SET THEORY

19

Set Theory is an important concept of Mathematics which is often asked in aptitude exams. There are two types of questions in this chapter:

1. Numerical questions on set theory based on Venn diagrams

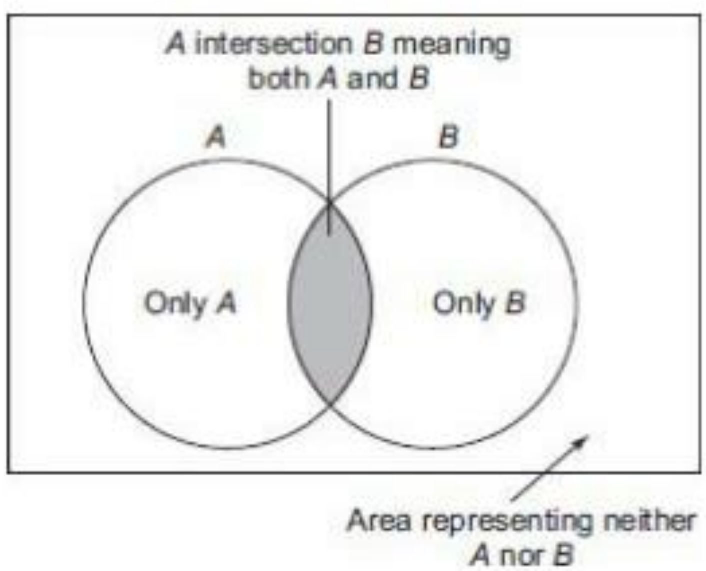
2. Logical questions based on set theory

Let us first take a look at some standard theoretical inputs related to set theory.

SET THEORY

Look at the following diagrams:

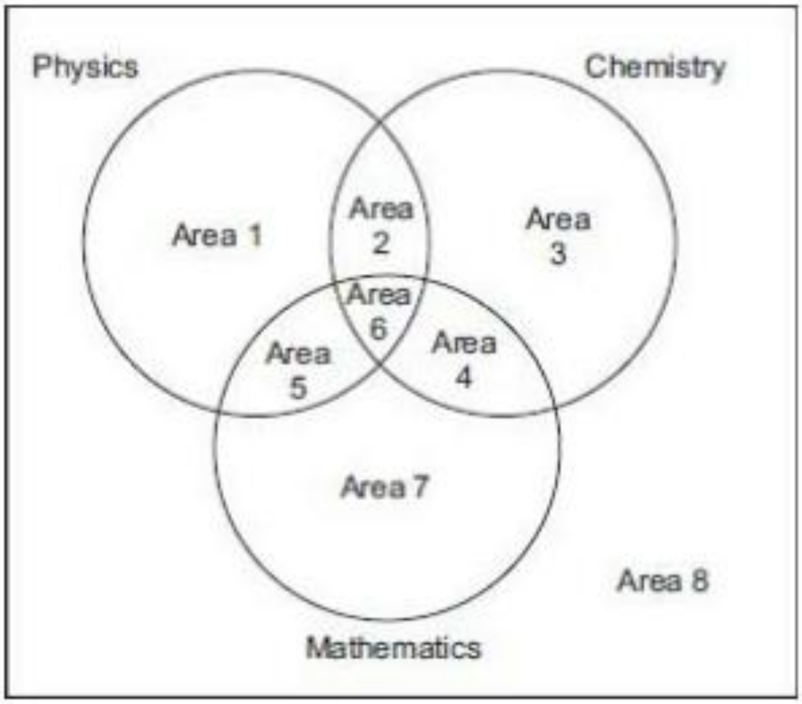
Figure 1: Refers to the situation where there are two attributes *A* and *B*. (Let's say *A* refers to people who passed in Physics and *B* refers to people who passed in Chemistry.) Then the shaded area shows the people who passed both in Physics and Chemistry.



In mathematical terms, the situation is represented as:

Total number of people who passed at least one subject = $A + B - A \cap B$

Figure 2: Refers to the situation where there are three attributes being measured. In the figure below, we are talking about people who passed Physics, Chemistry and/or Mathematics.



In the above figure, the following explain the respective areas:

Area 1: People who passed in Physics only

Area 2: People who passed in Physics and Chemistry only (in other words—people who passed Physics and Chemistry but not Mathematics)

Area 3: People who passed Chemistry only

Area 4: People who passed Chemistry and Mathematics only (also, can be described as people who passed Chemistry and Mathematics but not Physics)

Area 5: People who passed Physics and Mathematics only (also, can be described as people who passed Physics and Mathematics but not Chemistry)

Area 6: People who passed Physics, Chemistry and Mathematics

Area 7: People who passed Mathematics only

Area 8: People who passed in no subjects

Also take note of the following language which there is normally confusion about:

People passing Physics and Chemistry—Represented by the sum of areas 2 and 6

People passing Physics and Maths—Represented by the sum of areas 5 and 6

People passing Chemistry and Maths—Represented by the sum of areas 4 and 6

People passing Physics—Represented by the sum of the areas 1, 2, 5 and 6

In mathematical terms, this means:

Total number of people who passed at least one subject =

$P + C + M - P \cap C - P \cap M - C \cap M + P \cap C \cap M$

Let us consider the following questions and see how these figures work in terms of real-time problem-solving:

Illustration 1

At the birthday party of Sherry, a baby boy, 40 persons chose to kiss him and 25 chose to shake hands with him. Ten persons chose to both kiss him and shake hands with him. How many persons turned out at the party?

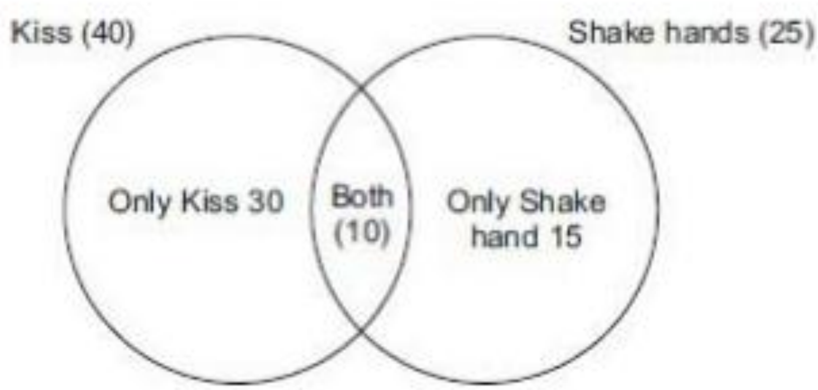
(a) 35

(b) 75

(c) 55

(d) 25

Solution:



From the figure, it is clear that the number of people at the party were 30 + 10 + 15 = 55.

We can, of course, solve this mathematically as below:

Let $n(A)$ = Number of persons who kissed Sherry = 40

$n(B)$ = Number of persons who shake hands with Sherry = 25

and $n(A \cap B)$ = Number of persons who shook hands with Sherry and kissed him both = 10

Then using the formula, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 40 + 25 - 10 = 55$

Illustration 2

Directions for Questions 1 to 4: Refer to the data below and answer the questions that follow:

In an examination 43% passed in Maths, 52% passed in Physics and 52% passed in Chemistry. Only 8% students passed in all the three. Fourteen per cent passed in Maths and Physics and 21% passed in Maths and Chemistry and 20% passed in Physics and Chemistry. Number of students who took the exam is 200.

1. How many students passed in Maths only?

   (a) 16

   (b) 32

   (c) 48

   (d) 80

2. Find the ratio of students passing in Maths only to the students passing in Chemistry only?

   (a) 16:37

   (b) 29:32

(c) 16:19

(d) 31:49

3. What is the ratio of the number of students passing in Physics only to the students passing in either Physics or Chemistry or both?

   (a) 34/46

   (b) 26/84

   (c) 49/32

   (d) None of these

4. A student is declared pass in the exam only if he/she clears at least two subjects. The number of students who were declared passed in this exam is?

   (a) 33

   (b) 66

   (c) 39

   (d) 78

Solution: Let *P* denote Physics, *C* denote Chemistry and *M* denote Maths.

% of students who passed in *P* and *C* only is given by:

% of students who passed in *P* and *C* - % of students who passed all three

= 20% – 8% = 12%

% of students who passed in *P* and *M* only is given by:

% of students who passed in *P* and *M* - % of students who passed all three

= 14% – 8% = 6%

% of students who passed in *M* and *C* only is given by:

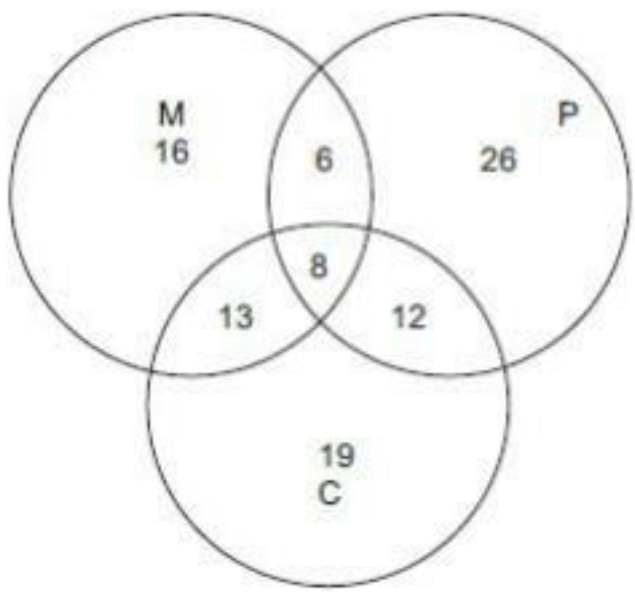
% of students who passed in C and M - % of students who passed all three = 21% – 8% = 13%

So, % of students who passed in P only is given by:

Total number passing in P – Number passing in P and C only – Number passing P and M only – Number passing in all three → 52% – 12% – 6% – 8% = 26%

% of students who passed in M only is given by:

Total number passing in M – Number passing in M and C only – Number passing P and M only – Number passing in all three → 43% – 13% – 6% – 8%- = 16%



% of students who passed in Chemistry only is given by:

Total number passing in C – Number passing in P and C only – Number passing in C and M only – Number passing in all three →

52% – 12% – 13% – 8%– = 19%

The answers are:

1. Only Maths =16% = 32 people. Option (b) is correct.

2. Ratio of only Maths to only Chemistry = 16:19. Option (c) is correct.

3. 26:84 is the required ratio. Option (b) is correct.

4. 39% or 78 people. Option (d) is correct.

Illustration 3

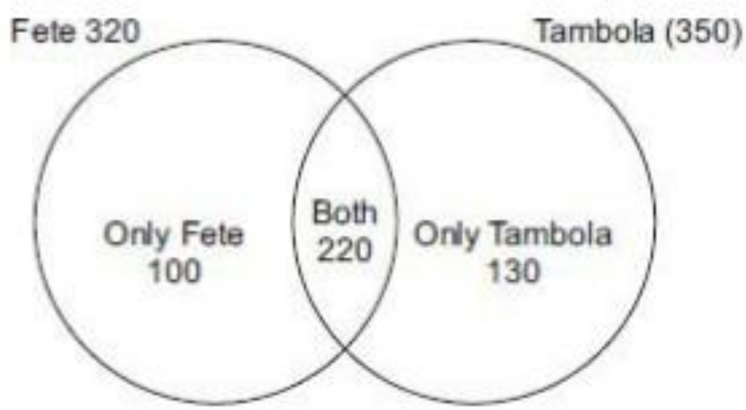
In the Mindworkzz club, all the members participate either in the Tambola or the Fete. In the fete, 320 participate,; 350 participate in the Tambola and 220 participate in both. How many members does the club have?

(a) 410

(b) 550

(c) 440

(d) None of these



The total number of people = 100 + 220 + 130 = 450

Option (d) is correct.

Illustration 4

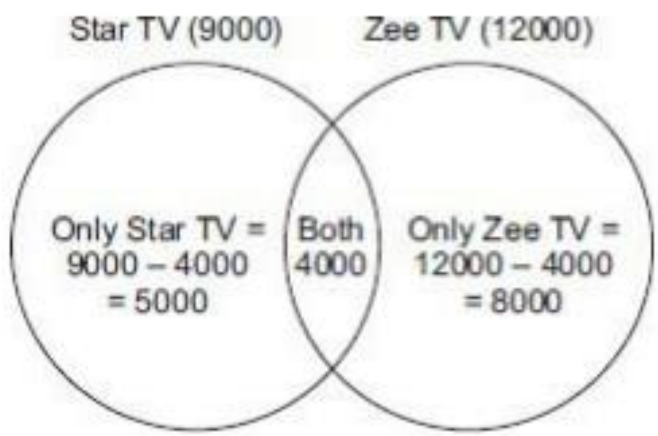
There are 20000 people living in Defence Colony, Gurugram. Out of them, 9000 subscribe to Star TV Network and 12000 to Zee TV Network. If 4000 subscribe to both, how many do not subscribe to any of the two?

(a) 3000

(b) 2000

(c) 1000

(d) 4000



The required answer would be 20000 – 5000 – 4000 – 8000 = 3000.

Illustration 5

Directions for Questions 1 to 3: Refer to the data below and answer the questions that follow.

Last year, there were three sections in the Catalyst, a mock CAT paper. Out of them, 33 students cleared the cut-off in Section 1, 34 students cleared the cut-off in Section 2 and 32 cleared the cut-off in Section 3. Ten students cleared the cut-off in Section 1 and Section 2, nine cleared the cut-off in Section 2 and Section 3, and eight cleared the cut-off in Section 1 and Section 3. The number of people who cleared each section alone was equal and was 21 for each section.

1. How many cleared all the three sections?

    (a) 3

    (b) 6

    (c) 5

(d) 7

2. How many cleared only one of the three sections?

(a) 21

(b) 63

(c) 42

(d) 52

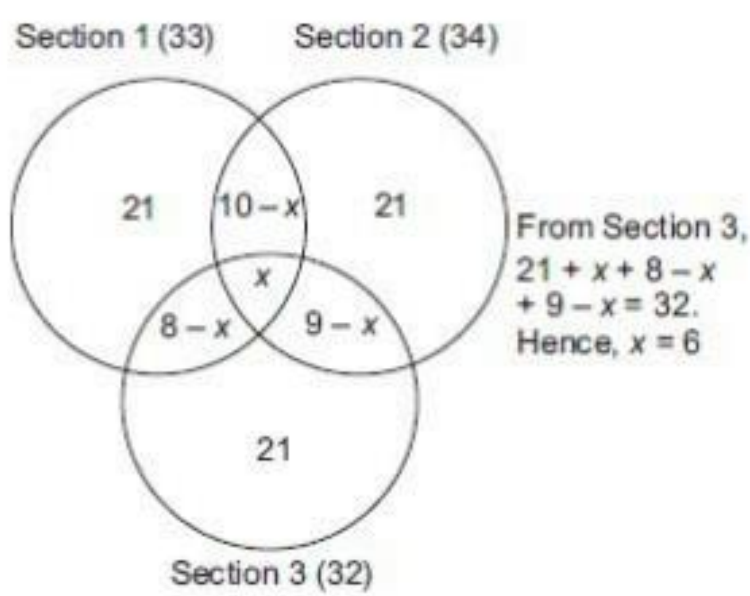
3. The ratio of the number of students clearing the cut-off in one or more of the sections to the number of students clearing the cut-off in Section 1 alone is?

(a) 78/21

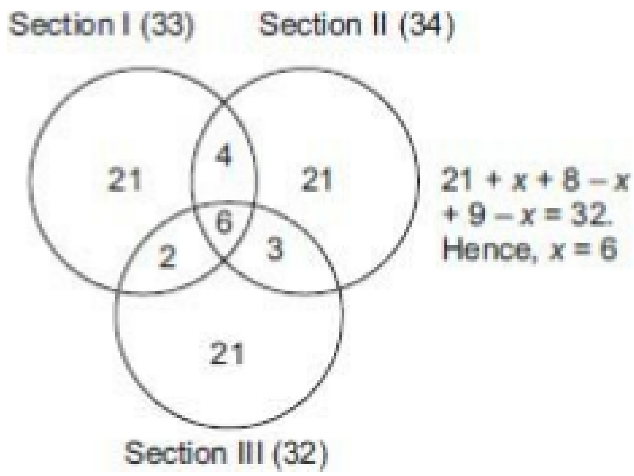
(b) 3

(c) 73/21

(d) None of these



Since, x = 6, the figure becomes:



The answers would be:

1. 6. Option (b) is correct.
2. 21 + 21 + 21 = 63. Option (b) is correct.
3. (21 + 21 + 21 + 6 + 4 + 3 + 2)/21 = 78/21. Option (a) is correct.

Illustration 6

In a locality having 1500 households, 1000 watch Zee TV, 300 watch NDTV and 750 watch Star Plus. Based on this information, answer the questions that follow:

1. The minimum number of households watching Zee TV and Star Plus is:

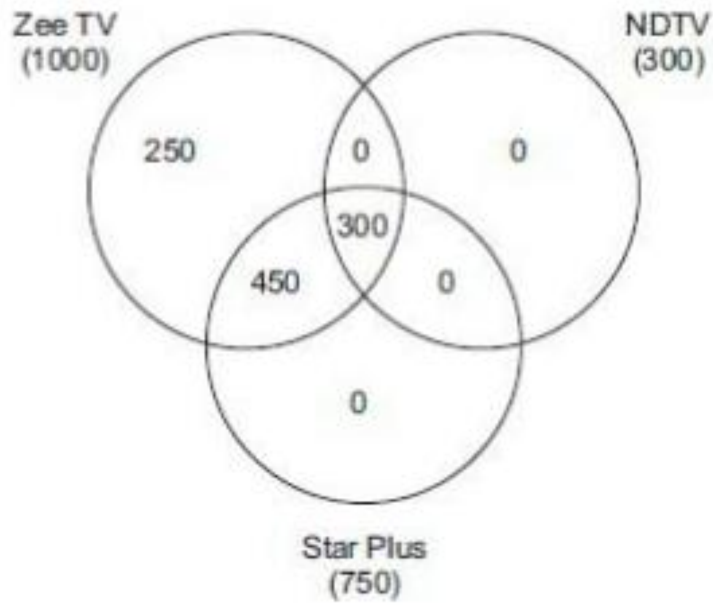
   Logic: If we try to consider each of the households watching Zee TV and Star Plus as independent of each other, we would get a total of 1000 + 750 = 1750 households. However, we have a total of only 1500 households in the locality and hence, there has to be a minimum interference of at least 250 households who would be watching both Zee TV and Star Plus. Hence, the answer to this question is 250.

2. The minimum number of households watching both Zee TV and NDTV is:

   In this case, the number of households watching Zee TV and NDTV can be separate from each other since there is no interference required between the households watching Zee TV and the households watching NDTV as their individual sum (1000 + 300) is smaller than the 1500 available households in the locality. Hence, the answer in this question is 0.

3. The maximum number of households who watch none of the three channels is:

For this to occur, the following situation would give us the required solution:



As you can clearly see from the figure, all the requirements of each category of viewers are fulfilled by the given allocation of 1000 households. In this situation, the maximum number of households who do not watch any of the three channels is visible as 1500 –1000 = 500.

Illustration 7

1. In a school, 90% of the students faced problems in Mathematics, 80% of the students faced problems in Computers, 75% of the students faced problems in Sciences, and 70% of the students faced problems in Social Sciences. Find the minimum percent of the students who faced problems in all four subjects.

Solution: In order to think about the minimum number of students who faced problems in all four subjects you would need to think of keeping the students who did not face a problem in any of the subjects separate from each other. We know that 30% of the students did not face problems in Social Sciences, 25% of the students did not face problems in Sciences, 20% students did not face problems in Computers and 10% students did not face problems in Mathematics. If each of these were separate from each other, we would have 30 + 25 + 20 + 10 = 85% people who did not face a problem in one of the four subjects. Naturally, the remaining 15% would be students who faced problems in all four subjects. This represents the minimum percentage of students who faced problems in all the four subjects.

2. For the above question, find the maximum possible percentage of students who could have problems in all four subjects.

   In order to solve this, you need to consider the fact that 100 (%) people are counted 315 (%) times, which means that there is an extra count of 215 (%). When you put a student into the 'has problems in each of the four subjects', he is one student counted four times — an extra count of 3. Since, 215/3 = 71 (quotient), we realise that if we have 71 students who have problems in all four subjects — we will have an extra count of 213 students. The remaining extra count of 2 more can be matched by putting one student in 'has problems in three subjects' or by putting two students in 'has problems in two subjects'. Thus, from the extra count angle, we have a limit of 71% students in the 'have problems in all four categories.'

   However, in this problem there is a constraint from another angle —

i.e. there are only 70% students who have a problem in Social Sciences — and hence, it is not possible for 71% students to have problems in all the four subjects. Hence, the maximum possible percentage of people who have a problem in all four subjects would be 70%.

3. In the above question if it is known that 10% of the students faced none of the above mentioned four problems, what would have been the minimum number of students who would have a problem in all four subjects?

If there are 10% students who face none of the four problems, we realise that these 10% would be common to students who face no problems in Mathematics, students who face no problems in Sciences, students who face no problems in Computers and students who face no problems in Social Sciences.

Now, we also know that overall there are 10% students who did not face a problem in Mathematics; 20% students who did not face a problem in Computers; 25% students who did not face a problem in Sciences and 30% students who did not face a problem in Social Sciences. The 10% students who did not face a problem in any of the subjects would be common to each of these four counts. Out of the remaining 90% students, if we want to identify the minimum number of students who had a problem in all four subjects we will take the same approach as we took in the first question of this set — i.e. we try to keep the students having problems in the individual subjects separate from each other. This would result in: 0% additional students having no problem in Mathematics; 10% additional students having no problem in Computers; 15% additional students having no problem in Sciences and 20% additional students having no problem in Social Sciences. Thus, we would get a total of

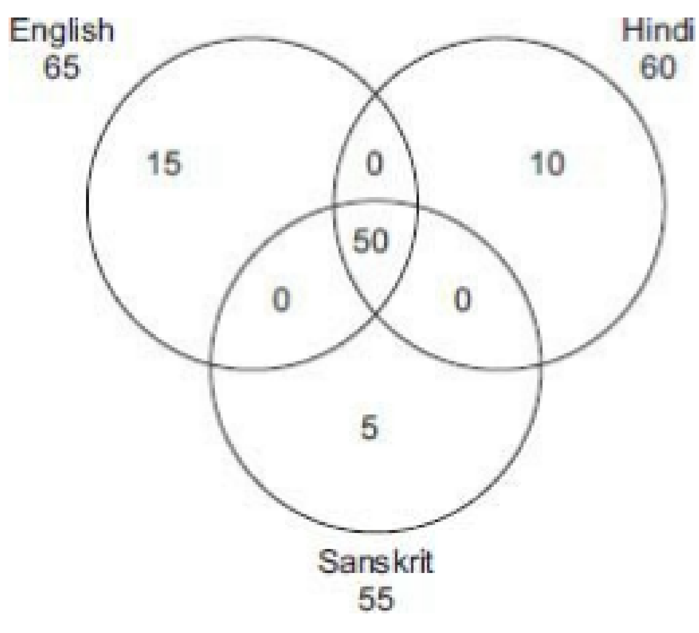
45% (0+10+15+20=45) students who would have no problem in one of the four subjects. Thus, the minimum percentage of students who had a problem in all four subjects would be 90 – 45 = 45%.

Illustration 8

In a class of 80 students, each of them studies at least one language — English, Hindi and Sanskrit. It was found that 65 studied English, 60 studied Hindi and 55 studied Sanskrit.

1. Find the maximum number of people who study all three languages.

   This question again has to be dealt with from the perspective of extra counting. In this question, 80 students in the class are counted 65 + 60 + 55 = 180 times — an extra count of 100. If we put 50 people in all the three categories as shown below, we would get the maximum number of students who study all three languages.

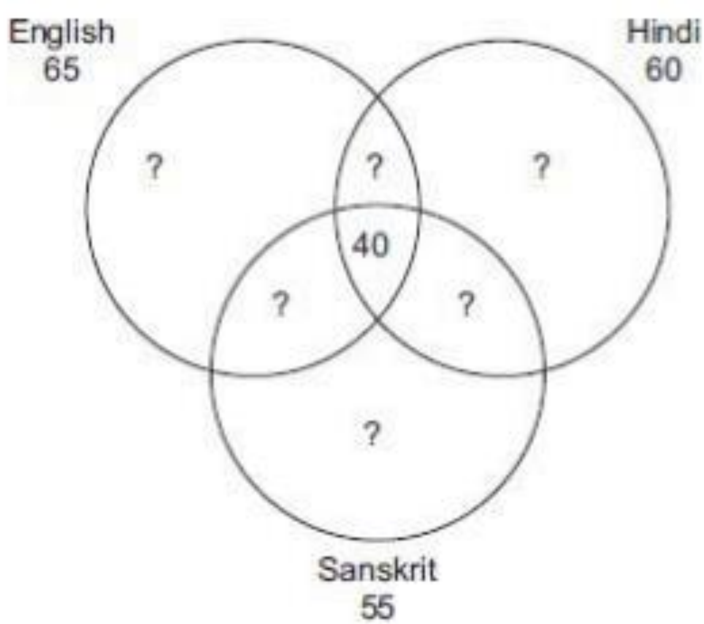


2. Find the minimum number of people who study all three languages.

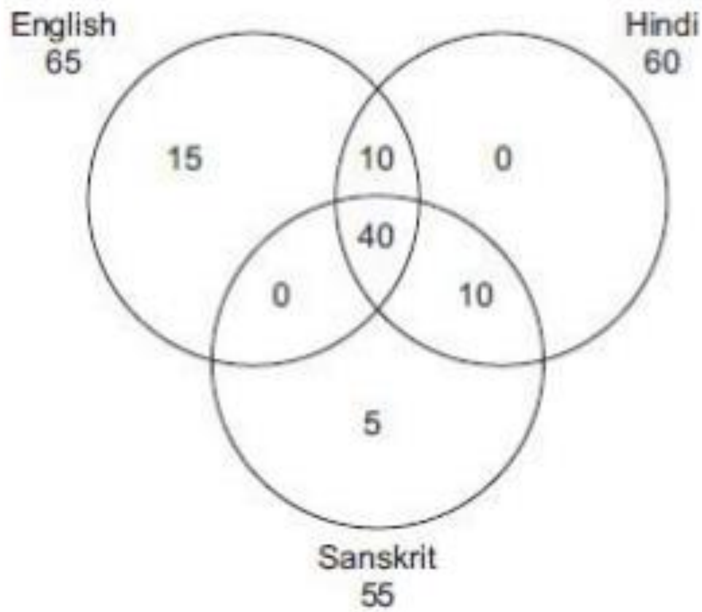
   In order to think about how many students are necessarily in the 'study all three languages' area of the figure (this thinking would lead us to the answer to the minimum number of people who study all three languages), we need to think about how many people we can shift out of the 'study all three category' for the previous question. When we try to do that, the following thought - process emerges:

Step 1: Let's take a random value for the all three categories (less than 50 of course) and see whether the numbers can be achieved. For this purpose we try to start with the value as 40 and see what happens. Before we move on, realise that the basic situation in the question remains the same — 80 students have been counted 180 times — which means that there is an extra count of 100 students and also realise that when you put an individual student in the all three categories, you get an extra count of 2, while at the same time when you put an individual student into the 'exactly two languages category', he/she is counted twice — hence an extra count of 1.

The starting figure we get, looks something like this:



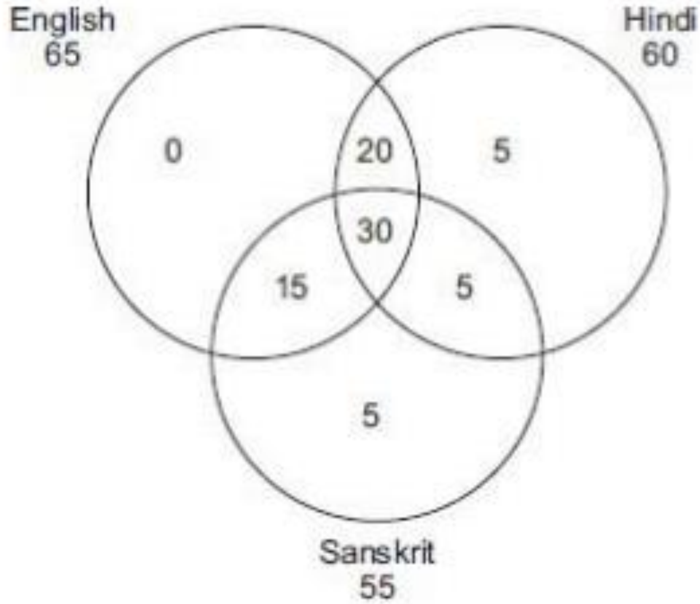
At this point, since we have placed 40 people in the all three categories, we have taken care of an extra count of 40 × 2 = 80. This leaves us with an extra count of 20 more to manage and as we can see in the above figure we have a lot of what can be described as 'slack' to achieve the required numbers. For instance, one solution we can think of from this point is as below:



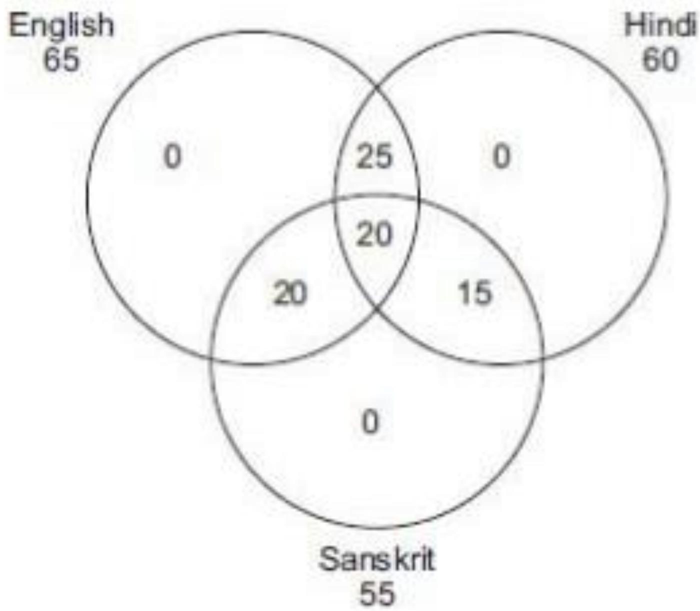
One look at this figure should tell you that the solution can be further optimised by reducing the middle value in the figure since there is still a lot of 'slack' in the figure — in the form of the number of students in the 'exactly one language category'. Also, you can easily see that there are many ways in which this solution could have been achieved with 40 in the middle. Hence, we go in search of a lower value in the middle.

So, we try to take an arbitrary value of 30 to see whether this is still achievable.

In this case we see the following as one of the possible ways to achieve this (again there is a lot of slack in this solution as the 'only Hindi' or the 'only Sanskrit' areas can be reallocated):



Trying the same solution for 20 in the middle, we get the optimum solution:



We realise that this is the optimum solution since there is no 'slack' in this solution and hence, there is no scope for re-allocating numbers from one area to another.

Author's note: You might be justifiably thinking, how do you do this kind of a random trial—and—error inside the exam! That is not the point of this question at this place. What I am trying to convey to you is that this is a critical thought structure which you need to have in your mind. Learn it here and do not worry about how you would think inside the exam — remember that in the CAT and in Aptitude tests, question setters have a responsibility to keep lengths limited and hence they would give you a question structure that is feasible to be solved within 3- 4 steps. Also, for this question, try to start by minimising the middle value as '0' and see how it works, feeling the slack and the movement in the question. On the Online resource center with this book, I have explained in details how to think in such a situation.

Illustration 9

In a group of 120 athletes, the number of athletes who can play Tennis, Badminton, Squash and Table Tennis is 70, 50, 60 and 30, respectively. What is the maximum number of athletes who can play none of the games?

