5}{2}\right)^4 \Rightarrow 24 < 39.0625$ which is correct.

But smallest positive integer n is 2.

Example: 2 Let $S(k) = 1+3+5+....... +(2k-1) = 3+k^2$. Then which of the following is true. **[AIEEE 2004]**

(a) Principle of mathematical induction can be used to prove the formula

(b) $S(k) \Rightarrow S(k+1)$

(c) $S(k) \not\Rightarrow S(k+1)$

(d) $S(1)$ is correct

Solution: (c) We have $S(k) = 1+3+5+...... +(2k-1) = 3+k^2$, $S(1) \Rightarrow 1 = 4$, Which is not true and $S(2) \Rightarrow 3 = 7$, Which is not true.

Hence induction cannot be applied and $S(k) \not\Rightarrow S(k+1)$

Example: 3 When P is a natural number, then $P^{n+1} + (P+1)^{2n-1}$ is divisible by **[IIT 1994]**

(a) P (b) $P^2 + P$ (c) $P^2 + P + 1$ (d) $P^2 - 1$

Solution: (c) For n =1, we get, $P^{n+1} + (P+1)^{2n-1} = P^2 + (P+1)^1 = P^2 + P + 1$,

Which is divisible by $P^2 + P + 1$, so result is true for n =1

Let us assume that the given result is true for $n = m \in N$

i.e. $P^{m+1} + (P+1)^{2m-1}$ is divisible by $P^2 + P + 1$ *i.e.* $P^{m+1} + (P+1)^{2m-1} = k(P^2 + P + 1) \quad \forall \quad k \in N$ .....(i)

Now, $P^{(m+1)+1} + (P+1)^{2(m+1)-1} = P^{m+2} + (P+1)^{2m+1} = P^{m+2} + (P+1)^2 (P+1)^{2m-1}$

$= P^{m+2} + (P+1)^2[k(P^2+P+1) - P^{m+1}]$ by using (i)

$= P^{m+2} + (P+1)^2 \cdot k(P^2+P+1) - (P+1)^2(P)^{m+1} = P^{m+1}[P-(P+1)^2] + (P+1)^2 \cdot k(P^2+P+1)$

$= P^{m+1}[P-P^2-2P-1] + (P+1)^2 \cdot k(P^2+P+1) = -P^{m+1}[P^2+P+1] + (P+1)^2 \cdot k(P^2+P+1)$

$= (P^2+P+1)[k \cdot (P+1)^2 - P^{m+1}]$

Which is divisible by p^2+p+1, so the result is true for $n = m+1$. Therefore, the given result is true for all $n \in N$ by induction.

Trick : For n = 2, we get, $P^{n+1} + (P+1)^{2n-1} = P^3 + (P+1)^3 = P^3 + P^3 + 1 + 3P^2 + 3P = 2P^3 + 3P^2 + 3P + 1$

Which is divisible by P^2+P+1. Given result is true for all $n \in N$

Example: 4 Given $U_{n+1} = 3U_n - 2U_{n-1}$ and $U_0 = 2$, $U_1 = 3$, the value of U_n for all $n \in N$ is

(a) $2^n - 1$ (b) $2^n + 1$ (c) 0 (d) None of these

Solution: (b) $\because U_{n+1} = 3U_n - 2U_{n-1}$ .....(i)

Step I : Given $U_1 = 3$

For n =1, $U_{1+1} = 3U_1 - 2U_0$, $U_2 = 3.3 - 2.2 = 5$

Option (b) $U_n = 2^n + 1$

For n = 1, $U_1 = 2^1 + 1 = 3$ which is true. For n = 2, $U_2 = 2^2 + 1 = 5$ which is true

Therefore, the result is true for n = 1 and n = 2

Step II : Assume it is true for $n = k$ then it is also true for $n = k - 1$

Then $U_k = 2^k + 1$ .....(ii) and $U_{k-1} = 2^{k-1} + 1$ .....(iii)

Step III : Putting $n = k$ in (i), we get

$U_{k+1} = 3U_k - 2U_{k-1} = 3[2^k+1] - 2[2^{k-1}+1] = 3.2^k + 3 - 2.2^{k-1} - 2 = 3.2^k + 1 - 2.2^{k-1}$

$\Rightarrow 3.2^k - 2^k + 1 = 2.2^k + 1 = 2^{k+1} + 1 \Rightarrow U_{k+1} = 2^{k+1} + 1$

This shows that the result is true for $n = k+1$, by the principle of mathematical induction the result is true for all $n \in N$.

6.2.4 Divisibility Problems

To show that an expression is divisible by an integer

(i) If a, p, n, r are positive integers, then first of all we write $a^{pn+r} = a^{pn}.a^r = (a^p)^n.a^r$.

(ii) If we have to show that the given expression is divisible by c.S

Then express, $a^p = [1 + (a^p - 1]$, if some power of $(a^p - 1)$ has c as a factor.

$a^p = [2 + (a^p - 2)]$, if some power of $(a^p - 2)$ has c as a factor.

$a^p = [K + (a^p - K)]$, if some power of $(a^p - K)$ has c as a factor.

Example: 5 $(1+x)^n - nx - 1$ is divisible by (where $n \in N$)

(a) $2x$ (b) x^2 (c) $2x^3$ (d) All of these

Solution: (b) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ... \Rightarrow (1+x)^n - nx - 1 = x^2\left[\frac{n(n-1)}{2!} + \frac{n(n-1)(n-3)}{3!}x + ...\right]$

From above it is clear that $(1+x)^n - nx - 1$ is divisible by x^2.

Trick : $(1+x)^n - nx - 1$. Put $n = 2$ and $x = 3$; Then $4^2 - 2.3 - 1 = 9$

Is not divisible by 6, 54 but divisible by 9. Which is given by option (c) = $x^2 = 9$.

Example: 6 The greatest integer which divides the number $101^{100} - 1$ is

(a) 100 (b) 1000 (c) 10000 (d) 100000

Solution: (c) $(1+100)^{100} = 1+100.100+\frac{100.99}{1.2}(100)^2+.... \Rightarrow 101^{100}-1 = 100.100\left[1+\frac{100.99}{1.2}+\frac{100.99.98}{3.2.1}100+.....\right]$

From above it is clear that, $101^{100}-1$ is divisible by $(100)^2 = 10000$

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Assignment

Expansion of Binomial theorem

Basic Level

1. The approximate value of $(1.0002)^{3000}$ is **[EAMCET 2002]**

(a) 1.6 (b) 1.4 (c) 1.8 (d) 1.2

2. If $(1+by)^n = (1+8y+24y^2+....)$, then the value of b and n are respectively

(a) 4, 2 (b) 2, – 4 (c) 2, 4 (d) – 2, 4

3. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$ then r is equal to **[Rajasthan PET 1991]**

(a) 5 (b) 4 (c) 3 (d) 2

4. If ${}^{m}C_1 = {}^{n}C_2$, then correct statement is **[Rajasthan PET 1994]**

(a) $2m = n$ (b) $2m = n(n+1)$ (c) $2m = n(n-1)$ (d) $2n = m(m-1)$

Advance Level

5. If $x+y=1$, then $\sum_{r=0}^{n} r^2\, {}^nC_r x^r y^{n-r}$ equals

(a) nxy (b) $nx(x+yn)$ (c) $nx(nx+y)$ (d) None of these

6. Let $f(x) = (\sqrt{x^2+1}+\sqrt{x^2-1})^6 + \left(\frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}}\right)^6$. Then

(a) $f(x)$ is a polynomial of the sixth degree in x (b) $f(x)$ has exactly two terms

(c) $f(x)$ is not a polynomial in x (d) Coefficient of x^6 is 64

7. In the expansion of $(x+a)^n$, the sum of odd terms is P and sum of even terms is Q, then the value of (P^2-Q^2) will be

[Rajasthan PET 1997; Pb. CET 1998]

(a) $(x^2+a^2)^n$ (b) $(x^2-a^2)^n$ (c) $(x-a)^{2n}$ (d) $(x+a)^{2n}$

8. $n^n\left(\frac{n+1}{2}\right)^{2n}$ is **[AMU 2001]**

(a) Less than $\left(\frac{n+1}{2}\right)^3$ (b) Greater than $\left(\frac{n+1}{2}\right)^3$ (c) Less than $(n!)^3$ (d) Greater than $(n!)^3$

9. The expression $(2+\sqrt{2})^4$ has value, lying between **[AMU 2001]**

(a) 134 and 135 (b) 135 and 136 (c) 136 and 137 (d) None of these

10. The positive integer just greater than $(1+0.0001)^{10000}$ is **[AIEEE 2002]**

(a) 4 (b) 5 (c) 2 (d) 3

11. $(\sqrt{2}+1)^6-(\sqrt{2}-1)^6=$ **[MP PET 1984]**

(a) 101 (b) $70\sqrt{2}$ (c) $140\sqrt{2}$ (d) $120\sqrt{2}$

12. The value of $(\sqrt{5}+1)^5-(\sqrt{5}-1)^5$ is **[MP PET 1985]**

(a) 252 (b) 352 (c) 452 (d) 532

13. The greatest integer less than or equal to $(\sqrt{2}+1)^6$ is **[Rajasthan PET 2000]**

(a) 196 (b) 197 (c) 198 (d) 199

14. The integer next above $(\sqrt{3}+1)^{2m}$ contains

(a) 2^{m+1} as a factor (b) 2^{m+2} as a factor (c) 2^{m+3} as a factor (d) 2^m as a factor

15. Let n be an odd natural number greater than 1. Then the number of zeros at the end of the sum 99^n+1 is

(a) 3 (b) 4 (c) 2 (d) None of these

General Term

Basic Level

16. 6^{th} term in expansion of $\left(2x^2-\frac{1}{3x^2}\right)^{10}$ is

(a) $\frac{4580}{17}$ (b) $-\frac{896}{27}$ (c) $\frac{5580}{17}$ (d) None of these

17. 16^{th} term in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$ is

(a) $136\,xy^7$ (b) $136\,xy$ (c) $-136\,xy^{15/2}$ (d) $-136\,xy^2$

18. In the binomial expansion of $(a-b)^n$, $n\geq 5$, the sum of the 5^{th} and 6^{th} terms is zero. Then $\frac{a}{b}$ is equal to

[IIT Screening 2001; Karnataka CET 2002]

(a) $\frac{1}{6}(n-5)$ (b) $\frac{1}{5}(n-4)$ (c) $\frac{5}{(n-4)}$ (d) $\frac{6}{(n-5)}$

