

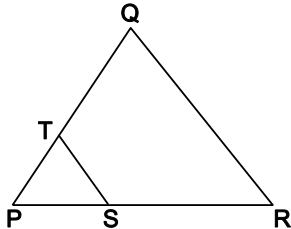
SAMPLE QUESTION PAPER
Class-X (2017–18)
Mathematics

Time allowed: 3 Hours

Max. Marks: 80

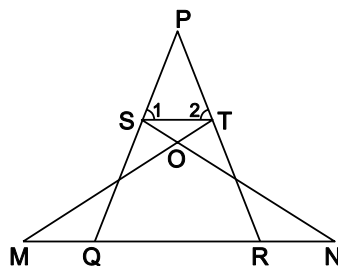
General Instructions:

- (i) *All questions are compulsory.*
- (ii) *The question paper consists of 30 questions divided into four sections A, B, C and D.*
- (iii) *Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each. Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.*
- (iv) *There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.*
- (v) *Use of **calculators** is not permitted.*

	Section A Question numbers 1 to 6 carry 1 mark each.
1.	Write whether the rational number $\frac{7}{75}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.
2.	Find the value(s) of k, if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has equal roots.
3.	Find the eleventh term from the last term of the AP: 27, 23, 19, ..., -65.
4.	Find the coordinates of the point on y-axis which is nearest to the point (-2, 5).
5.	<p>In given figure, $ST \parallel RQ$, $PS = 3$ cm and $SR = 4$ cm. Find the ratio of the area of ΔPST to the area of ΔPRQ.</p> 
6.	If $\cos A = \frac{2}{5}$, find the value of $4 + 4 \tan^2 A$

	<p style="text-align: center;">Section B</p> <p style="text-align: center;">Question numbers 7 to 12 carry 2 marks each.</p>
7.	<p>If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$; a, b are prime numbers, then verify:</p> <p>$\text{LCM}(p, q) \times \text{HCF}(p, q) = pq$</p>
8.	<p>The sum of first n terms of an AP is given by $S_n = 2n^2 + 3n$. Find the sixteenth term of the AP.</p>
9.	<p>Find the value(s) of k for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions.</p>
10.	<p>If $\left(1, \frac{p}{3}\right)$ is the mid-point of the line segment joining the points (2, 0) and $\left(0, \frac{2}{9}\right)$, then show that the line $5x + 3y + 2 = 0$ passes through the point $(-1, 3p)$.</p>
11.	<p>A box contains cards numbered 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is</p> <p>(i) a square number</p> <p>(ii) a multiple of 7</p>
12.	<p>A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the bag.</p>
	<p style="text-align: center;">Section C</p> <p style="text-align: center;">Question numbers 13 to 22 carry 3 marks each.</p>
13.	<p>Show that exactly one of the numbers n, n + 2 or n + 4 is divisible by 3.</p>
14.	<p>Find all the zeroes of the polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$ if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.</p>
15.	<p>Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.</p>
16.	<p>In what ratio does the x-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Find the co-ordinates of the point of division.</p> <p style="text-align: center;">OR</p> <p>The points A(4, -2), B(7, 2), C(0, 9) and D(-3, 5) form a parallelogram. Find the length of the altitude of the parallelogram on the base AB.</p>

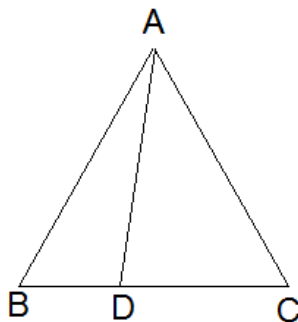
17. In given figure $\angle 1 = \angle 2$ and $\triangle NSQ \cong \triangle MTR$, then prove that $\triangle PTS \sim \triangle PRQ$.