Solution : In order to think of the maximum number of athletes who can play none of the games, we can think of the fact that since there are 70 athletes who play tennis, fundamentally there are a maximum of 50 athletes who would be possibly in the 'can play none of the games'. No other constraint in the problem necessitates a reduction of this number and hence, the answer to this question is 50.

LEVEL OF DIFFICULTY (I)

Directions for Questions 1 and 2: Refer to the data below and answer the questions that follow:

In the Indian athletic squad sent to the Olympics, 21 athletes were in the triathlon team; 26 were in the pentathlon team; and 29 were in the marathon team. Fourteen athletes can take part in triathlon and pentathlon; 12 can take part in marathon and triathlon; 15 can take part in pentathlon and marathon; and 8 can take part in all the three games.

1. How many players are there in all?

   (a) 35

(b) 43

(c) 49

(d) None of these

2. How many were in the marathon team only?

(a) 10

(b) 14

(c) 18

(d) 15

Directions for Questions 3 and 4: Refer to the data below and answer the questions that follow.

In a test in which 120 students appeared, 90 passed in History, 65 passed in Sociology and 75 passed in Political Science. Thirty students passed in only one subject and 55 students in only two. Five students passed no subjects.

3. How many students passed in all the three subjects?

(a) 25

(b) 30

(c) 35

(d) Data insufficient

4. Find the number of students who passed in at least two subjects.

(a) 85

(b) 95

(c) 90

(d) Data insufficient

Directions for Questions 5 to 8: Refer to the data below and answer the questions that follow.

Five per cent of the passengers who boarded Guwahati- New Delhi Rajdhani Express on 20 February, 2002 do not like coffee, tea and ice cream and 10% like all the three. Twenty per cent like coffee and tea, 25% like ice cream and coffee and 25% like ice cream and tea. Fifty-five per cent like coffee, 50% like tea and 50 % like ice cream.

5. The number of passengers who like only coffee is greater than the passengers who like only ice cream by

   (a) 50%

   (b) 100%

   (c) 25%

   (d) 0

6. The percentage of passengers who like both tea and ice cream but not coffee is

   (a) 15

   (b) 5

   (c) 10

   (d) 25

7. The percentage of passengers who like at least two of the three products is

   (a) 40

   (b) 45

   (c) 50

(d) 60

8. If the number of passengers is 180, then the number of passengers who like ice cream only is

(a) 10

(b) 18

(c) 27

(d) 36

Directions for Questions 9 to 15: Refer to the data below and answer the questions that follow.

In a survey among students at all the IIMs, it was found that 48% preferred coffee, 54% liked tea and 64% smoked. Of the total, 28% liked coffee and tea, 32% smoked and drank tea and 30% smoked and drank coffee. Only 6% did none of these. If the total number of students is 2000, then find the following.

9. The ratio of the number of students who like only coffee to the number who like only tea is

(a) 5:3

(b) 8:9

(c) 2:3

(d) 3:2

10. Number of students who like coffee and smoking but not tea is

(a) 600

(b) 240

(c) 280

(d) 360

11. The percentage of those who like coffee or tea but not smoking among those who like at least one of these is

(a) More than 30

(b) Less than 30
(c) Less than 25

(d) None of these

12. The percentage of those who like at least one of these is

(a) 100

(b) 90

(c) Nil

(d) 94

13. The two items having the ratio 1:2 are

(a) Tea only and tea and smoking only.

(b) Coffee and smoking only and tea only.

(c) Coffee and tea but not smoking and smoking but not coffee and tea.

(d) None of these

14. The number of persons who like coffee and smoking only and the number who like tea only bear a ratio

(a) 1:2

(b) 1:1

(c) 5:1

(d) 2:1

15. Percentage of those who like tea and smoking but not coffee is

(a) 14

(b) 14.9

(c) Less than 14

(d) More than 15

16. Thirty monkeys went to a picnic. Twenty-five monkeys chose to irritate cows while twenty chose to irritate buffaloes. How many chose to irritate both buffaloes and cows?

(a) 10

(b) 15

(c) 5

(d) 20

Directions for Questions 17 to 20: Refer to the data below and answer the questions that follow.

In the CBSE Board Exams last year, 53% passed in Biology, 61% passed in English, 60% in Social Studies, 24% in Biology and English, 35% in English and Social Studies, 27% in Biology and Social Studies and 5% in none.

17. Percentage of passes in all subjects is

(a) Nil

(b) 12

(c) 7

(d) 10

18. If the number of students in the class is 200, how many passed in only one subject?

(a) 48

(b) 46

(c) More than 50

(d) Less than 40

19. If the number of students in the class is 300, what will be the percentage change in the number of passes in only two subjects, if the original number of students is 200?

(a) More than 50%

(b) Less than 50%

(c) 50%

(d) None of these

20. What is the ratio of percentage of passes in Biology and Social Studies but not English in relation to the percentage of passes in Social Studies and English but not Biology?

(a) 5:7

(b) 7:5

(c) 4:5

(d) None of these

Directions for Questions 21 to 25: Refer to the data below and answer the questions that follow.

In the McGraw-Hill Mindworkzz Quiz held last year, participants were free to choose their respective areas from which they were asked questions. Out of 880 participants, 224 chose Mythology, 240 chose Science and 336 chose Sports, 64 chose both Sports and Science, 80 chose Mythology and Sports, 40 chose Mythology and Science and 24 chose all the three areas.

21. The percentage of participants who did not choose any area is

    (a) 23.59%

    (b) 30.25%

    (c) 37.46%

    (d) 27.27%

22. Of those participating, the percentage who choose only one area is

    (a) 60%

    (b) More than 60%

    (c) Less than 60%

    (d) More than 75%

23. Number of participants who chose at least two areas is

    (a) 112

    (b) 24

    (c) 136

    (d) None of these

24. Which of the following areas shows a ratio of 1:8?

(a) Mythology and Science but not Sports: Mythology only

(b) Mythology and Sports but not Science: Science only

(c) Science: Sports

(d) None of these

25. The ratio of students choosing Sports and Science but not Mythology to Science but not Mythology and Sports is

(a) 2:5

(b) 1:4

(c) 1:5

(d) 1:2

Directions for Questions 26 to 30: Refer to the data below and answer the questions that follow.

The table here gives the distribution of students according to professional courses.

Courses	*STUDENTS*			
	English		*Maths*	
	MALES	*FEMALES*	*MALES*	*FEMALES*
Part-time MBA	30	10	50	10
Full-time MBA only	150	8	16	6
CA only	90	10	37	3
Full time MBA and CA	70	2	7	1

26. The number of Maths students (for all courses) as a percentage of the number of English students (for all courses) is:

(a) 50.4

(b) 61.4

(c) 49.4

(d) None of these

27. The average number of females in all the courses is (count people doing full-time MBA and CA as a separate course)

(a) Less than 12

(b) Greater than 12

(c) 12

(d) None of these

28. The ratio of the number of girls to the number of boys is

(a) 5:36
(b) 1:9

(c) 1:7.2

(d) None of these

29. The percentage increase in students of full-time MBA only over CA only is

(a) Less than 20

(b) Less than 25

(c) Less than 30

(d) More than 30

30. The number of students doing full-time MBA or CA is

(a) 320

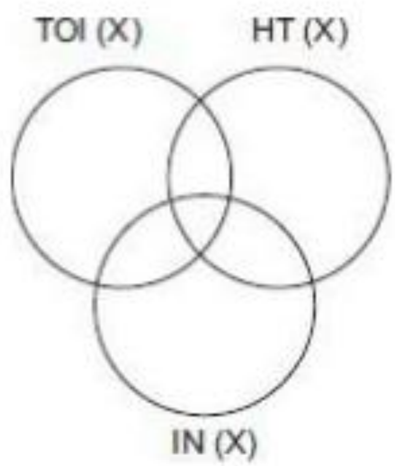
(b) 80

(c) 160

(d) None of these

Directions for Questions 31 to 34: Refer to the data below and answer the questions that follow:

A newspaper agent sells The TOI, HT and IN in equal numbers to 302 persons. Seven get HT and IN, twelve get The TOI and IN, nine get The TOI and HT and three get all the three newspapers. The details are given in the Venn diagram:



31. How many get only one paper?

(a) 280

(b) 327

(c) 109

(d) None of these

32. What percent get The TOI or The HT or both (but not The IN)? (Approximately)

(a) More than 65%

(b) Less than 60%

(c) 64%

(d) 72%

33. The number of persons buying The TOI and The HT only, The TOI and The IN only and The HT and The IN only are in the ratio of

(a) 6:4:9

(b) 6:9:4

(c) 4:9:6

(d) None of these

34. The difference between the number reading The HT and The IN only and HT only is

(a) 77

(b) 78

(c) 83

(d) None of these

35. A group of 78 people watch Zee TV, Star Plus or Sony. Of these, 36 watch Zee TV, 48 watch Star Plus and 32 watch Sony. If 14 people watch both Zee TV and Star Plus, 20 people watch both Star Plus and Sony, and 12 people watch both Sony and Zee TV find the ratio of the number of people who watch only Zee TV to the number of people who watch only Sony.

(a) 9:4

(b) 3:2

(c) 5:3

(d) 7:4

Directions for Questions 36 and 37: Answer the questions based on the following information.

The following data was observed from a study of car complaints received from 180 respondents at Colonel Verma's car-care workshop, viz., engine problem, transmission problem or mileage problem. Of those surveyed, there was no one who faced exactly two of these problems. There were 90 respondents who faced engine problems, 120 who faced transmission problems and 150 who faced mileage problems.

36. How many of them faced all the three problems?

(a) 45

(b) 60

(c) 90

(d) 20

37. How many of them faced either transmission problems or engine problems?

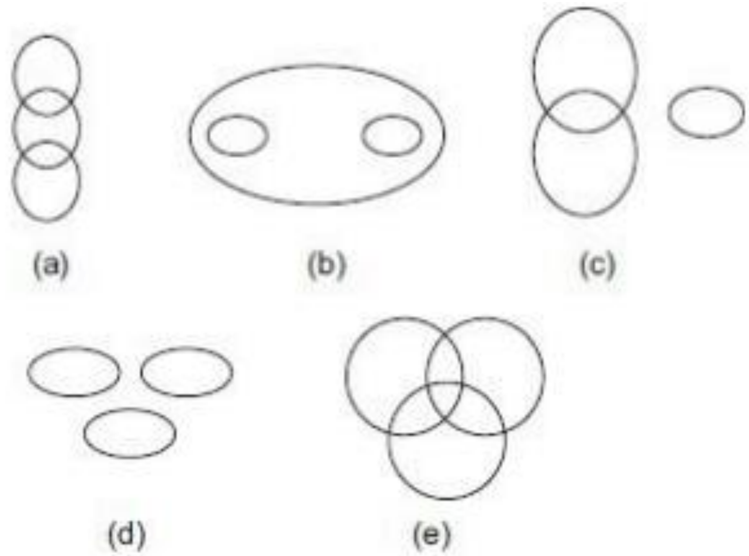
(a) 30

(b) 60

(c) 90

(d) 40

Directions for Questions 38 to 42: Given below are five diagrams one of which describes the relationship among the three classes given in each of the five questions that follow. You have to decide which of the diagrams is the most suitable for a particular set of classes.



38. Elephants, Tigers, Animals

39. Administrators, Doctors, Authors

40. Platinum, Copper, Gold

41. Gold, Platinum, Ornaments

42. Television, Radio, Mediums of Entertainment

43. Seventy percent of the employees in a multinational corporation have VCD players, 75 percent have microwave ovens, 80 percent have ACs and 85 percent have washing machines. At least what percentage of employees has all four gadgets?

(a) 15

(b) 5

(c) 10

(d) Cannot be determined

LEVEL OF DIFFICULTY (III)

1. At the Rosary Public School, there are 870 students in the senior secondary classes. The school is widely known for its Science education and there is the facility for students to do practical training in any of the four sciences – viz: Physics, Chemistry, Biology or Social Sciences. Some students in the school, however, have no interest in the sciences and hence do not undertake practical training in any of the four sciences. While considering the popularity of various choices for opting for practical training for one or more of the choices offered, Mr. Arvindaksham, the school principal noticed something quite extraordinary. He noticed that for every student in the school who opts for practical training in at least M sciences, there are exactly three students who opt for practical training in at least $(M - 1)$ sciences, for $M = 2, 3$ and 4. He also found that the number of students who opt for all four sciences was half the number of students who opt for none. Can you help him with the answer to "How many students in the school opt for exactly three sciences?"

   (a) 30

   (b) 60

   (c) 90

   (d) None of these

2. A bakery sells three kinds of pastries—pineapple, chocolate and black forest. On a particular day, the bakery owner sold the following number of pastries: 90 pineapple, 120 chocolate and 150 black forest. If none of the customers bought more than two pastries of each type, what is the minimum number of customers that must have visited the bakery that day?

(a) 80

(b) 75

(c) 60

(d) 90

3. Gauri Apartment housing society organised annual games, consisting of three games – viz: snooker, badminton and tennis. In all, 510 people were members in the apartments' society and they were invited to participate in the games — each person participating in as many games as he/she feels like. While viewing the statistics of the performance, Mr Kapoor realised the following facts. The number of people who participated in at least two games was 52% more than those who participated in exactly one game. The number of people participating in 1, 2 or 3 games respectively was at least equal to 1. Being a numerically inclined person, he further noticed an interesting thing — the number of people who did not participate in any of the three games was the minimum possible integral value with these conditions. What was the maximum number of people who participated in exactly three games?

(a) 298

(b) 300

(c) 303

(d) 304

4. A school has 180 students in its senior section where foreign languages are offered to students as part of their syllabus. The foreign languages offered are: French, German and Chinese and the numbers of people studying each of these subjects are 80, 90 and 100 respectively. The

number of students who study more than one of the three subjects is 50% more than the number of students who study all the three subjects. There are no students in the school who study none of the three subjects. Then how many students study all three foreign languages?

(a) 18

(b) 24

(c) 36

(d) 40

Directions for Questions 5 and 6: Answer the questions on the basis of the information given below.

In the second year, the Hampard Business School students are offered a choice of the specialisations, they wish to study from amongst only three specialisations —Marketing, Finance and HR. The number of students who have specialised in only Marketing, only Finance and only HR is all the numbers in an Arithmetic Progression— in no particular order. Similarly, the number of students specialising in exactly two of the three types of subjects are also numbers that form an Arithmetic Progression.

The number of students specialising in all three subjects is one-twentieth of the number of students specialising in only Finance which in turn is half of the number of students studying only HR. The number of students studying both Marketing and Finance is 15, whereas the number of students studying both Finance and HR is 19. The number of students studying HR is 120, which is more than the number of students studying Marketing (which is a two digit number above 50). It is known that there are exactly four students who opt for none of these specialisations and opt only for general subjects.

5. What is the total number of students in the batch?

(a) 223

(b) 233
(c) 237

(d) Cannot be determined

6. What is the number of students specialising in both Marketing and HR?

(a) 11

(b) 21

(c) 23

(d) Cannot be determined

Directions for Questions 7 to 9: In the Stafford Public School, students had an option to study none or one or more of three foreign languages, viz: French, Spanish and German. The total student strength in the school was 2116 students out of which 1320 students studied French and 408 students studied both French and Spanish. The number of people who studied German was found to be 180 higher than the number of students who studied Spanish. It was also observed that 108 students studied all three subjects.

7. What is the maximum possible number of students who did not study any of the three languages?

(a) 890

(b) 796
(c) 720

(d) None of these

8. What is the minimum possible number of students who did not study any of the three languages?

(a) 316

(b) 0

(c) 158

(d) None of these

9. If the number of students who used to speak only French was one more than the number of people who used to speak only German, then what could be the maximum number of people who used to speak only Spanish?

(a) 413

(b) 398

(c) 403

(d) 431

Directions for Questions 10 to 13: In the Vijayantkhand sports stadium, athletes choose from four different racquet games (apart from athletics which is compulsory for all). These are Tennis, Table Tennis, Squash and Badminton. It is known that 20% of the athletes practising there are not choosing any of the racquet sports. The four games given here are played by 460, 360, 360 and 440 students respectively. The number of athletes playing exactly two racquet games for any combination of two racquet games is 40. There are 60 athletes who play all the four games but in a strange coincidence, it was noticed that the number of people playing exactly three games was also equal to 20 for each combination of three games.

10. What is the number of athletes in the stadium?

(a) 1140

(b) 1040

(c) 1200

(d) 1300

11. What is the number of athletes in the stadium who play either only squash or only Tennis?

(a) 120
(b) 220

(c) 340

(d) 440

12. How many athletes in the stadium participate in only athletics?

(a) 160

(b) 1040

(c) 260

(d) 220

13. If all the athletes were compulsorily asked to add one game to their existing list (except those who were already playing all the four games) — then what will be the number of athletes who would be playing all four games after this change?

(a) 80

(b) 100

(c) 120

(d) 140

Directions for Questions 14 and 15: Answer the questions on the basis of the following information.

In the Pattabhiraman family, a clan of 192 individuals, each person has at least one of the three Pattabhiraman characteristics—blue eyes, blonde hair, and sharp mind. It is also known that:

(i) The number of family members who have only blue eyes is equal to the number of family members who have only sharp minds and this number is also equal to twice the number of family members who have blue eyes and sharp minds but not blonde hair.

(ii) The number of family members who have exactly two of the three features is 50.

(iii) The number of family members who have blonde hair is 62.

(iv) Among those who have blonde hair, 26 family members have at least two of the three characteristics.

14. If the number of family members who have blue eyes is the maximum amongst the three characteristics, then what is the maximum possible number of family members who have both sharp minds and blonde hair but do not have blue eyes?

(a) 11
(b) 10

(c) 12

(d) Cannot be determined

15. Which additional piece of information is required to find the exact number of family members who have blonde hair and blue eyes but not sharp minds?

(a) The number of family members who have exactly one of the three characteristics is 140

(b) Only two family members have all three characteristics.

(c) The number of family members who have sharp minds is 89.

(d) The number of family members who have only sharp minds is 52.

16. In a class of 97 students, each student plays at least one of the three games – Hockey, Cricket and Football. If 47 play Hockey, 53 play Cricket, 72 play Football and 15 play all the three games, what is the number of students who play exactly two games?

(a) 38

(b) 40

(c) 42

(d) 45

Directions for Questions 17 to 19: Answer the questions on the basis of the information given below.

In the ancient towns of Mohenjo Daro, a survey found that students were fond of three kinds of cold drinks (Pep, Cok and Thum). It was also found that there were three kinds of beverages that they liked (Tea, Cof and ColdCof).

The population of these towns was found to be 40,0000 people in all—and the survey was conducted on 10% of the population. Mr. Yadav, a data analyst observed the following things about the survey:

(i) The number of people in the survey who like exactly two cold drinks is five times the number of people who like all the three cold drinks

(ii) The sum of the number of people in the survey who like Pep and 42% of those who like Cok but not Pep is equal to the number of people who like Tea

(iii) The number of people in the survey who like Cof is equal to the sum of 3/8th of those who like Cok and ½ of those who like Thum. This number is also equal to the numbers who like ColdCof.

(iv) 18500 people surveyed like Pep

(v) 15000 like all the beverages and 3500 like all the cold drinks;

(vi) 14000 do not like Pep but like Thum

(vii) 11000 like Pep and exactly one more cold drink

(viii) 6000 like only Cok and the same number of people like Pep and Thum but not Cok

17. The number of people in the survey who do like at least one of the three cold drinks?

(a) 38,500

(b) 31,500

(c) 32,500

(d) 39,500

18. What is the maximum number of people in the survey who like none of the three beverages?

(a) 24,000

(b) 16,000

(c) 12,000

(d) Cannot be determined

19. What is the maximum number of people in the survey who like at least one of the three beverages?

(a) 7000

(b) 32,000

(c) 33,000

(d) Cannot be determined

20. In a certain class of students, the number of students who drink only tea, only coffee, both tea and coffee and neither tea nor coffee are x, $2x$, $3x$ and $4x$ respectively such that the value of x and $3x$ are both factors of 57. The number of people who drink coffee can be

(a) 41

(b) 40

(c) 59

(d) Both (a) and (c)

ANSWER KEY

Level of Difficulty (I)

1. (b)
2. (a)
3. (b)
4. (a)

5. (b)
6. (a)
7. (c)
8. (b)
9. (c)
10. (b)
11. (a)
12. (d)
13. (c)
14. (b)
15. (a)
16. (b)
17. (c)
18. (b)
19. (c)
20. (a)
21. (d)
22. (c)
23. (c)
24. (a)
25. (b)
26. (d)
27. (b)
28. (b)
29. (c)

30. (a)

31. (a)

32. (c)

33. (b)

34. (d)

35. (a)

36. (c)

37. (b)

38. (b)

39. (e)

40. (d)

41. (a)

42. (b)

43. (c)

Level of Difficulty (II)

1. (b)

2. (b)

3. (c)

4. (c)

5. (c)

6. (c)

7. (b)

8. (b)

9. (d)

10. (d)

11. (c)

12. (c)

13. (d)

14. (a)

15. (c)

16. (d)

17. (a)

18. (b)

19. (c)

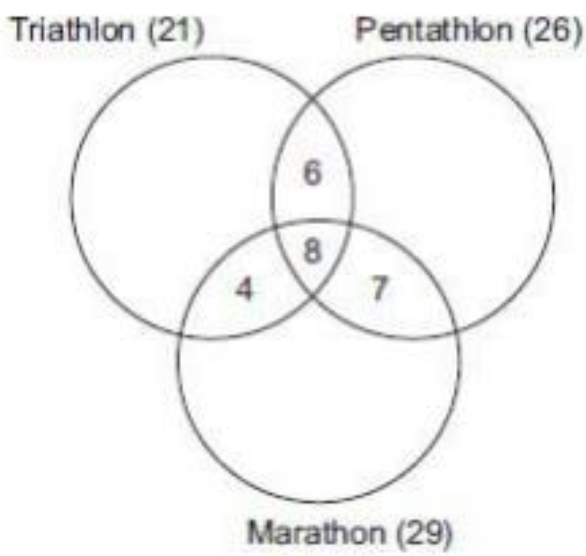
20. (d)

Solutions and Shortcuts

Level of Difficulty (I)

Solutions for Questions 1 and 2:

Since there are 14 players who are in triathlon and pentathlon, and there are 8 who take part in all three games, there will be 6 who take part in only triathlon and pentathlon. Similarly, only triathlon and marathon = 12 – 8 = 4 and only Pentathlon and Marathon = 15 – 8 = 7.



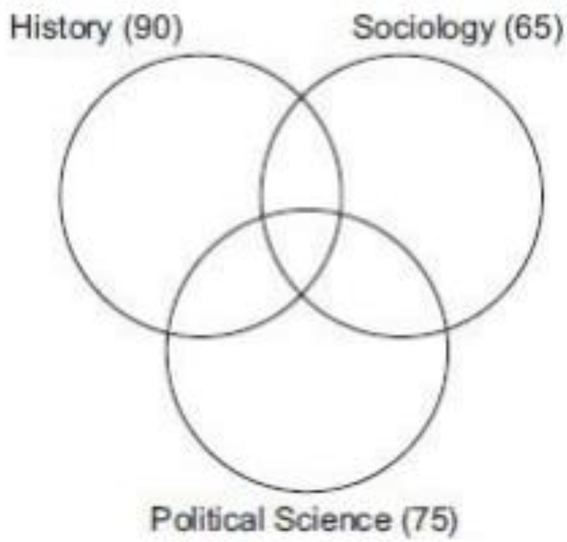
The figure above can be completed with values for each sport (only) plugged in:

The answers would be:

3 + 6 + 8 + 4 + 5 + 7 + 10 = 43. Option (b) is correct.

Option (a) is correct.

Solutions for Questions 3 and 4:



The given situation can be read as follows:

115 students are being counted 75 + 65 + 90 = 230 times.

This means that there is an extra count of 115. This extra count of 115 can be created in two ways.

1. By putting people in the 'passed exactly two subjects' category. In such a case each person would get counted two times (double counted), i.e., an extra count of 1.

2. By putting people in the 'all three' category, each person put there would be triple counted. One person counted three times – meaning an extra count of two per person.

The problem tells us that there are 55 students who passed exactly two subjects. This means an extra count of 55 would be accounted for. This would leave an extra count of 115 – 55 = 60 more to be accounted for by 'passed all three' category. This can be done by using thirty people in the 'all 3' category.

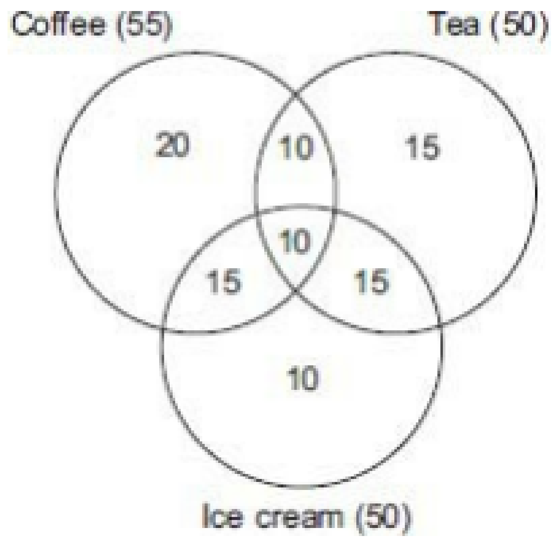
Hence, the answers are:

3. Option (b)

4. Option (a)

Solutions for Questions 5 to 8:

Based on the information provided, we would get the following figure:



The answers could be read off the figure as:

5. [(20 – 10)/10] * 100 = 100%. Option (b) is correct.

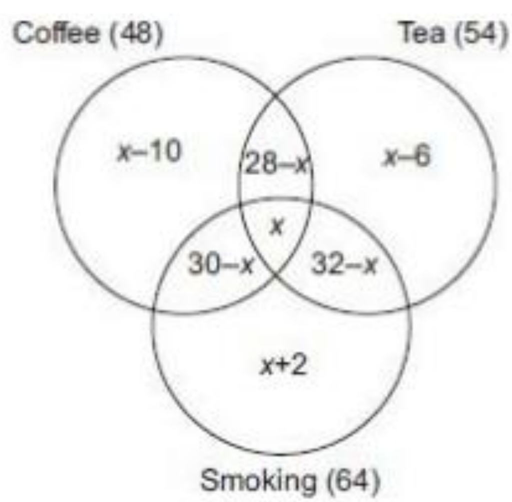
6. 15% (from the figure). Option (a) is correct.

7. 10+10+15+15=50%. Option (c) is correct.

8. Only ice cream is 10% of the total. Hence, 10% of 180 =18. Option (b) is correct.

Solutions for Questions 9 to 15:

If you try to draw a figure for this question, the figure would be something like:



We can then solve this as:

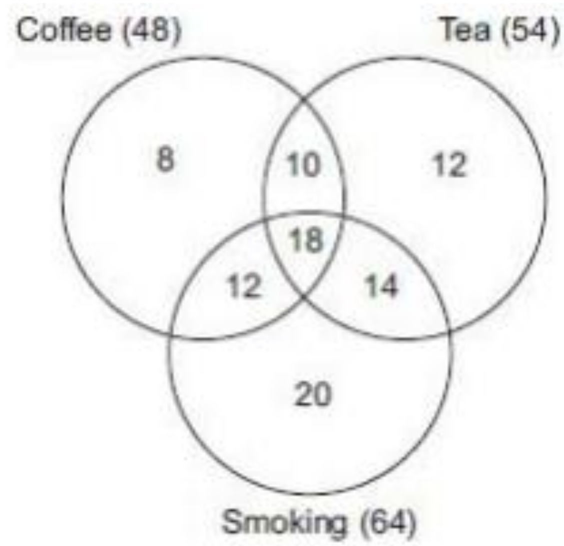
$x - 10 + 28 - x + x + 30 - x + x + 2 + 32 - x + x - 6 = 94 \rightarrow x + 76 = 94 \rightarrow x = 18.$

(**Note:** In this question, since all the values for the use of the set theory formula are given, we can find the missing value of students who liked all three as follows:

94 = 48 + 54 + 64 – 28 – 32 – 30 + All three → All three = 18

As you can see, this is a much more convenient way of solving this question, and the learning you take away for the three circle situation is that whenever you have all the values known and the only unknown value is the centre value – it is wiser and more efficient to solve for the unknown using the formula rather than trying to solve through a Venn diagram.)

Based on this value of *x*, we get the diagram completed as:



The answers then are:

9. 8:12 = 2:3 → Option (c) is correct.

10. 12 % of 2000 = 240. Option (b) is correct.

11. 30/94 → more than 30%. Option (a) is correct.

12. 94%. Option (d) is correct.

13. Option (c) is correct as the ratio turns out to be 10:20 in that case.

14. 12:12 = 1:1 → Option (b) is correct.

15. 14%. Option (a) is correct.

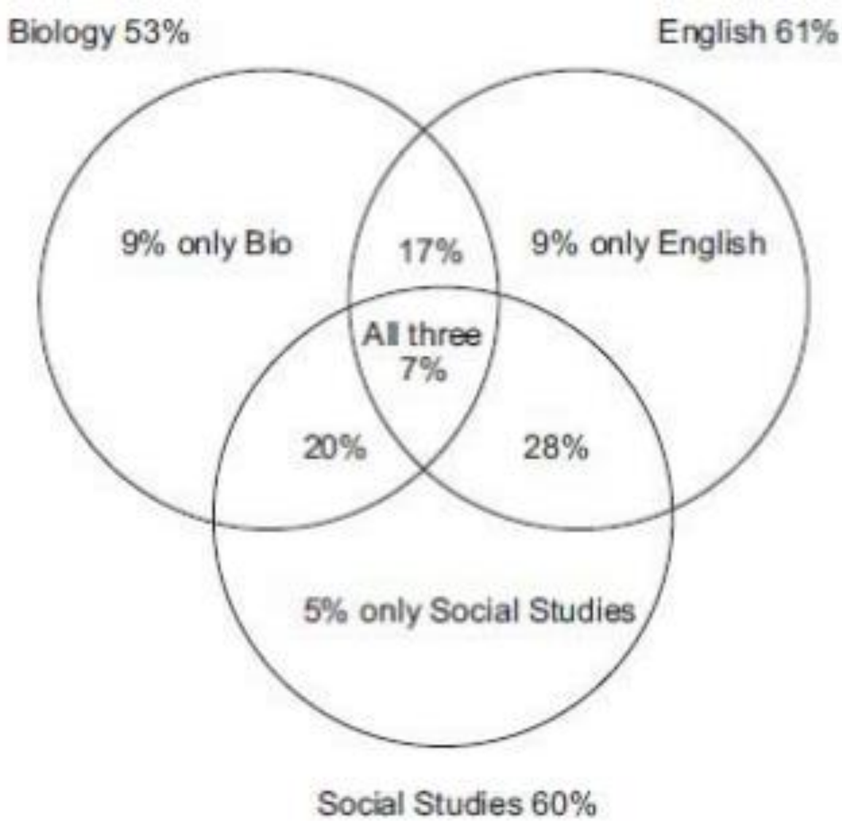
16. $30 = 25 + 20 - x \rightarrow x = 15$. Option (b) is correct.

Solutions for Questions 17 to 20:

Let people who passed all three be x. Then:

$53 + 61 + 60 - 24 - 35 - 27 + x = 95 \rightarrow x = 7$.