19. The first 3 terms in the expansion of $(1+ax)^n\,(n\neq 0)$ are 1, $6x$ and $16x^2$. Then the value of a and n are respectively

[Kerala (Engg.) 2002]

(a) 2 and 9 (b) 3 and 2 (c) 2/3 and 9 (d) 3/2 and 6

20. If the third term in the expansion of $\left(\frac{1}{x}+x^{\log_{10}x}\right)^5$ is 1000, then the value of x is

(a) 10 (b) 100 (c) 1 (d) None of these

21. If the ratio of the 7th term from the beginning to the seventh term from the end in the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^x$ is $\frac{1}{6}$, then x is

(a) 9 (b) 6 (c) 12 (d) None of these

22. The last term in the binomial expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$. Then the 5th term from the beginning is

(a) ${}^{10}C_6$ (b) $2\cdot{}^{10}C_4$ (c) $\frac{1}{2}\cdot{}^{10}C_4$ (d) None of these

23. In the expansion of $(1+x)^n$, $\frac{T_{r+1}}{T_r}$ is equal to

(a) $\frac{n+1}{r}x$ (b) $\frac{n+r+1}{r}x$ (c) $\frac{n-r+1}{r}x$ (d) $\frac{n+r}{r+1}x$

24. If 6^{th} term in the expansion of $\left(\frac{1}{x^{8/3}}+x^2\log_{10}x\right)^8$ is 5600, then x is equal to

(a) 8 (b) 9 (c) 10 (d) None of these

Advance Level

25. The value of x in the expression $[x+x^{\log_{10}(x)}]^5$, if the third term in the expansion is 10,00,000 **[Roorkee 1992]**

(a) 10 (b) 11 (c) 12 (d) None of these

26. If $T_0, T_1, T_2, ... T_n$ represent the terms in the expansion of $(x+a)^n$, then $(T_0 - T_2 + T_4 - ....)^2 + (T_1 - T_3 + T_5 - ....)^2 =$

(a) (x^2+a^2) (b) $(x^2+a^2)^n$ (c) $(x^2+a^2)^{1/n}$ (d) $(x^2+a^2)^{-1/n}$

27. The value of x, for which the 6th term in the expansion of $\left\{2^{\log_2\sqrt{(9^{x-1}+7)}}+\frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right\}^7$ is 84, is equal to **[Pb. CET 1992]**

(a) 4 (b) 3 (c) 2 (d) 1

28. Given that 4th term in the expansion of $\left(2+\frac{3}{8}x\right)^{10}$ has the maximum numerical value, the range of value of x for which this will be true is given by **[Roorkee 1994]**

(a) $-\frac{64}{21}<x<-2$ (b) $-\frac{64}{21}<x<2$ (c) $\frac{64}{21}<x<4$ (d) None of these

29. If the $(r+1)^{th}$ term in the expansion of $\left(\sqrt[3]{\frac{a}{\sqrt{b}}}+\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{21}$ has the same power of a and b, then the value of r is

(a) 9 (b) 10 (c) 8 (d) 6

30. If the 6th term in the expansion of the binomial $\left[\sqrt{2^{\log(10-3^x)}}+\sqrt[5]{2^{(x-2)\log 3}}\right]^m$ is equal to 21 and it is known that the binomial coefficients of the 2nd, 3rd and 4th terms in the expansion represent respectively the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10), then $x =$ **[Roorkee 1993]**

(a) 0 (b) 1 (c) 2 (d) 3

31. If the fourth term of $\left(\sqrt{x^{\left(\frac{1}{1+\log_{10}x}\right)}}+\sqrt[12]{x}\right)^6$ is equal to 200 and $x>1$, then x is equal to

(a) $10\sqrt{2}$ (b) 10 (c) 10^4 (d) $10/\sqrt{2}$

Independent Term

Basic Level

32. To make the term ${}^{3n}C_r(-1)^r x^{3n-r}$ free from x, necessary condition is

(a) ${}^{3n}C_r = 0$ (b) $x^{3n-r}=0$ (c) $3n = r$ (d) None of these

33. In the expansion of $\left(2x+\frac{1}{3x^2}\right)^9$, the term independent of x is

(a) ${}^9C_3\,8$ (b) $\frac{1792}{9}$ (c) ${}^9C_3\,64$ (d) ${}^9C_3\,\frac{1}{81}$

34. The term independent of y in the expansion of $(y^{-1/6}-y^{1/3})^9$ is **[BIT Ranchi 1980]**

(a) 84 (b) 8.4 (c) 0.84 (d) – 84

35. The term independent of x in the expansion of $\left(\frac{1}{2}x^{1/3}+x^{-1/5}\right)^8$ will be **[Roorkee 1985]**

(a) 5 (b) 6 (c) 7 (d) 8

36. In the expansion of $\left(x-\frac{1}{x}\right)^6$, the constant term is **[AMU 1982; MP PET 1984; MNR 1979]**

(a) – 20 (b) 20 (c) 30 (d) – 30

37. The term independent of x in the expansion of $\left(x^2-\frac{1}{x}\right)^9$ is **[EAMCET 1982; MP PET 2003]**

(a) 1 (b) – 1 (c) – 48 (d) None of these

38. In the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term independent of x is **[MP PET 1993]**

(a) ${}^{15}C_6\,2^6$ (b) ${}^{15}C_5\,2^5$ (c) ${}^{15}C_4\,2^4$ (d) ${}^{15}C_8\,2^8$

39. In the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$, the term independent of x is **[MNR 1981; AMU 1983; Rajasthan PET 1996; JMIEE 2001]**

(a) ${}^9C_3\cdot\frac{1}{6^3}$ (b) ${}^9C_3\left(\frac{3}{2}\right)^3$ (c) ${}^9C_3$ (d) None of these

40. The term independent of x in $\left(2x-\frac{1}{2x^2}\right)^{12}$ is **[Rajasthan PET 1985]**

(a) – 7930 (b) – 495 (c) 495 (d) 7920

41. The term independent of x in $\left[\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{x^2}\right]^{10}$ is **[EAMCET 1984; Rajasthan PET 2000]**

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$ (c) $\frac{4}{3}$ (d) None of these

42. The term independent of x in $\left(\sqrt{x}-\frac{2}{x}\right)^{18}$ is **[EAMCET 1990]**

(a) ${}^{18}C_6\,2^6$ (b) ${}^{18}C_6\,2^{12}$ (c) ${}^{18}C_{18}\,2^{18}$ (d) None of these

43. The ratio of the coefficient of x^{15} to the term independent of x in $\left(x^2+\frac{2}{x}\right)^{15}$ is

(a) 1 : 32 (b) 32 : 1 (c) 1 : 16 (d) 16 : 1

44. The term independent of x in the expansion of $\left(2x+\frac{1}{3x}\right)^6$ is **[MNR 1995]**

(a) $\frac{160}{9}$ (b) $\frac{80}{9}$ (c) $\frac{160}{27}$ (d) $\frac{80}{3}$

45. The term independent of x in the expansion of $\left(x^2-\frac{1}{3x}\right)^9$ is **[Roorkee 1981; Rajasthan PET 1990, 95; Pb. CET 2000]**

(a) $\frac{28}{81}$ (b) $\frac{28}{243}$ (c) $-\frac{28}{243}$ (d) $-\frac{28}{81}$

46. The term independent of x in the expansion of $\left(x^2 - \frac{3\sqrt{3}}{x^3}\right)^{10}$ is **[Rajasthan PET 1999]**

(a) 153090 (b) 150000 (c) 150090 (d) 153180

47. The term independent of x in the expansion of $\left(2x - \frac{3}{x}\right)^6$ is **[Pb. CET 1999]**

(a) 4320 (b) 216 (c) – 216 (d) – 4320

48. In the expansion of $\left(x - \frac{3}{x^2}\right)^9$, the term independent of x is **[Karnataka CET 2001]**

(a) Not existent (b) $^9C_2$ (c) 2268 (d) – 2268

49. In the expansion of $\left(x + \frac{1}{x}\right)^{2n}$ $(n \in N)$, the term independent of x is **[Rajasthan PET 1995]**

(a) $\frac{1.3.5.......(2n-1)}{n!}2^n$ (b) $\frac{(2n)!}{n!}$ (c) $\frac{(2n)!}{n!}2^n$ (d) $\frac{n!}{(2n)!}$

Advance Level

50. The sum of the coefficients in the binomial expansion of $\left(\frac{1}{x} + 2x\right)^n$ is equal to 6561. The constant term in the expansion is

(a) $^8C_4$ (b) $16 \cdot {}^8C_4$ (c) $^6C_4 \cdot 2^4$ (d) None of these

51. The greatest value of the term independent of x in the expansion of $(x\sin\alpha + x^{-1}\cos\alpha)^{10}, \alpha \in R$, is

(a) 2^5 (b) $\frac{10!}{(5!)^2}$ (c) $\frac{1}{2^5} \cdot \frac{10!}{(5!)^2}$ (d) None of these

Coefficients of any power of x

Basic Level

52. If the coefficients of p^{th}, $(p+1)^{th}$ and $(p+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P., then

(a) $n^2 - 2np + 4p^2 = 0$ (b) $n^2 - n(4p+1) + 4p^2 - 2 = 0$ (c) $n^2 - n(4p+1) + 4p^2 = 0$ (d) None of these

53. The coefficient of two consecutive terms in the expansion of $(1+x)^n$ will be equal, if

(a) n is any integer (b) n is an odd integer (c) n is an even integer (d) None of these

54. In the expansion of $\left(\frac{a}{x} + bx\right)^{12}$, the coefficient of x^{-10} will be

(a) $12a^{11}$ (b) $12b^{11}a$ (c) $12a^{11}b$ (d) $12a^{11}b^{11}$

55. If the ratio of the coefficient of third and fourth term in the expansion of $\left(x - \frac{1}{2x}\right)^n$ is 1 : 2, then the value of n will be

(a) 18 (b) 16 (c) 12 (d) – 10

56. In the expansion of $\left(x^3 + \frac{1}{x^2}\right)^8$, the term containing x^4 is

(a) $70x^4$ (b) $60x^4$ (c) $56x^4$ (d) None of these

57. If the coefficients of r^{th} term and $(r+4)^{th}$ term are equal in the expansion of $(1+x)^{20}$, then the value of r will be **[Rajasthan PET 1985, 97; Kerala (Engg.) 2001; MP PET 2002]**

(a) 7 (b) 8 (c) 9 (d) 10

58. In the expansion of $\left(y^2+\frac{c}{y}\right)^5$, the coefficient of y will be **[MNR 1983]**

(a) 20 c (b) 10 c (c) $10\,c^3$ (d) $20\,c^2$

59. If p and q be positive, then the coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ will be**[MNR 1983; AIEEE 2002]**