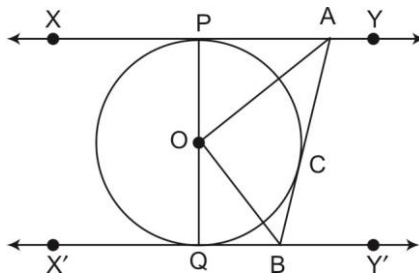
OR

In an equilateral triangle ABC, D is a point on the side BC such that

$$BD = \frac{1}{3} BC. \text{ Prove that } 9AD^2 = 7AB^2$$



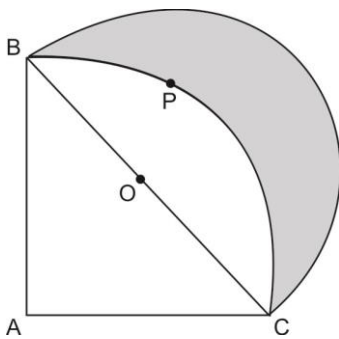
18. In given figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.




19. Evaluate: $\frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\cot^2 66^\circ + \sec^2 27^\circ} + \frac{\sin^2 63^\circ + \cos 63^\circ \sin 27^\circ + \sin 27^\circ \sec 63^\circ}{2(\operatorname{cosec}^2 65^\circ - \tan^2 25^\circ)}$

OR

If $\sin \theta + \cos \theta = \sqrt{2}$, then evaluate: $\tan \theta + \cot \theta$

20.	<p>In given figure ABPC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region</p> 												
21.	<p>Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?</p> <p style="text-align: center;">OR</p> <p>A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the remaining solid after the cone is carved out.</p>												
22.	<p>Find the mode of the following distribution of marks obtained by the students in an examination:</p> <table border="1" data-bbox="412 970 1315 1075"><tr><td>Marks obtained</td><td>0-20</td><td>20-40</td><td>40-60</td><td>60-80</td><td>80-100</td></tr><tr><td>Number of students</td><td>15</td><td>18</td><td>21</td><td>29</td><td>17</td></tr></table> <p>Given the mean of the above distribution is 53, using empirical relationship estimate the value of its median.</p>	Marks obtained	0-20	20-40	40-60	60-80	80-100	Number of students	15	18	21	29	17
Marks obtained	0-20	20-40	40-60	60-80	80-100								
Number of students	15	18	21	29	17								
	<p style="text-align: center;">Section D</p> <p style="text-align: center;">Question numbers 23 to 30 carry 4 marks each.</p>												
23.	<p>A train travelling at a uniform speed for 360 km would have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train.</p> <p style="text-align: center;">OR</p> <p>Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots and if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.</p>												
24.	<p>An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three terms is 429. Find the AP.</p>												
25.	<p>Show that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.</p> <p style="text-align: center;">OR</p> <p>Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.</p>												

26.	Draw a triangle ABC with side BC = 7 cm, $\angle B = 45^\circ$, $\angle A = 105^\circ$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of $\triangle ABC$.																																
27.	Prove that $\frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} = \operatorname{cosec} \theta + \cot \theta$																																
28.	The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60° , respectively. Find the height of the tower and also the horizontal distance between the building and the tower.																																
29.	<p>Two dairy owners A and B sell flavoured milk filled to capacity in mugs of negligible thickness, which are cylindrical in shape with a raised hemispherical bottom. The mugs are 14 cm high and have diameter of 7 cm as shown in given figure. Both A and B sell flavoured milk at the rate of ₹ 80 per litre. The dairy owner A uses the formula $\pi r^2 h$ to find the volume of milk in the mug and charges ₹ 43.12 for it. The dairy owner B is of the view that the price of actual quantity of milk should be charged. What according to him should be the price of one mug of milk? Which value is exhibited by the dairy owner B? $\left(\text{use } \pi = \frac{22}{7}\right)$</p> <div></div>																																
30.	<p>The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency k.</p> <table><tr><td>Daily pocket allowance (in ₹)</td><td>11–13</td><td>13–15</td><td>15–17</td><td>17–19</td><td>19–21</td><td>21–23</td><td>23–25</td></tr><tr><td>Number of children</td><td>3</td><td>6</td><td>9</td><td>13</td><td>k</td><td>5</td><td>4</td></tr></table> <p style="text-align: center;">OR</p> <p>The following frequency distribution shows the distance (in metres) thrown by 68 students in a Javelin throw competition.</p> <table><tr><td>Distance (in m)</td><td>0–10</td><td>10–20</td><td>20–30</td><td>30–40</td><td>40–50</td><td>50–60</td><td>60–70</td></tr><tr><td>Number of students</td><td>4</td><td>5</td><td>13</td><td>20</td><td>14</td><td>8</td><td>4</td></tr></table> <p>Draw a less than type Ogive for the given data and find the median distance thrown using this curve.</p>	Daily pocket allowance (in ₹)	11–13	13–15	15–17	17–19	19–21	21–23	23–25	Number of children	3	6	9	13	k	5	4	Distance (in m)	0–10	10–20	20–30	30–40	40–50	50–60	60–70	Number of students	4	5	13	20	14	8	4
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Marking Scheme