The Venn diagram in this case would become:



17. Option (c) is correct.

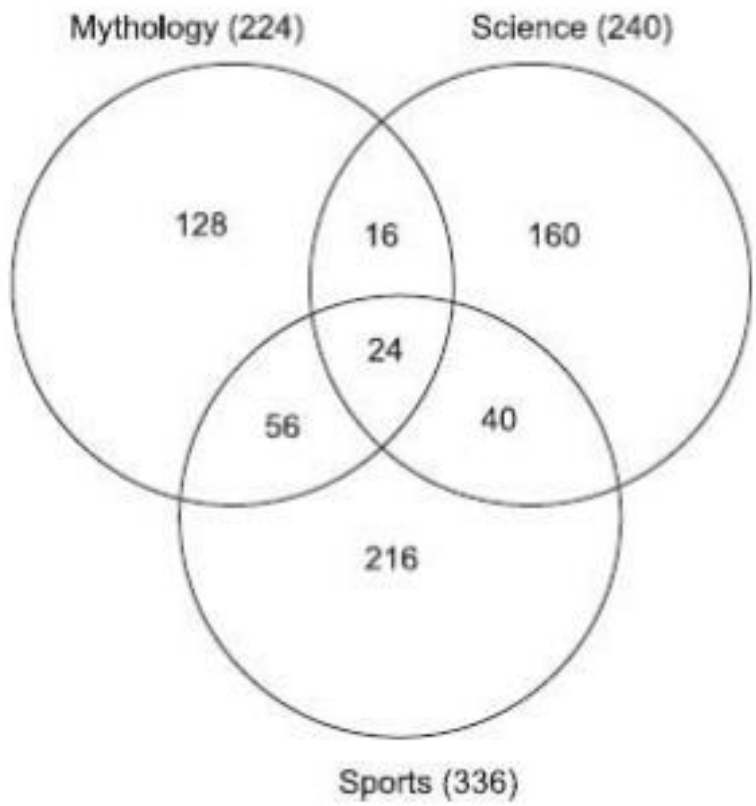
18. 23% of 200 = 46. Option (b) is correct.

19. If the number of students is increased by 50%, the number of students in each category would also be increased by 50%. Option (c) is correct.

20. 20:28 = 5:7. Option (a) is correct.

Solutions for Questions 21 to 25:

The following figure would emerge on using all the information in the question:

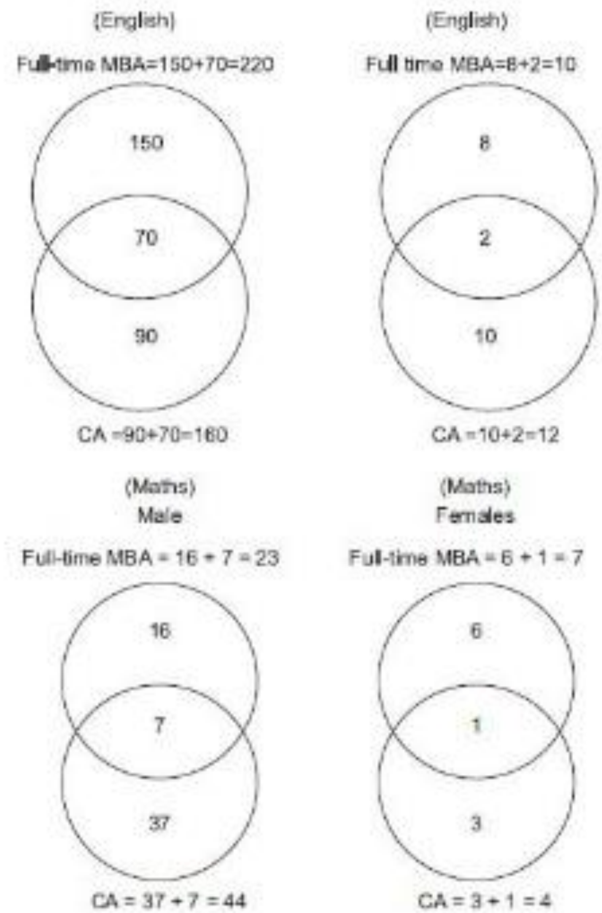


The answers would then be:

21. 240/880 = 27.27%. Option (d) is correct.
22. 504/880 = 57.27%. Hence, less than 60. Option (c) is correct.
23. 40 + 16 + 56 + 24 = 136. Option (c) is correct.
24. Option (a) gives us 16:128 = 1:8. Option (a) is hence correct.
25. 40:160 → 1:4. Option (b) is correct.

Solutions for Questions 26 to 30:

The following Venn diagrams would emerge:



26. Maths Students = 130. English Students =370 130/370 = 35.13%. Option (d) is correct.

27. Number of female students = 10 + 8 + 10 + 2 + 10 + 6 + 3 + 1 = 50. Average number of females per course = 50/3 = 16.66. Option (b) is correct.

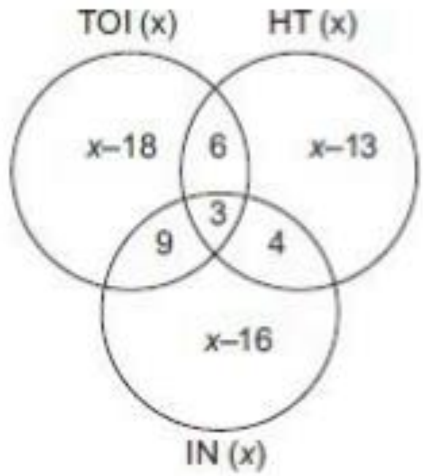
28. 50:450 = 1:9. Option (b) is correct.

29. 40/140 → 28.57%. Option (c) is correct.

30. From the figures, this value would be 150 + 8 + 90 + 10 + 16 + 6 + 37 + 3 = 320. Option (a) is correct.

Solutions for Questions 31 to 34:

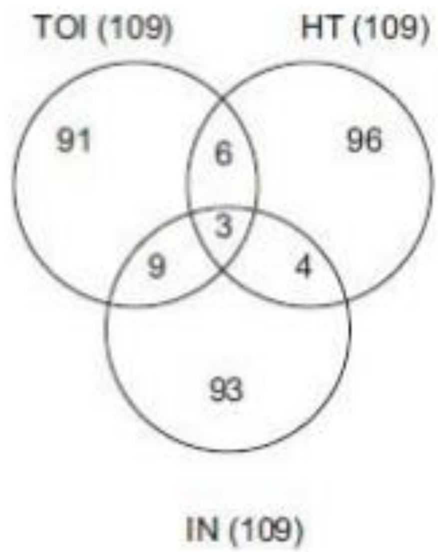
The following figure would emerge-



Based on this figure, we have:

$x + x - 13 + 4 + x - 16 = 302 \rightarrow 3x - 25 = 302 \rightarrow x = 109.$

Consequently, the figure becomes:



The answers are:

31. $91 + 93 + 96 = 280$. Option (a) is correct.

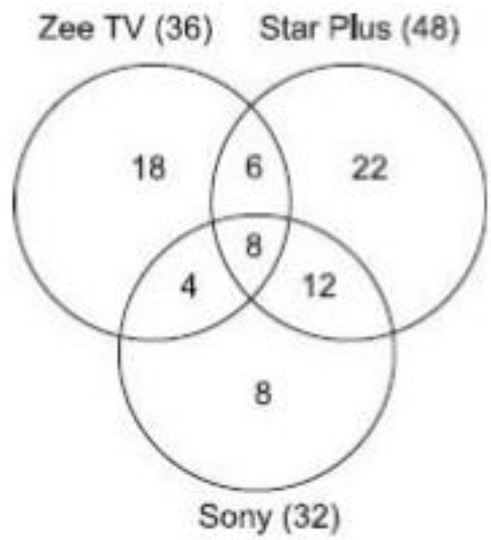
32. 193/302 @ 64%

33. 6:9:4 is the required ratio. Option (b) is correct.

34. $96 - 4 = 92$. Option (d) is correct.

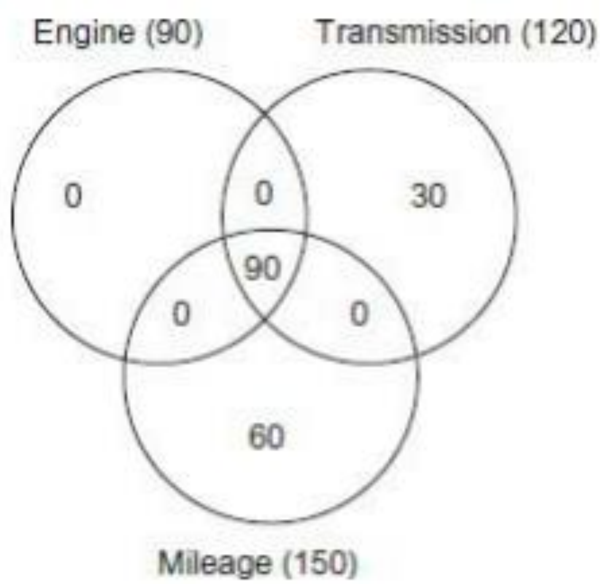
35. $78 = 36 + 48 + 32 - 14 - 20 - 12 + x \rightarrow x = 8.$

The figure for this question would become:



Required ratio is 18:8 → 9:4. Option (a) is correct.

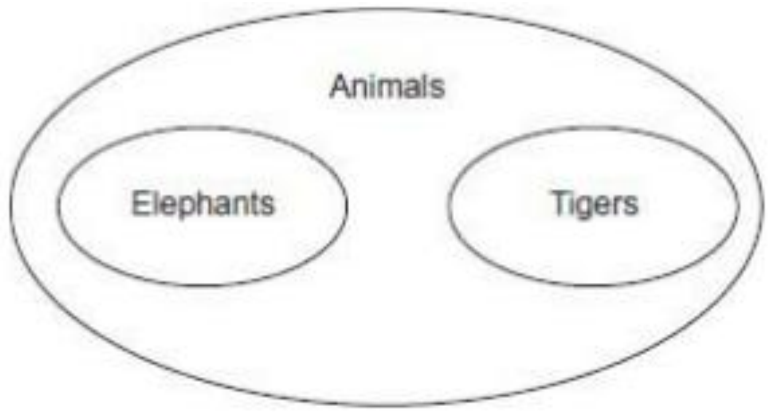
36. Option (c)



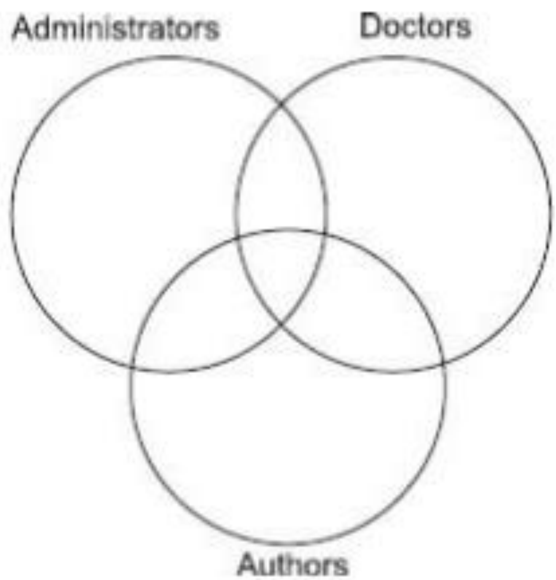
37. There are thirty such people. Option (b) is correct.

Solutions for Questions 38 to 42:

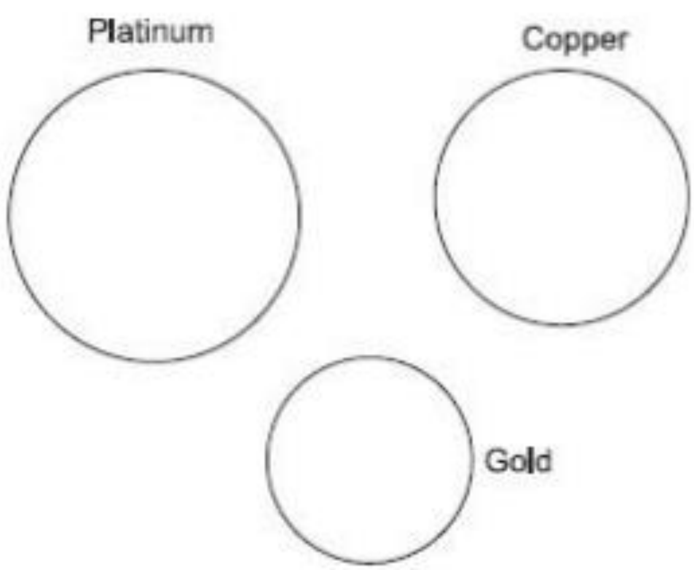
38. Option (b) is correct



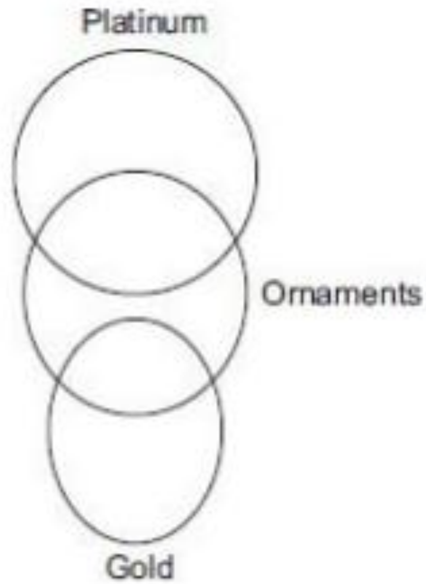
39. Option (e) is correct.



40. Option (d) is correct.



41. Option (a) is correct.



42. Option (b) is correct

Solutions for Question 43:

43. The least percentage of people with all four gadgets would happen if all the employees who are not having any one of the four objects is mutually exclusive.

    Thus, 100 – 30 – 25 – 20 – 15 = 10

    Option (c) is correct.

Level of Difficulty (II)

1. The key to think about this question is to understand what is meant by the statement —"for every student in the school who opts for practical training in at least *M* sciences, there are exactly three students who opt for practical training in at least (*M* – 1) sciences, for *M* = 2, 3 and 4".

   What this statement means is that if there are *x* students who opt for practical training in all four sciences, there would be 3x students who would opt for practical training in at least three sciences. Since opting for at least 3 sciences includes those who opted for exactly 3 sciences and those who opted for exactly 4 sciences—we can conclude from this that: The number of students who opted for exactly 3 sciences = Number of students who opted for at least 3 sciences – Number of students who opted for all 4 sciences = $3x - x = 2x$

Thus, the number of students who opted for various number of Science practicals can be summarised as below:

	Number of students who opted for at least n subjects	*Number of students who opted for exactly n subjects*
$n = 4$	x	x
$n = 3$	$3x$	$2x$
$n = 2$	$9x$	$6x$
$n = 1$	$27x$	$18x$

Also, number of students who opt for none of the sciences = twice the number of students who opt for exactly 4 sciences = $2x$.

Based on these deductions, we can clearly identify that the number of students in the school would be: $x + 2x + 6x + 18x + 2x = 29x = 870 \rightarrow x = 30$.

Hence, number of students who opted for exactly three sciences = $2x = 60$

2. (b) In order to estimate the minimum number of customers we need to assume that each customer, must have bought the maximum number of pastries possible for him to purchase.

Since, the maximum number of pastries an individual could purchase is constrained by the information that no one bought more than two pastries of any one kind—this would occur under the following situation —First 45 people would buy two pastries of all three kinds, which would completely exhaust the 90 pineapple pastries and leave the bakery with 30 chocolate and 60 black forest pastries. The next 15 people would buy two pastries each of the available kinds and after this; we would be left with 30 black forest pastries. Fifteen people would buy these pastries, each person buying two pastries each.

Thus, the total number of people (minimum) would be: 45 + 15 + 15 = 75.

3. (c) Let the number of people who participated in 0, 1, 2 and 3 games is A, B, C, D respectively. Then from the information, we have:

$C + D = 1.52 \times B$ (Number of people who participate in at least two games is 52% higher than the number of people who participate in exactly one game)

$A + B + C + D = 510$ (Number of people invited to participate in the games is 510)

This gives us: $A + 2.52B = 510 \rightarrow B = \frac{25}{63}(510 - A)$

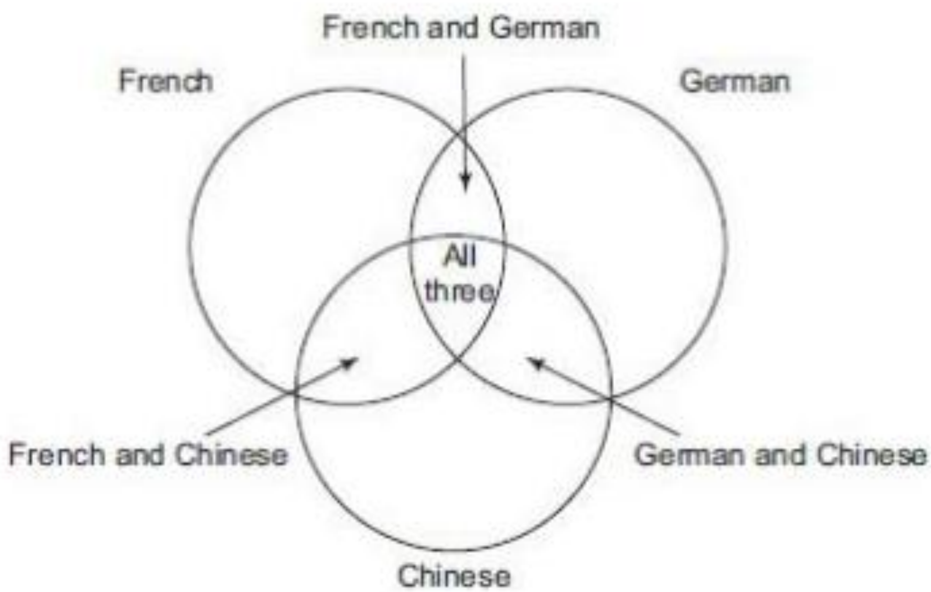
For A to be minimum, $510 - A$ should give us the largest multiple of 63. Since, $63 \times 8 = 504$, we have $A = 6$.

Also, $2.52B = 504$, so $B = 200$ and $C + D = 1.52B = 304$.

For number of people participating in exactly three games to be maximum, the number of people participating in exactly two games has to be minimised and made equal to 1. Thus, the required answer = 304 – 1 = 303.

4. (c) In order to think about this question, the best way is to use the process of slack thinking. In this question, we have 180 students counted 270 times. This means that there is an extra count of 90 students. In a three circle Venn diagram, extra counting can occur only due to exactly two regions (where 1 individual student would be counted in two subjects leading to an extra count of 1) and the exactly three region (where 1 individual student would be counted in 3 subjects leading to an extra count of 2).

This can be visualised in the figure below:



A student placed in all the three areas will be counted three times when you count the number of students studying French, the number of students studying German and the number of students studying Chinese independently. Hence, he/she would be counted three times—leading to an extra count of 2 for each individual places here.

A person placed in any of the three 'Exactly two' areas would be counted two times when we count the number of students studying French, the number of students studying German and the number of students studying Chinese independently. Hence, he/she would be counted two times—leading to an extra count of 1 for each individual placed in any of these three areas.

The thought-chain leading to the solution would go as follows:

(i) 180 students are counted 80 + 90 + 100 = 270 times

(ii) This means that there is an extra count of 90 students

(iii) Extra counts can fundamentally occur only from the 'exactly two' areas or all the three areas in the figure

(iv) We also know that 'The number of students who study more than one of the three subjects is 50% more than the number of students who study all the three subjects', hence, we know that if there are a total of 'n' students studying all three subjects, there would be $1.5n$ students studying more than one subject. This in turn means that there must be $0.5n$ students who study two subjects.

(Since, number of students studying more than one subject = number of students studying two subjects + number of students studying three subjects.
i.e. $1.5n = n$ + number of students studying 2 subjects → number of students studying 2 subjects = $1.5n - n = 0.5n$)

(v) The extra counts from the n students studying three subjects would amount to $n \times 2 = 2n$ – since each student is counted twice extra when he/she studies all three subjects.

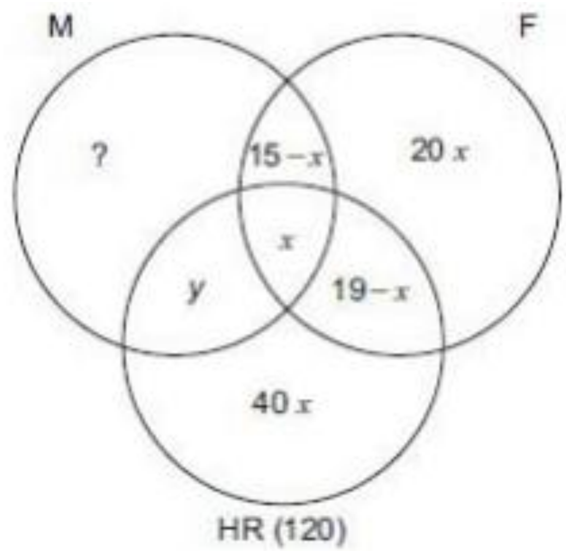
(vi) The extra counts from the $0.5n$ students who study exactly two subjects would be equal to $0.5n \times 1 = 0.5\,n$.

(vii) Thus, extra count = $90 = 2n + 0.5n \rightarrow n = 90/2.5 = 36$.

(viii) Hence, there must be 36 people studying all three subjects.

Solutions for Questions 5 and 6:

The following would be the starting Venn diagram encapsulating the basic information:

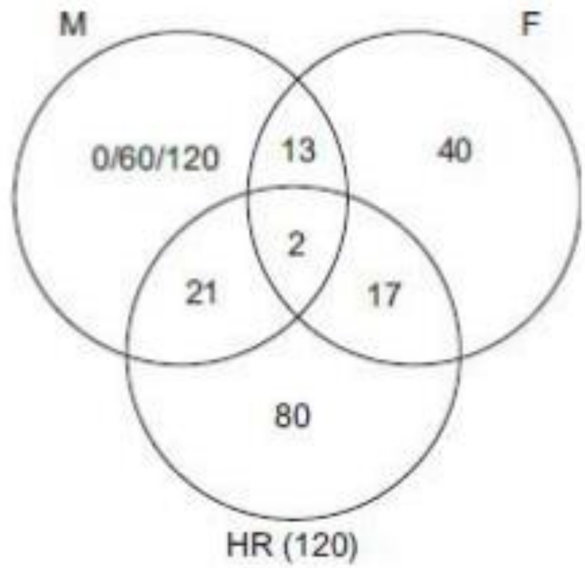


From this figure, we get the following equation:

$40x + (19 - x) + x + y = 120$. This gives us $40x + y = 101$

Thinking about this equation, we can see that the value of x can be either 1 or 2. In case we put x as 1, we get $y = 61$ and then we have to also meet the additional condition that $15-x$, $19-x$ and y should form an AP which is obviously not possible (since it is not possible practically to build an AP having two positive terms below 19 and the third term being 61. Hence, this option is rejected).

Moving forward, the other possible value of x from the equation is $x = 2$ in which case we get, $y = 21$ and $15 - x = 13$ and $19 - x = 17$. Thus, we get the AP 13, 17, 21 which satisfies the given conditions. Putting $x = 2$ and $y = 21$ in the figure, the Venn diagram evolves to:



In this figure, the value that only Marketing takes can either be 0, 60 or 120 (to satisfy the AP condition). However, since the total number of students in Marketing is a two-digit number above 50; the number of people studying only marketing would be narrowed down to the only possibility which remains – viz. 60.

Thus, the number of students studying in the batch = 120 + 40 + 60 + 13 + 4 = 237.

The number of students specialising in both Marketing and HR is 21 + 2 = 23.

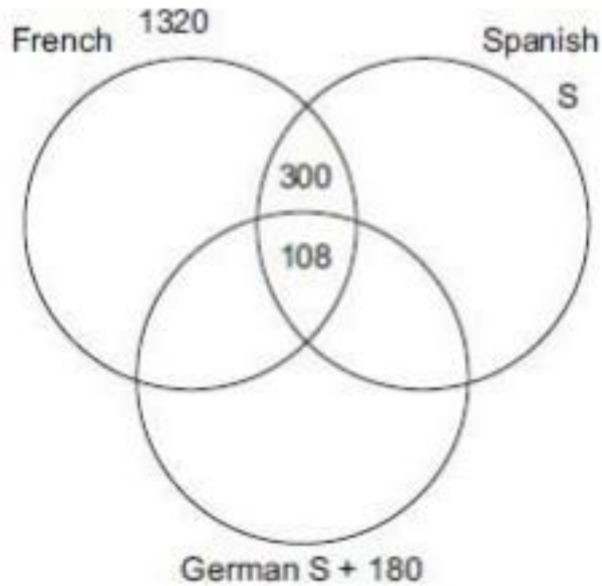
5. (d) The total number of students is 237.

6. (c) The number of students studying both Marketing and HR is 23.

Solutions for Questions 7 to 9:

7 and 8: In order to think about the possibility of the maximum and/or the minimum number of people who could be studying none of the three languages, you need to first think of the basic information in the question. The basic informa-

tion in the question can be encapsulated by the following Venn diagram:



At this point, we have the flexibility to try to put the remaining numbers into this Venn diagram while maintaining the constraints the question has placed on the relative numbers in the figure. In order to do this, we need to think of the objective with which we have to fill in the remaining numbers in the figure. At this stage, you have to keep two constraints in mind while filling the remaining numbers:

(a) The remaining part of the French circle has to total 1320 – 408 = 912;

(b) The German circle has to be 180 more than the Spanish circle.

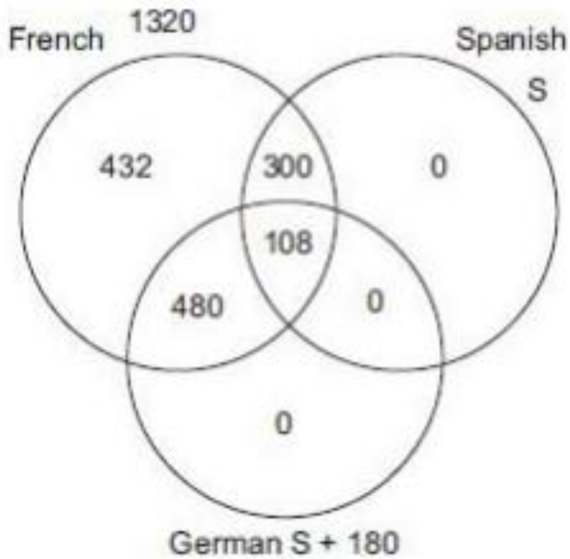
When we try to fill in the figure for making the number of students who did not study any of the three subjects maximum:

You can think of first filling the French circle by trying to think of how you would want to distribute the remaining 912 in that circle. When we want to maximise the number of students who study none of the three, we would need to use the minimum number of people inside the three circles—while making sure that all the constraints are met.

Since we have to forcefully fit in 912 into the remaining areas of the

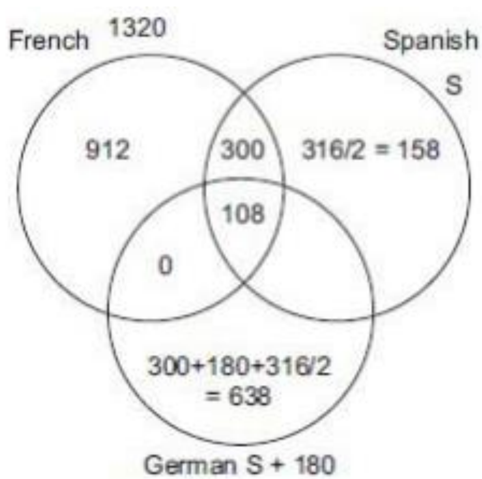
French circle, we need to see whether while doing the same, we can also meet the second constraint.

This thinking would lead you to see the following solution possibility:



In this case, we have ensured that the German total is 180 more than the Spanish total (as required) and at the same time the French circle has also reached the desired 1320. Hence, the number of students who study none of the three can be 2116–1320 = 796 (at maximum).

When minimising the number of students who have studied none of the three subjects, the objective would be to use the maximum number of students who can be used in order to meet the basic constraints. The answer in this case can be taken to as low as zero in the following case:



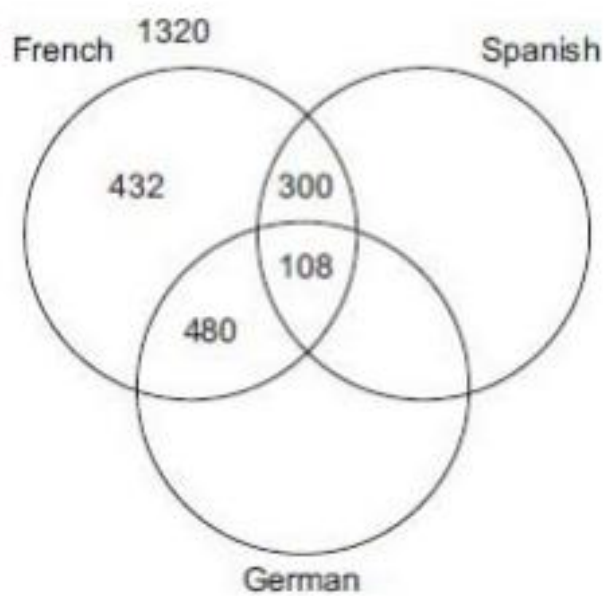
(**Note:** While thinking about the numbers in this case, we first use the 912 in the 'only French' area. At this point, we have 796 students left to be allocated. We first make the German circle 180 more than the Spanish circle (by taking the only German as 300 +180 to start with; this is accomplished). At this point, we are left with 316 more students, who can be allocated equally as 316 ÷ 2 for both the 'only German' and the 'only Spanish' areas.)

Thus, the minimum number of students who study none of the three is 0.

Solution for Question 9:

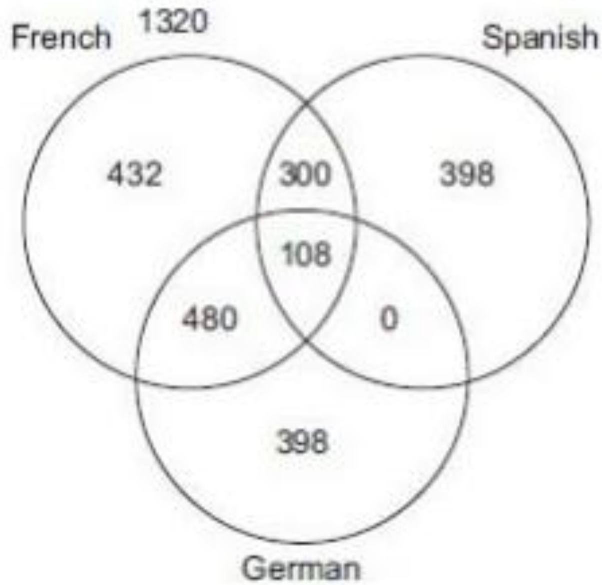
In order to think about this question, let us first see the situation we had in order to maintain all constraints.

If we try to fit in the remaining constraints in this situation, we would get:

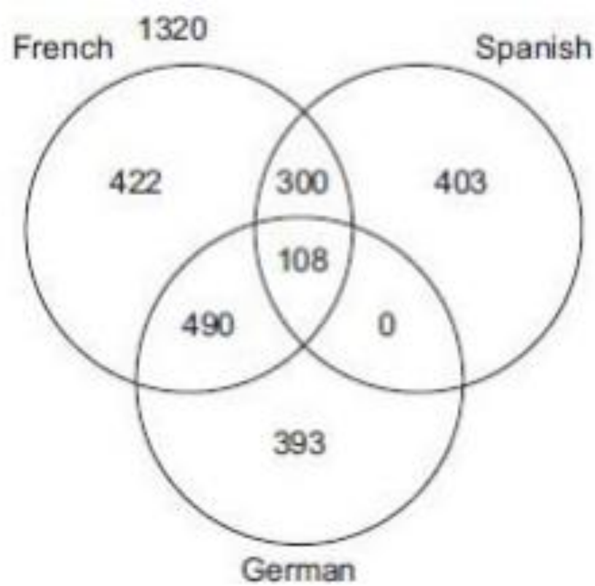


This leaves us with a slack of 796 people which would need to be divided equally since we cannot disturb the equilibrium of German being exactly 180 more than Spanish.