(a) Equal (b) Equal in magnitude but opposite in sign

(c) Reciprocal to each other (d) None of these

60. If the coefficients of 5th, 6th and 7th terms in the expansion of $(1+x)^n$ be in A.P., then n =**[Roorkee 1984; Pb. CET 1999]**

(a) 7 only (b) 14 only (c) 7 or 14 (d) None of these

61. Two consecutive terms in the expansion of $(3+2x)^{74}$ whose coefficients are equal, are

(a) 29th and 30th (b) 30th and 31st (c) 31st and 32nd (d) None of these

62. The coefficient of x^7 in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^8$ is **[MNR 1975]**

(a) – 56 (b) 56 (c) – 14 (d) 14

63. If for positive integers $r > 1$, $n > 2$, the coefficient of the $(3r)^{\text{th}}$ and $(r+2)^{\text{th}}$ powers of x in the expansion of $(1+x)^{2n}$ are equal, then

[IIT 1983; BIT 1990; Kurukshetra CEE 1992; DCE 2000; UPSEAT 1998, 2002; AIEEE 2002]

(a) $n = 2r$ (b) $n = 3r$ (c) $n = 2r + 1$ (d) None of these

64. In the expansion of $(x^2-2x)^{10}$, the coefficient of x^{16} is **[MP PET 1985]**

(a) – 1680 (b) 1680 (c) 3360 (d) 6720

65. In the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$, the coefficient of x^4 is**[IIT 1983; EAMCET 1985; DCE 2000; Rajasthan PET 2001; UPSEAT 20**

(a) $\frac{405}{256}$ (b) $\frac{504}{259}$ (c) $\frac{450}{263}$ (d) None of these

66. The coefficient of x^{32} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is **[MP PET 1994; Karnataka CET 2003]**

(a) ${}^{15}C_5$ (b) ${}^{15}C_6$ (c) ${}^{15}C_4$ (d) ${}^{15}C_7$

67. If coefficient of $(2r+3)^{\text{th}}$ and $(r-1)^{\text{th}}$ terms in the expansion of $(1+x)^{15}$ are equal, then value of r is

[Rajasthan PET 1995, 2003; UPSEAT 2001]

(a) 5 (b) 6 (c) 4 (d) 3

68. If x^4 occurs in the r^{th} term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then r = **[MP PET 1995]**

(a) 7 (b) 8 (c) 9 (d) 10

69. If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is

[EAMCET 1983; Kurukshetra CEE 1998; DCE 2000; Rajasthan PET 2001; UPSEAT 2001]

(a) 56 (b) 55 (c) 45 (d) 15

70. If coefficients of $(2r+1)^{\text{th}}$ term and $(r+2)^{\text{th}}$ term are equal in the expansion of $(1+x)^{43}$, then the value of r will be **[UPSEAT 1999]**

(a) 14 (b) 15 (c) 13 (d) 16

71. If the coefficient of 4^{th} term in the expansion of $(a+b)^n$ is 56, then n is **[AMU 2000]**

(a) 12 (b) 10 (c) 8 (d) 6

72. If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a is **[DCE 2001]**

(a) $-\frac{7}{9}$ (b) $-\frac{9}{7}$ (c) $\frac{7}{9}$ (d) $\frac{9}{7}$

73. The coefficient of x^3 in the expansion of $\left(x-\frac{1}{x}\right)^7$ is **[MP PET 1997]**

(a) 14 (b) 21 (c) 28 (d) 35

74. If the coefficient of $(2r+4)^{th}$ and $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal, then r = **[MP PET 1997]**

(a) 12 (b) 10 (c) 8 (d) 6

75. If x^m occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{2n}$, then the coefficient of x^m is **[UPSEAT 1999]**

(a) $\frac{(2n)!}{(m)!(2n-m)!}$ (b) $\frac{(2n)!3!3!}{(2n-m)!}$ (c) $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ (d) None of these

76. If coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the binomial expansion of $(1+x)^n$ are in A.P., then n^2-9n is equal to **[Rajastha**

(a) – 7 (b) 7 (c) 14 (d) – 14

77. The coefficient of x^{-9} in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^9$ is **[Kerala (Engg.) 2001]**

(a) 512 (b) – 512 (c) 521 (d) 251

78. If the coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270, then k = **[EAMCET 2002]**

(a) 1 (b) 2 (c) 3 (d) 4

79. In the expansion of $(1+x)^n$ the coefficient of p^{th} and $(p+1)^{th}$ terms are respectively p and q. Then $p+q$ = **[EAMCET 200**

(a) $n+3$ (b) $n+1$ (c) $n+2$ (d) n

80. The coefficient of x^{39} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is **[MP PET 2001]**

(a) – 455 (b) – 105 (c) 105 (d) 455

81. If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in A.P., then r = **[Pb. CET 2002]**

(a) 6 (b) 7 (c) 8 (d) 9

82. In the expansion of $(1+x)^n$, coefficients of 2^{nd}, 3^{rd} and 4^{th} terms are in A.P., then n is equal to

[IIT 1994; UPSEAT 2002; Rajasthan PET 2002]

(a) 7 (b) 9 (c) 11 (d) None of these

83. Coefficient of x^2 in the expansion of $\left(x-\frac{1}{2x}\right)^8$ is **[UPSEAT 2002]**

(a) $\frac{1}{7}$ (b) $\frac{-1}{7}$ (c) – 7 (d) 7

84. The coefficient of x^5 in the expansion of $(x+3)^6$ is **[DCE 2002]**

(a) 18 (b) 6 (c) 12 (d) 10

85. If A and B are coefficients of x^r and x^{n-r} respectively in the expansion of $(1+x)^n$, then

(a) $A = B$ (b) $A \neq B$ (c) $A = \lambda B$ for some λ (d) None of these

86. If the r^{th} term in the expansion of $(x/3 - 2/x^2)^{10}$ contains x^4, then r is equal to **[Roorkee 1992]**

(a) 2 (b) 3 (c) 4 (d) 5

87. The coefficient of x^3 in $\left(\sqrt{x^5} + \frac{3}{\sqrt{x^3}}\right)^6$ is **[EAMCET 1994]**

(a) 0 (b) 120 (c) 420 (d) 540

88. If the coefficient of $(r + 1)$th term in the expansion of $(1+x)^{2n}$ be equal to that of $(r + 3)$th term, then

(a) $n - r + 1 = 0$ (b) $n - r - 1 = 0$ (c) $n + r + 1 = 0$ (d) None of these

89. x^{-26} occurs in the expansion of $\left(x^2 - \frac{1}{x^4}\right)^{11}$ in

(a) T_8 (b) T_9 (c) T_{10} (d) None of these

90. In the expansion of $(1+ax)^n$, $n \in N$, the coefficient of x and x^2 are 8 and 24 respectively. Then

(a) $a = 2, n = 4$ (b) $a = 4, n = 2$ (c) $a = 2, n = 6$ (d) $a = -2, n = 4$

Advance Level

91. The coefficient of the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is **[DCE 1994]**

(a) $\frac{1}{3}$ (b) $\frac{19}{54}$ (c) $\frac{17}{54}$ (d) $\frac{1}{4}$

92. The coefficient of $\frac{1}{x}$ in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is

(a) $\frac{n!}{(n-1)!(n+1)!}$ (b) $\frac{(2n)!}{(n-1)!(n+1)!}$ (c) $\frac{(2n)!}{(2n-1)!(2n+1)!}$ (d) None of these

93. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is **[MNR 1993; DCE 1998; Rajasthan PET 2001]**

(a) ${}^nC_4$ (b) ${}^nC_4 + {}^nC_2$ (c) ${}^nC_4 + {}^nC_2 + {}^nC_4 . {}^nC_2$ (d) ${}^nC_4 + {}^nC_2 + {}^nC_1 . {}^nC_2$

94. The coefficient of x^{53} in the following expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} . 2^m$ is **[IIT 1992]**

(a) ${}^{100}C_{47}$ (b) ${}^{100}C_{53}$ (c) $-{}^{100}C_{53}$ (d) $-{}^{100}C_{100}$

95. The sum of the coefficients of even power of x in the expansion of $(1 + x + x^2 + x^3)^5$ is **[EAMCET 1988]**

(a) 256 (b) 128 (c) 512 (d) 64

96. The coefficient of x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + ... + (1+x)^{30}$ is **[UPSEAT 2001]**

(a) ${}^{51}C_5$ (b) ${}^9C_5$ (c) ${}^{31}C_6 - {}^{21}C_6$ (d) ${}^{30}C_5 + {}^{20}C_5$

97. The coefficient of t^{32} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is **[IIT Screening 2003]**

(a) ${}^{12}C_6 + 2$ (b) ${}^{12}C_5$ (c) ${}^{12}C_6$ (d) ${}^{12}C_7$

98. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and – 6 respectively, then m is**[IIT 1999; MP PET 20**

(a) 6 (b) 9 (c) 12 (d) 24

99. In the expansion of the following expression $1 + (1+x) + (1+x)^2 + ... + (1+x)^n$, the coefficient of x^k $(0 \le k \le n)$ is

[Rajasthan PET 2000]

(a) ${}^{n+1}C_{k+1}$ (b) ${}^nC_k$ (c) ${}^nC_{n-k-1}$ (d) None of these

100. If there is a term containing x^{2r} in $\left(x + \frac{1}{x^2}\right)^{n-3}$, then

(a) $n - 2r$ is a positive integral multiple of 3 (b) $n - 2r$ is even
(c) $n - 2r$ is odd (d) None of these

101. If the binomial coefficients of 2nd, 3rd and 4th terms in the expansion of $\left[\sqrt{2^{\log_{10}(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_{10}3}}\right]^m$ are in A.P. and the 6th term is 21, then the value(*s*) of *x* is (are)

(a) 1, 3 (b) 0, 2 (c) 4 (d) – 1

102. The coefficient of $x^r (0 \le r \le (n-1))$ in the expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+....+(x+2)^{n-1}$ is

(a) ${}^nC_r(3^r-2^n)$ (b) ${}^nC_r(3^{n-r}-2^{n-r})$ (c) ${}^nC_r(3^r+2^{n-r})$ (d) None of these