Mathematics Class X (2017-18)

Section A

S.No.	Answer	Marks
1.	Non terminating repeating decimal expansion.	[1]
2.	$k = \pm 4$	[1]
3.	$a_{11} = -25$	[1]
4.	$(0, 5)$	[1]
5.	$9 : 49$	[1]
6.	25	[1]

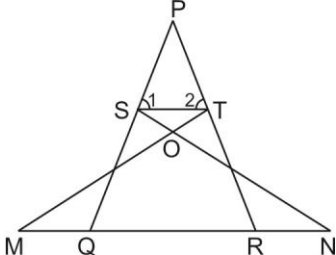
Section B

7.	$\text{LCM}(p, q) = a^3b^3$ $\text{HCF}(p, q) = a^2b$ $\text{LCM}(p, q) \times \text{HCF}(p, q) = a^5b^4 = (a^2b^3)(a^3b) = pq$	[1/2] [1/2] [1]
8.	$S_n = 2n^2 + 3n$ $S_1 = 5 = a_1$ $S_2 = a_1 + a_2 = 14 \Rightarrow a_2 = 9$ $d = a_2 - a_1 = 4$ $a_{16} = a_1 + 15d = 5 + 15(4) = 65$	[1/2] [1/2] [1/2] [1/2]
9.	For pair of equations $kx + 1y = k^2$ and $1x + ky = 1$ We have: $\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$ For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\therefore \frac{k}{1} = \frac{1}{k} \Rightarrow k^2 = 1 \Rightarrow k = 1, -1 \quad \dots(i)$ and $\frac{1}{k} = \frac{k^2}{1} \Rightarrow k^3 = 1 \Rightarrow k = 1 \quad \dots(ii)$ From (i) and (ii), $k = 1$	[1/2] [1/2] [1/2] [1/2]
10.	Since $\left(1, \frac{p}{3}\right)$ is the mid-point of the line segment joining the points $(2, 0)$ and $\left(0, \frac{2}{9}\right)$ therefore, $\frac{p}{3} = \frac{0 + \frac{2}{9}}{2} \Rightarrow p = \frac{1}{3}$ The line $5x + 3y + 2 = 0$ passes through the point $(-1, 1)$ as $5(-1) + 3(1) + 2 = 0$	[1] [1]
11.	(i) $P(\text{square number}) = \frac{8}{113}$ (ii) $P(\text{multiple of } 7) = \frac{16}{113}$	[1] [1]

12.	<p>Let number of red balls be = x</p> <p>$\therefore P(\text{red ball}) = \frac{x}{12}$</p> <p>If 6 more red balls are added:</p> <p>The number of red balls = $x + 6$</p> <p>$P(\text{red ball}) = \frac{x+6}{18}$</p> <p>Since, $\frac{x+6}{18} = 2\left(\frac{x}{12}\right) \Rightarrow x = 3$</p> <p>$\therefore$ There are 3 red balls in the bag.</p>	<p>[1/2]</p> <p>[1]</p> <p>[1/2]</p>
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Section C