This gives us the following figure:



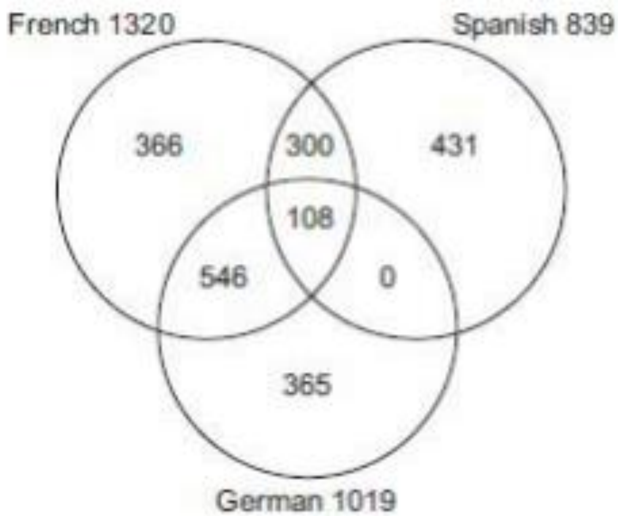
When you think about this situation, you realise that it is quite possible to increase Spanish if we reduce the only French area and reallocate the reduction into the 'only Spanish' and German area. A reduction of 10 from the 'only French' area can be visualised as follows:



In this case, as you can see from the figure above, the number of students who study only Spanish has gone up by 5 (which is half of 10).

Since, there is still some gap between the 'only German' and the 'only French' areas in the figure; we should close that gap by reducing the 'only French' area as much as possible.

The following solution figure would emerge when we think that way:



Hence, the maximum possible for the only Spanish area is 431.

Solutions for Questions 10 to 13:

The information given in the question can be encapsulated in the following way:

Game	Only that game	2 games combination 1	2 games combination 2	2 games combination 3	3 games combination 1	3 games combination 2	3 games combination 3	All 4 games
Tennis (460)	220	40	40	40	20	20	20	60
TT (360)	120	40	40	40	20	20	20	60
Squash (360)	120	40	40	40	20	20	20	60
Badminton (440)	200	40	40	40	20	20	20	60

From the above table, we can draw the following conclusions,, which can then be used to answer the questions asked.

The total number of athletes who play at least one of the four games =

$220 + 120 + 120 + 200 + 40\times6 + 20\times4 + 60 = 1040$.

(**Note:** In doing this calculation, we have used 40×6 for calculating how many unique people would be playing exactly two games—where 40 for each combination is given and there are ${}_4C_2 = 6$ combinations of exactly two sports that exist. Similar logic applies to the 20×4 calculation for number of athletes playing exactly three sports.)

Also, since we know that the number of athletes who participate in none of these four games is 20% of the total number of athletes, we can calculate the total number of athletes who practise in the stadium as 5 × 1040 ÷ 4 = 1300.

Thus, the questions can be answered as follows:

10. The number of athletes in the stadium = 1300.

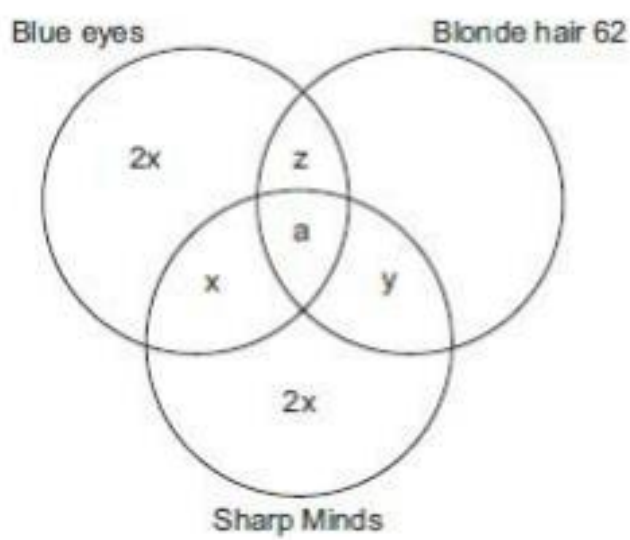
11. Only squash + only tennis = 120 + 220 = 340 (from the table)

12. Only athletics means none of the four games = total number of athletes – number of athletes who play at least one game = 1300 – 1040 = 260.

13. In case, all the three game athletes would add one more game they would become four game athletes. Hence, the number of athletes who play all four games would be: Athletes playing 3 games earlier + athletes playing all 4 games earlier = 80 + 60 = 140.

Solutions for Questions 14 and 15:

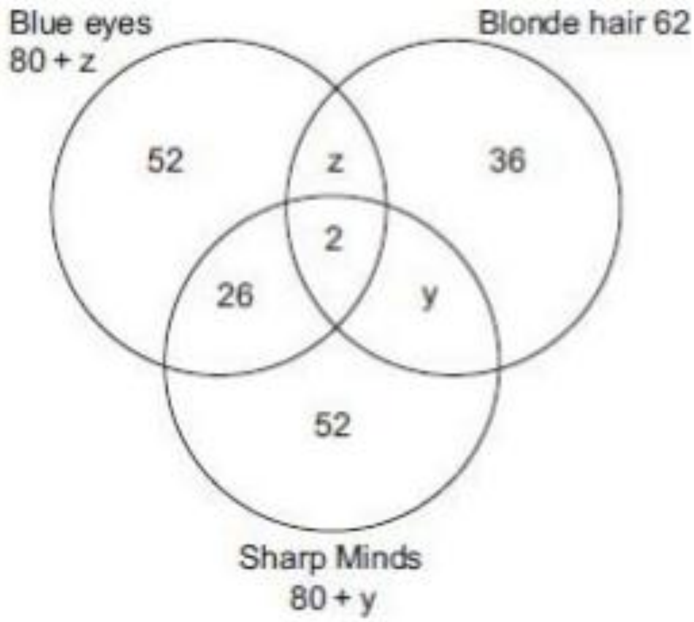
The starting figure based on the information given in the question would look something as below:



From this figure, we see a few equations:

$x + y + z = 50; a + y + z = 26 \rightarrow x - 24 = a$

Also, since, $5x + 62 = 192$, we get the value of x as 26. The figure would evolve as follows.



Based on this, we can deduce the answer to the two questions as:

14. For the number of family members with blue eyes to be maximum, the family members with both sharp minds and blonde hair, but not blue eyes (represented by 'y' in the figure), would be at maximum 11 because we would need to keep $z > y$. Hence, option (a) is the correct answer.
15. If we are given the information in option (c), we know the value of y would be 9 and hence, the value of z would be determined as 15. Hence, option (c) provides us the information to determine the exact number of family members who have blonde hair and blue eyes but not sharp minds. Notice here that the information in each of the other options is already known to us.

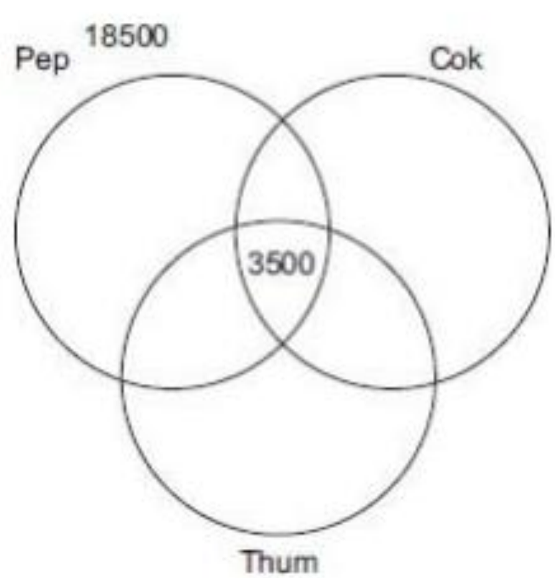
16. Solve this again using slack thinking by using the following thought process:

97 students are counted 47 + 53 + 72 = 172 times— which means that there is an extra count of 75 students (172 – 97 = 75). Now, since there are 15 students who are playing all the three games, they would be counted 45 times—hence, they take care of an extra count of 15 × 2 = 30. (**Note:** in a three circle Venn diagram situation, any person placed in the all three areas is counted thrice—hence he/she is counted two extra times).

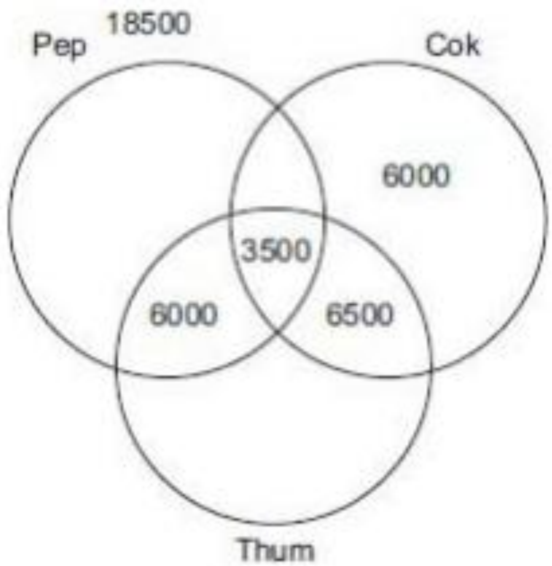
This leaves us with an extra count of 45 to be managed—and the only way to do so is to place people in the exactly two areas. A person placed in the 'exactly two games area' would be counted once extra. Hence, with each student who goes into the 'exactly two games' areas it would be counted once extra. Thus, to manage an extra count of 45, we need to put 45 people in the 'exactly two' area. Hence, option (d) is correct.

Solutions for Questions 17 to 19:

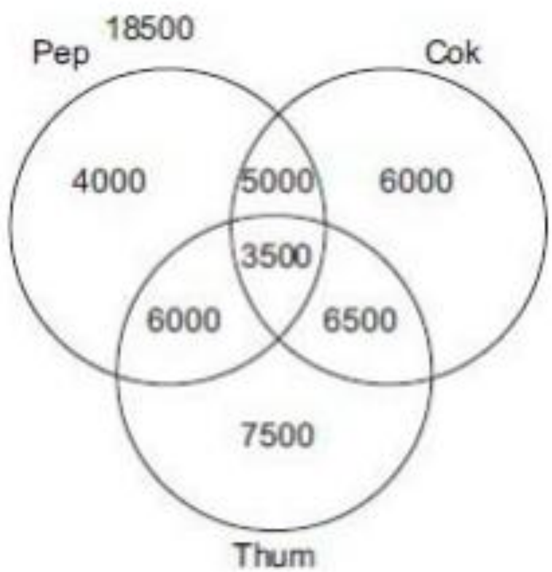
Once you fill in the basic information into the Venn diagram, you reach the following position:



At this point, we know that since the 'all three area' is 3500, the value of the 'exactly two areas' would be 5 × 3500 = 17,500. Also, we know that "11,000 like Pep and exactly one more cold drink" which means that the area for Cok and Thum but not Pep is equal to 17,500 – 11000 = 6500. Further, when you start adding the information: "6000 like only Cok and the same number of people like Pep and Thum but not Cok,", the Venn Diagram transforms to the following:



Filling in the remaining gaps in the picture, we get:



(**Note:** we have used the following info here:

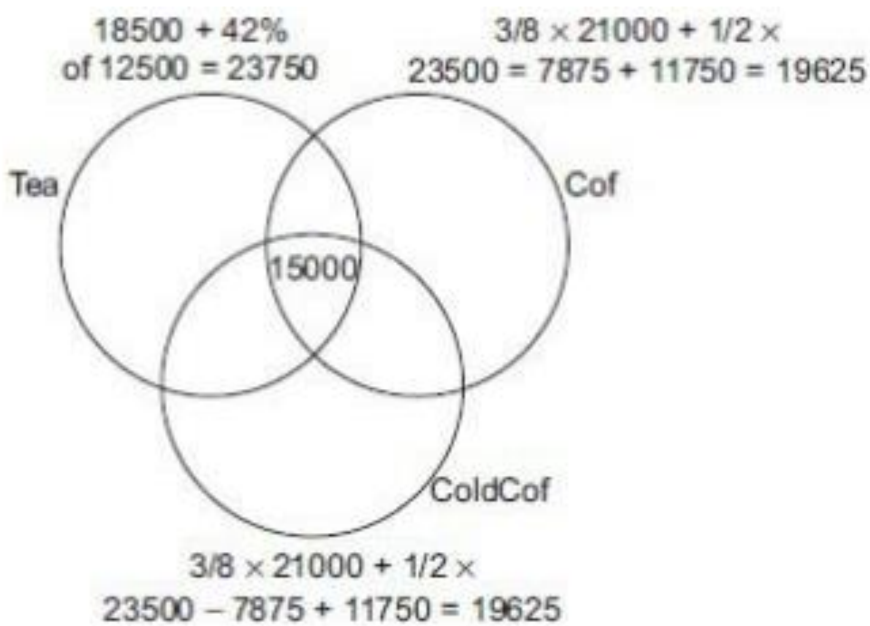
Thum but not Pep is 14,000 and since we already know that Thum and Cok but not Pep is 6500, the value of 'only Thum' would be 14,000 – 6500 = 7500.

We also know that the 'exactly two' areas add up to 17500 and we know that two of these three areas are 6500 and 6000 respectively. Thus, Pep and Cok but not Thum is 17500 – 6000 – 6500 = 5000.

Finally, the 'only Pep' area would be 18500 – 5000 – 6000 – 3500 = 4000.

Once we have created the Venn diagram for the cold drinks, we can focus our attention to the Venn diagram for the beverages.

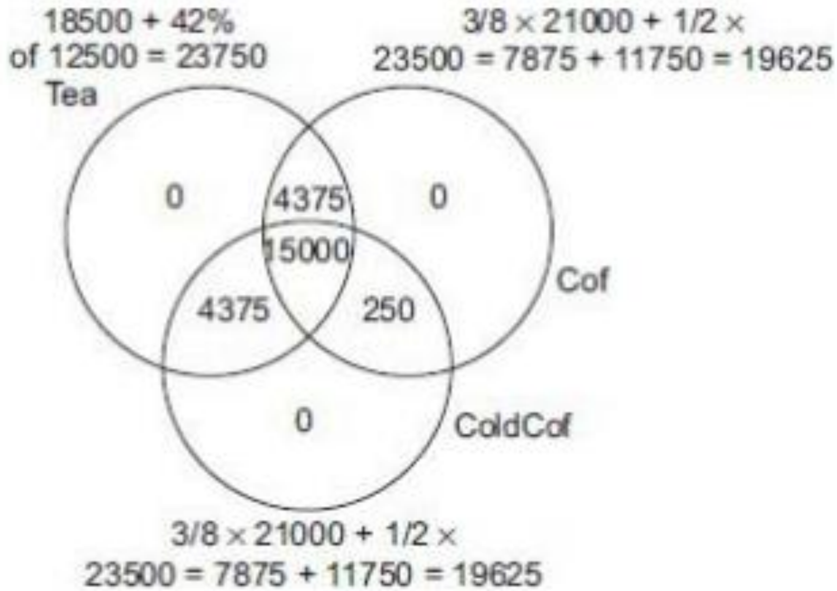
Based on the information provided, the following diagram can be created.)



Based on these figures, the questions asked can be solved as follows:

17. Option (a) is correct as the number of people who like at least one of the cold drinks is the sum of 18500 + 6000 + 6500 + 7500 = 38500.

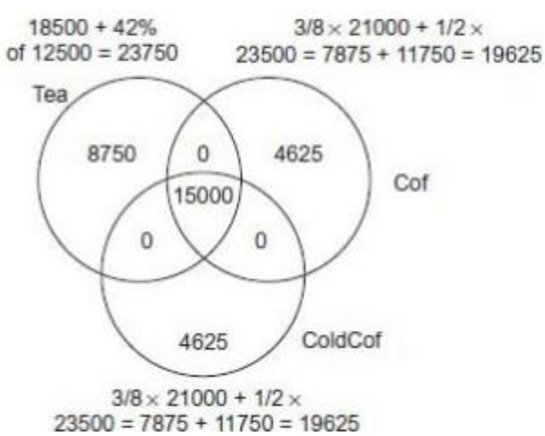
18. For the number of people who do not like any of the beverages to be maximum, we have to ensure that the number of people used in order to meet the situation described by the beverage's Venn diagram should be minimum. This can be done by filling values in the inner areas of this Venn diagram:



In this situation, the number of people used inside the Venn diagram to match upto all the values for this figure = 15000 + 4375 + 4375 + 250 = 24000.

Naturally, in this case the number of people who do not like any of the beverages is maximised at 40000 – 24000 = 16000. Option (b) is correct.

19. The solution for this situation would be given by the following figure:



The number of people who like at least one of the three beverages is:

15000 + 8750 +4625 + 4625 = 33000. Hence, option (c) is correct.

20. The number of people cannot be a fraction in any situation. We can deduce that the values of x and $3x$ have to be factors of 57. This gives us that the values of x can only be either 1 or 19 (for both x and $3x$ to be a factor of 57).

So, the number of people who drink coffee is equal to $2x + 57/x$ which can be 59 (if $x = 1$) or 41 (if $x = 19$).

Hence, Option (d) is correct.

TRAINING GROUND FOR BLOCK VI

HOW TO THINK IN PROBLEMS ON BLOCK VI?

1. The probability that a randomly chosen positive divisor of 10^{29} is an integer multiple of 10^{23} is: a^2/b^2, then '$b - a$' would be:

(a) 8

(b) 15

(c) 21

(d) 23

(e) 45

Solution: The number $10^{29} = 2^{29} \times 5^{29}$.

Factors or divisors of such a number would be of the form: $2^a \times 5^b$ where the values of a and b can be represented as $0 \leq a, b \leq 29$, i.e. there are $30 \times 30 = 900$ possibilities when we talk about randomly selecting a positive divisor of 1029.

Next, we need to think of numbers which are integral multiples of 1023. Such numbers would be of the form $2^x \times 5^y$ such that $x, y \geq 23$.

Hence, the number of values possible when the chosen divisor would also be an integer multiple of 10^{23} would be when $23 \leq x, y \leq 29$. There would be $7 \times 7 = 49$ such combinations.

Thus, the required probability is $49 \div 900$. In the context of $a^2 \div b^2$, the values of a and b would come out as 7 and 30, respectively. The required difference between a and b is 23. Hence, option (d) is correct.

2. Aditya has a total of 18 red and blue marbles in two bags (each bag has marbles of both colours). A marble is randomly drawn from the first bag followed by another randomly drawn from the second bag; the probability of both being red is 5/16. What is the probability of both marbles being blue?

(a) 1/16

(b) 2/16

(c) 3/16

(d) 4/16

(e) None of these

Solution: The problem most students face in such situations is to understand how to place how many balls of each colour in each bag. Since there is no directive given in the question that tells us how many balls are there and/or how many balls are placed in any bag the next thing that a mathematically oriented mind would do will be to try to assume some variables to represent the number of balls in each bag. However, if you try to do so, on your own, you would realise that it would be the wrong way to solve this question as it would lead to extreme complexity while solving the problem. So how can we think alternately? Is there a smarter way to think about this question?

Yes! indeed there is. Let me explain it to you here. In order to think about this problem, you would need to first think about how a fraction like 5/16 would emerge. The value of 5/16 = 10/32 = 15/48 = 20/64 = 25/80 = 30/96 and so on. Next, you need to understand that there are a total of 18 balls and this 18 has to be broken into two parts such that their product is one of the above denominators. Scanning the denominators, we see the opportunity that the number 80 = 10 × 8 and hence, we realise that the probability of both balls being red would happen in a situation where the structure of the calculation would look something like: $(r_1/10) \times (r_2/8)$. Next, to get 25 as the corresponding numerator with 80 as the denominator, the values of r_1 and r_2 should both be 5. This means that there are five red balls out of ten in the first bag and five red balls out of eight in the second bag. This further means that the number of blue balls would be five out of ten and three out of eight. Thus, the correct answer would be: (5/10) × (3/8) = 15/80 = 3/16. Hence, option (c) is the correct answer.

3. The scheduling officer for a local police department is trying to schedule additional patrol units in each of two neighbourhoods – southern and northern. She knows that on any given day, the probabilities of major crimes and minor crimes being committed in the northern neighbourhood were 0.418 and 0.612, respectively, and that the corresponding probabilities in the southern neighbourhood were 0.355 and 0.520. Assuming that all the crimes occur independent of each other and likewise that crime in the two neighbourhoods are independent of each other, what is the probability that no crime of either type is committed in either neighbourhood on any given day?

(a) 0.069

(b) 0.225

(c) 0.690

(d) 0.775

(e) None of these

Solution: The key to solving this correctly is to look at the event definition. A major crime not occurring in the northern neighbourhood is the non-event for a major crime occurring in the northern neighbourhood on any given day. Its probability would be (1–0.418) = 0.582.

The values of minor crime not occurring in the northern neighbourhood and a major crime not occurring in the southern neighbourhood and a minor crime not occurring in the northern neighbourhood would be (1–0.612); (1–0.355) and (1–0.520), respectively. The value of the required probability would be the probability of the event:

Major crime does not occur in the northern neighbourhood and minor crime does not occur in the northern neighbourhood and major crime does not occur in the southern neighbourhood and minor crime does not occur in the northern neighbourhood =

$(1-0.418) \times (1-0.612) \times (1-0.355) \times (1-0.520)$. Option (a) is the closest answer.

4. There are four machines in a factory. At exactly 8 PM, when the mechanic is about to leave the factory, he is informed that two of the four machines are not working properly. The mechanic is in a hurry, and decides that he will identify the two faulty machines before going home, and repair them next morning. It takes him twenty minutes to walk to the bus stop.

The last bus leaves at 8:32 PM. If it takes six minutes to identify whether a machine is defective or not, and if he decides to check the machines at random, what is the probability that the mechanic will be able to catch the last bus?

(a) 0

(b) 1/6

(c) 1/4

(d) 1/3

(e) 1

Solution: The first thing you look for in this question, is that obviously the mechanic has only 12 minutes to check the machines before he leaves to catch the bus. In twelve minutes, he can at best check two machines. He will be able to identify the two faulty machines under the following cases:

(The first machine checked is faulty AND the second machine checked is also faulty) OR (The first machine checked is working fine AND the second machine checked is also working fine)

Required probability = $\frac{2}{4}\times\frac{1}{3}+\frac{2}{4}\times\frac{1}{3}=\frac{1}{3}$

5. Little Pika who is five-and-half years old has just learnt addition. However, he does not know how to carry. For example, he can add 14 and 5, but he does not know how to add 14 and 7. How many pairs of consecutive integers between 1000 and 2000 (both 1000 and 2000 included) can Little Pika add?

(a) 150

(b) 155

(c) 156

(d) 258

(e) None of these

Solution: If you try to observe the situations under which the addition of two consecutive four-digit numbers between 1000 and 2000 would come through without having a carryover value in the answer you would be able to identify the following situations – each of which differs from the other due to the way it is structured with respect to the values of the individual digits.

Category 1: 1000 + 1001; 1004 +1005, 1104 + 1105 and so on. A little bit of introspection should show you that in this case, the two numbers are 1abc and 1abd where $d = c + 1$. Also, for the sum to come out without any carry-overs, the values of a, b and c should be between 0 and 4 (including both). Thus, each of a, b and c gives us five values each – giving us a total of $5 \times 5 \times 5 = 125$ such situations.

Category 2: 1009 + 1010; 1019 + 1020; 1029 + 1030; 1409 + 1410. The general form of the first number here would be 1ab 9 with the values of a and b being between 0 and 4 (including both). Thus, each of a, b gives us 5 values each – giving us a total of $5 \times 5 = 25$ such situations.

Category 3: 1099 + 1100; 1199 + 1200; 1299 + 1300; 1399 + 1400 and 1499 + 1500. There are only 5 such pairs. Note that 1599 + 1600 would not work in this case as the addition of the hundreds' digit would become more than 10 and lead to a carry-over.

Category 4: 1999 + 2000 is the only other situation where the addition would not lead to a carry-over calculation. Hence, **1 more situation is possible.**

The required answer = 125 + 25 + 5 + 1 = 156. Option (c) is the correct answer.

6. In the country of Twenty, there are exactly twenty cities, and there is exactly one direct road between any two cities. No two direct roads have an overlapping road segment. After the election dates are announced,

candidates from their respective cities start visiting the other cities. The following are the rules that the election commission has laid down for the candidates:

- Each candidate must visit each of the other cities exactly once.
- Each candidate must use only the direct roads between two cities for going from one city to another.
- Candidate must return to his own city at the end of the campaign.
- No direct road between two cities would be used by more than one candidate.

The maximum possible number of candidates is

(a) 5

(b) 6

(c) 7

(d) 8

(e) 9

Solution: The key to understanding this question is from two points.

1. Since there is exactly one direct road between any pair of two cities – there would be a toal of $20C_2$ roads = 190 roads.

2. The other key condition in this question is the one which talks about each candidate must visit each of the other cities exactly once and 'No direct road between two cities would be used by more than one candidate.' This means two things. (i) Since each candidate visits each city exactly once, if there are '*c*' candidates, there would be a total of 20c roads used and since no road is repeated, it means that the 20 roads Candidate *A* uses will be different from the 20 roads Candidate *B* uses and so on. Thus, the value

different from the 20 roads Candidate *B* uses and so on. Thus, the value of $20c \leq 190$ should be an inequality that must be satisfied.This gives us a maximum possible value of *c* as 9. Hence, option (e) is correct.

7. In a bank, the account numbers are all eight-digit numbers, and they all start with the digit 2. So, an account number can be represented as $2x_1x_2x_3x_4x_5x_6x_7$. An account number is considered to be a 'magic' number if $x_1x_2x_3$ is exactly the same as $x_4x_5x_6$ or $x_5x_6x_7$ or both, x_i can take values from 0 to 9, but 2 followed by seven 0s is not a valid account number. What is the maximum possible number of customers having a 'magic' account number?

   (a) 9989

   (b) 19980

   (c) 19989

   (d) 19999

   (e) 19990

Solution: In order to solve this question, we need to think of the kinds of numbers which would qualify as magic numbers. Given the definition of a magic number in the question, a number of the form 2*mnpmnpq* would be a magic number
which would qualify as magic numbers. Given the definition of a magic number in the question, a number of the form 2*mnpmnpq* would be a magic number
while at the same time a number of the form 2*mnpqmnp* would also qualify as a magic number. In this situation, each of *m*, *n* and *p* can take any of the ten digit values from 0 to 9. Also, *q* would also have ten different possibilities from 0 to 9. Thus, the total number of numbers of the form 2*mnpmnpq* would be 10^4 =

10000. Similarly, the total number of numbers of the form 2*mnpqmnp* would also be 104= 10000. This gives us a total of 20000 numbers. However, in this count the numbers like 21111111, 22222222, 23333333, 24444444, etc., have been counted under both the categories. Hence, we need to remove these numbers once each (a total of 9 reductions). Also, the number 20000000 is not a valid number according to the question. This number needs to be removed from both the counts.

Hence, the final answer = 20000 – 9 – 2 = 19989.

8. If all letters of the word "*CHCJL*" be arranged in an English dictionary, what will be the 50th word?

   (a) HCCLJ

   (b) LCCHJ

   (c) LCCJH

   (d) JHCLC

   (e) None of these

Solution: In the English dictionary, the ordering of the words would be in alphabetical order. Thus, words starting with *C* would be followed by words starting with *H*, followed by words starting with *J* and finally words starting with *L*. Words starting with *C* = 4! = 24; words starting with *H* = 4! ÷ 2! = 12 words; words starting with *J* = 4! ÷ 2! = 12 words. This gives us a total of 48 words. The 49th and the 50th words would start with *L*. The 49th word would be the first word starting with *L* (=*LCCHJ*) and the 50th word would be the 2nd word starting with L – which would be *LCCJH*. Option (c) is correct.

9. The supervisor of a packaging unit of a milk plant is being pressurised to finish the job closer to the distribution time, thus giving the production staff more leeway to cater to last minute demand. He has the option of running the unit at normal speed or at 110% of normal – "fast speed". He estimates that he will be able to run at the higher speed 60% of time. The packet is twice as likely to be damaged at the higher speed which would mean temporarily stopping the process. If a packet on a randomly selected packaging runs has probability of 0.112 of damage, what is the probability that the packet will not be damaged at normal speed?

(a) 0.81

(b) 0.93

(c) 0.75

(d) 0.60

(e) None of these

Solution: Let the probability of the package being damaged at normal speed be 'p'. This means that the probability of the damage of a package when the unit is running at a fast speed is '$2p$'. Since, he is under pressure to complete the production quickly; we would need to assume that he runs the unit at fast speed for the maximum possible time (60% of the time).

Then, we have

Probability of damaged packet in all packaging runs

$= 0.6 \times 2p + 0.4 \times p = 0.112.$

or $p = 0.07$

Probability of non-damaged packets at normal speed

$= 1 - p = 1 - 0.07 = 0.93$. Option (b) is correct.

10. Let X be a four-digit positive integer such that the unit digit of X is prime and the product of all digits of X is also prime. How many such integers are possible?

(a) 4

(b) 8

(b) 12

(d) 24

(e) None of these

Solution: This one is an easy question as all you need to do is understand that given the unit digit is a prime number, it would mean that the number can only be of the form $abc2$; $abc3$ or $abc5$ or $abc7$. Further, for each of these, the product of the four digits $a \times b \times c \times$ units digit has to be prime. This can occur only if $a = b = c = 1$. Thus, there are only four such numbers viz: 1112, 1113, 1115 and 1117. Hence, option (a) is correct.

11. The chance of India winning a cricket match against Australia is 1/6. What is the minimum number of matches India should play against Australia so that there is a fair chance of winning at least one match?

(a) 3

(b) 4

(c) 5

(d) 6

(e) None of these

Solution: A fair chance is defined when the probability of an event goes to above 0.5. If India plays three matches, the probability of at least one win will be given by the non-event of losing all matches. This would be:

$1 - (5/6)^3 = 1 - 125/216 = 91/216$ which is less than 0.5. Hence, option (a) is rejected.

For four matches, the probability of winning at least one match would be:

$1 - (5/6)^4 = 1 - 625/1296 = 671/1296$ which is more than 0.5. Hence, option (b) is correct.

12. Two teams *Arrogant* and *Overconfident* are participating in a cricket tournament. The odds that team *Arrogant* will be champion is 5 to 3, and the odds that team *Overconfident* will be the champion is 1 to 4. What are the odds that either *Arrogant* or team *Overconfident* will become the champion?

(a) 3 to 2

(b) 5 to 2

(c) 6 to 1

(d) 7 to 1

(e) 33 to 7

Solution: You need to be clear about what odds for an event mean in order to solve this. Odds for team *Arrogant* to be champion being 5 to 3 means that the probability of team *Arrogant* being champion is 5/8. Similarly, the probability of team *Overconfident* being champion is 1/5 (based on odds of team *Overconfident* being champion being 1 to 4). Thus, the probability that either of the teams would be the champion would be

$$= \frac{5}{8} + \frac{1}{5} = \frac{33}{40}$$

This means that in 40 times, 33 times the event of one of the teams being champion would occur. Hence, the odds for one of the two given teams to be the champion would be 33 to 7.

So, the required odds will be 33 to 7. Hence, option (e) is correct.

13. Let X be a four-digit number with exactly three consecutive digits being same and is a multiple of 9. How many such X's are possible?

(a) 12

(b) 16

(c) 19

(d) 21

(e) None of these

Solution: Since the number has to be a multiple of 9, the sum of the digits would be either 9 or 18 or 27. Also, the number would either be in the form *aaab* or *baaa*. For the sum of the digits to be 9, we would have the following cases:

a = 1 and b = 6, for the numbers 1116 and 6111;

a = 2 and b = 3, for the numbers 2223 and 3222;

a = 3 and b = 0, for the number 3330 and

b = 9 and a = 0, for the number 9000. We get a total of 6 such numbers.