103. The coefficient of a^8b^{10} in the expansion of $(a+b)^{18}$ is

(a) ${}^{18}C_8$ (b) ${}^{18}P_{10}$ (c) 2^{18} (d) None of these

104. The coefficient of x^{65} in the expansion of $(1+x)^{131}(x^2-x+1)^{130}$ is

(a) ${}^{130}C_{65}+{}^{129}C_{66}$ (b) ${}^{130}C_{65}+{}^{129}C_{55}$ (c) ${}^{130}C_{66}+{}^{129}C_{65}$ (d) None of these

105. The coefficient of x^{13} in the expansion of $(1-x)^5(1+x+x^2+x^3)^4$ is

(a) 4 (b) – 4 (c) 0 (d) None of these

106. The coefficient of x^{17} in the expansion of $(x - 1)(x - 2)....(x - 18)$ is

(a) 171 (b) – 171 (c) 342 (d) 171/2

107. In the expansion of $(1+x+x^3+x^4)^{10}$, the coefficient of x^4 is **[MP PET 2000]**

(a) ${}^{40}C_4$ (b) ${}^{10}C_4$ (c) 210 (d) 310

Number of terms in the expansion of (a + b)ⁿ, (a + b + c)ⁿ and (a + b + c + d)ⁿ

Basic Level

108. The number of non-zero terms in the expansion of $(1+3\sqrt{2}x)^9+(1-3\sqrt{2}x)^9$ is **[EAMCET 1991]**

(a) 9 (b) 0 (c) 5 (d) 10

109. The number of terms in the expansion of $(a+b+c)^n$ will be

(a) $n + 1$ (b) $n + 3$ (c) $\frac{(n+1)(n+2)}{2}$ (d) None of these

110. The total number of terms in the expansion of $(x+y)^{100}+(x-y)^{100}$ after simplification is

(a) 50 (b) 51 (c) 202 (d) None of these

111. The expression $[x+(x^3-1)^{1/2}]^5+[x-(x^3-1)^{1/2}]^5$ is a polynomial of degree **[IIT 1992]**

(a) 5 (b) 6 (c) 7 (d) 8

112. The number of terms in the expansion of $[(x-3y)^2(x+3y)^2]^3$ is

(a) 6 (b) 7 (c) 8 (d) None of these

113. If *n* is a negative integer and |*x*| < 1 then the number of terms in the expansion of $(1+x)^n$ is

(a) $n + 1$ (b) $n + 2$ (c) 2^n (d) Infinite

114. The number of terms in the expansion of $(1+3x+3x^2+x^3)^6$ is

(a) 18 (b) 9 (c) 19 (d) 24

115. The number of terms whose values depend on *x* in the expansion of $\left(x^2-2+\frac{1}{x^2}\right)^n$ is

(a) $2n + 1$ (b) $2n$ (c) n (d) None of these

116. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}, n \in N, x > 0$, is

(a) n (b) $n + 1$ (c) $n - 1$ (d) $2n$

117. In the expansion of $(x+\sqrt{x^2-1})^6 + (x-\sqrt{x^2-1})^6$, the number of terms is

(a) 7 (b) 14 (c) 6 (d) 4

118. The number of distinct terms in the expansion of $(x+2y-3z+5w-7u)^n$ is

(a) $n + 1$ (b) ${}^{n+4}C_4$ (c) ${}^{n+4}C_n$ (d) $\frac{(n+1)(n+2)(n+3)(n+4)}{24}$

119. In how many terms in the expansion of $(x^{1/5}+y^{1/10})^{55}$ do not have fractional power of the variable **[Pb. CET 1992]**

(a) 6 (b) 7 (c) 8 (d) 10

Middle Term

Basic Level

120. If the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924\,x^6$, then n =

(a) 10 (b) 12 (c) 14 (d) None of these

121. The middle term in the expansion of $\left(\frac{x}{a}+\frac{a}{x}\right)^{20}$ is

(a) ${}^{20}C_{11}\frac{x}{a}$ (b) ${}^{20}C_{11}\frac{a}{x}$ (c) ${}^{20}C_{10}$ (d) None of these

122. The middle term in the expansion of $\left(\frac{a}{x}+bx\right)^{12}$ will be

(a) $924\,a^6b^6$ (b) $924\,\frac{a^6b^6}{x}$ (c) $924\,\frac{a^6b^6}{x^2}$ (d) $924\,a^6b^6x^2$

123. The coefficient of middle term in the expansion of $(1+x)^{10}$ is **[UPSEAT 2001]**

(a) $\frac{10\,!}{5!6!}$ (b) $\frac{10\,!}{(5!)^2}$ (c) $\frac{10!}{5!7!}$ (d) None of these

124. The middle term in the expansion of $(1+x)^{2n}$ is **[DCE 2002]**

(a) $\frac{(2n)!}{n\,!}x^2$ (b) $\frac{(2n)!}{n!(n-1)!}x^{n+1}$ (c) $\frac{(2n)!}{(n\,!)^2}x^n$ (d) $\frac{(2n)!}{(n+1)!(n-1)!}x^n$

125. The middle term in the expansion of $\left(\frac{2x}{3}-\frac{3}{2x^2}\right)^{2n}$ is

(a) ${}^{2n}C_n$ (b) $(-1)^n\frac{(2n)!}{(n!)^2}\cdot x^{-n}$ (c) ${}^{2n}C_n\cdot\frac{1}{x^n}$ (d) None of these

126. The middle terms in the expansion of $(x^2-a^2)^5$ is

(a) $10x^6a^4, -10x^4a^6$ (b) $-10x^6a^4, 10x^4a^6$ (c) $10x^6a^4, 10x^4a^6$ (d) $-10x^6a^4, -10x^4a^6$

Advance Level

127. The middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$ is **[MP PET 1995]**

(a) $\frac{1.3.5.....(2n-3)}{n!}$ (b) $\frac{1.3.5.....(2n-1)}{n!}$ (c) $\frac{1.3.5.....(2n+1)}{n!}$ (d) None of these

128. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is p and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are q and r, then

(a) $p + q = r$ (b) $p + r = q$ (c) $p = q + r$ (d) $p + q + r = 0$

129. Middle term in the expansion of $(1+3x+3x^2+x^3)^6$ is **[MP PET 1997]**

(a) 4^{th} (b) 3^{rd} (c) 10^{th} (d) None of these

130. The coefficient of each middle term in the expansion of $(1+x)^n$, when n is odd, is

(a) $\frac{1.3.5....(n-1)}{2.4.6....n}2^n$ (b) $\frac{1.3.5.....n}{2.4.6....n}2^n$ (c) $\frac{1.3.5....(n+1)}{2.4.6....n}2^n$ (d) $\frac{1.3.5....n}{2.4.6....(n+1)}2^n$

131. If the rth term is the middle term in the expansion of $\left(x^2-\frac{1}{2x}\right)^{20}$ then the $(r + 3)$th term is

(a) ${}^{20}C_{14}\cdot\frac{1}{2^{14}}\cdot x$ (b) ${}^{20}C_{12}\cdot\frac{1}{2^{12}}\cdot x^2$ (c) $-\frac{1}{2^{13}}\cdot{}^{20}C_7\cdot x$ (d) None of these

132. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals

[AIEEE 2004]

(a) $\frac{3}{5}$ (b) $\frac{10}{3}$ (c) $\frac{3}{10}$ (d) $\frac{-3}{10}$

Greatest term and Greatest coefficient

Basic Level

133. The sum of the coefficients in the expansion of $(x+y)^n$ is 4096. The greatest coefficient in the expansion is
[Kurukshetra CEE 1998; AIEEE 2002]

(a) 1024 (b) 924 (c) 824 (d) 724

134. The greatest coefficient in the expansion of $(1+x)^{2n+1}$ is **[Rajasthan PET 1997]**

(a) $\frac{(2n+1)!}{n!(n+1)!}$ (b) $\frac{(2n+2)!}{n!(n+1)!}$ (c) $\frac{(2n+1)!}{[(n+1)!]^2}$ (d) $\frac{(2n)!}{(n!)^2}$

135. If the sum of the coefficients in the expansion of $(x+y)^n$ is 1024, then the value of the greatest coefficient in the expansion is

[Orissa JEE 2003]

(a) 356 (b) 252 (c) 210 (d) 120

136. If n is even, then the greatest coefficient in the expansion of $(x+a)^n$ is

(a) ${}^nC_{\frac{n}{2}+1}$ (b) ${}^nC_{\frac{n}{2}-1}$ (c) ${}^nC_{\frac{n}{2}}$ (d) None of these

137. If $x = 1/3$, then the greatest term in the expansion of $(1+4x)^8$ is

(a) $56\left(\frac{3}{4}\right)^4$ (b) $56\left(\frac{4}{3}\right)^5$ (c) $56\left(\frac{3}{4}\right)^5$ (d) $56\left(\frac{2}{5}\right)^4$

138. The numerically greatest term of $(2+3x)^9$ when $x = 3/2$ is

(a) T_6 (b) T_7 (c) T_8 (d) None of these

139. If the sum of the coefficients in the expansion of $(x-2y+3z)^n$ is 128, then the greatest coefficient in the expansion of $(1+x)^n$ is

(a) 35 (b) 20 (c) 10 (d) None of these

140. If the coefficient of the 5th term be the numerically greatest coefficient in the expansion of $(1-x)^n$ then the positive integral value of n is

(a) 9 (b) 8 (c) 7 (d) 10

141. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is **[IIT 1966]**

(a) T_7 (b) T_8 (c) T_9 (d) None of these

Advance Level

142. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also, is

(a) $\frac{n}{n+2} < x < \frac{n+2}{n}$ (b) $\frac{n+1}{n} < x < \frac{n}{n+1}$ (c) $\frac{n}{n+4} < x < \frac{n+4}{4}$ (d) None of these

143. The interval in which x must lie so that the numerically greatest term in the expansion of $(1-x)^{21}$ has the numerically greatest coefficient is

(a) $\left[\frac{5}{6}, \frac{6}{5}\right]$ (b) $\left(\frac{5}{6}, \frac{6}{5}\right)$ (c) $\left(\frac{4}{5}, \frac{5}{4}\right)$ (d) $\left[\frac{4}{5}, \frac{5}{4}\right]$

Properties of Binomial coefficients

Basic Level

144. $\binom{n}{0}+2\binom{n}{1}+2^2\binom{n}{2}+......... +2^n\binom{n}{n}$ is equal to **[AMU 2000]**

(a) 2^n (b) 0 (c) 3^n (d) None of these

145. In the expansion of $(1+x)^{50}$, the sum of the coefficient of odd powers of x is **[UPSEAT 2001]**

(a) 0 (b) 2^{49} (c) 2^{50} (d) 2^{51}

146. $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}}$ is equal to

(a) $\frac{n}{2}$ (b) $\frac{n+1}{2}$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2(n+1)}$