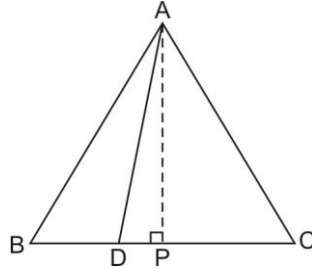
13.	<p>Let $n = 3k, 3k + 1$ or $3k + 2$.</p> <p>(i) When $n = 3k$:</p> <p>n is divisible by 3.</p> <p>$n + 2 = 3k + 2 \Rightarrow n + 2$ is not divisible by 3.</p> <p>$n + 4 = 3k + 4 = 3(k + 1) + 1 \Rightarrow n + 4$ is not divisible by 3.</p> <p>(ii) When $n = 3k + 1$:</p> <p>n is not divisible by 3.</p> <p>$n + 2 = (3k + 1) + 2 = 3k + 3 = 3(k + 1) \Rightarrow n + 2$ is divisible by 3.</p> <p>$n + 4 = (3k + 1) + 4 = 3k + 5 = 3(k + 1) + 2 \Rightarrow n + 4$ is not divisible by 3.</p> <p>(iii) When $n = 3k + 2$:</p> <p>n is not divisible by 3.</p> <p>$n + 2 = (3k + 2) + 2 = 3k + 4 = 3(k + 1) + 1 \Rightarrow n + 2$ is not divisible by 3.</p> <p>$n + 4 = (3k + 2) + 4 = 3k + 6 = 3(k + 2) \Rightarrow n + 4$ is divisible by 3.</p> <p>Hence exactly one of the numbers $n, n + 2$ or $n + 4$ is divisible by 3.</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p>
14.	<p>Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the two zeroes therefore, $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2 - 5)$</p> <p>is a factor of given polynomial.</p> <p>We divide the given polynomial by $3x^2 - 5$.</p> $ \begin{array}{r} \overline{3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{ \pm 3x^4 } \\ 6x^3 + 3x^2 - 10x - 5 \\ \underline{ \pm 6x^3 } \\ 3x^2 - 5 \\ \underline{ \pm 3x^2 } \\ 0 \end{array} $ <p>For other zeroes, $x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0, x = -1, -1$</p> <p>$\therefore$ Zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p>

15.	<p>Let the ten's and the units digit be y and x respectively. So, the number is $10y + x$. The number when digits are reversed is $10x + y$. Now, $7(10y + x) = 4(10x + y) \Rightarrow 2y = x \quad \dots(i)$ Also $x - y = 3 \quad \dots(ii)$ Solving (1) and (2), we get $y = 3$ and $x = 6$. Hence the number is 36.</p>	<p>[1/2] [1/2] [1] [1/2] [1/2]</p>
16.	<p>Let x-axis divides the line segment joining $(-4, -6)$ and $(-1, 7)$ at the point P in the ratio $1 : k$. Now, coordinates of point of division $P\left(\frac{-1-4k}{k+1}, \frac{7-6k}{k+1}\right)$ Since P lies on x-axis, therefore $\frac{7-6k}{k+1} = 0$ $\Rightarrow 7 - 6k = 0$ $\Rightarrow k = \frac{7}{6}$ Hence the ratio is $1 : \frac{7}{6} = 6 : 7$ Now, the coordinates of P are $\left(\frac{-34}{13}, 0\right)$. <p style="text-align: center;">OR</p> Let the height of parallelogram taking AB as base be h. Now $AB = \sqrt{(7-4)^2 + (2+2)^2} = \sqrt{3^2 + 4^2} = 5$ units. Area (ΔABC) = $\frac{1}{2}[4(2-9) + 7(9+2) + 0(-2-2)] = \frac{49}{2}$ sq units. Now, $\frac{1}{2} \times AB \times h = \frac{49}{2}$ $\Rightarrow \frac{1}{2} \times 5 \times h = \frac{49}{2}$ $\Rightarrow h = \frac{49}{5} = 9.8$ units.</p>	<p>[1/2] [1] [1/2] [1] [1] [1] [1]</p>
17.	<p>$\angle SQN = \angle TRM$ (CPCT as $\Delta NSQ \cong \Delta MTR$)</p>  <p>Since, $\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$ (Angle sum property) $\Rightarrow \angle 1 + \angle 2 = \angle PQR + \angle PRQ$ $\Rightarrow 2\angle 1 = 2\angle PQR$ (as $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$) $\angle 1 = \angle PQR$</p>	<p>[1] [1]</p>