Similarly for the sum of the digits to be 18, we will get:

3339, 9333; 4446, 6444; 5553, 3555; 6660; we get a total of 7 such numbers.

For the sum of the digits to be 27, we will get:

6669, 9666; 7776, 6777; 8883, 3888 and 9990; thus, we get a total of 7 such numbers. Hence, the total number of numbers is 20. Hence, option (e) is correct.

14. A shop sells two kinds of rolls—egg roll and mutton roll. Onion, tomato, carrot, chilli sauce and tomato sauce are the additional ingredients. You can have any combination of additional ingredients, or have standard rolls without any additional ingredients subject to the following constraints:

(i) You can have tomato sauce if you have an egg roll, but not if you have a mutton roll.

(ii) If you have onion or tomato or both you can have chilli sauce, but not otherwise.

How many different rolls can be ordered according to these rules?

(a) 21

(b) 33

(c) 40

(d) 42

(e) None of these

Solution: Let the five additional ingredients onion, tomato, carrot, chilli sauce and tomato sauce are denoted by *O*, *T*, *C*, *CS*, *TS*, respectively.

Number of ways of ordering the egg roll:

For the egg roll, there are a total of 32 possibilities (with each ingredient being either present or not present – there being five ingredients, the total number of possibilities of the combinations of the egg rolls would be equal to $2 \times 2 \times 2 \times 2 \times 2 = 32$ ways).

However, out of these 32 instances, the following combinations are not possible due to the constraint given in statement (b) which tells us that to have CS in the roll either of onion or tomato must be present (or both should be present). The combinations which are not possible are:

(CS) (CS, TS) (CS, C) (CS, C, TS)

Total number of ways, egg roll can be ordered

= 32 – 4 = 28

Number of ways of ordering the mutton roll:

Total number of cases for mutton roll without any constraints = 2 × 2 × 2 × 2 = 16 ways. Cases rejected due to constraint given in statement (b): (CS); (CS, C) → 16 – 2 = 14 cases.

Total number of ways or ordering a roll = 28 + 14 = 42. Hence, option (d) is correct.

15. Steel Express stops at six stations between Howrah and Jamshedpur. Five passengers board at Howrah. Each passenger can get down at any station till Jamshedpur. The probability that all five persons will get down at different stations is:

(a) $\frac{^6P_5}{6^5}$

(b) $\frac{^6C_5}{6^5}$

(c) $\frac{^7P_5}{7^5}$

(d) $\frac{^6C_5}{7^5}$

(e) None of these

Solution: The required probability would be given by:

$$\frac{\left(\begin{array}{c}\text{Total number of ways in which 5 people can get down}\\ \text{at 5 different stations from amongst 7 stations}\end{array}\right)}{}$$

$$\left(\begin{array}{c}\text{Total number of ways in which 5 people can get down} \\ \text{at 7 stations}\end{array}\right)$$

The value of the numerator would be ${}_7P_5$, while the value of the denominator would be 7^5. The correct answer would be option (c).

16. In how many ways can 53 identical chocolates be distributed amongst three children– C_1, C_2 and C_3 – such that C_1 gets more chocolates than C_2, and C_2 gets more chocolates than C_3?

(a) 468

(b) 344

(c) 1404

(d) 234

Solution: Fifty-three identical chocolates can be distributed amongst three children in ${}^{55}C_2$ ways = 1485 ways (${}^{n+r-1}C_{r-1}$ formula). Out of these ways of distributing 53 chocolates, the following distributions methods are not possible as they would have two values equal to each other– (0, 0, 53); (1, 1, 51); (2, 2, 49).... (26, 26, 1).

There are 27 such distributions, but when allocated to C_1, C_2 and C_3 respectively, each of these distributions can be allocated in three ways amongst them. Thus, C_1 = 0, C_2 = 0 and C_3 = 53 is counted differently from C_1 = 0, C_2 = 53 and C_3 = 0 and also from C_1 = 53, C_2 = 0 and C_3 = 0. This will remove 27 × 3 = 81 distributions from 1485, leaving us with 1404 distributions. These 1404 distributions are those where all three numbers are different from each other. However, whenever we have three different values allocated to three children, there can be 3! = 6 ways of allocating the three different values amongst the three people. For instance, the distribution of 10, 15 and 48 can be seen as follows:

C_1	C_2	C_3	
48	15	10	Only case which meets the problems' requirement
48	10	15	
15	48	10	
15	10	48	
10	15	48	
10	48	15	

Hence, out of every six distributions counted in the 1404 distributions we currently have, we need to count only one. The answer can be arrived at by dividing 1404 ÷ 6 = 234. Hence, option (d) is correct.

17. In a chess tournament at the ancient Olympic Games of Reposia, it was found that the number of European participants was twice the number of non-European participants. In a round robin format, each player played every other player exactly once. The tournament rules were such that no match ended in a draw – any conventional draws in chess were resolved in favour of the player who had used up the lower time. While analysing the results of the tournament, K. Gopal, the tournament referee observed that the number of matches won by the non-European players was equal to the number of matches won by the European players. Which of the following can be the total number of matches in which a European player defeated a non-European player?

(a) 57

(b) 58

(c) 59

(d) 60

Solution: If we assume the number of non-European players to be n, the number of European players would be $2n$. Then there would be three kinds of matches played –

Matches between two Europan players – a total of ${}_{2n}C_2$ matches – which would yield a European winner.

Matches between two non-European players – a total of ${}_nC_2$ matches, – which would yield a non-European winner.

Matches, between a European and a non-European player = $2n^2$. These matches would have some European wins and some non-European wins. Let the number of European wins amongst these matches be x, then the number of non-European wins = $2n^2 - x$.

Now, the problem clearly states that the number of European wins = Number of non-European wins

$$\Rightarrow \quad \frac{2n(2n-1)}{2} + x = \frac{n(n-1)}{2} + 2n^2 - x$$

$$\Rightarrow \quad n(n+1) = 4x$$

This means that the value of four times the number of wins for a European player over a non-European player should be a product of two consecutive natural numbers (since n has to be a natural number).

Among the options, $x = 60$ is the only possible value as the value of $4 \times 60 = 15 \times 16$.

Hence, option (d) is correct.

18. A man, starting from a point M in a park, takes exactly eight equal steps. Each step is in one of the four directions – East, West, North and South. What is the total number of ways in which the man ends up at point M after the eight steps?

(a) 4200

(b) 2520

(c) 4900

(d) 5120

Solution: For the man to reach back to his original point, the number of steps North should be equal to the number of steps South. Similarly, the number of steps East should be equal to the number of steps West.

The following cases would exist:

4 steps North and 4 steps South = 8!/(4! × 4!) = 70 ways;

3 steps North, 3 steps South, 1 step East and 1 step West = 8!/(3! × 3!) = 1120 ways;

2 steps North, 2 steps South, 2 steps East and 2 steps West = 8!/(2! × 2! × 2! × 2!) = 2520 ways;

1 step North, 1 step South, 3 steps East and 3 steps West = 8!/(3! × 3!) = 1120 ways;

4 steps East and 4 steps West =8!/(4! × 4!) = 70 ways;

Thus, the total number of ways = 70 × 2 + 1120 × 2 + 2520 = 140 + 2240 + 2520 = 4900 ways.

Option (c) is correct.

REVIEW CAT SCAN

REVIEW CAT Scan 1

1. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side. Determine the number of ways in which the sitting arrangements can be made.

(a) ${}_{11}P_n \times (9!)_2$

(b) ${}_{11}C_5 \times (9!)_2$

(c) ${}_{11}P_6 \times (9!)_2$

(d) None of these

2. If m parallel lines in a plane are intersected by a family of n parallel lines, find the number of parallelograms that can be formed.

(a) $m_2 \times n_2$

(b) $m(_{m+1})n(_{n+1})/4$

(c) ${}_mC_2 \times {}_nC_2$

(d) None of these

3. A father with eight children takes three at a time to the zoological garden, as often as he can without taking the same three children together more than once. How often will he go and how often will each child go?

(a) ${}_8C_3$, ${}_7C_3$

(b) ${}_8C_3$, ${}_7C_2$

(c) ${}_8P_3$, ${}_7C_3$

(d) ${}_8P_3$, ${}_7C_2$

4. A candidate is required to answer seven questions out of two questions which are divided into two groups, each containing six questions. He is not permitted to attempt more than five questions from either group. In how many different ways can he choose the seven questions?

(a) 390

(b) 520

(c) 780

(d) None of these

5. Find the sum of all five digit numbers formed by the digits 1, 3, 5, 7, 9 when no digit is being repeated.

(a) 4444400

(b) 8888800

(c) 13333200

(d) 6666600

6. Consider a polygon of n sides. Find the number of triangles, none of whose sides is the side of the polygon.

(a) $nC_3 - n - n \times (n - 4)C_1$

(b) $n(n - 4)(n - 5)/3$

(c) $n(n - 4)(n - 5)/6$

(d) $n(n - 1)(n - 2)/3$

7. The number of four-digit numbers that can be formed using the digits 0, 2, 3, 5 without repetition is

(a) 18

(b) 20

(c) 24

(d) 20

8. Find the total number of words that can be made by using all the letters from the word MACHINE, using them only once.

(a) 7!

(b) 5020

(c) 6040

(d) 7!/2

9. What is the total number of words that can be made by using all the letters of the word REKHA, using each letter only once?

(a) 240

(b) 4!

(c) 124

(d) 5!

10. How many different five-digit numbers can be made from the first five natural numbers, using each digit only once?

(a) 240

(b) 4!

(c) 124

(d) 5!

11. There are seven seats in a row. Three persons take seats at random. What is the probability that the middle seat is always occupied and no two persons are sitting on consecutive seats?

(a) 7/70

(b) 14/35

(c) 8/70

(d) 4/35

12. Let $N = 33x$, where x is any natural number. What is the probability that the unit digit of N is 3?

(a) 1/4

(b) 1/3

(c) 1/5

(d) 1/2

13. Find the probability of drawing one ace in a single draw of one card out of 52 cards.

(a) 1/(52 × 4)

(b) 1/4

(c) 1/52

(d) 1/13

14. In how many ways can a committee of four persons be made from a group of ten people?

(a) 10! / 4!

(b) 210

(c) 10! / 6!

(d) None of these

15. In Question 14, what is the number of ways of forming the committee, if a particular member must be there in the committee?

(a) 12

(b) 84

(c) 9!/ 3!

(d) None of these

16. A polygon has 54 diagonals. The numbers of sides of this polygon are

(a) 12

(b) 84

(c) 3. 3!

(d) 4. 4!

17. An anti-aircraft gun can take a maximum of four shots at an enemy plane moving away from it, the probabilities of hitting the plane at first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane?

(a) 0.7654

(b) 0.6976

(c) 0.3024

(d) 0.2346

18. Seven white balls and three black balls are placed in a row at random. Find the probability that no two black balls are adjacent.

(a) 2/15

(b) 7/15

(c) 8/15

(d) 4/15

19. The probability that A can solve a problem is 3/10 and that B can solve is 5/7. If both of them attempt to solve the problem, what is the probability that the problem can be solved?

(a) 3/5

(b) 1/4

(c) 2/3

(d) 4/5

20. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. Find the number of triangles that can be constructed using these points as vertices.

(a) 180

(b) 105

(c) 205

(d) 280

21. From six gentlemen and four ladies, a committee of five is to be formed. In how many ways can this be done if there is no restriction in its formation?

(a) 256

(b) 246

(c) 252

(d) 260

22. From four officers and eight *jawans* in how many ways can six be chosen to include exactly one officer?

(a) $_{12}C_6$

(b) 1296

(c) 1344

(d) 224

23. From four officers and eight *jawans* in how many ways can six be chosen to include atleast one officer?

(a) 868

(b) 924

(c) 896

(d) None of these

24. Two cards are drawn one after another from a pack of 52 ordinary cards. Find the probability that the first card is an ace and the second drawn is an honour card if the first card is not replaced while drawing the second.

(a) 12/13

(b) 12/51

(c) 1/663

(d) None of these

25. The probability that Andrews will be alive fifteen years from now is 7/15 and that Bill will be alive fifteen years from now is 7/10. What is the probability that both Andrews and Bill will be dead fifteen years from now?

(a) 12/150

(b) 24/150

(c) 49/150

(d) 74/150

REVIEW CAT Scan 2

1. A group consists of 100 people; 25 of them are women and 75 men; 20 of them are rich and the remaining poor; 40 of them are employed. The probability of selecting an employed rich woman is

   (a) 0.05

   (b) 0.04

   (c) 0.02

   (d) 0.08

2. Out of thirteen job applicants, there are five boys and eight men. It is desired to choose two applicants for the job. The probability that at least one of the selected applicants will be a boy is

   (a) 5/13

   (b) 14/39

   (c) 25/39

   (d) 10/13

3. Fours dogs and three pups stand in a queue. The probability that they will stand in alternate positions is

   (a) 1/34

(b) 1/35

(c) 1/17

(d) 1/68

4. Asha and Vinay play a number game where each is asked to select a number from 1 to 5. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is

(a) 1/25

(b) 24/25

(c) 2/25

(d) None of these

5. The number of ways in which six British and five French can dine at a round table if no two French are to sit together is given by:

(a) $6! \times 5!$

(b) $5! \times 4!$

(c) 30

(d) $7! \times 5!$

6. A cricket team of eleven players is to be formed from twenty players including six bowlers and three wicketkeepers. Find the number of ways in which a team can be formed having exactly four bowlers and two wicketkeepers

(a) 20790

(b) 6930

(c) 10790

(d) 360

7. Three boys and three girls are to be seated around a circular table. Among them one particular boy Rohit does not want any girl neighbour and one particular girl Shaivya does not want any boy neighbour. How many such arrangements are possible?

(a) 5

(b) 6

(c) 4

(d) 2

8. Words with five letters are formed from ten different letters of an alphabet. Then the number of words which have at least one letter repeated is

(a) 19670

(b) 39758

(c) 69760

(d) 99748

9. Sunil and Kapil toss a coin alternatively till one of them gets a head and wins the game. If Sunil starts the game, the probability that he (Sunil) will win is

(a) 0.66

(b) 1

(c) 0.33

(d) None of these

10. The number of parallelograms that can be formed if seven parallel horizontal lines intersect six parallel vertical lines, is

(a) 42

(b) 294

(c) 315

(d) None of these

11. 1.3.5…$(2n - 1)$/2.4.6…$(2n)$ is equal to

(a) $(2n)! \div 2_n(n!)$)2

(b) $(2n)! \div n!$

(c) $(2n - 1) \div$) $n - 1)!$

(d) 2_n

12. How many four-digit numbers, each divisible by four can be formed using the digits 1, 2, 3, 4 and 5 (repetitions allowed)?

(a) 100

(b) 150

(c) 125

(d) 75

13. A student is to answer ten out of thirteen questions in a test such that he/she must choose at least four from the first five questions. The number of choices available to him is

(a) 140

(b) 280

(c) 196

(d) 346

14. The number of ways in which a committee of three ladies and four gentlemen can be appointed from a meeting consisting of eight ladies and seven gentlemen; if Mrs. Pushkar refuses to serve in a committee if Mr. Modi is its member, is

(a) 1960

(b) 3240

(c) 1540

(d) None of these

15. A room has three lamps. From a collection of ten light bulbs of which six are not good, a person selects three at random and puts them in a socket. The thatprobability that he will have light is

(a) 5/6

(b) 1/2

(c) 1/6

(d) None of these

16. Two different series of a question booklet for an aptitude test are to be given to twelve students. In how many ways can the students be placed in two rows of six each so that there should be no identical series side by side and that the students sitting one behind the other should have the same series?

(a) $2 \times {}_{12}C_6 \times (6!)2$

(b) 6! × 6!

(c) 7! × 7 ×

(d) None of these

17. The letters of the word PROMISE are arranged so that no two of the vowels should come together. The total number of arrangements is:

(a) 49

(b) 1440

(c) 7

(d) 1898

18. Find the remainder left after dividing 1! + 2! + 3! + … + 1000! by 7.

(a) 0

(b) 5

(c) 21

(d) 14

19. In the McGraw-Hill Mindworkzz mock test paper; there are two sections, each containing four questions. A candidate is required to attempt five questions but not more than three questions from any section. In how many ways can five questions be selected?

(a) 24

(b) 48

(c) 72

(d) 96

20. A bag contains ten balls out of which three are pink and rest are orange. In how many ways can a random sample of six balls be drawn from the bag so that at the most two pink balls are included in the sample and no sample has all the six balls of the same colour?

(a) 105

(b) 168

(c) 189

(d) 120

ANSWER KEY

REVIEW CAT Scan 1

1. (b)
2. (c)
3. (b)
4. (c)
5. (d)
6. (a)
7. (a)
8. (a)
9. (d)
10. (d)
11. (d)
12. (a)
13. (d)
14. (b)

15. (b)
16. (a)
17. (b)
18. (b)
19. (d)
20. (c)
21. (c)
22. (d)
23. (c)
24. (d)
25. (b)

REVIEW CAT Scan 2

1. (c)
2. (c)
3. (b)
4. (b)
5. (b)
6. (a)
7. (c)
8. (c)
9. (c)
10. (c)
11. (a)
12. (c)
13. (a)
14. (d)

15. (d)

16. (b)

17. (b)

18. (b)

19. (b)

20. (b)

TASTE OF THE EXAMS—BLOCK VI

CAT

1. Ten points are marked on a straight-line and 11 points are marked on another straight-line. How many triangles can be constructed with vertices from among the above points? **(CAT 1999)**

   (a) 495

   (b) 550

   (c) 1045

   (d) 2475

2. For a scholarship, at the most n candidates out of $2n + 1$ can be selected. If the number of different ways of selection of at least one candidate is 63, the maximum number of candidates that can be selected for the scholarship is **(CAT 1999)**

   (a) 3

   (b) 4

   (c) 6

(d) 5

3. In a survey of political preferences, 78% of those asked were in favour of at least one of the proposals: I, II and III. 50% of those asked favoured proposal I, 30% favoured proposal II and 20% favoured proposal III. If 5% of those asked favoured all three of the proposals, what percentage of those asked favoured more than one of the three proposals? **(CAT 1999)**

(a) 10

(b) 12

(c) 17

(d) 22

4. Sam has forgotten his friend's seven-digit telephone number. He remembers the following: the first three digits are either 635 or 674, the number is odd, and the number 9 appears once. If Sam were to use a trial and error process to reach his friend, what is the minimum number of trials he has to make before he can be certain to succeed? **(CAT 2000)**

(a) 10,000

(b) 2,430

(c) 3,402

(d) 3,006

5. There are three cities: *A*, *B* and *C*. Each of these cities is connected with the other two cities by at least one direct road. If a traveller wants to go from one city (origin) to another city (destination), she can do so either by traversing a road connecting the two cities directly, or by traversing

two roads, the first connecting the origin to the third city and the second connecting the third city to the destination. In all there are 33 routes from *A* to *B* (including those via *C*). Similarly, there are 23 routes from *B* to *C* (including those via *A*). How many roads are there from *A* to *C* directly? **(CAT 2000)**

(a) 6

(b) 3

(c) 5

(d) 10

6. One red flag, three white flags and two blue flags are arranged in a line such that: **(CAT 2000)**

I. No two adjacent flags are of the same colour.

II. The flags at the two ends of the line are of different colours.

In how many different ways can the flags be arranged?

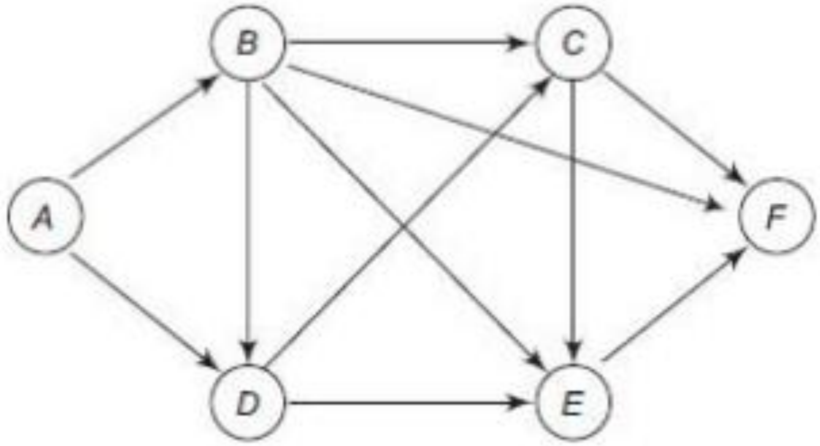
(a) 6

(b) 4

(c) 10

(d) 2

7. The figure below shows the network connecting cities *A*, *B*, *C*, *D*, *E* and *F*. The arrows indicate permissible direction of travel. What is the number of distinct paths from *A* to *F*?



(a) 9

(b) 10

(c) 11

(d) None of these

8. Let n be the number of different five-digit numbers, divisible by 4 with the digits 1, 2, 3, 4, 5 and 6, no digit being repeated in the numbers. What is the value of n? **(CAT 2001)**

(a) 144

(b) 168

(c) 192

(d) None of these

Directions for Questions 9 and 10: Answer the questions based on the following information. Each of the 11 letters $A, H, I, M, O, T, U, V, W, X$ and Z appears same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters. **(CAT 2002)**

9. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?

(a) 7,920

(b) 330

(c) 14,640

(d) 4,19,430

10. How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

(a) 990

(b) 2,730

(c) 12,870

(d) 15,600

11. In how many ways is it possible to choose a white square and a black square on a chessboard so that the squares must not lie in the same row or column? **(CAT 2002)**

(a) 56

(b) 896

(c) 60

(d) 768

12. How many numbers greater than 0 and less than a million can be formed with the digits 0, 7 and 8? **(CAT 2002)**

(a) 486

(b) 1,084

(c) 728

(d) None of these

13. Ten straight lines, no two of which are parallel and no three of which pass through any common point, are drawn on a plane. The total number of regions (including finite and infinite regions) into which the plane would be divided by the lines is **(CAT 2002)**

(a) 56

(b) 255

(c) 1024

(d) not unique

14. Each family in a locality has at most two adults, and no family has fewer than 3 children. Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families. Then, the minimum possible number of families in the locality is **(CAT 2004)**

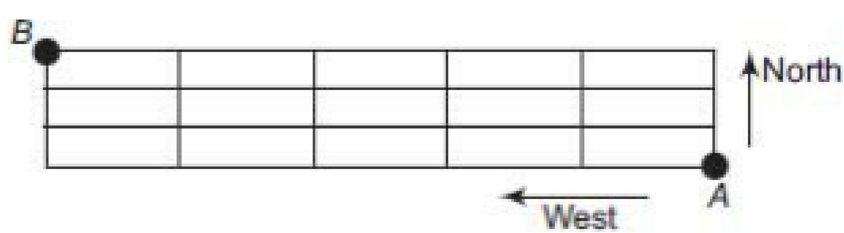
(a) 4

(b) 5

(c) 2

(d) 3

15. In the adjoining figure, the lines represent one-way roads allowing travel only northwards or only westwards. Along how many distinct routes can a car reach point *B* from point *A*? **(CAT 2004)**



(a) 15

(b) 56

(c) 120

(d) 336

16. Three Englishmen and three Frenchmen work for the same company. Each of them knows a secret not known to others. They need to exchange these secrets over person-to-person phone calls so that eventually each person knows all six secrets. None of the Frenchmen knows English, and only one Englishman knows French. What is the minimum number of phone calls needed for the above purpose? **(CAT 2005)**

(a) 5

(b) 10

(c) 9

(d) 15

17. In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is **(CAT 2005)**

(a) 200

(b) 216

(c) 235

(d) 256

18. Let *S* be the set of five-digit numbers formed by digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd positions are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in *S*? **(CAT 2005)**

(a) 228

(b) 216

(c) 294

(d) 192

19. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done? **(CAT 2006)**

(a) 144

(b) 180

(c) 192

(d) 360

(e) 716

20. A survey was conducted of 100 people to find out whether they had read recent issues of Golmal, a monthly magazine. The summarized information regarding readership in 3 months is given below:

Only September: 18; September but not August: 23;

September and July: 8; September: 28; July: 48;

July and August: 10; None of the three months: 24.

What is the number of surveyed people who have read exactly two consecutive issues (out of the three)? **(CAT 2006)**

(a) 7

(b) 9

(c) 12

(d) 14

(e) 17

21. How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3 and 4, if repetition of digits is allowed? **(CAT 2008)**

(a) 499

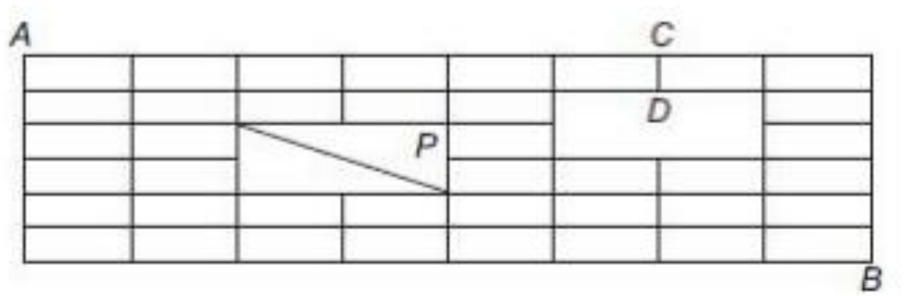
(b) 500

(c) 375

(d) 376

(e) 501

Directions for Questions 22 and 23: The figure below shows the plan of a town. The streets are at right angles to each other. *A* rectangular park (*P*) is situated inside the town with a diagonal road running through it. There is also a prohibited region (*D*) in the town. **(CAT 2008)**



22. Neelam rides her bicycle from her house at A to her office at B, taking the shortest path. Then the number of possible shortest paths that she can choose is

(a) 60

(b) 75

(c) 45

(d) 90

(e) 72

23. Neelam rides her bicycle from her house at A to her club at C, via B taking the shortest path. Then the number of possible shortest paths that she can choose is

(a) 1170

(b) 630

(c) 792

(d) 1200

(e) 936

24. In how many ways can 8 identical pens be distributed among Amal, Bimal and Kamal so that Amal gets atleast 1 pen, Bimal gets atleast two pens and Kamal gets atleast 3 pens? **(CAT 2017)**

25. In how many ways can 7 identical erasers be distributed among 4 kids in such a ways that each kid gets atleast one eraser and no kid gets more than 3 erasers? **(CAT 2017)**

26. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with telephone lines such that every two towns are connected with three direct Iines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?

(a) 72

(b) 90

(c) 96

(d) 144

27. A survey on a sample of 25 new cars being sold at a local auto-dealer was conducted to see which of the three popular options (air conditioning, ratio and power windows) were already installed. The survey found:

- 15 had air conditioning
- 2 had air conditioning and power windows but no radios
- 12 had radio
- 6 had air conditioning and radio but no power windows
- 11 had power windows
- 4 had radio and power windows
- 3 had all three options.

What is the number of cars that had none of the options? **(CAT 2003)**

(a) 4

(b) 3

(c) 1

(d) 2

Directions for Questions 28 and 29: Answer the questions on the basis of information given below.

A string of three English letters is formed as per the following rules:

(a) The first letter is any vowel.

(b) The second letter is *m*, *n* or *p*.

(c) If the second letter is *m*, then the third letter is any vowel which is different from the first letter.

(d) If the second letter is *n*, then the third letter is *e* or *u*.

(e) If the second letter is *p*, then the third letter is the same as the first letter.

28. How many strings of letters can possibly be formed using the above rules? **(CAT 2003)**

(a) 40
(b) 45

(c) 30

(d) 35

29. How many strings of letters can possibly be formed using the above rules such that the third letter of the string is *e*? **(CAT 2003)**

(a) 8

(b) 9

(c) 10

(d) 11

30. 70 percent of the employees in a multi-national corporation have VCD players, 75 percent have microwave ovens, 80 percent have ACs and 85 percent have washing machines. At least what percentage of employees have all the four gadgets? **(CAT 2003)**

(a) 15

(b) 5

(c) 10

(d) Cannot be determined

31. *N* persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-minute song, one pair after the other. If the total time taken for singing is 28 minutes, what is *N*? **(CAT 2004)**

(a) 5

(b) 7

(c) 9

(d) None of these

32. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent stripes have the same colour is: **(CAT 2004)**

(a) 12×81

(b) 16×192

(c) 20×125

(d) 24 × 216

33. What is the number of distinct terms in the expansion of $(a + b + c)^{20}$? **(CAT 2008)**

(a) 231

(b) 253

(c) 242

(d) 210

(e) 228

Directions for Questions 34 and 35: Answer the questions on the basis of the information given below:

New Age Consultants has three consultants—Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together in six projects. Gyani works with Medha in 14 projects. Buddhi has two projects with Medha but without Gyani, and three projects with Gyani but without Medha. The total number of projects for New Age Consultants is one less than twice the number of projects in which more than one consultant is involved.

34. What is the number of projects in which Gyani alone is involved? **(CAT 2003 LEAKED)**

(a) Uniquely equal to zero

(b) Uniquely equal to 1

(c) Uniquely equal to 4

(d) Cannot be determined uniquely

35. What is the number of projects in which Medha alone is involved? **(CAT 2003 LEAKED)**

(a) Uniquely equal to zero

(b) Uniquely equal to 1

(c) Uniquely equal to 4

(d) Cannot be determined

36. Each of 74 students in a class studies at least one of the three subjects H, E and P. Ten students study all three subjects, while twenty study H and E, but not P. Every student who studies P also studies H or E, or both. If the number of students studying H equals that studying E, then the number of students studying H is **(CAT 2018)**

37. If among 200 students, 105 like pizza and 134 like burger, then the number of students who like only burger can possibly be **(CAT 2018)**

(a) 26

(b) 23

(c) 96

(d) 93

38. For two sets A and B, let $A \Delta B$ denote the set of elements which belong to A or B but not both. If $P = \{1,2,3,4\}$, $Q = \{2,3,5,6,\}$, $R = \{1,3,7,8,9\}$, $S = \{2,4,9,10\}$, then the number of elements in $(P \Delta Q) \Delta (R \Delta S)$ is **(CAT 2018)**

(a) 7

(b) 8

(c) 9

(d) 6

39. If $A = \{6^{2n} - 35n - 1 : n = 1,2,3,...\}$ and $B = \{35(n - 1) : n = 1,2,3,...\}$ then which of the following is true? **(CAT 2018)**

(a) Neither every member of A is in B, nor is every member of B in A

(b) Every member of A is in B and at least one member of B is not in A

(c) Every member of B is in A

(d) At least one member of A is not in B

40. In a tournament, there are 43 junior-level and 51 senior-level participants. Each pair of juniors plays one match. Each pair of seniors plays one match. There is no junior versus senior match. The number of girls versus girl matches in junior level is 153, while the number of boys versus boy matches in senior level is 276. The number of matches a boy plays against a girl is **(CAT 2018)**

41. A club has 256 members of whom 144 can play football, 123 can play tennis, and 132 can play cricket. Moreover, 58 members can play both football and tennis, 25 can play both cricket and tennis, while 63 can play both football and cricket. If every member can play at least one game, then the number of members who can play only tennis is **(CAT 2019)**

(a) 32

(b) 43

(c) 38

(d) 45

ANSWER KEY

1. (c)
2. (a)
3. (c)
4. (c)
5. (a)
6. (a)
7. (b)
8. (c)
9. (a)
10. (c)
11. (d)
12. (c)
13. (a)
14. (d)
15. (b)
16. (c)
17. (a)
18. (b)
19. (a)
20. (b)
21. (d)
22. (d)
23. (a)
24. 6

25. 16

26. (b)

27. (d)

28. (d)

29. (c)

30. (c)

31. (b)

32. (a)

33. (a)

34. (d)

35. (b)

36. 52

37. (d)

38. (a)

39. (b)

40. 1098

41. (b)

Solutions

1. A triangle can be formed when we take two points from any of the lines and meet those points from any point of the second line.Number of triangles = ${}^{10}C_2 \times 11 + {}^{11}C_2 \times 10 = 45 \times 11 + 55 \times 10 = 1045$.