147. If P_n denotes the product of all the coefficients in the expansion of $(1+x)^n$, then $\frac{P_{n+1}}{P_n}$ is equal to

(a) $\frac{(n+2)^n}{n!}$ (b) $\frac{(n+1)^{n+1}}{(n+1)!}$ (c) $\frac{(n+1)^{n+1}}{n!}$ (d) $\frac{(n+1)^n}{(n+1)!}$

148. ${}^nC_0 - \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 - ..... + (-1)^n \frac{{}^nC_n}{n+1} =$

(a) n (b) $1/n$ (c) $\frac{1}{n+1}$ (d) $\frac{1}{n-1}$

149. $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n =$ **[BIT Ranchi 1986]**

(a) $\frac{(2n)!}{(n-r)!(n+r)!}$ (b) $\frac{n!}{(-r)!(n+r)!}$ (c) $\frac{n!}{(n-r)!}$ (d) None of these

150. If n is odd, then $C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^n C_n^2 =$

(a) 0 (b) 1 (c) ∞ (d) $\frac{n!}{(n/2)^2!}$

151. ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9 =$ **[MP PET 1982]**

(a) 2^9 (b) 2^{10} (c) $2^{10} - 1$ (d) None of these

152. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + ... + 15\frac{C_{15}}{C_{14}} =$ **[IIT 1962]**

(a) 100 (b) 120 (c) – 120 (d) None of these

153. $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + ... =$ **[Rajasthan PET 1999]**

(a) $\frac{2^{n+1}}{n+1}$ (b) $\frac{2^{n+1}-1}{n+1}$ (c) $\frac{2^n}{n+1}$ (d) None of these

154. $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ... =$

(a) $\frac{2^n}{n!}$ (b) $\frac{2^{n-1}}{n!}$ (c) 0 (d) None of these

155. The sum of $C_0^2 - C_1^2 + C_2^2 - ...... + (-1)^n C_n^2$ where n is an even integer, is

(a) ${}^{2n}C_n$ (b) $(-1)^n\ {}^{2n}C_n$ (c) ${}^{2n}C_{n-1}$ (d) None of these

156. In the expansion of $(1+x)^n$ the sum of coefficients of odd powers of x is **[MP PET 1986, 93, 2003]**

(a) $2^n + 1$ (b) $2^n - 1$ (c) 2^n (d) 2^{n-1}

157. $C_0 - C_1 + C_2 - C_3 + ... + (-1)^n C_n$ is equal to **[MNR 1991; Rajasthan PET 1995; UPSEAT 2000]**

(a) 2^n (b) $2^n - 1$ (c) 0 (d) 2^{2n-1}

158. The value of ${}^{15}C_0^2 - {}^{15}C_1^2 + {}^{15}C_2^2 - .... - {}^{15}C_{15}^2$ is **[MP PET 1996]**

(a) 15 (b) – 15 (c) 0 (d) 51

159. If $C_0, C_1, C_2, ..., C_n$ are the binomial coefficients, then $2.C_1 + 2^3.C_3 + 2^5.C_5 + ...$ equals **[AMU 1999]**

(a) $\frac{3^n + (-1)^n}{2}$ (b) $\frac{3^n - (-1)^n}{2}$ (c) $\frac{3^n + 1}{2}$ (d) $\frac{3^n - 1}{2}$

160. If m, n, r are positive integers such that $r < m, n$, then ${}^mC_r + {}^mC_{r-1}\ {}^nC_1 + {}^mC_{r-2}\ {}^nC_2 + ... + {}^mC_1\ {}^nC_{r-1} + {}^nC_r$ equals

(a) $({}^nC_r)^2$ (b) ${}^{m+n}C_r$ (c) ${}^{m+n}C_r + {}^mC_r + {}^nC_r$ (d) None of these

161. The value of $\frac{1}{81^n} - \frac{10}{81^n}\ {}^{2n}C_1 + \frac{10^2}{81^n}\ {}^{2n}C_2 - \frac{10^3}{81^n}\ {}^{2n}C_3 + ... + \frac{10^{2n}}{81^n}$ is

(a) 2 (b) 0 (c) 1/2 (d) 1

162. The value of $\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + ...$ is

(a) $\frac{2^{n-2}}{(n-1)!}$ (b) $\frac{2^{n-1}}{n!}$ (c) $\frac{2^n}{n!}$ (d) $\frac{2^n}{(n-1)!}$

163. The sum of $(n + 1)$ terms of $({}^nC_0)^2 + 3.({}^nC_1)^2 + 5.({}^nC_2)^2 + ...$ is

(a) ${}^{2n-1}C_{n-1}$ (b) ${}^{2n-1}C_n$ (c) $2(n+1).{}^{2n-1}C_n$ (d) None of these

164. If sum of all the coefficients in the expansion of $(x^{3/2} + x^{-1/3})^n$ is 128, then the coefficient of x^5 is

(a) 35 (b) 45 (c) 7 (d) None of these

165. The sum of 12 terms of the series ${}^{12}C_1 \cdot \frac{1}{3} + {}^{12}C_2 \cdot \frac{1}{9} + {}^{12}C_3 \cdot \frac{1}{27} + ...$ is

(a) $\left(\frac{4}{3}\right)^{12} - 1$ (b) $\left(\frac{3}{4}\right)^{12} - 1$ (c) $\left(\frac{3}{4}\right)^{12} + 1$ (d) None of these

166. The sum of the coefficients of all the integral powers of x in the expansion of $(1+2\sqrt{x})^{40}$ is

(a) $3^{40}+1$ (b) $3^{40}-1$ (c) $\frac{1}{2}(3^{40}-1)$ (d) $\frac{1}{2}(3^{40}+1)$

167. If $(1+2x+x^2)^n=\sum_{r=0}^{2n} a_r x^r$, then $a_r=$ **[EAMCET 1994]**

(a) $({}^nC_r)^2$ (b) ${}^nC_r\cdot{}^nC_{r+1}$ (c) ${}^{2n}C_r$ (d) ${}^{2n}C_{r-1}$

Advance Level

168. If $(1+x)^n=C_0+C_1x+C_2x^2+.....+C_nx^n$, then $C_0C_2+C_1C_3+C_2C_4+C_{n-2}C_n$ equals **[Rajasthan PET 1996]**

(a) $\frac{(2n)!}{(n+1)!(n+2)!}$ (b) $\frac{(2n)!}{(n-2)!(n+2)!}$ (c) $\frac{(2n)!}{(n)!(n+2)!}$ (d) $\frac{2n!}{(n-1)!(n+2)!}$

169. If $(1+x)^n=C_0+C_1x+C_2x^2+....+C_nx^n$, then the value of $C_0+C_2+C_4+......$ is **[Rajasthan PET 1997]**

(a) 2^{n-1} (b) 2^n-1 (c) 2^n (d) $2^{n-1}-1$

170. If a_r is the coefficient of x^r, in the expansion of $(1+x+x^2)^n$, then $a_1-2a_2+3a_3-.....-2n\,a_{2n}=$ **[EAMCET 2003]**

(a) 0 (b) n (c) $-n$ (d) $2n$

171. If $(1+x)^n=C_0+C_1x+C_2x^2+......+.....\ C_nx^n$, then $C_0^2+C_1^2+C_2^2+C_3^2....+C_n^2=$

[MP PET 1985; Karnataka CET 1995; UPSEAT 1999]

(a) $\frac{n!}{n!\,n!}$ (b) $\frac{(2n)!}{n!\,n!}$ (c) $\frac{(2n)!}{n!}$ (d) None of these

172. If a and d are two complex numbers, then the sum to $(n+1)$ terms of the following series $aC_0-(a+d)C_1+(a+2d)C_2-.....+.....$ is

(a) $\frac{a}{2^n}$ (b) na (c) 0 (d) None of these

173. If $(1+x)^{15}=C_0+C_1x+C_2x^2+....+C_{15}x^{15}$, then $C_2+2C_3+3C_4+.....+14C_{15}=$ **[IIT 1966]**

(a) 14.2^{14} (b) $13.2^{14}+1$ (c) $13.2^{14}-1$ (d) None of these

174. The sum of the series $\sum_{r=0}^{n}(-1)^r\,{}^nC_r\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+....m\text{ terms}\right)$ is **[IIT 1985]**

(a) $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$ (b) $\frac{2^{mn}-1}{2^n-1}$ (c) $\frac{2^{mn}+1}{2^n+1}$ (d) None of these

175. If n is a positive integer and $C_k={}^nC_k$, then the value of $\sum_{k=1}^{n}k^3\left(\frac{C_k}{C_{k-1}}\right)^2=$ **[Roorkee 1991]**

(a) $\frac{n(n+1)(n+2)}{12}$ (b) $\frac{n(n+1)^2}{12}$ (c) $\frac{n(n+2)^2(n+1)}{12}$ (d) None of these

176. The sum of the series $\sum_{r=0}^{10}{}^{20}C_r$ is

(a) 2^{20} (b) 2^{19} (c) $2^{19}+\frac{1}{2}\,{}^{20}C_{10}$ (d) $2^{19}-\frac{1}{2}\,{}^{20}C_{10}$

177. If $(1+x-2x^2)^6=1+C_1x+C_2x^2+C_3x^3+....+C_{12}x^{12}$, then the value of $C_2+C_4+C_6+....+C_{12}$ is

(a) 30 (b) 32 (c) 31 (d) None of these

178. If $C_0, C_1, C_2,....., C_n$ denote the binomial coefficient in the expansion of $(1+x)^n$, then the value of $aC_0+(a+b)C_1+(a+2b)C_2+....+(a+nb)C_n$ is

(a) $(a+nb)2^n$ (b) $(a+nb)2^{n-1}$ (c) $(2a+nb)2^{n-1}$ (d) $(2a+nb)2^n$

179. If $C_r={}^nC_r$ and $(C_0+C_1)(C_1+C_2).......(C_{n-1}+C_n)=k\dfrac{(n+1)^n}{n!}$, then the value of k is

(a) $C_0\,C_1C_2....C_n$ (b) $C_1^2\,C_2^2.....C_n^2$ (c) $C_1+C_2+.....+C_n$ (d) None of these

180. ${}^{n-1}C_r=(K^2-3).\,{}^nC_{r+1}$, if $K\in$ **[IIT Screening 2004]**

(a) $[-\sqrt{3},\sqrt{3}]$ (b) $(-\infty,-2)$ (c) $(2,\infty)$ (d) $(\sqrt{3},2)$