Also $\angle 2 = \angle PRQ$
 And $\angle SPT = \angle QPR$ (common)
 $\Delta PTS \sim \Delta PRQ$ (By AAA similarity criterion)

[1]

OR



Construction: Draw $AP \perp BC$

[1/2]

In ΔADP , $AD^2 = AP^2 + DP^2$

[1/2]

$AD^2 = AP^2 + (BP - BD)^2$

[1/2]

$AD^2 = AP^2 + BP^2 + BD^2 - 2(BP)(BD)$

$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$

[1]

$AD^2 = \frac{7}{9}AB^2$ ($\because BC = AB$)

$9AD^2 = 7AB^2$

[1/2]

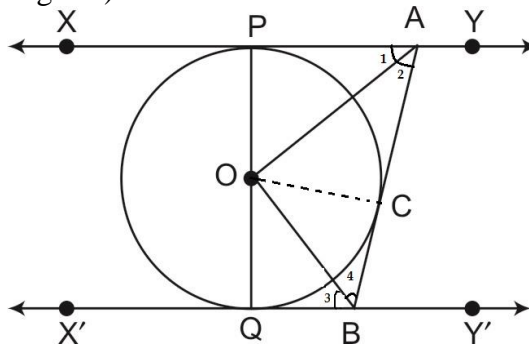
18.

Join OC

In ΔOPA and ΔOCA

$OP = OC$ (radii of same circle)

$PA = CA$ (length of two tangents)



[1]

$AO = AO$ (Common)

$\therefore \Delta OPA \cong \Delta OCA$ (By SSS congruency criterion)

Hence, $\angle 1 = \angle 2$ (CPCT)

[1]

Similarly $\angle 3 = \angle 4$

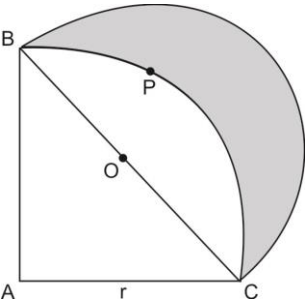
Now, $\angle PAB + \angle QBA = 180^\circ$

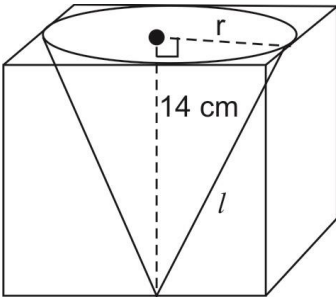
$\Rightarrow 2\angle 2 + 2\angle 4 = 180^\circ$

[1]

$\Rightarrow \angle 2 + \angle 4 = 90^\circ$

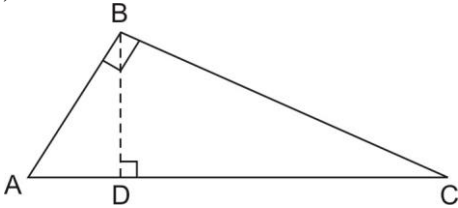
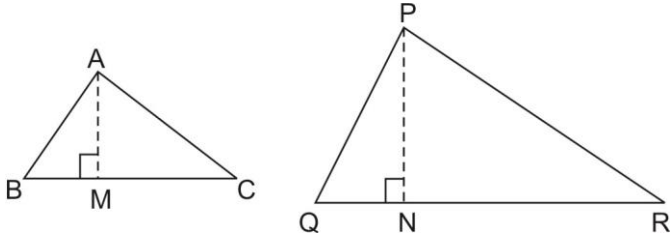
$\Rightarrow \angle AOB = 90^\circ$ (Angle sum property)