2. If we have *n* candidates who can be selected at the maximum, naturally, the answer to the question would also represent *n*. Also, we see that the options are also giving us values for *n*.

Hence, we check for the first option. If $n = 3$, then $2n + 1 = 7$ and it means that there are 7 candidates to be chosen from. Since it is given that the number of ways of selection of at least 1 candidate is 63, we should try to see, whether selecting 1, 2 or 3 candidates from 7 indeed adds up to 63 ways. If it does this would be the correct answer.

$_7C_1 + {_7C_2} + {_7C_3} = 7 + 21 + 35 = 63$. Thus, the first option fits the situation and is correct.

3. 78% favoured at least one. Also, the count of the three values given for favouring I, favouring II and favouring III is 100. Thus, we have an extra count of 22 – i.e. 78 people are counted 100 times, so 22 people are extra because they would be counted twice or thrice (People who favoured 2 proposals would be counted twice – so once extra; people who favoured three proposals would be counted thrice – so twice extra).
We are also given that 5% people are counted thrice (Those who favour all three proposals). Hence, they would account for 10 extra counts (5×2). Naturally, the remaining 12 extra counts should happen due to the people who favoured 2 proposals. Thus, we can see that 12% people were counted twice. The percentage who favoured more than one of the proposals would be $12 + 5 = 17$.

Alternate solution: Formula is - Favouring I + Favouring II + Favouring III - (favoured exactly 2 proposals) – 2(favoured exactly 3 proposals) = 78

$50 + 30 + 20 - (F) - 2 \times 5 = 100$

By solving we get, $F = 12$
People who favoured more than one of the three proposals = People who favoured two proposals + People who favoured three proposals = 12 + 5 = 17%

4. There are two possible cases.

Case 1: The number 9 comes at the end.

Case 2: 9 comes at position 4, 5, or 6.

In both these cases, the blanks can be occupied by any of the available 9 digits (0, 1, 2, ..., 8).

For the first case, the number 9 can occupy the last position

Thus, total possible numbers would be 2 × (9 × 9 × 9) = 1458.

For the second case, the number 9 can occupy any of the given position 4, 5, or 6, and there shall be an odd number at position 7. Thus, the total number of ways shall be 2 × [3 × (9 × 9 × 4)] = 1944.

Hence, total ways = 1458 + 1944 = 3402

5. We need to go through the options and use the MNP rule relating to Permutations and Combinations.

If the first option is true—i.e. there are 6 routes between *A* to *C*:

Then, the number of routes between *C* to *B* and consequently the number of routes between *A* to *B* would be given by the following possibilities:

Routes Between A–C	*C to B (possibilities)*	*Routes from A–C–B*	*Direct routes from A to B*	*Total routes from A to B*
6	5	30	3	33
	4	24	9	33
	3	18	15	33
	2	12	21	33
	1	6	27	33

We also know that there are 23 routes between *B* to *C*. If the given numbers in the first possibility of the table are true, we see that *B–A–C* we would have 3 × 6 = 18 routes and from *B* to *C* we would have 5 routes – a total of 18 + 5 as required. Hence, this option fits the situation perfectly. Option (a) is correct.

6. This problem can be approached by putting the white flags in their possible positions. There are essentially 4 possibilities for placing the 3 white flags based on the condition that two flags of the same color cannot be together:

   1, 3, 5; 1, 3, 6; 1, 4, 6 and 2, 4, 6.

   Out of these 4 possible arrangements for the 3 white flags we cannot use 1, 3, 6 and 1, 4, 6 as these have the same colored flag at both ends – something, which is not allowed according to the question. Thus there are only 2 possible ways of placing the white flags – 1, 3, 5 or 2, 4, 6. In each of these 2 ways, there are a further 3 ways of placing the 1 red flag and the 2 blue flags. Thus, we get a total of 6 combinations.

7. Ten ways can be counted in this case: 7 through *AB* – (*CF*, *CEF*, *BF*, *EF*, *DCF*, *DCEF*, *DEF*) and three through *AD* (*EF*, *CF* and *CEF*).

8. Three cases are possible

   Case 1: When the unit's digit is 2

   Total possible numbers = $\underline{2} \times \underline{3} \times \underline{4} \times \underline{3} \times \underline{1} = 72$

   Case 2: When the unit's digit is 4

   Total possible numbers = $\underline{2} \times \underline{3} \times \underline{4} \times \underline{2} \times \underline{1} = 48$

   Case 3: When the unit's digit is 6

Total possible numbers = $\underline{2} \times \underline{3} \times \underline{4} \times \underline{3} \times \underline{1} = 72$

Total possible numbers = 72 + 72 + 48 = 192

9. Total four-letter computer passwords that can be formed using only the symmetric letters (no repetition allowed) = $11 \times 10 \times 9 \times 8 = 7920$

10. Total possible numbers = Total number of passwords using all letters – Total number of passwords using no symmetric letters= $(26 \times 25 \times 24) - (15 \times 14 \times 13) = 12870$

11. A white square can be selected in 32 ways and once the white square is selected, 8 black squares become ineligible for selection (4 in the same row and 4 in the same column). So the number of black squares available = 24.

    Hence the required number of ways = $32 \times 24 = 768$

12. Total numbers = 1 digit numbers + 2 digit numbers + 3 digit numbers + 4 digit numbers + 5 digit numbers + 6 digit numbers

    $= 2 + 2 \times 3 + 2 \times 3^2 + 2 \times 3^3 + 2 \times 3^4 + 2 \times 3^5$

    $= 2\,(1 + 3 + 3^2 + 3^3 + 3^4 + 3^5) = 728$

13. Let the number of lines be 'x'.

    The number of regions would be given by $= \dfrac{x(x+1)}{2} + 1$,

    Now put $x = 10$ in the above equation, we get number of regions = $\dfrac{10 \times 11}{2} + 1 = 56$

14. Two families together have at most $2 \times 2 = 4$ adults and at least 6 children.

Number of adults (4) > Number of boys > Number of girls > 2 (Not possible).

Three families together have at most 2 × 3 = 6 adults and at least 9 children.

Number of families (3) < Number of girls (4) < Number of boys (5) < Number of adults (6)

With three families all conditions are satisfied. Hence, Option (d) is correct.

15. From A to B, there are 8 one-way roads. Out of these 8 roads, 3 roads are in Northwards and 5 roads are westwards. The required number of routes would be given by ${}_8C_3$

Therefore the required number of distinct routes = $\frac{8!}{3!5!} = 56$

16. Let the Englishmen be *A*, *B* and *C*. Out of these, let *C* know French. Also, the French can be assumed to be *D*, *E* and *F*. First of all, let *A* and *B* call *C* so that *C* knows all the three secrets with the Englishmen. Also, then let *D* and *E* call *F*, so that *F* knows all the three secrets with the French. Then, let *C* call *F* to exchange all secrets. At this point *C* and *F* would know all 6 secrets. They then need to transmit it to *A*, *B* and *D*, *E* respectively. So, C must call *A* and *B*. Also, *F* must call *D* and *E*. Thus, there will be a total of 2 + 2 + 1 + 2 + 2 = 9 calls, option (c).

17. Let there be '*x*' boys and '*y*' girls

$$^{y}C_2 = 45 \Rightarrow \frac{y(y-1)}{2} = 45 \Rightarrow y(y-1) = 90 \Rightarrow y = 10$$

$$^{x}C_2 = 190 \Rightarrow \frac{x(x-1)}{2} = 190 \Rightarrow x(x-1) = 380 \Rightarrow x = 20$$

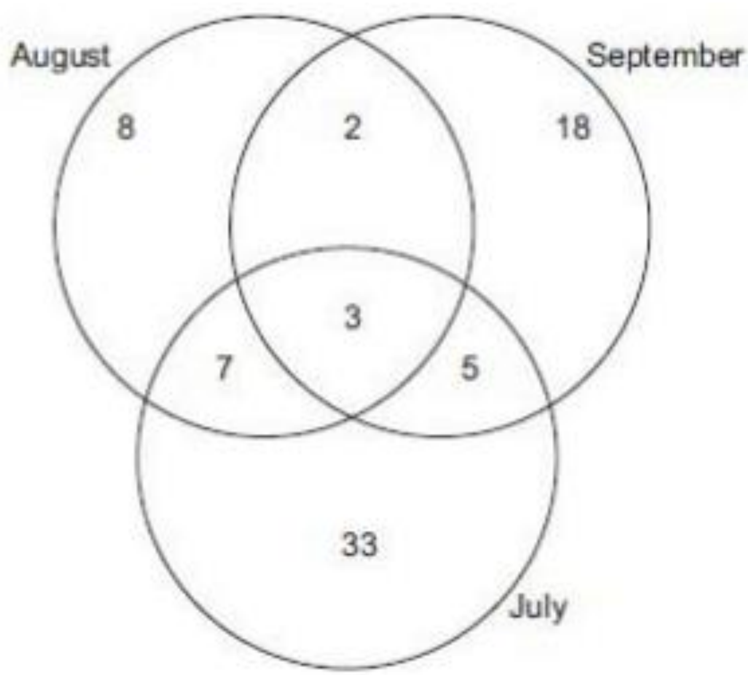
Number of games between one boy and one girl = ${}_{20}C_1 \times {}_{10}C_1 = 10 \times 20 = 200$.

18. The number of numbers formed with 5 as the ending digit would be ${}_2C_1 * {}_2C_1 * 2! * 2! = 16$. Similarly, the number of numbers formed with 1 and 3 in the units digit would be 16 and 16 respectively. Also, the number of numbers formed with 2 and 4 in the units place respectively, would be given by ${}_3C_2 \times 2! \times 2! = 12$ each.

    Thus, the sum of the rightmost digits would be $16 \times 5 + 16 \times 3 + 16 \times 1 + 12 \times 2 + 12 \times 4 = 216$. Option (b) is correct.

19. Task 2 can be assigned in 2 ways and Task 1 can be assigned in 3 ways and Tasks 3 to 6 can be assigned in 4! ways. Thus, the required answer would be: $2 \times 3 \times 4! = 2 \times 3 \times 4 \times 3 \times 2 \times 1 = 144$ ways.

20. Total people reading the newspaper in consecutive months (July and August and August and September) is $2 + 7 = 9$ people.



21. The integers can be identified as follows:

4-digit numbers starting with 1: 5 × 5 × 5 = 125

4-digit numbers starting with 2: 5 × 5 × 5 = 125

4-digit numbers starting with 3: 5 × 5 × 5 = 125 + the number 4000.

Hence, the answer would be 125 + 125 + 125 + 1 = 376, option (d).

22. Neelam has to take the diagonal of the rectangular park compulsorily if she wants to take the shortest route. Hence, the key becomes to get her from vertex *A* to the start of the diagonal of the rectangular field and from the end of the diagonal vertex *B*.

The solution to this question would be given by the algorithm $^4C_2 \times {}^6C_2$ = 6 × 15 = 90

option (d).

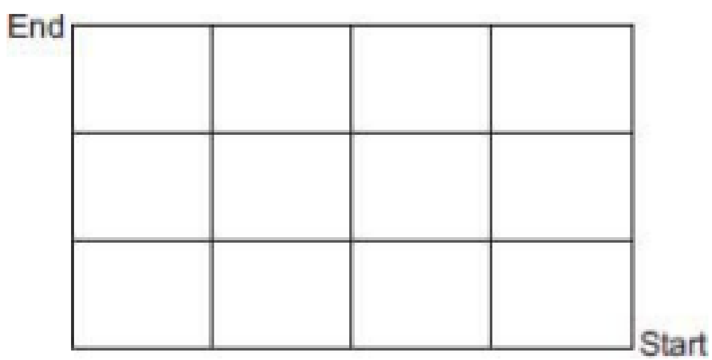
Note: This algorithm is again one of the “either you know it or you are dead” algorithms that the CAT paper regularly presents. What I mean to say by this is that if you do not know this algorithm before-hand then it is highly unlikely that you would be able to do this question derive this logic especially under the pressure situation of the CAT.

Theory Point:

To understand this logic let us look at the following examples where we are required to find the number of ways of going from one corner of a grid to the opposite corner by traversing along the lines of the sides of the rectangles:

In this case the total number of ways is 6. You are welcome to verify the number by physically counting the number of ways. (Note that 6 can also be given by ${}_4C_2$!)

Consider a few more examples before we conclude about an algorithm for the same.



In the above figure the number of ways of traversing the grid would be given by ${}_7C_3 = 35$.

In order to explain the logic, it is simply given by the formula:

(NUMBER OF ROWS + NUMBER OF COLUMNS)C(NUMBER OF ROWS)

This formula will always work.

(**Note:** If you know this algorithm, you would do this question in less than 30 seconds. If not, you would require at least 3–4 minutes to get a wrong answer — wrong because it is unlikely that you would be able to count correctly under the CAT pressure.)

23. The answer to this question has to be a multiple of 90 (answer to the previous question). Hence, there are only two possible answers. The correct answer would be given by:

90 × number of ways in which Neelam can go from B to point C.

It can be easily verified that the number of ways of going from B to C is greater than 7. Hence, the option 630 would be rejected and you can go ahead and mark 1170 as the correct answer option. Thus, we will mark option (a).

24. After distributing 1, 2 and 3 pens to Amal, Bimal and Kamal we are left with only 2 pens.
Now we can distribute 2 (n) pens among 3 (r) persons in ${}^{n+r-1}C_{r-1}$ ways. Hence, we have ${}^{4}C_{2}$ ways = 6 ways.

25. After distributing one eraser to each of the four kids we are left only with 3 erasers.

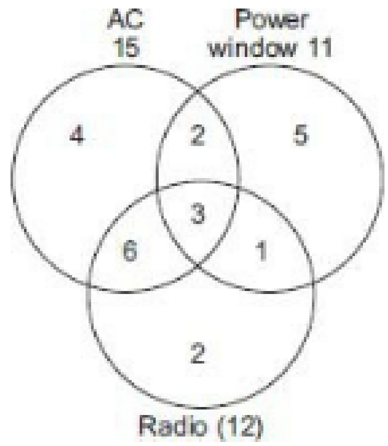
We can distribute these 3 erasers in the following ways:

Case 1: When one kid gets 2 erasers and one kid gets 1 eraser. We can do this in 12 ways.

Case 2: When three kids get 1 eraser each. We can do this in 4 ways.

Hence the required number of ways = 12 + 4 = 16.

26. There will be 9 lines within each zone giving a total of 36 intra zonal lines. Also let us say that the towns are (A, B, C); (D, E, F); (G, H, I) and (J, K and L). Then, each of A, B and C would have 9 inter-zonal lines (total 27) D, E, F would have 6 each for a total of 18 and G, H, I would have 3 each for a total of 9 lines. Adding all these up, we would get: 36 + 27 + 18 + 9 = 90.

27. 

From the figure, it is clear that Option (d) is the correct answer.

Solutions for Questions 28 and 29:

First of all make a structure of what are the possible combinations:

With *m* as the middle letter – (vowel) – *m* – (another vowel)

With *n* as the middle letter – (vowel) – *n* – (*e* or *u*)

With *p* as the middle letter – (vowel) – *p* – (same vowel)

28. The number of possible letter strings are—

With *m*: 5 × 4 = 20, with *n* = 5 × 2 = 10, with *p*: 5 × 1 = 5

Thus, a total of 35 possible strings can possibly be formed, option (d).

29. With *m*: 4, with *n*: 5, with *p*: 1. Thus a total of 10 strings can be formed with *e* as the third letter, option (c).

30. The least percentage of people with all the four gadgets would happen if all the employees who are not having any one of the four objects is mutually exclusive.

Thus, 100 – 30 – 25 – 20 – 15 = 10. Option (c).

31. The number of pairs is given by NC2–N. Since it takes 28 minutes, there must be 14 pairs. $_7C_2 - 7 = 14$. Hence, Option (b) is correct.

32. This can be solved using the mnp rule. For the first vertical stripe we can use any of 4 colours, for the second we would have only 3 options, same for the third to the sixth stripe. Hence, the required answer would be 4 × 3 × 3 × 3 × 3 × 3 = 12 × 81. Option (a).

33. The question is representative of $(a + b + c)(a + b + c) \ldots (a + b + c)$—twenty times. What you should realise at the outset while solving this question is that for each term, the sum of all powers of a, b and c would be 20. Hence, with this insight when you start solving the problem, you would make the following realisations:

The first term would be $a^{20} b^0 c^0$

For the next series of terms, you would get with a^{19} two ways in which you could distribute the remaining power as $a^{19} b^1 c^0$ or $a^{19} b^0 c^1$.

When you go for a^{18}, the remaining two power values can be distributed in three ways: viz: $a^{18}b^2 c^0$ or $a^{18}b^1 c^1$ or $a^{18}b^0 c^2$.

So, where is this leading you to? A little bit of thought and previous experience with triangular numbers (incidentally one of the favourite logics for CAT paper setters) will make you realise at this point that all we are doing in this question is adding the first 21 natural numbers. i.e., 1 + 2 + 3 + 4 + ... 21 = 231.

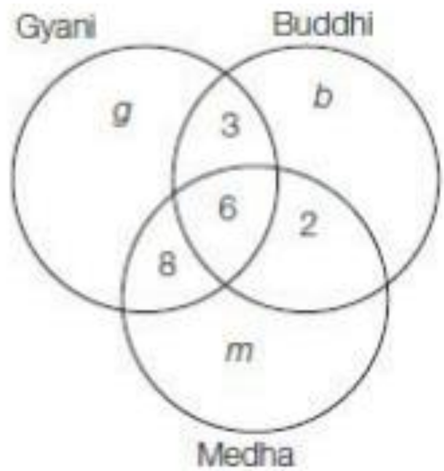
Of course, you can also solve it with the logic of distributing 20 identical objects amongst 3 people, with any number allowed to anyone including 0. Using the ${}^{(N + R - 1)}C_{(R - 1)}$ formula, we get ${}^{22}C_2 = 231$.

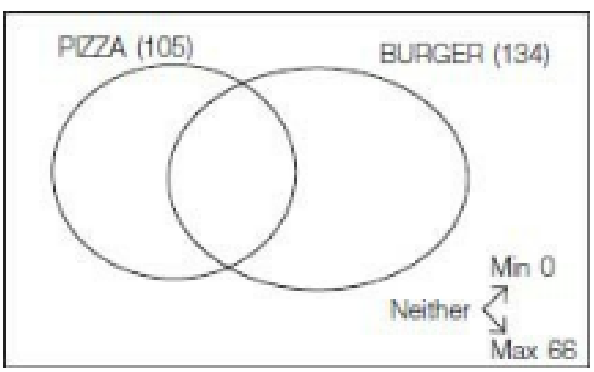
Hence, we will mark option (a).

Solutions for Questions 34 and 35:

If we consider the number of projects for Gyani alone as 'g' and Buddhi alone as 'b', we will get the equation: $g + b = 17$. Hence, we cannot determine uniquely the value of g or b.

However, for question 35, we know that $g + b + m = 18$. Since $g + b = 17$, $m = 1$. Hence, the answers are:



37. 

In this question, the key thought is that we do not know how many like neither. From the given circumstances, it can be seen that if 0 people like neither, then the central value x would become equal to 39. (Use the principle of double counting to get only pizza: 66, pizza and burger: 39 and only burger = 95).

The other extreme is when we maximise the number of people who like neither. From the given situation, we can see that out of 200 people, since 134 like burger, the maximum who would like neither would be 200 – 134 = 66. In such a case, we will get the following break up: Only pizza = 0, pizza and burger = 105, only burger = 29.

As we move the numbers who like neither from 0 to 66, the numbers for who like only burgers drops from 95 to 29. Hence, amongst the options, the only possible value that only burgers could have is 93. Hence, Option (d) is correct.

38. $(P\Delta Q) = \{1, 4, 5, 6\}$. $(R\Delta S) = \{1, 2, 3, 4, 7, 8, 10\}$

$(P\Delta Q)\Delta(R\Delta S) = \{1, 4, 5, 6\}\Delta\{1, 2, 3, 4, 7, 8, 10\} = \{2, 3, 5, 6, 7, 8, 10\}$. Hence, the number of elements in the given expression is 7. Option (a) is correct.

39. It can be easily seen and understood that the set B, is the set of all multiples of 35 (starting with 0 (for $n = 1$). Hence, set B {0, 35, 70, 105, 140, … and so on till infinity}

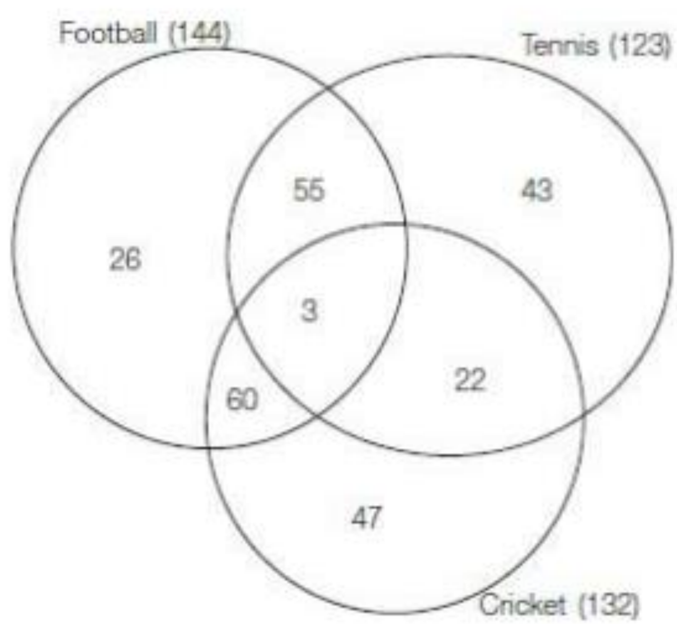
The first two elements of set A can be calculated using $n = 1$ and 2 respectively to get these elements as 35 and 1225. It can be observed that both these are also multiples of 35. However, the set A seems to be containing only a few multiples of 35 (and not all multiples of 35). If we can conclude that all the elements of set A are indeed multiples of 35, we can select the Option (b) which says that 'Every member of A is in B and at least one member of B is not in A'.

We can use the remainder theorem to confirm the same. The remainder of $6^{2n} \div 35$, would always be 1, while the remainder of $-1 \div 35$ would always be –1. Also, since $-35n$ would always be divisible by 35, we can conclude that the remainder of $(6^{2n} - 35n - 1) \div 35$ would always be 0. Hence, we can conclude that Option (b) is correct.

40. Let there be 'g' girls and 'b' boys in the junior level tournament. Also, let there be 'G' girls and 'B' boys in the senior-level tournament. From the statement: 'The number of girls versus girl matches in junior level is 153' we get the ${}^{g}C_2 = 153 \to g = 18$. We are given that $g + b = 43$. Hence, $b = 25$. Next, from the statement: the number of boys versus boy matches in senior level is 276,we get that ${}^{B}C_2 = 276 \to B = 24$ and since we are given that $B + G = 51$, it means that $G = 27$. The number of matches a boy plays against a girl is obtained by: Number of boy versus girl matches in the junior level + Number of boy versus girl matches in the senior level = ${}^{g}C_1 \times {}^{b}C_1 + {}^{G}C_1 \times {}^{B}C_1 = {}^{18}C_1 \times {}^{25}C_1 + {}^{27}C_1 \times {}^{24}C_1 = 450 + 648 = 1098$.

41. We can straightaway use the formula for a three-set intersection as: $F \cup T \cup C = 256 = F + T + C - F \cap T - C \cap T - F \cap C + F \cap C \cap T \to 256 = 144 + 123 + 132 - 58 - 25 - 63 + F \cap C \cap T$. Solving this, we get: $F \cap C \cap T = 3$.

This leads to the following Venn diagram:



CAT 2020 EXAMINATION PAPER

CAT 2020 SLOT 1

1. How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7?

2. The mean of all 4-digit even natural numbers of the form 'aabb', where a > 0, is

   (a) 5544

   (b) 4466

   (c) 4864

   (d) 5050

3. If $f(5 + x) = f(5 - x)$ for every real x, and $f(x) = 0$ has four distinct real roots, then the sum of these roots is

   (a) 20

   (b) 0

   (c) 40

   (d) 10

4. A circle is inscribed in a rhombus with diagonals 12 cm and 16 cm. The ratio of the area of circle to the area of rhombus is

   (a) $\frac{2\pi}{15}$

   (b) $\frac{5\pi}{18}$

(c) $\frac{3\pi}{25}$

(d) $\frac{6\pi}{25}$

5. The area of the region satisfying the inequalities $|x| - y \le 1, y \ge 0$ and $y \le 1$ is

6. If $x = (4096)^{7+4\sqrt{3}}$, then which of the following equals 64?

(a) $\frac{\frac{7}{x^{2}}}{\frac{4}{x^{\sqrt{3}}}}$

(b) $\frac{\frac{7}{x^{2}}}{x^{2\sqrt{3}}}$

(c) $\frac{x^{7}}{x^{4\sqrt{3}}}$

(d) $\frac{x^{7}}{x^{2\sqrt{3}}}$

7. Let A, B and C be three positive integers such that the sum of A and the mean of B and C is 5. In addition, the sum of B and the mean of A and C is 7. Then the sum of A and B is

(a) 5

(b) 4

(c) 7

(d) 6

8. A straight road connects points A and B. Car 1 travels from A to B and Car 2 travels from B to A, both leaving at the same time. After meeting each other, they take 45 minutes and 20 minutes, respectively, to complete their journeys. If Car 1 travels at the speed of 60 km/hr, then the speed of Car 2, in km/hr, is

(a) 100

(b) 80

(c) 70

(d) 90

9. Leaving home at the same time, Amal reaches office at 10:15 am if he travels at 8km/hr, and at 9:40 am if he travels at 15 km/hr. Leaving home at 9:10 am, at what speed, in km/hr, must he travel so as to reach office exactly at 10 am?

(a) 11

(b) 12

(c) 14

(d) 13

10. A solid right circular cone of height 27 cm is cut into two piece along a plane parallel to its base a height of 18 cm from the base. If the difference in volume of the two pieces is 225 cc, the volume, in cc, of the original cone is

(a) 256

(b) 232

(c) 243

(d) 264

11. Among 100 students, x_1 have birthdays in January, x_2 have birthdays in February, and so on. If x_0 = max (x_1, x_2, … x_{12}), then the smallest possible value of x_0 is

unpainted is two–thirds of the painted area then the perimeter of the rectangle in inches is

(a) $3\sqrt{\pi}\left(5+\frac{12}{\pi}\right)$

(b) $5\sqrt{\pi}\left(3+\frac{9}{\pi}\right)$

(c) $3\sqrt{\pi}\left(\frac{5}{2}+\frac{6}{\pi}\right)$

(d) $4\sqrt{\pi}\left(3+\frac{9}{\pi}\right)$

14. If a, b and c are positive integers such that $ab = 432$, $bc = 96$ and $c < 9$, then the smallest possible value of $a + b + c$ is

(a) 49

(b) 46

(c) 59

(d) 56

15. If y is a negative number such that $2^{y^2 \log_3 5} = 5^{\log_2 3}$, then y equals

(a) $-\log_2 (1/5)$

(b) $-\log_2 (1/3)$

(b) $\log_2 (1/3)$

(d) $\log_2 (1/5)$

16. Veeru invested ₹10000 at 5% simple annual interest, and exactly after two years, Joy invested ₹8000 at 10% simple annual interest. How many years after Veeru's investment, will their balances, i.e., principal plus accumulated interest, be equal?

16. Veeru invested ₹10000 at 5% simple annual interest, and exactly after two years, Joy invested ₹8000 at 10% simple annual interest. How many years after Veeru's investment, will their balances, i.e., principal plus accumulated interest, be equal?

17. A solution, of volume 40 litres, has dye and water in the proportion 2 : 3. Water is added to the solution to change this proportion to 2 : 5. If one–fourths of this diluted solution is taken out, how many litres of dye must be added to the remaining solution to bring the proportion back to 2 : 3?

18. A gentleman decided to treat a few children in the following manner. He gives half of his total stock of toffees and one extra to the first child, and then the half of the remaining stock along with one extra to the second and continues giving away in this fashion.

His total stock exhausts after he takes care of 5 children. How many toffees were there in his stock initially?

19. Two persons are walking beside a railway track at respective speeds of 2 and 4 km per hour in the same direction. A train came from behind them and crossed them in 90 and 100 seconds, respectively. The time, in seconds, taken by the train to cross an electric post is nearest to

(a) 82

(b) 87

(c) 78

(d) 75

20. In a group of people, 28% of the members are young while the rest are old. If 65% of the members are literates, and 25% of the literates are young, then the percentage of old people among the illiterates is nearest to

(a) 66

(b) 55

(c) 59

(d) 62

21. The number of real-valued solutions of the equation $2^x + 2^{-x} = 2 - (x - 2)^2$ is

(a) 2

(b) infinite

(c) 0

(d) 1

22. If $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$ then y equals

23. A train travelled at one-thirds of its usual speed, and hence reached the destination 30 minutes after the scheduled time. On its return journey, the train initially travelled at its usual speed for 5 minutes but then stopped for 4 minutes for an emergency. The percentage by which the train must now increase its usual speed so as to reach the destination at the scheduled time, is nearest to

(a) 67

(b) 61

(c) 50

(d) 58

24. The number of distinct real roots of the equation $\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)+2=0$ equals

25. A person spent ₹50000 to purchase a desktop computer and a laptop computer. He sold the desktop at 20% profifit and the laptop at 10% loss. If overall he made a 2% profifit then the purchase price, in rupees, of the desktop is

26. How many distinct positive integer–valued solutions exist to the equation $(x^2-7x+11)^{(x^2-13x+42)}=1$?

(a) 6

(b) 2

(c) 8

(d) 4

CAT 2020 SLOT 2

1. Let C be a circle of radius 5 meters having center at O. Let PQ be a chord of C that passes through points A and B where A is located 4 meters north of O and B is located 3 meters east of O. Then, the length of PQ, in meters, is nearest to

(a) 7.8

(b) 7.2

(c) 8.8

(d) 6.6

3. The distance from B to C is thrice that from A to B. Two trains travel from A to C via B. The speed of train 2 is double that of train 1 while traveling from A to B and their speeds are interchanged while traveling from B to C. The ratio of the time taken by train 1 to that taken by train 2 in travelling from A to C is

(a) 4:1

(b) 5:7

(c) 7:5

(d) 1:4

4. The number of integers that satisfy the equality $(x^2-5x+7)^{x+1}=1$ is

(a) 3

(b) 5

(c) 2

(d) 4

5. John takes twice as much time as Jack to finish a job. Jack and Jim together take one–thirds of the time to finish the job than John takes working alone. Moreover, in order to finish the job, John takes three days more than that taken by three of them working together. In how many days will Jim finish the job working alone?