181. The coefficient of x^n in the polynomial $(x+{}^{2n+1}C_0)(x+{}^{2n+1}C_1)(x+{}^{2n+1}C_2)....(x+{}^{2n+1}C_n)$ is

(a) 2^{n+1} (b) $2^{2n+1}-1$ (c) 2^{2n} (d) $2^{2n+1}+1$

182. If n is positive integer then the sum of $\left[\left({}^nC_0-{}^nC_1\dfrac{1+x}{1+nx}+{}^nC_2\cdot\dfrac{1+2x}{(1+nx)^2}-{}^nC_3\cdot\dfrac{1+3x}{(1+nx)^3}+....\right)\right]$ is equal to

(a) 0 (b) $2\left(\dfrac{nx}{1+nx}\right)^n$ (c) $\left(\dfrac{2nx}{1+nx}\right)^n$ (d) None of these

183. The value of ${}^{4n}C_0+{}^{4n}C_4+{}^{4n}C_8+.....+{}^{4n}C_{4n}$ is

(a) $2^{4n-2}+(-1)^n\,2^{2n-1}$ (b) $2^{4n-2}+2^{2n-1}$ (c) $2^{2n-1}+(-1)^n\,2^{4n-2}$ (d) None of these

184. The sum to $(n+1)$ terms of the following series $3C_0-8C_1+13C_2-18C_3+.....$ is

(a) 0 (b) 1 (c) – 1 (d) None of these

185. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$, then the value of $1^2C_1+2^2C_2+3^2C_3+....+n^2C_n$ is

(a) $n(n+1)2^{n-2}$ (b) $n(n+1)2^{n-1}$ (c) $n(n+1)2^n$ (d) None of these

186. The value of $\sum_{k=0}^{n}{}^nC_k.\sin(kx)\cos(n-k)x$ is

(a) $2^{n-1}.\sin(nx)$ (b) $2^n\sin(nx)$ (c) $2^{n-1}.\cos(nx)$ (d) $2^{n-1}\sin(nx)\cos x$

187. Let n be an odd integer. If $\sin n\theta=\sum_{r=0}^{n}b_r\sin^r\theta$ for every value of θ, then

(a) $b_0=1, b_1=3$ (b) $b_0=0, b_1=n$ (c) $b_0=-1, b_1=n$ (d) $b_0=1, b_1=n^2-3n+3$

188. $\sum_{r=0}^{2n}a_r(x-2)^r=\sum_{r=0}^{2n}b_r(x-3)^r$ and $a_k=1$ for all $k\ge n$, then **[IIT 1992]**

(a) $b_n={}^{2n}C_n$ (b) $b_n={}^{2n+1}C_{n-1}$ (c) $b_n={}^{2n+1}C_{n+1}$ (d) None of these

189. Let $n\in N$. If $(1+x)^n=a_0+a_1x+a_2x^2+.....+a_nx^n$, and $a_{n-3}, a_{n-2}, a_{n-1}$ are in A.P. then

(a) a_1, a_2, a_3 are in A.P (b) a_1, a_2, a_3 are in H.P (c) $n = 7$ (d) $n = 14$

190. If $(1+2x+3x^2)^{10}=a_0+a_1x+a_2x^2+....+a_{20}x^{20}$, then a_1 equals

(a) 10 (b) 20 (c) 210 (d) 420

191. If $(2x-3x^2)^6=a_0+a_1x+..........+a_{12}x^{12}$, then value of a_0 and a_6 are

(a) 0, 6 (b) $0, 2^6$ (c) 1, 6 (d) 0

192. If a_1, a_2, a_3 are in A.P. and $(1+x^2)^2(1+x)^n=\sum_{k=0}^{n+4}a_kx^k$, then n is equal to

(a) 2 (b) 3 (c) 4 (d) All of these

193. If $(1+x)^{10} = a_0 + a_1x + a_2x^2 + ...a_{10}x^{10}$ then $(a_0 - a_2 + a_4 - a_6 + a_8 - a_{10})^2 + (a_1 - a_3 + a_5 - a_7 + a_9)^2$ is equal to

(a) 3^{10} (b) 2^{10} (c) 2^9 (d) None of these

194. If $(1+x)^{2n} = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ then

(a) $a_0 + a_2 + a_4 + ... = \frac{1}{2}(a_0 + a_1 + a_2 + a_3 + ...)$ (b) $a_{n+1} < a_n$

(c) $a_{n-3} = a_{n+3}$ (d) None of these

Sum of Coefficients

Basic Level

195. The sum of the coefficients in the expansion of $(1+x-3x^2)^{2163}$ will be **[IIT 1982]**

(a) 0 (b) 1 (c) – 1 (d) 2^{2163}

196. The sum of all the coefficients in the binomial expansion of $(x^2+x-3)^{319}$ is **[Bihar CEE 1994]**

(a) 1 (b) 2 (c) – 1 (d) 0

197. The sum of the coefficients in $(x+2y+z)^{10}$ is

(a) 2^{10} (b) 3^{10} (c) 2 (d) None of these

198. If the sum of the coefficients in the expansion of $(\alpha x^2 - 2x + 1)^{35}$ is equal to the sum of the coefficients in the expansion of $(x - \alpha y)^{35}$, then $\alpha =$

(a) 0 (b) 1 (c) May be any real number (d) No such value exist

199. The sum of coefficients in the expansion of $(x+2y+3z)^8$ is **[Rajasthan PET 2000]**

(a) 3^8 (b) 5^8 (c) 6^8 (d) None of these

200. If the sum of the coefficients in the expansion of $(1-3x+10x^2)^n$ is *a* and if the sum of the coefficients in the expansion of $(1+x^2)^n$ is *b*, then **[UPSEAT 2001]**

(a) *a* = 3*b* (b) $a = b^3$ (c) $b = a^3$ (d) None of these

201. The sum of coefficients in $(1+x-3x^2)^{2134}$ is **[Kurukshetra CEE 2001]**

(a) – 1 (b) 1 (c) 0 (d) 2^{2134}

202. The sum of coefficients in the expansion of is $(1+x+x^2)^n$ **[EAMCET 2002]**

(a) 2 (b) 3^n (c) 4^n (d) 2^n

203. If $n \in N$, then the sum of the coefficients in the expansion of the binomial $(5x-4y)^n$ is

(a) 1 (b) – 1 (c) 2 (d) 0

204. In the expansion of $(1+x)^n(1+y)^n(1+z)^n$, the sum of the coefficients of the terms of degree *r* is

(a) $({}^nC_r)^3$ (b) $3.{}^nC_r$ (c) ${}^{3n}C_r$ (d) ${}^nC_{3r}$

205. The sum of the numerical coefficients in the expansion of $\left(1+\frac{x}{3}+\frac{2y}{3}\right)^{12}$ is

(a) 1 (b) 2 (c) 2^{12} (d) None of these

206. The sum of the coefficients in the expansion of $(1+x-3x^2)^{2148}$ is **[Karnataka CET 2003]**

(a) 7 (b) 8 (c) – 1 (d) 1

Binomial theorem for any Index

Basic Level

207. If $y = 3x + 6x^2 + 10x^3 + ...$, then the value of x in terms of y is

(a) $1-(1-y)^{-1/3}$ (b) $1-(1+y)^{1/3}$ (c) $1+(1+y)^{-1/3}$ (d) $1-(1+y)^{-1/3}$

208. The coefficient of x in the expansion of $[\sqrt{1+x^2}-x]^{-1}$ in ascending powers of x, when $|x| < 1$, is **[MP PET 1996]**

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

209. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is **[AIEEE 2003]**

(a) 7^{th} term (b) 5^{th} term (c) 8^{th} term (d) 6^{th} term

210. The approximate value of $(7.995)^{1/3}$ correct to four decimal places is **[MNR 1991; UPSEAT 2000]**

(a) 1.9995 (b) 1.9996 (c) 1.9990 (d) 1.9991

211. Cube root of 217 is

(a) 6.01 (b) 6.04 (c) 6.02 (d) None of these

212. If $|x| < 1$, then in the expansion of $(1+2x+3x^2+4x^3+....)^{1/2}$, the coefficient of x^n is

(a) n (b) $n + 1$ (c) 1 (d) – 1

213. If $|x| < 1$, then the value of $1+n\left(\frac{2x}{1+x}\right)+\frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2+.......\infty$ will be **[AMU 1983]**

(a) $\left(\frac{1+x}{1-x}\right)^n$ (b) $\left(\frac{2x}{1+x}\right)^n$ (c) $\left(\frac{1+x}{2x}\right)^n$ (d) $\left(\frac{1-x}{1+x}\right)^n$

214. The sum of $1+n\left(1-\frac{1}{x}\right)+\frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2+.....\infty$, will be **[Roorkee 1975]**

(a) x^n (b) x^{-n} (c) $\left(1-\frac{1}{x}\right)^n$ (d) None of these

215. The first four terms in the expansion of $(1-x)^{3/2}$ are **[Rajasthan PET 1989]**

(a) $1-\frac{3}{2}x+\frac{3}{8}x^2-\frac{1}{16}x^3$ (b) $1-\frac{3}{2}x-\frac{3}{8}x^2-\frac{x^3}{16}$ (c) $1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{x^3}{16}$ (d) None of these

216. The coefficient of x^n in the expansion of $(1-9x+20x^2)^{-1}$ is

(a) $5^n - 4^n$ (b) $5^{n+1}-4^{n+1}$ (c) $5^{n-1}-4^{n-1}$ (d) None of these

217. If the third term in the binomial expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m is

(a) 2 (b) 1/2 (c) 3 (d) 4

218. $\frac{1}{\sqrt{5+4x}}$ can be expanded by binomial theorem, if

(a) $x < 1$ (b) $|x| < 1$ (c) $|x| < \frac{5}{4}$ (d) $|x| < \frac{4}{5}$

219. $(r + 1)^{th}$ term in the expansion of $(1-x)^{-4}$ will be

(a) $\frac{x^r}{r!}$ (b) $\frac{(r+1)(r+2)(r+3)}{6}x^r$ (c) $\frac{(r+2)(r+3)}{2}x^r$ (d) None of these

220. If $|x| < 1$, then the coefficient of x^n in the expansion of $(1+x+x^2+.....)^2$ will be **[Pb. CET 1989]**

(a) 1 (b) n (c) $n + 1$ (d) None of these

221. The general term in the expansion of $(1-2x)^{3/4}$ is

(a) $\frac{-3}{2^r r!}x^2$ (b) $\frac{-3^r}{2^r r!}x^r$ (c) $\frac{-3^r}{2^r (2r)!}x^r$ (d) None of these