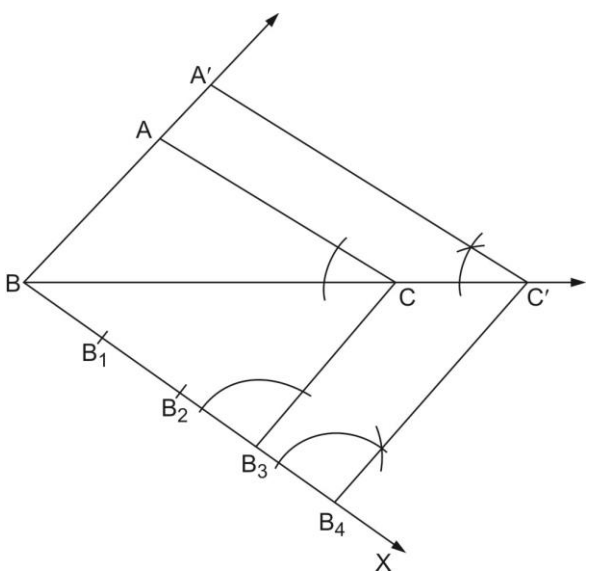
19.	$\frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\cot^2 66^\circ + \sec^2 27^\circ} + \frac{\sin^2 63^\circ + \cos 63^\circ \sin 27^\circ + \sin 27^\circ \sec 63^\circ}{2(\operatorname{cosec}^2 65^\circ - \tan^2 25^\circ)}$ $= \frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\tan^2(90^\circ - 66^\circ) + \operatorname{cosec}^2(90^\circ - 27^\circ)} + \frac{\sin^2 63^\circ + \cos 63^\circ \cos(90^\circ - 27^\circ) + \sin 27^\circ \operatorname{cosec}(90^\circ - 63^\circ)}{2[\operatorname{cosec}^2 65^\circ - \cot^2(90^\circ - 25^\circ)]}$ $= \frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\tan^2 24^\circ + \operatorname{cosec}^2 63^\circ} + \frac{\sin^2 63^\circ + \cos^2 63^\circ + \sin 27^\circ \operatorname{cosec} 27^\circ}{2(\operatorname{cosec}^2 65^\circ - \cot^2 65^\circ)}$ $= 1 + \frac{1+1}{2(1)} = 2$ <p style="text-align: center;">OR</p> $\sin \theta + \cos \theta = \sqrt{2}$ $\Rightarrow (\sin \theta + \cos \theta)^2 = (\sqrt{2})^2$ $\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2$ $\Rightarrow 1 + 2 \sin \theta \cos \theta = 2$ $\Rightarrow \sin \theta \cos \theta = \frac{1}{2} \quad \dots(i)$ <p>we know, $\sin^2 \theta + \cos^2 \theta = 1 \quad \dots(ii)$</p> <p>Dividing (ii) by (i) we get</p> $\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{1/2}$ $\Rightarrow \tan \theta + \cot \theta = 2$	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1/2]</p> <p>[1]</p> <p>[1/2]</p> <p>[1]</p>
20.	<p>We know, $AC = r$</p> <p>In ΔACB, $BC^2 = AC^2 + AB^2$</p> $\Rightarrow BC = AC\sqrt{2} \quad (\because AB = AC)$ $\Rightarrow BC = r\sqrt{2}$ <div style="text-align: center;">  </div> <p>Required area = ar(ΔACB) + ar(semicircle on BC as diameter) – ar(quadrant ABPC)</p> $= \frac{1}{2} \times r \times r + \frac{1}{2} \times \pi \times \left(\frac{r\sqrt{2}}{2}\right)^2 - \frac{1}{4} \pi r^2$ $= \frac{r^2}{2} + \frac{\pi r^2}{4} - \frac{\pi r^2}{4}$ $= \frac{r^2}{2} = \frac{196}{2} \text{ cm}^2 = 98 \text{ cm}^2$	<p>[1]</p> <p>[1]</p> <p>[1]</p>