6. Students in a college have to choose at least two subjects from chemistry, mathematics and physics. The number of students choosing all three subjects is 18, choosing mathematics as one of their subjects is 23 and choosing physics as one of their subjects is 25. The smallest possible number of students who could choose chemistry as one of their subjects

is

(a) 19

(b) 21

(c) 20

(d) 22

7. If x and y are positive real number satisfying $x + y = 102$, then the minimum possible value of $2601\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)$ is

8. The sum of the perimeters of an equilateral triangle and a rectangle is 90 cm. The area, T, of the triangle and the area, R, of the rectangle, both in sq cm, satisfy the relationship $R = T^2$. If the sides of the rectangle are in the ratio 1:3, then the length, in cm, of the longer side of the rectangle, is

(a) 18

(b) 24

(c) 21

(d) 27

9. A and B are two points on a straight line. Ram runs from A to B while Rahim runs from B to A. After crossing each other, Ram and Rahim reach their destinations in one minute and four minutes, respectively. If they start at the same time, then the ratio of Ram's speed to Rahim's speed is

(a) $\sqrt{2}$

(b) $2\sqrt{2}$

(c) $\frac{1}{2}$

(d) 2

10. In a group of 10 students, the mean of the lowest 9 scores is 42 while the mean of the highest 9 scores is 47. For the entire group of 10 students, the maximum possible mean exceeds the minimum possible mean by

(a) 3

(b) 6

(c) 4

(d) 5

11. Anil buys 12 toys and labels each with the same selling price. He sells 8 toys initially at 20% discount on the labeled price. Then he sells the remaining 4 toys at an additional 25% discount on the discounted price. Thus, he gets a total of ₹2112, and makes a 10% profit. With no discounts, his percentage of profit would have been

(a) 60

(b) 55

(c) 50

(d) 54

12. Let the mth and nth terms of a geometric progression be $\frac{3}{4}$ and 12, respectively, where $m < n$. If the common ratio of the progression is an integer r, then smallest possible value of $r + n - m$ is

(a) 2

(b) −2

(c) –4

(d) 6

13. For the same principal amount, the compound interest for two years at 5% per annum exceeds the simple interest for three years at 3% per annum by ₹1125. Then the principal amount in rupees is

14. For real x, the maximum possible value of $\frac{x}{\sqrt{1+x^4}}$ is

(a) $\frac{1}{\sqrt{2}}$

(b) 1

(c) $\frac{1}{\sqrt{3}}$

(d) $\frac{1}{2}$

15. Let C1 and C2 be concentric circles such that the diameter of C1 is 2 cm longer than that of C2. If a chord of C1 has length 6cm and is tangent to C2, then the diameter, in cm, of C1 is

16. A sum of money is split among Amal, Sunil and Mita so that the ratio of the shares of Amal and Sunil is 3 : 2, while the ratio of the shares of Sunil and Mita is 4 : 5. If the difference between the largest and the smallest of these three shares is ₹400, then Sunil's share, in rupees, is

17. The value of $\log_a\left(\frac{a}{b}\right)+\log_b\left(\frac{b}{a}\right)$, for $1 < a \le b$ cannot be equal to

(a) –0.5

(b) 1

(c) 0

(d) –1

18. From an interior point of an equilateral, triangle, perpendiculars are drawn on all three sides. The sum of the lengths of the three perpendiculars is s. Then the area of the triangle is

(a) $\frac{2s^2}{\sqrt{3}}$

(b) $\frac{s^2}{\sqrt{3}}$

(c) $\frac{s^2}{2\sqrt{3}}$

(d) $\frac{\sqrt{3}s^2}{2}$

19. If x and y are non–negative integers such that $x + 9 = z$, $y + 1 = z$ and $x + y < z + 5$, then the maximum possible value of $2x + y$ equals

20. In how many ways can a pair of integers (x, a) be chosen such that $x2 - 2|x| + |a - 2| = 0$?

(a) 4

(b) 6

(c) 7

(d) 5

21. How many 4-digit numbers, each greater than 1000 and each having all four digits distinct, are there with 7 coming before 3?

22. Aron bought some pencils and sharpeners. Spending the same amount of money as Aron, Aditya bought twice as many pencils and 10 less sharpeners. If the cost of one sharpener is ₹2 more than the cost of a pencil, then the minimum possible number of pencils bought by Aron and Aditya together is

(a) 27

(b) 36

(c) 33

(d) 30

23. Two circular tracks T1 and T2 of radii 100 m and 20 m, respectively touch at a point A. Starting from A at the same time, Ram and Rahim are walking on track T1 and track T2 at speeds 15 km/hr and 5 km/hr respectively. The number of full rounds that Ram will make before he meets Rahim again for the first time is

(a) 2

(b) 5

(c) 3

(d) 4

24. The number of pairs of integers (x, y) satisfying $x \geq y \geq -20$ and $2x + 5y = 99$ is

25. In May, John bought the same amount of rice and the same amount of wheat as he had bought in April, but spent ₹150 more due to price increase of rice and wheat by 20% and 12%, respectively. If John had spent ₹450 on rice in April, then how much did he spend on wheat in May?

(a) ₹580

(b) ₹590

(c) ₹560

(d) ₹570

26. Let $f(x) = x^2 + ax + b$ and $g(x) = f(x + 1) - f(x - 1)$. If $f(x) \geq 0$ for all real x, and $g(20) = 72$, then the smallest possible value of b is

(a) 1

(b) 16

(c) 4

(d) 0

CAT 2020 SLOT 3

1. Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if

(a) $|k| = 2$

(b) $k \neq 2$

(c) $k = 2$

(d) $|k| \neq 2$

2. Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point?

(a) 4.6

(b) 5.2

(c) 4.8

(d) 4.2

3. How many of the integers 1, 2, … , 120, are divisible by none of 2, 5 and 7?

(a) 43

(b) 40

(c) 41

(d) 42

4. A and B are two railway stations 90 km apart. A train leaves A at 9:00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10:30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at

(a) 11:20 am

(b) 11:00 am

(c) 10:45 am

(d) 11:45 am

5. A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to

(a) 35

(b) 25

(c) 31

(d) 22

6. In a trapezium ABCD, AB is parallel to DC, BC is perpendicular to DC and $\angle BAD = 45°$, If DC = 5cm, BC = 4 cm, the area of the trapezium in sq cm is

7. If $x_1 = -1$ and $x_m = x_{m+1} + (m+1)$ for every positive integer m, then x_{100} equals

(a) −5150

(b) −5051

(c) −5050

(d) −5151

8. $\frac{2 \times 4 \times 8 \times 16}{(\log_2 4)^2 (\log_4 8)^3 (\log_8 16)^4}$ equals

9. The points (2, 1) and (−3, −4) are opposite vertices of a parallelogram. If the other two vertices lie on the line $x + 9y + c = 0$, then c is

(a) 13

(b) 14

(c) 12

(d) 15

10. If a, b, c are non-zero and $14_a = 36_b = 84_c$, then $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ is equal to

11. Let N, x and y be positive integers such that N = $x + y$, $2 < x < 10$ and $14 < y < 23$. If N > 25, then how many distinct values are possible for N?

12. A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became ₹18522. The amount, in rupees, that the person had invested is

13. Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is

14. The vertices of a triangle are (0, 0), (4, 0) and (3, 9). The area of the circle passing through these three points is

(a) $\frac{12\pi}{5}$

(b) $\frac{205\pi}{9}$

(c) $\frac{123\pi}{7}$

(d) $\frac{14\pi}{3}$

15. Vimla starts for office every day at 9 am and reaches exactly on time if she drives at her usual speed of 40 km/hr. She is late by 6 minutes if she drives at 35 km/hr. One day, she covers two-thirds of her distance to office in one-thirds of her usual time to reach office, and then stops for 8 minutes. The speed, in km/hr, at which she should drive the remaining distance to reach office exactly on time is

(a) 26

(b) 27

(c) 28

(d) 29

16. How many integers in the set {100, 101, 102, ..., 999} have at least one digit repeated?

17. The area, in sq. units, enclosed by the lines $x = 2, y = |x-2|+4,$ the x-axis is equal to

(a) 8

(b) 6

(c) 12

(d) 10

18. In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is

(a) 417

(b) 399

(c) 439

(d) 357

19. A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time?

20. Let m and n be positive integers, If $x^2 + mx + 2n = 0$ and $x^2 + 2nx + m = 0$ have real roots, then the smallest possible value of $m + n$ is

(a) 8

(b) 7

(c) 6

(d) 5

21. If $f(x + y) = f(x) f(y)$ and $f(5) = 4$, then $f(10) - f(-10)$ is equal to

(a) 0

(b) 3

(c) 15.9375

(d) 14.0625

22. How many pairs (a, b) of positive integers are there such that $a \le b$ and $ab = 4_{2017}$?

(a) 2020

(b) 2019

(c) 2017

(d) 2018

23. Two alcohol solutions, A and B, are mixed in the proportion 1 : 3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is

(a) 92%

(b) 94%

(c) 90%

(d) 89%

24. Let $\log_a 30 = A$, $\log_a \frac{5}{3} = -B$ and $\log_2 a = \frac{1}{3}$, then $\log_3 a$ equals

(a) $\frac{2}{A+B-3}$

(b) $\frac{A+B-3}{2}$

(c) $\frac{A+B}{2}-3$

(d) $\frac{2}{A+B}-3$

25. Let m and n be natural numbers such that n is even and $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$. Then $m - 2n$ equals

(a) 3

(b) 4

(c) 1

(d) 2

26. A batsman played $n + 2$ innings and got out on all occasions. His average score in these $n + 2$ innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first n innings. In these n innings, his average score was 30 runs and lowest score was x runs. The smallest possible value of x is

(a) 1

(b) 4

(c) 2

(d) 3

ANSWER KEY

CAT 2020 SLOT 1

1. 21
2. (a)
3. (a)
4. (d)
5. 3
6. (b)
7. (d)
8. (d)
9. (b)
10. (c)
11. (b)
12. (d)
13. (a)
14. (b)
15. (c)
16. 12 Years
17. 8
18. 62
19. (a)
20. (a)
21. (c)
22. 36
23. (a)
24. 1

25. 20000
26. (a)

CAT 2020 SLOT 2

1. (c)
2. (b)
3. (b)
4. (a)
5. 4
6. (c)
7. 2704
8. (d)
9. (d)
10. (c)
11. (c)
12. (b)
13. 90000
14. (a)
15. 10
16. 400
17. (b)
18. (b)
19. 23
20. (c)
21. 315
22. (c)
23. (c)
24. 7

25. (c)
26. (c)

CAT 2020 SLOT 3

1. (d)
2. (c)
3. (c)
4. (b)
5. (b)
6. 28
7. (c)
8. 24
9. (b)
10. 3
11. 6
12. 16000
13. 18 Years
14. (b)
15. (c)
16. 252
17. (d)
18. (b)
19. 40 Men
20. (c)
21. (c)
22. (d)

23. (a)

24. (a)

25. (c)

26. (c)

Solutions and Shortcuts

CAT 2020 SLOT 1

1. We are looking for 3 digit numbers, whose product of digits is either 3,4,5 or 6. For product to be 3, the number has to be formed using the digits 1,1,3 ⇒ 3 numbers (113,131 and 311); For product = 4, the digits used should be 2,2,1 or 4,1,1. There will be a total of 3 + 3 = 6 such numbers. For product to be 5, the number has to be formed using the digits 1,1,5 ⇒ 3 numbers (115,151 and 511). For product = 6, the digits used should be 6,1,1 or 1,2,3. There will be a total of 3 + 6 = 9 such numbers. Total 3 + 6 + 3 + 9 = 21 numbers.
2. The numbers will be 1100, 1122, 1144, 1166, 1188, 2200, 2222, ...9988. There will be a total of 100 numbers of this type. The average value of the last two digits will be the average of 00, 22, 44, 66 and 88, which is 44 (since these numbers will appear an equal number of times in the last two digits of the numbers formed). The average of the first two digits of the number would be the average of 11,22,33,44,55,..99 ⇒ which is 55. Hence, the correct answer is 5544. Option (a) is correct.
3. Since the value of $f(5 + x)$ and $f(5 - x)$ are equal, it means that if a root exists at $f(6)$, it will also exist at $f(4)$. This is because the function can be thought of as $f(6) = f(4)$ when $x = 1$; $f(7) = f(3)$ when $x = 2$; $f(8) = f(4)$ when $x = 3$. Further, the function will also equate itself for decimal values of x. For instance, at $x = 1.5$, we can see that $f(6.5) = f(3.5)$. Thus, whenever we discover a root at $f(5 + x)$, there is also a root at $f(5 - x)$. This means that

each pair of roots will have their arguments adding to 10. Since, there are a total of 4 roots, it means that there are two pairs of roots. Thus, the sum of the roots is 10 + 10 = 20

4. In the figure, AB = 10, since it is the hypotenuse of the right triangle AOB, with AO = 6 and OB = 8. (**Note:** AO = 6, since AC = 12, while OB = 8, since BD = 16). Let, OT = r = radius of the inscribed circle.

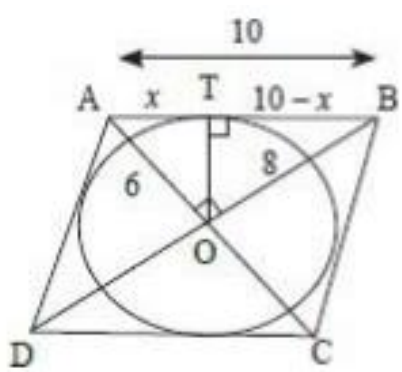
   In right triangle OTB, $r^2 + (10 - x)^2 = 8^2$, while in right triangle AOT, $x^2 + r^2 = 36$. Subtracting the second equation from the first we get $20x = 72$ and hence, $x = 3.6$. Consequently, from the second equation, we have $r^2 = 36 - x^2 = 36 - (3.6)^2 = 23.04$. Also, the area of the rhombus is given by: $\frac{1}{2} \times$ product of diagonals

   $\frac{1}{2} \times 192 = 96.$

   Required ratio = $\frac{23.04\pi}{96} = \frac{23.04\pi \times 25}{96 \times 25}$

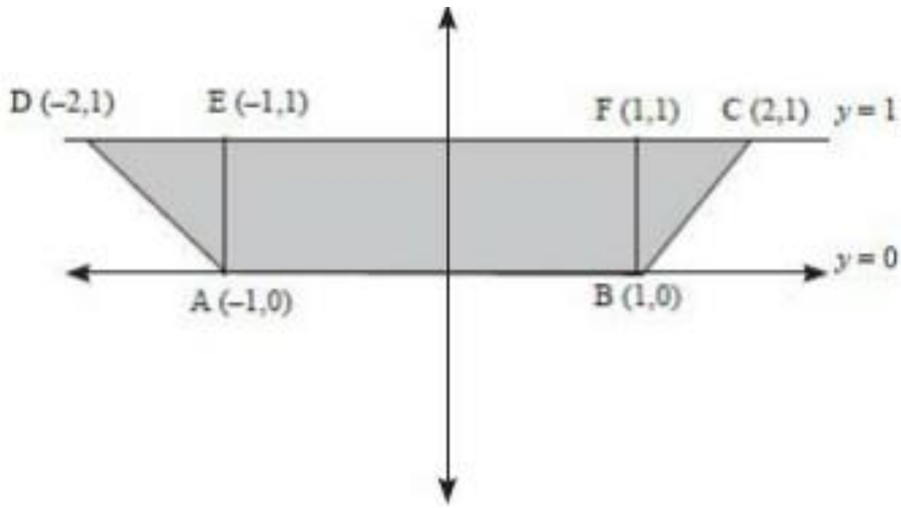
   $= \frac{576\pi}{96 \times 25} = \frac{6\pi}{25}$

   Option (d) is correct.



5. The area formed is the area shown by the shaded region ABCD in the figure. It can be calculated using

   Required area = (Area of triangle ADE + Area of triangle BCF + Area of rectangle ABFE) = 0.5 + 0.5 + 2 = 3. The correct answer is 3.



6. $x = 4096^{7+4\sqrt{3}} = (2^{12})^{7+4\sqrt{3}} = 2^{84+48\sqrt{3}}$.

Checking the options we see that

$x^{7/2} = 2^{294+168\sqrt{3}}$. Also $x^{2\sqrt{3}} = 2^{168\sqrt{3}+288}$

Option (b) gives us : $\frac{2^{294+168\sqrt{3}}}{2^{168\sqrt{3}+288}} = 64$.

7. It is given to us that:

$$A + \frac{B+C}{2} = 5 \Rightarrow 2A + B + C = 10.$$

Also, $2B + A + C = 14$

Thus, we get: B – A = 4 and also 3A + 3B + 2C = 24. Checking the options, if we choose A + B = 6 from option (d), we get: 2C = 6 ⇒ C = 3. Also, since B – A = 4, we get B = 5 and A = 1. These values satisfy all the equations. Hence, this is the correct option. If we try to use a wrong option (say A + B = 7 from option (c)), we get: 2C = 3 ⇒ C = 1.5, which is not possible, since A,B,C are integers. Option (a) also gets rejected, since we get C as a decimal value, which is not possible. If we tried option (b), A + B = 4, we get C = 6. This gives us B = 4 and A = 0, which is again not possible, since A,B,C are positive integers. Hence, Option (d) is correct.

8. Since the speeds of the cars are constant we have

from the A to M journey: $\frac{\text{Speed of car 1}}{\text{Speed of car 2}} = \frac{20}{t}$

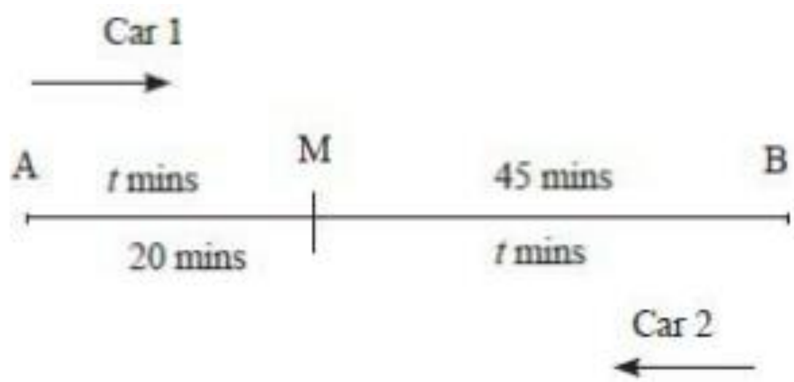
From the M to B journey, we have:

$$\frac{\text{Speed of car 1}}{\text{Speed of car 2}} = \frac{t}{45}$$

Hence, $\frac{\text{Speed of car 1}}{\text{Speed of car 2}} = \frac{20}{t} = \frac{t}{45}$

$\Rightarrow t = 30$ mins.

Hence, the ratio of $\frac{\text{Speed of car 1}}{\text{Speed of car 2}} = \frac{2}{3}$. Hence, if Car 1 travels at 60 kmph, Car 2 travels at 90 kmph. Option (d) is correct.



9. From a speed of 8 kmph, to 15 kmph, Amal, has increased his speed by $\frac{7}{8}$. This means that his time will go down by $\frac{7}{15}$ (since the distance is constant). The time actually has gone down by 35 minutes. Hence, if the original time required is t mins, then $\frac{7}{15} \Rightarrow t = 75$ mins. Hence, since he reaches office at 10:15, it follows that he must have left home at 9 AM. Thus, the distance = 8 × 1.25 = 10 kms. If he leaves at 9:10 and has to reach at 10 AM, he has to reach in 50 minutes or $\frac{5}{6}$ of an hour. The speed required should be such that: $s \times \frac{5}{6} = 10 \Rightarrow s = 12$ kmph.

Option (b) is correct.

10. When you cut the solid circular cone at a height of 18 from the base, it means you are cutting it at a height of 9 from the top. Since, the height of the new cone thus formed is 1/3rd the height of the original cone, the volume of the new cone will be $\frac{1}{27}$ of the volume of the original cone and the volume of the frustum of the cone formed, would be $\frac{26}{27}$ of the original cone. Assume the volume of the original cone is 27V, then the volume of the frustum would be 26V and the volume of the new cone (smaller one with height 9), would be V. The difference between 26V and V = 25V is given to us as 225.

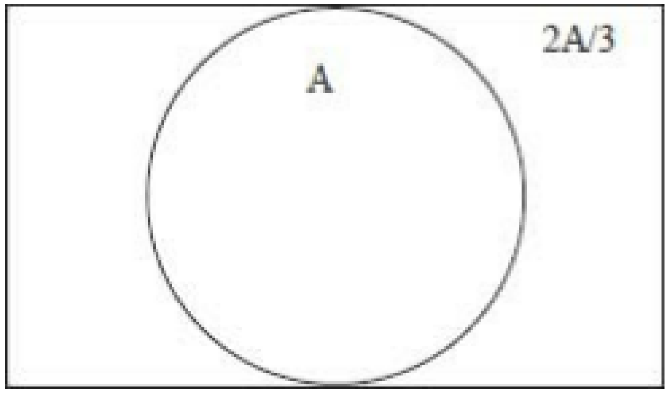
Hence, V = $\frac{225}{25}$ = 9. Thus, the volume of the original cone = 27V = 279 = 243.

Option (c) is correct.

11. This question is based on the pigeon hole principle. The required answer will be derived if we equally place 8 students having birthdays in each month from January to December. This would account for the birthday of 96 children. The birthday of the remaining 4 children can be distributed as 1 each to 4 different months amongst January to December. Thus, there will be a pattern that will look like: x_0 = Max (8, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9). The required maximum is 9. Hence, option (b) is correct.

12. The weights of the three alloys would be in the ratio: 3 × 4 : 4 × 2 : 7 × 6 = 15:8:42. Since, the total weight is 130 kg, it means that the weight of the heaviest piece would be 42 × 2 = 84 kg. Option (d) is correct.

13. The given sheet of metal will look as shown in the figure.



Since, the total area of the rectangular sheet is 135, we get that the area of the circle A = 81 and $\frac{2A}{3}$ = 54. We can calculate the radius of the circle and consequently find the height of the rectangle by doubling the radius (since the height of the rectangular sheet would be equal to the diameter of the circle). $\pi \times r^2 = 81$

$$\Rightarrow r = \frac{9}{\sqrt{\pi}}$$

Hence, the height of the rectangle

= diameter of the circle = $\frac{18}{\sqrt{\pi}}$.

We are also given the area of the rectangular sheet as 135. Using:

Area of a rectangle = $b \times h$

$$\Rightarrow 135 = \frac{18}{\sqrt{\pi}} \times b$$

$$\Rightarrow b = \frac{15\sqrt{\pi}}{2}.$$

The perimeter of the rectangle would be:

$$2 \times \frac{18}{\sqrt{\pi}} + 2 \times \frac{15\sqrt{\pi}}{2}$$

$$= \frac{36}{\sqrt{\pi}} + 15\sqrt{\pi} = 3\sqrt{\pi}\left(5 + \frac{12}{\pi}\right)$$

Hence, option (a) is correct.

14. We can form the following possibilities for *a*,*b*,*c*. **Note:** We start with $bc = 96$ to find pairs of values of b and c, using $c < 9$ as a constraint and then try to calculate the value of a using $ab = 432$

a	b	c	$a + b + c$
Not integral	96	1	NA
9	48	2	59
Not integral	32	3	NA
18	24	4	46
27	16	6	49
36	12	8	56

Hence, option (b) is correct.

15. Taking logs to base 3 on both sides, we get: $y^2\log_3 5 \times \log_3 2 = \log_2 3 \times \log_3 5$

$\Rightarrow y^2 = (\log_2 3)^2$.

Thus, $y = \pm\log_2 3$. Since, it is given to us that y is a negative number, it follows that we should consider $y = -\log_2 3 = \log_2(1/3)$. Option (c) is correct.

16. After 2 years, Veeru's amount would be 10000 + 1000 = 11000. Further his interest earned per year will continue to be 5% of 10000 = 500 (since, it is simple interest). Joy, invests 8000 and 10% pa simple interest. This means that he would earn 800 per year simple interest. Thus, Joy would catch up with Veeru's amount at the rate of 300 rupees a year. He will take 10 years to catch up. Hence, the total time required to catch up after Veeru's investment would be 2 + 10 = 12 years.

17. Initially the solution has 16 dye and 24 water. To this we have to add 16 water to make the dye to water equal a ratio of 2:5. At this stage there will be 16 dye and 40 water, a total of 56 liters. When one fourth of this

diluted mixture is withdrawn, 14 liters would be withdrawn out of the 56 liters of mixture. Dye and water will come out in the same proportion (i.e.2:5). Thus, dye withdrawn = 4 and water withdrawn = 10. Dye left = 16 – 4 = 12, water left = 40 – 10 = 30. We would need to add 8 liters of water to make it's dye and water ratio as 20:30 = 2:3. Hence, the correct answer is 8.

18. Before the 5th child is given toffees, the gentleman would have 2 toffees. Only then, would he be left with 0, if he gives half of his stock and one extra. Prior to the 4th child, the gentleman would be left with 6 toffees – from 6 if you were to give half the stock + one extra you would give out 4 toffees and be left with 2. Prior to the 3rd child the gentleman would have 14 toffees (14/2 = 7 plus 1 = 8 and 14 – 8 = 6). The series of numbers in reverse order for toffees left would be:

Toffees at the end	0
Toffees left after 4th child	2
Toffees left after 3rd child	6
Toffees left after 2nd child	14
Toffees left after 1st child	30
Toffees left before 1st child	62

Hence, the correct answer is 62.

19. Let the speed of the train be S kmph. Then, according to the condition given in the question we have:

$(S - 2) \times 90 = (S - 4) \times 100$
$\Rightarrow S = 22$ kmph

Total length of train = $(22 - 2) \times \frac{5}{18} \times 90$

= 500 meters.

Hence, the time required to cross an electric post would be: $\frac{500}{22 \times \frac{5}{18}} \approx 82$ seconds.

20. You can think of the numbers as shown here:

Young (28)		Old (72)	
Literate	Illiterate	Literate	Illiterate
16.25 (= 25% of 65)	11.75 (=28 – 16.25)		23.25 (= 35 – 11.75)

The required percentage of old people among the illiterates is $\frac{23.25}{35} \approx 66$ percent

21. The given expression on the LHS is of the form:

$$2^x + \frac{1}{2^x}.$$

The minimum value of the LHS is 2 and occurs at $x = 0$. The RHS, at $x = 0$ is 2 –(4) = –2. The RHS becomes equal to 2 only at $x = 2$, but at that value the LHS is not equal to 2. Hence, since the LHS ≥ 2 and the RHS ≤ 2, and they are not equal to 2 at the same time, it means that there is no solution where the LHS and RHS of the given equation are equal. Hence, Option (c) is correct.

22. $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$

$$\Rightarrow \log_4 5 = \frac{(\log_4 y)(\log_4 \sqrt{5})}{\log_4 6}$$

$$= \frac{1}{2} \times \frac{(\log_4 y)(\log_4 5)}{(\log_4 6)}$$

Canceling $\log_4 5$ from both sides, we get:

$\log_4 y = 2 \log_4 6$

$\Rightarrow \log_4 y = \log_4 36$.

Hence, the value of $y = 36$.

23. From the first statement, we get that the train must have taken thrice the time it normally takes, since it traveled at 1/3rd of its speed for the same distance. Thus, if 't' is the original time it takes, on this day it took time '$3t$'. Extra time taken = $3t - t = 2t$ = 30 minutes

$\Rightarrow$ Normal time = 15 minutes.

On the return journey, in 5 minutes at normal speed, the train would cover 1/3rd the distance. It would be left with a journey of 10 minutes at normal speed. However, it has to cover this distance in 6 minutes $\Rightarrow$ since it stopped for 4 minutes. This entails a 40% decrease in time taken for the same distance – which, means a 66.66% increase in speed would be required. Option (a) is correct.

Note: Alternately, we could have thought of this as: drop in time: $\frac{2}{5}$;

Hence, increase in speed = $\frac{2}{3}$.

24. Think of the expression as $A^2 - 3A + 2 = 0$.

The value of A = $\frac{3 \pm \sqrt{9-8}}{2}$ = 1 or 2.

However, an expression like $x + \frac{1}{x}$ cannot be equal to 1, since it's minimum value is 2. Hence, there is only one real root of the equation, when $x + \frac{1}{x} = 2$, viz: $x = 1$. The correct answer is that there is only one distinct real root for the equation.

25. Using the concept of alligation, you can work out that the ratio of the price of the desktop to the laptop is 2:3. **Note:** In the case of profit and loss situations, the average profit percentage created on the sale of two items of different costs, is the weighted average of the individual profits on the two items. The weights to be used in such a case would be the ratio of the costs of the two items. In this question, we are given that he makes a profit of 20% and a loss of 10% on the desktop and laptop respectively. Also that he makes an overall profit of 2% ⇒ which is what gives us that the ratio of the price of the desktop to the laptop is 2:3. Hence, the price of the desktop is 20000.

26. An expression of the form A^B equals a value of 1, in three cases: Case 1: B = 0, A can take any value except 0, as then $A^0 = 1$; Case 2: A = 1, B can take any value; Case 3: A = – 1, B can take any even value.

Checking the given expression for each of the three cases.

Checking for Case 1

B = 0: $x^2 - 13x + 42 = 0$

⇒ $x = 6,7$. Two solutions.

Checking for Case 2

A = 1 and

B ≠ 0: $x^2 - 7x + 11 = 1$

⇒ $x^2 - 7x + 10 = 0$

⇒ $x = 2,5$ and

we already know that B = 0 is occurring at $x = 6,7$. Hence, we can count both these values as solutions.

Checking for Case 3

A = – 1 and B is even: $x^2 - 7x + 11 = -1$

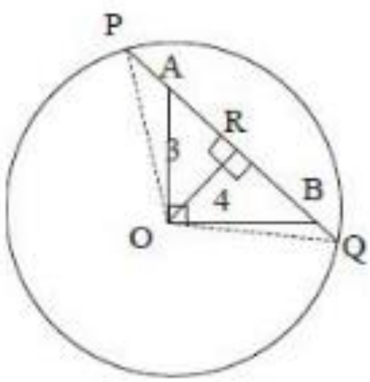
$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x = 3,4$ and

we can check at $x = 3$ and $x = 4$, the value of $x^2 - 13x + 42$ is even. Hence, we can count both these values as solutions.

Thus, there are a total of 6 solutions in this case.

CAT 2020 SLOT 2

1. 

O is the center of the circle and the radius of the circle = 5

In triangle AOB, AB = 5 (Pythagoras theorem).

Drop a perpendicular from O to R as shown. Using the property of right angled triangles,

$OR = \frac{3 \times 4}{5} = 2.4$. OR being dropped from the centre as a perpendicular to the chord, means that it bisects the chord. Thus, we can find the length of PR and double it to get PQ.

In triangle ORP, $OP^2 = OR^2 + PR^2$, we get: $OR^2 = 19.24$, or $OR = \sqrt{19.24} \approx 4.4$

Hence, PQ ≈ 8.8. Option (c) is correct.

2. Let the race be for a distance of '*d*'. Then according to the question, we have $\frac{d}{d-50} = \frac{d-45}{d-90}$.

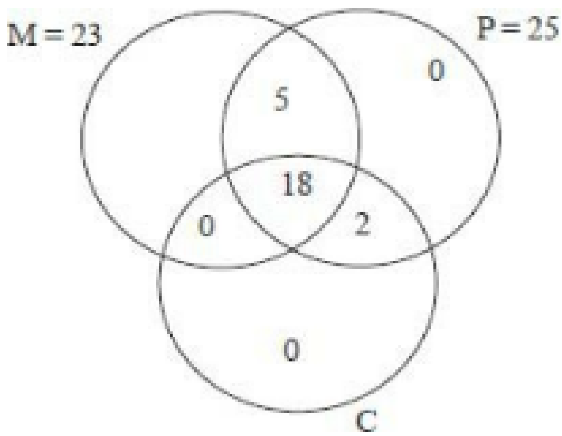
Inserting values for *d* from the options, we see that *d* = 450, satisfies this equation. Hence, Option (b) is correct.

3. Let the distance from A to B be 50 km and from B to C be 150 km. Let the initial speeds of the two trains be 25 and 50 kmph. Train 1 will cover the distance in: $\frac{50}{25} + \frac{150}{50} = 5$ hours; Train 2 will cover the distance in: $\frac{50}{50} + \frac{150}{25} = 7$hours. Required ratio of time taken is 5:7. Option (b) is correct.