222. The coefficient of x^2 in $(1+3x)^{1/2}(1-2x)^{-1/3}$ is

(a) 6/13 (b) 55/72 (c) 7/19 (d) 2/9

223. The coefficient of x^n in the expansion of $\frac{1}{(1-x)(1-2x)}$ is

(a) $1-2^{n+1}$ (b) $2^{n+1}-1$ (c) (2^n-1) (d) $2^{n-1}-1)$

224. The coefficient of x^4 in $\frac{1+2x+3x^2}{(1-x)^2}$ is

(a) 13 (b) 15 (c) 20 (d) 22

225. The value of $2(x+x^3+x^5+....)$ is

(a) $(1-x)^{-1}+(1+x)^{-1}$ (b) $(1-x)^{-1}-(1+x)^{-1}$ (c) $(1+x)^{-1}-(1-x)^{-1}$ (d) $(2+x)^{-1}-(2-x)^{-1}$

226. The coefficient of x^r in the expansion of $(1+3x+6x^2+10x^3+....)^2$ is

(a) $\frac{(r+1)(r+2)(r+3)}{5!}$ (b) $\frac{(r+2)(r+3)(r+4)}{5!}$

(c) $\frac{(r+1)(r+2)(r+3)(r+4)(r+5)}{5!}$ (d) None of these

227. The coefficient of x^{50} in the expression $(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+....+1001x^{1000}$ is

(a) ${}^{1000}C_{50}$ (b) ${}^{1001}C_{50}$ (c) ${}^{1002}C_{50}$ (d) ${}^{1000}C_{51}$

228. The value of $\sum_{r=0}^{\infty}(-1)^r(r+1)x^r$ is

(a) $(1+x)^{-1}$ (b) $(1-x)^{-1}$ (c) $(1+x)^{-2}$ (d) $(1-x)^{-2}$

229. The coefficient of x^n in $(1+x+2x^2+3x^3+....+nx^n)^2$ is

(a) $\frac{n(n^2+11)}{6}$ (b) $\frac{n(n^2+10)}{6}$ (c) $\frac{n(n^2+11)}{4}$ (d) $\frac{n(n^2+10)}{4}$

230. The coefficient of x in the expansion of $(1-ax)^{-1}(1-bx)^{-1}(1-cx)^{-1}$ is

(a) $a+b+c$ (b) $a-b-c$ (c) $-a+b+c$ (d) $a-b+c$

231. If x be so small that its 2 and higher power may be neglected, then $(1+2x)^{1/2}+(1-4x)^{-5/2}$ is equal to**[Rajasthan PET 1984**

(a) $2+x$ (b) $2+10x$ (c) $1-2x$ (d) $2+11x$

232. $\left(1+\frac{1}{3}x+\frac{1.4}{3.6}x^2+\frac{1.4.7}{3.6.9}x^3+....\right)^3$ is equal to **[Rajasthan PET 1986]**

(a) $(1+x)^{-1}$ (b) $(1+x)^{-2}$ (c) $(1-x)^{-1}$ (d) $(1-x)^{-2}$

233. The fourth term in the expansion of $(1-2x)^{3/2}$ will be **[Rajasthan PET 1989]**

(a) $-\frac{3}{4}x^4$ (b) $\frac{x^3}{2}$ (c) $-\frac{x^3}{2}$ (d) $\frac{3}{4}x^4$

234. $1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2\cdot 5}{3\cdot 6}\left(\frac{1}{2}\right)^2+\frac{2\cdot 5\cdot 8}{3\cdot 6\cdot 9}\cdot\left(\frac{1}{2}\right)^3+......=$

(a) $2^{1/3}$ (b) $3^{1/4}$ (c) $4^{1/3}$ (d) $3^{1/3}$

235. If $(a+bx)^{-2}=\frac{1}{4}-3x+......$, then (a, b) = **[UPSEAT 2002]**

(a) (2, 12) (b) (– 2, 12) (c) (2, – 12) (d) None of these

236. $\left(\frac{a}{a+x}\right)^{\frac{1}{2}}+\left(\frac{a}{a-x}\right)^{\frac{1}{2}}=$ **[DCE 1994; Pb. CET 2002; AIEEE 2002]**

(a) $2+\frac{3x^2}{4a^2}+...$ (b) $1+\frac{3x^2}{8a^2}+...$ (c) $2+\frac{x}{a}+\frac{3x^2}{4a^2}+.....$ (d) $2-\frac{x}{a}+\frac{3x^2}{4a^2}-.....$

Advance Level

237. The coefficient of x^n in the expansion of $\frac{(1+x)^2}{(1-x)^3}$ is

(a) n^2+2n+1 (b) $2n^2+n+1$ (c) $2n^2+2n+1$ (d) n^2+2n+2

238. $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+......=$ **[Rajasthan PET 1996; EAMCET 2001]**

(a) $\sqrt{2}$ (b) $1/\sqrt{2}$ (c) $\sqrt{3}$ (d) $1/\sqrt{3}$

239. $\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3}+\frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3}+\frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+3^3}+.....n$ terms = **[EAMCET 2000]**

(a) $\left(\frac{n}{n+1}\right)^2$ (b) $\left(\frac{n}{n+1}\right)^3$ (c) $\left(\frac{n}{n+1}\right)$ (d) $\left(\frac{1}{n+1}\right)$

240. The sum of the series $1+\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+....$ is equal to **[Roorkee 1998]**

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{\frac{5}{3}}$ (d) $\sqrt{5}$

241. If $\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{\sqrt{4-x}}$ is approximately equal to $a + bx$ for small values of x, then (a, b) =

(a) $\left(1,\frac{35}{24}\right)$ (b) $\left(1,-\frac{35}{24}\right)$ (c) $\left(2,\frac{35}{12}\right)$ (d) $\left(2,-\frac{35}{12}\right)$

242. In the expansion of $\left(\frac{1+x}{1-x}\right)^2$, the coefficient of x^n will be

(a) $4n$ (b) $4n - 3$ (c) $4n + 1$ (d) None of these

243. The coefficient of x^3 in the expansion of $\frac{(1+3x)^2}{1-2x}$ will be

(a) 8 (b) 32 (c) 50 (d) None of these

244. If $(1-x)^{-n}=a_0+a_1x+a_2x^2+....+a_rx^r+......$, then $a_0+a_1+a_2+.....+a_r$ is equal to

(a) $\frac{n(n+1)(n+2).....(n+r)}{r!}$ (b) $\frac{(n+1)(n+2).....(n+r)}{r!}$ (c) $\frac{n(n+1)(n+2).....(n+r-1)}{r!}$ (d) None of these

245. If p is nearly equal to q and $n > 1$, such that $\frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}=\left(\frac{p}{q}\right)^k$, then the value of k is

(a) n (b) $\frac{1}{n}$ (c) $n + 1$ (d) $\frac{1}{n+1}$

246. If x is very small compared to 1, then $(1-7x)^{1/3}(1+2x)^{-3/4}$ is equal to

(a) $1+\frac{23x}{6}$ (b) $1-\frac{23x}{6}$ (c) $1-\frac{25x}{6}$ (d) $1+\frac{25x}{6}$

247. If x is very small and $\frac{\left(1+\frac{3x}{4}\right)^{-4}\sqrt{16-3x}}{(8+x)^{2/3}}=P+Qx$, then

(a) $P=1, Q=\frac{305}{96}$ (b) $P=1, Q=-\frac{305}{96}$ (c) $P=2, Q=\frac{305}{48}$ (d) $P=2, Q=-\frac{305}{48}$

248. If a, b are approximately equal then the approximate value of $\left(\frac{b+2a}{a+2b}\right)$ is

(a) $(b/a)^{1/3}$ (b) $(a/b)^{1/3}$ (c) 1 (d) 9/3*b*

249. If *x* is nearly equal to 1, then the approximate value of $\frac{px^q - qx^p}{x^q - x^p}$ is

(a) $\frac{p+q}{1-x}$ (b) $\frac{1}{1-x}$ (c) $\frac{1}{1+x}$ (d) $\frac{p+q}{1+x}$

250. The coefficient of x^{20} in the expansion of $(1+x^2)^{40} \cdot \left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$ is

(a) ${}^{30}C_{10}$ (b) ${}^{30}C_{25}$ (c) 1 (d) None of these

Problem regarding three/four Consecutive terms or Coefficients

Basic Level

251. If in the expansion of $(1+x)^n$, *a*, *b*, *c* are three consecutive coefficients, then *n* =

(a) $\frac{ac+ab+bc}{b^2+ac}$ (b) $\frac{2ac+ab+bc}{b^2-ac}$ (c) $\frac{ab+ac}{b^2-ac}$ (d) None of these

252. If *n* is a positive integer and three consecutive coefficients in the expansion of $(1+x)^n$ are in the ratio 6 : 33 : 110, then *n* =

(a) 4 (b) 6 (c) 12 (d) 16

253. If the three consecutive coefficients in the expansion of $(1+x)^n$ are 28, 56 and 70, then the value of *n* is **[MP PET 1985]**

(a) 6 (b) 4 (c) 8 (d) 10

254. The coefficients of three successive terms in the expansion of $(1+x)^n$ are 165, 330 and 462 respectively, then the value of *n* will be

[UPSEAT 1999]

(a) 11 (b) 10 (c) 12 (d) 8

Multinomial theorem

Basic Level

255. The coefficient of x^3 in the expansion of $(1-x+x^2)^5$ is

(a) 10 (b) – 20 (c) – 50 (d) – 30

256. The coefficient of $a^8b^6c^4$ in the expansion of $(a+b+c)^{18}$ is

(a) ${}^{18}C_{14} \cdot {}^{14}C_8$ (b) ${}^{18}C_{10} \cdot {}^{10}C_6$ (c) ${}^{18}C_6 \cdot {}^{12}C_8$ (d) ${}^{18}C_4 \cdot {}^{14}C_6$

257. The coefficient of $x^3.y^4.z$ in the expansion of $(1+x+y-z)^9$ is

(a) $2.{}^9C_7.{}^7C_4$ (b) $-2.{}^9C_2.{}^7C_3$ (c) ${}^9C_7.{}^7C_4$ (d) None of these