4. In general the value of an expression like A^B can be equal to 1 for 3 cases. First: If B = 0, the value of A does not matter, since A^0 is always equal to 1; Second: If A = 1, the value of B does not matter since $1^B = 1$, no matter what value B takes. Also, for A = –1, B should be even for A^B to be 1. In this situation, we have: for $x = -1$, the first case takes place. For the second case to happen, $x^2 - 5x + 7 = 1 \rightarrow x^2 - 5x + 6 = 0 \rightarrow x = 2$ and 3. If we look for the third case, we want to get integer values of x, for $x^2 - 5x + 7 = -1 \rightarrow x^2 - 5x + 8 = 0$. But this equation has imaginary roots and hence the third case would not occur here. Thus there are a total of 3 integer values of x. Option (a) is correct.

5. If John takes time $2t$, then Jack takes time t for the same work. This means that the work ratio of John and Jack are in the ratio 1:2. Also, it is given to us that Jack and Jim finish the work in one third the time that John takes to finish the work. This means that the work rate of Jack and Jim (combined) is thrice the work rate of John alone. Hence, the ratio of work

rates of John, Jack and Jim is 1:2:1. If the total work is taken as 4 units, and their individual rates of work are taken as 1 unit, 2 units and 1 unit per day, we get that together they will take 1 day, while John will take 4 days (i.e. 3 days more than they take together). This satisfies the given condition and hence, Jim will finish the work in $\frac{4}{1} = 4$ days. The correct answer is 4.

6. 

Since, we are given the value of all 3 as 18 and that to keep choosing Chemistry as minimum, it means that we need to maximise the number of people who chose Physics and Mathematics. This can be maximised at 5. Hence, the minimum number of people who chose Chemistry is 18 + 2 = 20.

7. To find the minimum possible value keep x and y as close as possible

So, $x = y = 51$

$$= 2601\left(1 + \frac{1}{51}\right)\left(1 + \frac{1}{51}\right) = 52 \times 52 = 2704.$$

8. Let the lengths of the sides of the rectangle be 'l' and '$3l$' respectively. The area of the rectangle R = $3l^2 = T^2 \Rightarrow T = l\sqrt{3}$ = area of the triangle.

Also, area of an equilateral triangle having area 'a' is given by:

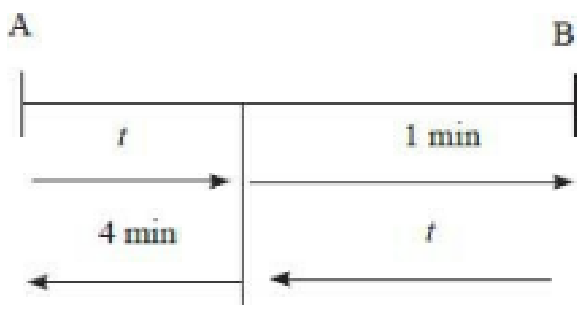
$$\frac{\sqrt{3}}{4}a^2 = l\sqrt{3} \Rightarrow a = 2\sqrt{l}$$

The sum of the perimeters

$= 6\sqrt{l} + 8l = 90 \Rightarrow l = 9.$

Hence, the length of the longer side of the rectangle = 27.

Option (d) is correct.

9. 

Distance is constant, So ratio of their time taken is same

$\frac{t}{4} = \frac{1}{t}.$

$t_2 = 4$

$t = 2$

Ram takes 3 minutes and Rahim takes 6 minutes

So Ratio of Ram's speed : Rahim's speed

= 2 : 1 = 2

Option (d) is correct.

10. Let the numbers be *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i* and *j*, such that a is the smallest number and *j* the largest. The average of the first 9 numbers is given as 42, hence the sum of the first 9 numbers (*a* to *i*) would be 42 × 9 = 378. Likewise, the sum of the last 9 numbers (*b* to *j*) would be 47 × 9 = 423. Thus, *j* – *a* = 45. The instance of the maximum average would be if we take a to *i* as 42 each and *j* as 87. In such a case, the sum of the 10 num-

bers is 378 + 87 = 465 and their average is 46.5. The other extreme is the minimum average, this would occur when the values of a = 2 and b to j is 47 each. In this case, the total is 423 + 2 = 425 and the minimum average is 42.5.

(**Note:** we cannot take a = 0 and i = 45 as in such a case, there will be at least 1 number greater than j to achieve an average of 47 for the last 9 numbers).

The required difference of averages is

46.5 – 42.5 = 4

Option (c) is correct.

11. Assume, he buys each toy and labels at ₹ 1 each. Thus, the labeled price of 12 toys would be a total of rupees 12. He sells, 8 toys at 0.8 and 4 toys at 0.6. Hence, he recovers a total of 6.4 + 2.4 = ₹ 8.8. This represents a profit of 10% on his cost, which means that his cost is $\frac{8.8}{1.1} = 8$rupees. If he were able to sell all his toysat the labeled price, he would have recovered ₹ 12 on a cost of 8 rupees. This would mean a profit percentage of 50%.

    Option (c) is correct.

12. The GP to minimise $r + n - m$, can be visualised as $\frac{3}{4}, -3, 12$. In this case, r = –4, m = 1 and n = 3. $r + n - m = -4 + 3 - 1 = -2$.

    Option (b) is correct.

13. P $(1.05)_2 - P$ = CI

    $$P \times \frac{3 \times 3}{100} = SI$$

    CI = 0.1025 P

SI = 0.09P

CI – SI = 0.0125P = $\left(\frac{125}{10000}\right)P$ = 1125

P = 90000

14. Dividing both numerator and denominator by x,

$$\frac{1}{\sqrt{\frac{1+x^4}{x^2}}}=\frac{1}{\sqrt{\frac{1}{x^2}+x^2}}$$

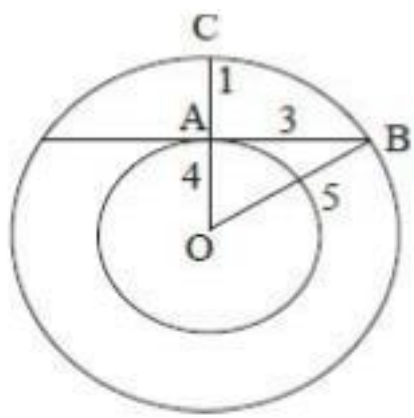
$$\frac{1}{x^2}+x^2\geq 2$$

$$\sqrt{\left[\frac{1}{x^2}+x^2\right]}\geq\sqrt{2}$$

$$\frac{1}{\sqrt{\frac{1}{x^2}+x^2}}\leq\frac{1}{\sqrt{2}}$$

15. We can visualise the 3-4-5 Pythagoras triplet as shown in the figure.

Hence, the radius of C_1 = 5 and it's diameter is 10.



16. The ratio of money split between Amal, Sunil and Mita will be 6:4:5 or $6x$, $4x$ and $5x$. The difference between the largest and the smallest, i.e. $6x$ and $4x$ is given as 400.

Hence, $2x$ = 400

⇒ $x = 200$ and hence, Sunil will get ₹ 800.

17. The given expression can be written as:

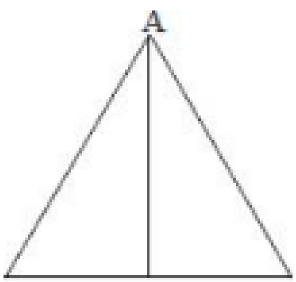
$(\log_a a - \log_a b) + (\log_b b - \log_b a)$.

This simplifies to: $2 - \left\{\log_a b + \frac{1}{\log_a b}\right\}$.

The value of $\left(x + \frac{1}{x}\right)$ is always ≥ 2. Hence, the expression $2 - \left\{\log_a b + \frac{1}{\log_a b}\right\}$,

can never be equal to 1. It can be 0 or negative.

Hence, Option (b) is correct.

18. 

From any point on an equilateral triangle, if we draw three perpendiculars on other sides those three perpendiculars will add up to the Altitude of the equilateral triangle. We know that the value of the altitude of an equilateral triangle with side 'a' is $\frac{\sqrt{3}}{2}a$

Hence, $s = \frac{\sqrt{3}}{2}a$

⇒ ⇒ $a = \frac{2s}{\sqrt{3}}$.

Thus, the area of the triangle

$$= \frac{\sqrt{3}}{4}a \times a = \frac{\sqrt{3}}{4} \times \frac{2s}{\sqrt{3}} \times \frac{2s}{\sqrt{3}} \rightarrow \frac{2s^2}{\sqrt{3}}.$$

Hence, option (b) is correct.

19. If we assume, $x = 1$, z will equal 10 and $y = 9$. In this case checking for $x + y < z + 5 \Rightarrow 10 < 15$. We are asked to find the maximum value of $2x + y$. Hence, we should look for larger values of x and y.

Value of x	Value of z	Value of y	Value of $x + y$	Value of $z + 5$	Is $x + y < z + 5$
2	11	10	12	16	Yes
3	12	11	14	17	Yes
4	13	12	16	18	Yes
5	14	13	18	19	Yes
6	15	14	20	20	No

Hence, the maximum value of $2x + y$

$= 10 + 13 = 23$.

20. $x2 - 2|x| + |a - 2| = 0$

For the given expression to be equal to 0, we see that for $x = 1$, $-2|x| = -2$ and hence $x2 - 2|x| = -1$.

At $x = 2$, the value of $x2 - 2|x| = 0$. Values of x above 2 will not work, since the expression cannot be equal to 0, if $x2 - 2|x|$ is greater than 0. Also, the same conditions will be met for $x = -1$ and $x = -2$. Besides, at $x = 0$, we will get $x2 - 2|x| = 0$ and there can be a solution of this expression for that value of x too.

For $x = 1$, since $x2 - 2|x|$ is equal to -1, we get $|a - 2| = 1 \Rightarrow a = 3$ or $a = 1$;

For $x = 2$, since $x2 - 2|x|$ is equal to 0, we get $|a - 2| = 0 \Rightarrow a = 2$;

For $x = -1$, since $x2 - 2|x|$ is equal to -1, we get $|a - 2| = 1 \Rightarrow a = 3$ or $a = 1$;

For $x = -2$, since $x2 - 2|x|$ is equal to 0, we get $|a - 2| = 0 \Rightarrow a = 2$;

For $x = 0$, since $x^2 - 2|x|$ is equal to 0, we get $|a - 2| = 0 \Rightarrow a = 2$

We can thus see a total of 7 solutions for the integer pair (x, a).

Hence, option (c) is correct.

21. Each of these numbers would have 7 and 3 in them. Also, the order of using 7 and 3 is fixed. Thus, there will be 6 kinds of numbers having 3 after 7 in a 4 digit number. We can think of these as:

	Thousands Place	Hundreds Place	Tens Place	Units Place	Number of cases for a and b
Case 1	7	3	a	b	8 × 7
Case 2	7	a	3	b	8 × 7
Case 3	7	a	b	3	8 × 7
Case 4	a	7	3	b	7 × 7
Case 5	a	7	b	3	7 × 7
Case 6	a	b	7	3	7 × 7
				Total cases	315

22. Let Aron buy P pencils and S sharpeners at prices of x and $(x + 2)$. Then, we have Aditya would buy 2P pencils and $S - 10$ sharpeners at the same prices. It is also given to us that the amount spent by both are equal. Hence,

$$Px + S(x + 2) = 2Px + (S - 10)(x + 2)$$

$$\Rightarrow Px + Sx + 2S = 2Px + Sx + 2S - 10x - 20$$

$$\Rightarrow Px = 10x + 20$$

$$x(P - 10) = 20$$

The, minimum value of P for this expression to hold would be $P = 11$.

Hence, the combined number of pencils they would have bought would be 3P = 33. Hence, Option (c) is correct.

23. R_1 = 100 m and R_2 = 20m

Speed of Ram = 15 km/hr and

Rahim's speed = 5 km/hr

Time taken by Ram = $2\pi\frac{(100)}{15}$

Time taken by Rahim = $2\pi\frac{(20)}{5}$

Ratio of the time taken by Ram : Rahim to cover the round = 5 : 3. Thus, when after 15 units of time, Ram and Rahim would be at the starting point again. In this situation, Ram would have done 3 rounds and Rahim would have done 5 rounds. Option (c) is correct.

24. The first set of values that comes to mind that satisfies $2x + 5y = 99$ would be $x = 2$ and $y = 19$. However, this value would not be counted since we are given that $x \geq y \geq -20$.

The next values for (x, y) would be (7, 17); (12, 15) both not counted. The first feasible pair when $x > y$ is (17, 13) followed by (22, 11); (27, 9)….(?, –19). The number of pairs can be counted using the arithmetic progression that y takes from 13, 11, 9, 7…–19. This has 17 terms and hence the correct answer would be 17.

25. Expense of Rice in April = 450 (given)

Increase in 20% in expense of rice = + 90

Since 150 more is the amount he spent in May, + 60 should be the extra expense on wheat.

So 12% of amount spent on Wheat in April is = Rs. 60

Hence, amount spent on wheat in April = 500

So amount spent on Wheat in May = 500 + 60

= 560.

26. $f(x + 1) = (x + 1)^2 + a(x + 1) + b$

$f(x - 1) = (x - 1)^2 + a(x - 1) + b$

$g(x) = (x + 1)^2 + a(x + 1) + b - (x - 1)^2 - a(x - 1) - b = 4x + 2a.$

Since it is given to us that $g(20) = 72$; we can use $g(x) = 4x + 2a = 72$. With $x = 20$, we get $a = -4$.

Also, it is given to us that $f(x) \geq 0$, which means that for the quadratic expression $x^2 + ax + b$, the value of the discriminant viz $a^2 - 4b \leq 0$. With $a = -4$, this occurs at a minimum value of $b = 4$. Hence, Option (c) is correct.

CAT 2020 SLOT 3

1. It can be seen that at $k = 2$, the two equations become inconsistent since, we get $2x + y = 3$ and $4x + 2y = 4$ (which essentially is $2x + y = 2$). These two equations cannot co-exist at the same time. Hence, the value of $k = 2$ is not possible.

   Also, at $k = -2$, we get: $-2x + y = 3$ and $4x + -2y = 4$ (which essentially is $-2x + y = -2$). These two equations cannot co-exist at the same time. Hence, the value of $k = -2$ is not possible.

   Hence, the correct answer is option (d).

2. Anil will take 12 minutes to cover a round, while Sunil will take 18 minutes to do so. Hence, they will first meet at the starting point after 36 minutes = 0.6 hours. In this time, distance covered by Ravi will be $8 \times 0.6 = 4.8$ kms.

Option (c) is correct.

3. The required answer will be got using the logic:

   All numbers – (Number of numbers divisible by 2) – (Number of numbers divisible by 5, but not 2) – (Number of numbers divisible by 7, but not by 2 or 5).

   In this case, all numbers = 120 and number of numbers divisible by 2 = number of terms in the series 2, 4, 6, 8,…120 = 60.

   The Number of numbers divisible by 5, but not 2 = number of terms in the series: 5, 15, 25, 35, 45,…115 = 12.

   The number of numbers divisible by 7, but not by 2 or 5 = 7, 21, 49, 63, 77, 91 and 119 = 7.

   Hence, the required answer = 120 – 60 – 12 – 7 = 41. Option (c) is correct.

4. By 10:30 AM, the train from A, would have already covered 60 km. Thus, the distance between the two trains would be 30 km. Since, they are traveling towards each other, with speeds of 40 and 20 kmph respectively, it means that they would be approaching each other with a relative speed of 60 kmph. At that speed they would take half an hour more to meet. Hence, the time at which they would meet would be 11 AM. Option (b) is correct.

5. Assume that he bought the sugar at ₹1 per kg. Hence, his total purchase price is ₹35. His marked price will be 1.2. According to the conditions in the problem, he sells 5 kg at the marked price and 15 kgs at a 10% discount on the marked price. He also loses 3 kgs as wastage. Hence, his total recovery for these 5 + 15 + 3

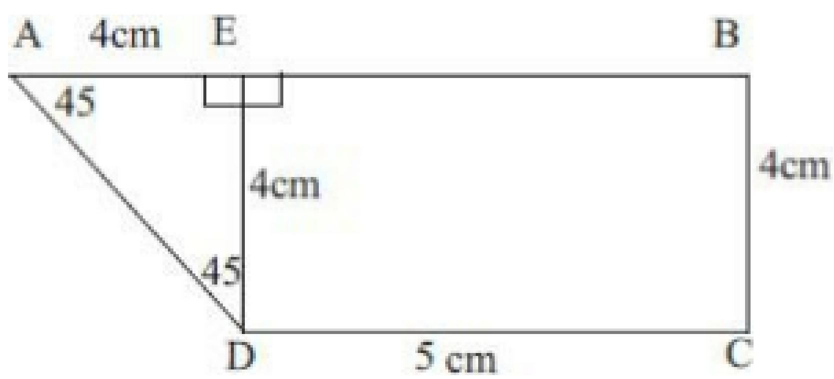
   = 23 kgs would be:

$5 \times 1.2 + 15 \times 1.08 + 3 \times 0 = 22.2$.

In order to gain an overall 15%, he needs to recover a total of $35 \times 1.15 = 40.25$.

Thus, he has to sell the remaining 12 kgs for $40.25 – 22.2$ = ₹18.05. The selling price has to be approximately 1.5. From a marked price of 1.2, he needs approximately a 25% markup to get to a selling price of 1.5. Hence, Option (b) is correct.

6. We can make the following figure for the given trapezium:



The required area would be Area of rectangle BCDE + Area of Triangle ADE = 20 + 8 = 28.

7. $x_m = x_{m+1} + (m + 1)$

$\Rightarrow x_{m+1} = x_m – (m + 1)$

Since it is given to us that $x_1 = – 1$, we can calculate $x_2 = x_1 – (2) = – 3$;

$x_3 = x_2 – (3) = – 3 – 3 = – 6$;

$x_4 = x_3 – 4 = – 6 – 4 = – 10$ and so on.

We realise that the value of x_n in general is just the negative outcome of the addition of the first n natural numbers.

Thus, we have $x_{100} = – (1 + 2 + 3 + … + 100)$.

$$x_{100} = -\frac{100\times101}{2} = -5050.$$

Option (c) is correct.

8. If we evaluate each component of the denominator separately, we will get:

$$(\log_2 4)^2 = 2^2 = 4;\ (\log_4 8)^3 = \left(\frac{1}{2}\times3\right)^3 = \frac{27}{8};$$

$$(\log_8 16)^4 = \left(\frac{1}{3}\times\log_2 16\right)^4 = \frac{4\times4\times4\times4}{3\times3\times3\times3}.$$

The given expression transforms to:

$$\frac{2\times4\times8\times16}{\frac{4\times27\times256}{8\times81}} = \frac{8\times81}{27} = 24$$

9. The diagonals of a parallelogram bisect each other. Hence, the midpoint of the diagonal across the given vertices, should also lie on the other diagonal, which is given to us as $x + 9y + c = 0$. The mid – point of the line connecting (2, 1) and (–3, –4) would be given by: x coordinate $= \frac{2-3}{2} = -0.5$; y coordinate $= \frac{1-4}{2} = -1.5$

$x = -0.5$ and $y = -1.5$ should equate LHS and RHS in the equation $x + 9y + c = 0$. Inserting these values, we get:

$-0.5 - 13.5 + c = 0 \Rightarrow c = 14.$

Option (b) is correct.

10. Let $14^a = 36^b = 84^c = k$. Using the definition of logs, we can write:

$\log_{14}k = a; \log_{36}k = b; \log_{84}k = c.$

$$6b\left(\frac{1}{c}-\frac{1}{a}\right) = 6\log_{36}k\,(\log_k 84 - \log_k 14)$$

$$\Rightarrow \frac{6}{\log_k 36} \times \log 6 = \frac{6}{2 \times \log_k 6} \times (\log_k 6) = 3$$

11. Since, the value of N > 25, The values that N can be taken to can be visualized if we keep y maximized at 22. For $y = 22$, if we take x as 4, 5, 6, 7, 8 or 9, we will get values of N as 26, 27, 28, 29, 30, 31. If we try to take values of y as 21, x with values of 5, 6, 7, 8 or 9 will only be able to reach values like 26, 27, 28, 29, 30. We should realise through this thinking that there are only 6 values that N can take. Hence, the correct answer is 6.

12. The correct answer can be simply derived using:

    $$\frac{18522}{1.05 \times 1.05 \times 1.05} = 16000.$$

    Note: You are advised to do such calculations on the calculator in the CAT.

13. Let Tom = t, Dick = $3t$ and Harry = $6t$. Average age = $10t/3 = 3.33t$.

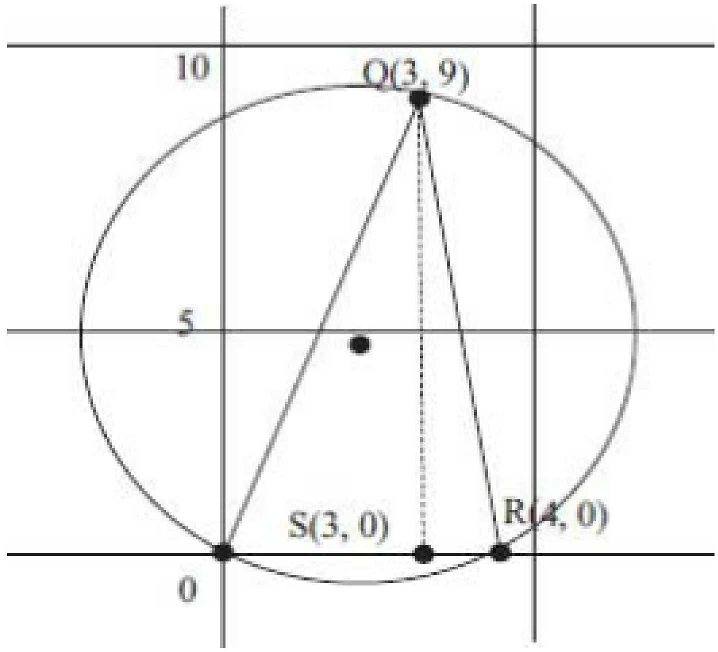
    Difference between average age and Dick's age = $0.33t$ = 1year ⇒ $t = 3$.

    Hence, the age of Harry is 18 years.

14. We are asked to find the area of this circle

    We know that the area of a triangle with sides a, b and c, inscribed in a circle of radius R is given by

    Area = $abc/4R$ where a, b, c are the lengths of the sides of the triangle and R is the radius of the circle.



With the help of the co-ordinates of the triangle it is easy to find the height of the triangle, which is QS = 9 units.

Hence, the area of the triangle

$A = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times PR \times QS$

$= \frac{1}{2} \times 4 \times 9 = 18$ sq. units.

The sides of the triangle can be calculated using the distance formula as:

$a = PR = 4$ units

$b = QR = \sqrt{(4-3)^2 + (9-0)^2} = \sqrt{82}$

$c = PQ = \sqrt{(3-0)^2 + (9-0)^2} = \sqrt{90}$

Thus, we have: $18 = \frac{4\sqrt{82} \times \sqrt{90}}{4R}$

$\Rightarrow \Rightarrow \qquad R = \sqrt{\frac{205}{3}}$

Area of the circle = $\pi \times R_2 = 205\frac{\pi}{9}$.

Hence, Option (b) is correct.

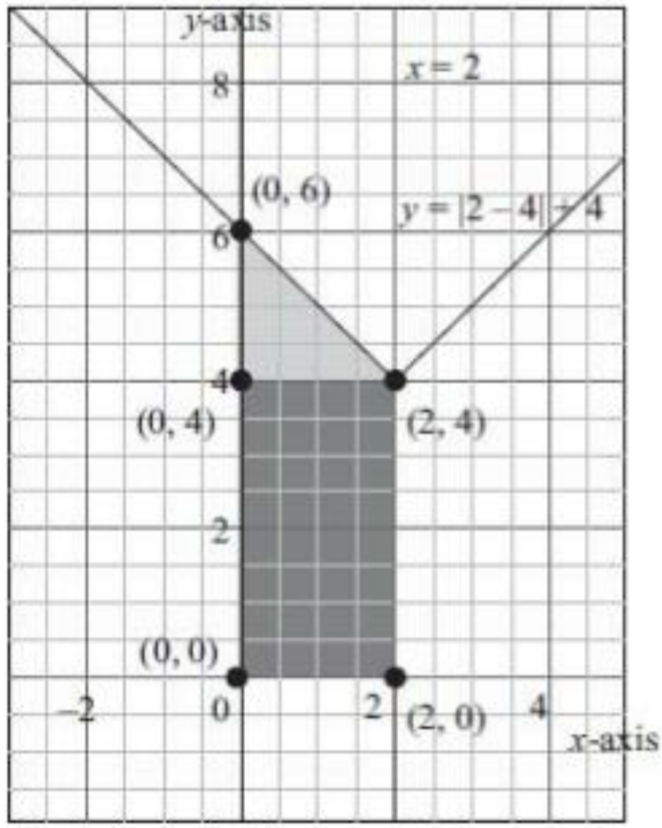
15. This problem has to be solved in two parts.

First part: When, Vimla reduces her speed by 1/8th, her time would go up by 1/7th (since distance traveled is constant). It is given to us that her time increases by 6 minutes. Thus, 6 minutes is 1/7th of the time she normally takes to reach her office. Hence, at 40 kmph, she would take 42 minutes (or 0.7 hours). Hence, the distance to her office is 40 × 0.7 = 28 kms.

Second part: It is given to us that: 'she covers two – thirds of her distance to office in one – thirds of her usual time to reach office, and then stops for 8 minutes.' This essentially means that she has covered 2/3rd of 28 kms = 18.66 kms and has consumed 14 + 8 = 22 mins to do so. She has 20 more minutes to reach office on time and in that time she has to cover 9.33 kms more. Her speed in kmph = 28 kmph in order to cover 9.33 km in 20 minutes (use unitary method thinking here). Option (c) is correct.

16. There will be 28 numbers with at least 1 digit repeated between 100 to 199. Some of these numbers are 100, 101, 110 to 119, 121, 122, 131, 133…191, 199. This pattern will continue from 200 to 299 and so also between 300 to 399, till 900 to 999. Thus, there will be a total of 9 sets of 28 numbers each that have the characteristic of at least 1 digit repeated. Hence, the required answer is 28 × 9 = 252.

17. We can visualise the following figure for the given enclosed area.



Now the enclosed area is the sum of areas of the rectangle and the triangle.

Enclosed Area = $2 \times 4 + \frac{1}{2} \times 2 \times 2 = 8 + 2 = 10$.

18. The difference between Bishnu's and Asha's percentage score is 12%. This difference is also given to us as 23 + 34 = 57 (Interpretation of the second line of the question). Hence, we can deduce that since 12% is 57, 100% will be 475. (using unitary method). Geeta's score would be 84% of 475 = 399. Option (b) is correct.

19. The contractor finishes 1.5 km in 60 days with 140 people.

That means, he finishes, 0.5 km in 20 days with 140 people.

He has 200 – 60 = 140 days left to finish the job.

Also, since he has to do a total of 6 km, he needs to do triple the work already done. The work already done was 140 × 60 (number of men × number of days).

Work to be done in next 140 days = 3 × 140 × 60 = 180 × 140. Since the number of days left is 140, the number of men required is 180. He has to hire 180 – 140 = 40 men.

20. Using the property of real roots for a quadratic equation, we get: $m^2 - 8n > 0$ and $4n^2 - 4m \geq 0$. Since, we have to minimise the value of $m + n$, we start to think with small values. In the first inequality, m as 3 and n as 1 work, but these values do not work in the second inequality. Taking $m = 4$ and $n = 2$, we see that it works for both the inequalities. Hence, the minimum value of $m + n = 6$. Option (c) is correct.

21. We are told that $f(x + y) = f(x) \times f(y)$

$f(x) = f(x + 0) = f(x) \times f(0)$

$f(x) = f(x) \times f(0)$

Assuming that $f(x)$ is non-zero;

This implies: $f(0) = 1$

$f(5) = 4$ (Given)

$f(10) = f(5 + 5) = f(5) \times f(5) = 4 \times 4 = 16$

$f(10) = 16$

We know that $f(0) = 1$

$f(0) = f(10 - 10) = f(10 + (-10))$

$= f(10) \times f(-10) = 1$

$16 \times f(-10) = 1$

$f(-10) = \frac{1}{16} = 0.0625$

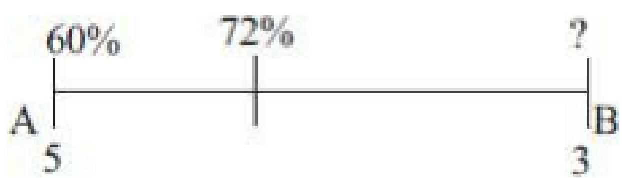
$f(10) - f(-10) = 16 - 0.0625 = 15.9375.$

Option (c) is correct.

22. $4^{2017} = 2^{4034}$. Pairs of a, b can be visualized as 2^0 & 2^{4034}; 2^1 & 2^{4033}; 2^2 & 2^{4032}…2^{2017}& 2^{2017}. Hence, there are a total of 2018 pairs possible for the values of a, b. Option (d) is correct.

Note: When we take the value of $a = 2^0$, the value of a turns out to be 1 and hence it obeys the requirement that a and b are positive integers. A lot of students made the mistake of missing 20 as a counting case and came up with an answer of 2017 and got this question incorrect.

23. Initially the ratio of A:B is 1:3. Then, if the mixture is doubled by adding A, it means that we are adding 4 of A, to make the effective A:B ratio as: 5:3. The percentage of alcohol in solution B can be worked out using alligations as shown:



The value of B works out to 92%. Option (a) is correct.

24. Since all the options have a component of A + B, we should first look at what A + B, yields us.

$A + B = \log_a 30 - \log_a\left(\frac{5}{3}\right)$

$= \log_a 18 = 2\log_a 3 + \log_a 2$.

We are told that $\log_2 a = \frac{1}{3}$

$\log_2 a = \frac{1}{3}$ implies $\log_a 2 = 3$

Hence, $A + B = 2\log_a 3 + 3$.

Hence, $A + B - 3 = 2\log_a 3$

Going for option (a) we see that:

$$\frac{2}{A+B-3} = \frac{2}{2\log_a 3} = \frac{1}{\log_a 3} = \log_3 a.$$

Hence, Option (d) is correct.

25. $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$

$$\frac{1}{5} < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < \frac{1}{2}$$

$$\frac{1}{5} < \frac{m}{20} < \frac{1}{2} \Rightarrow 5, 6, 7, 8, 9$$

Since, at $m = 4$, the LHS inequality is not obeyed and for $m = 10$, the RHS inequality is disobeyed. So, the possible values that m can take = {5, 6, 7, 8, 9}

Thinking, in the same way, we get that the possible values of $n = 3, 4, 5$.

However, since n is even, we have $n = 4$.

If $n = 4 \frac{n}{m}$ will be greater than $\frac{1}{5}$ if m is less than 20.

Similarly at $n = 4$; $\frac{n}{m}$ will be lesser than $\frac{1}{2}$ if m is greater than 8.

So, for $n = 4$; $8 \le m \le 20$.

m can only take one of the values in {5, 6, 7, 8, 9}

Hence $m = 9$.

$m - 2n = 9 - 2(4) = 9 - 8 = 1$.

Option (c) is correct.

26. The average score of the last 2 innings

$$= \frac{(38 + 15)}{2} = 26.5$$

The average of the first n innings = 30

The average of all the $n + 2$ innings = 29.

$29 \times (n + 2) = n \times 30 + 2 \times 26.5$

$n = 5$

The average of the first 5 innings = 30 and in none of the innings did the batsman score 38 or above. The minimum score for an innings will happen if we maximise the scores of the first 4 innings at 37 each. Hence, the minimum possible score in any one inning = $150 - 37 \times 4 = 150 - 148 = 2$.