Terms free from radical signs in the expansion of $(a^{1/p} + b^{1/q})$

Basic Level

258. The number of terms which are free from radical signs in the expansion of $(y^{1/5} + x^{1/10})^{55}$ is

(a) 5 (b) 6 (c) 7 (d) None of these

259. The number of integral terms in the expansion of $(5^{1/2} + 7^{1/6})^{642}$ is **[Kurukshetra CEE 1996]**

(a) 106 (b) 108 (c) 103 (d) 109

260. In the expansion of $[7^{1/3} + 11^{1/9}]^{6561}$, the number of terms free from radicals is

(a) 730 (b) 715 (c) 725 (d) 750

261. The number of rational terms in the expansion of $(1+\sqrt{2}+\sqrt[3]{3})^6$ is

(a) 6 (b) 7 (c) 5 (d) 8

262. The number of terms with integral coefficients in the expansion of $(7^{1/3}+5^{1/2}.x)^{600}$ is

(a) 100 (b) 50 (c) 101 (d) None of these

263. The sum of the rational terms in the expansion of $(\sqrt{2}+\sqrt[5]{3})^{10}$ is

(a) 32 (b) 9 (c) 41 (d) None of these

Miscellaneous

Basic Level

264. If $(8+3\sqrt{7})^n = P+F$, where *P* is an integer and *F* is a proper fraction then

(a) *P* is an odd integer (b) *P* is an even integer (c) $F(P+F)=1$ (d) $(1-F)(P+F)=1$

265. If [*x*] denotes the greatest integer less than or equal to *x*, then $[(6\sqrt{6}+14)^{2n+1}]$

(a) Is an even integer (b) Is an odd integer (c) Depends on *n* (d) None of these

266. Which of the following expansion will have term containing x^2

(a) $(x^{-1/5}+2x^{3/5})^{25}$ (b) $(x^{3/5}+2x^{-1/5})^{24}$ (c) $(x^{3/5}-2x^{-1/5})^{23}$ (d) $(x^{3/5}+2x^{-1/5})^{22}$

267. If the second term in the expansion $\left(\sqrt[13]{a}+\frac{a}{\sqrt{a^{-1}}}\right)^n$ is $14a^{5/2}$, then the value of ${}^nC_3 / {}^nC_2$ is

(a) 4 (b) 3 (c) 12 (d) 6

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Assignment

Mathematical Induction

Basic Level

1. $10^n + 3.4^{n+2} + 5, \forall n \in N$ is divisible by

(a) 5 (b) 7 (c) 9 (d) 11

2. For every natural number n, $n(n-1)(2n-1)$ is divisible by

(a) 6 (b) 12 (c) 24 (d) 5

3. For every natural number n, $3^{2n+2} - 8n - 9$ is divisible by **[IIT 1977]**

(a) 16 (b) 128 (c) 256 (d) None of these

4. $49^n + 16n - 1$ is divisible by **[Kurukshetra CEE 2001]**

(a) 3 (b) 19 (c) 64 (d) 29

5. For all positive integral values of n, $2^{4n} - 1$ is divisible by

(a) 8 (b) 16 (c) 24 (d) None of these

6. For all positive integral values of n, $3^{2n} - 2n + 1$ is divisible by

(a) 2 (b) 4 (c) 8 (d) 12

7. If $n \in N$, then $x^{2n-1} + y^{2n-1}$ is divisible by

(a) $x + y$ (b) $x - y$ (c) $x^2 + y^2$ (d) $x^2 + xy$

8. For each $n \in N$, $2^{3n} - 7n - 1$ is divisible by

(a) 23 (b) 41 (c) 49 (d) 98

9. For each $n \in N$, $x^n - y^n$ is divisible by

(a) $x + y$ (b) $x - y$ (c) $x^2 + y^2$ (d) $x^2 - y^2$

10. If $n \in N$, then the greatest integer which divides $n(n - 1)(n - 2)$ is

(a) 2 (b) 3 (c) 6 (d) 8

11. If $n \in N$, then $7^{2n} + 2^{3n-3}.\ 3^{n-1}$ is always divisible by **[IIT 1982]**

(a) 25 (b) 35 (c) 45 (d) None of these

12. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by **[Roorkee 1982]**

(a) 113 (b) 123 (c) 133 (d) None of these

13. For every natural number n, $n(n^2 - 1)$ is divisible by **[Rajasthan PET 1991]**

(a) 4 (b) 6 (c) 10 (d) None of these

14. The difference between an integer and its cube is divisible by **[MP PET 1999]**

(a) 4 (b) 6 (c) 9 (d) None of these

15. For every natural number n

(a) $n > 2^n$ (b) $n < 2^n$ (c) $n \geq 2^n$ (d) $n \leq 2^n$

16. For each $n \in N$, the correct statement is

(a) $2^n < n$ (b) $n^2 > 2n$ (c) $n^4 < 10^n$ (d) $2^{3n} > 7n+1$

17. For natural number n, $2^n (n-1)! < n^n$, if

(a) $n < 2$ (b) $n > 2$ (c) $n \geq 2$ (d) Never

18. If n is a natural number then $\left(\frac{n+1}{2}\right)^n \geq n!$ is true when

(a) $n > 1$ (b) $n \geq 1$ (c) $n > 2$ (d) $n \geq 2$

19. For positive integer n, $10^{n-2} > 81n$, if

(a) $n > 5$ (b) $n \geq 5$ (c) $n < 5$ (d) $n > 6$

20. For every positive integer n, $2^n < n!$ when

(a) $n < 4$ (b) $n \geq 4$ (c) $n < 3$ (d) None of these

21. For every positive integral value of n, $3^n > n^3$ when

(a) $n > 2$ (b) $n \geq 3$ (c) $n \geq 4$ (d) $n < 4$

22. For natural number n, $(n!)^2 > n^n$, if

(a) $n > 3$ (b) $n > 4$ (c) $n \geq 4$ (d) $n \geq 3$

23. The value of the n natural numbers n such that the inequality $2^n > 2n+1$ is valid is **[MNR 1994]**

(a) For $n \geq 3$ (b) For $n < 3$ (c) For mn (d) For any n

24. Let $P(n)$ denote the statement that $n^2 + n$ is odd. It is seen that $P(n) \Rightarrow P(n+1)$, P_n is true for all **[IIT JEE 1996]**

(a) $n > 1$ (b) n (c) $n > 2$ (d) None of these

25. If $\{x\}$ denotes the fractional part of x then $\left\{\frac{3^{2n}}{8}\right\}, n \in N$, is

(a) 3/8 (b) 7/8 (c) 1/8 (d) None of these

26. If p is a prime number, then $n^p - n$ is divisible by p when n is a

(a) Natural number greater than 1 (b) Irrational number

(c) Complex number (d) Odd number

27. $x(x^{n-1} - na^{n-1}) + a^n(n-1)$ is divisible by $(x-a)^2$ for

(a) $n > 1$ (b) $n > 2$ (c) All $n \in N$ (d) None of these

28. Let $P(n)$ be a statement and let $P(n) \Rightarrow p(n+1)$ for all natural numbers n, then $P(n)$ is true

(a) For all n (b) For all $n > 1$

(c) For all $n > m$, m being a fixed positive integer (d) Nothing can be said

29. If $P(n) = 2 + 4 + 6 + \ldots + 2n$, $n \in N$, then $P(k) = k(k+1) + 2 \Rightarrow P(k+1) = (k+1)(k+2) + 2$ for all $k \in N$. So we can conclude that $P(n) = n(n+1) + 2$ for

(a) All $n \in N$ (b) $n > 1$ (c) $n > 2$ (d) Nothing can be said

30. For every natural number n, $n(n + 1)$ is always

(a) Even (b) Odd (c) Multiple of 3 (d) Multiple of 4

31. The statement $P(n)$ "$1\times1!+2\times2!+3\times3!+.....+n\times n!=(n+1)!-1$" is

(a) True for all $n > 1$ (b) Not true for any n (c) True for all $n \in N$ (d) None of these

32. If $A=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$, then for any $n \in N$, A^n equals

(a) $\begin{pmatrix}\cos^n\theta & \sin^n\theta\\ -\sin^n\theta & \cos^n\theta\end{pmatrix}$ (b) $\begin{pmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{pmatrix}$ (c) $\begin{pmatrix}n\cos\theta & n\sin\theta\\ -n\sin\theta & n\cos\theta\end{pmatrix}$ (d) None of these

33. The least remainder when 17^{30} is divided by 5 is **[Karnataka CET 2003]**

(a) 1 (b) 2 (c) 3 (d) 4

34. The remainder when 5^{99} is divided by 13 is

(a) 6 (b) 8 (c) 9 (d) 10

35. 2^{60} when divided by 7 leaves the remainder

(a) 1 (b) 6 (c) 5 (d) 2

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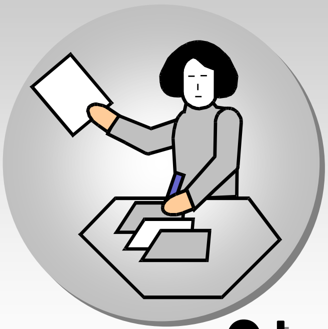

Answer Sheet

Binomial — *Assignment (Basic and Advance Level)*

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	c	c	c	c	a,b,d	b	b	b	d	c	b	b	a	c	b	c	b	c	b
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
a	a	c	c	a	b	c,d	a	a	a	b	c	b	d	c	a	d	b	a	d
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	a	a	c	b	a	d	d	a	b	c	b	b	c	d	a	c	c	a	c
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
b	c	c	c	a	c	a	c	b	a	c	d	b	d	c	d	b	c	b	a
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
d	a	c	a	a	b	d	b	b	a	c	b	d	c	c	c	a	c	a	a
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
b	c	a	d	a	b	d	c	c	b	c	b	d	c	b	a	d	b,c,d	a	b
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
c	a	b	c	b	a	b	c	c	d	c	d	b	a	b	c	b	b	a	b
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
b	a	b	c	b	a	b	c	a	a	a	b	c	b	d	d	c	c	b	b
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
d	b	c	a	a	d	c	b	a	c	b	c	b	a	d	c	c	c	a	d
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
c	a	a	a	a	a	b	c	c	b	b	d	b	a,b,c	c	c	d	b	c	b
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
b	b	a	c	c	d	d	d	c	b	a	c	a	a	c	b	b	c	b	c
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
d	b	b	d	d	c	c	c	a	a	d	c	b	c	a	a	c	a	c	c
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
b	a	c	b	b	b	b	b	b	b	b	c	c	a	d	a,b,d	b	b	b	a
261	**262**	**263**	**264**	**265**	**266**	**267**													
b	c	c	a	a	d	a													

Mathematical — *Assignment (Basic and Advance Level)*

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
c	a	a	c	d	a	a	c	b	c	a	c	b	b	b	c	b	b	b	b
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**					
c	d	a	d	c	a	c	d	d	a	c	b	d	b	a					