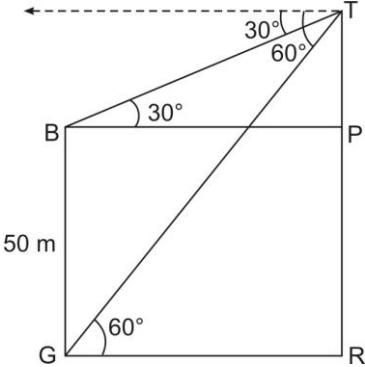
21.	<p>Let the area that can be irrigated in 30 minute be $A \text{ m}^2$.</p> <p>Water flowing in canal in 30 minutes = $\left(10,000 \times \frac{1}{2}\right) \text{ m} = 5000 \text{ m}$</p> <p>Volume of water flowing out in 30 minutes = $(5000 \times 6 \times 1.5) \text{ m}^3 = 45000 \text{ m}^3 \dots(i)$</p> <p>Volume of water required to irrigate the field = $A \times \frac{8}{100} \text{ m}^3$</p> <p>...(ii)</p> <p>Equating (i) and (ii), we get</p> $A \times \frac{8}{100} = 45000$ $A = 562500 \text{ m}^2.$ <p style="text-align: center;">OR</p> $l = \sqrt{7^2 + 14^2} = 7\sqrt{5}$ <p>Surface area of remaining solid = $6l^2 - \pi r^2 + \pi rl$, where r and l are the radius and slant height of the cone.</p>  $= 6 \times 14 \times 14 - \frac{22}{7} \times 7 \times 7 + \frac{22}{7} \times 7 \times 7\sqrt{5}$ $= 1176 - 154 + 154\sqrt{5}$ $= (1022 + 154\sqrt{5}) \text{ cm}^2$	<p>[1/2]</p> <p>[1]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1]</p> <p>[1/2]</p> <p>[1]</p> <p>[1]</p>
22.	$\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ $= 60 + \left(\frac{29 - 21}{58 - 21 - 17}\right) \times 20$ $= 68$ <p>So, the mode marks is 68.</p> <p>Empirical relationship between the three measures of central tendencies is:</p> <p>3 Median = Mode + 2 Mean</p> <p>3 Median = 68 + 2 × 53</p> <p>Median = 58 marks</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p>

Section D

23.	<p>Let original speed of the train be x km/h.</p> <p>Time taken at original speed = $\frac{360}{x}$ hours</p> <p>Time taken at increased speed = $\frac{360}{x+5}$ hours</p> <p>Now, $\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$</p> <p>$\Rightarrow 360 \left[\frac{1}{x} - \frac{1}{x+5} \right] = \frac{4}{5}$</p> <p>$\Rightarrow x^2 + 5x - 2250 = 0$</p> <p>$\Rightarrow x = 45$ or -50 (as speed cannot be negative)</p> <p>$\Rightarrow x = 45$ km/h</p> <p style="text-align: center;">OR</p> <p>Discriminant = $b^2 - 4ac = 36 - 4 \times 5 \times (-2) = 76 > 0$</p> <p>So, the given equation has two distinct real roots</p> <p>$5x^2 - 6x - 2 = 0$</p> <p>Multiplying both sides by 5.</p> <p>$(5x)^2 - 2 \times (5x) \times 3 = 10$</p> <p>$\Rightarrow (5x)^2 - 2 \times (5x) \times 3 + 3^2 = 10 + 3^2$</p> <p>$\Rightarrow (5x - 3)^2 = 19$</p> <p>$\Rightarrow 5x - 3 = \pm\sqrt{19}$</p> <p>$\Rightarrow x = \frac{3 \pm \sqrt{19}}{5}$</p> <p>Verification:</p> <p>$5 \left(\frac{3 + \sqrt{19}}{5} \right)^2 - 6 \left(\frac{3 + \sqrt{19}}{5} \right) - 2 = \frac{9 + 6\sqrt{19} + 19}{5} - \frac{18 + 6\sqrt{19}}{5} - \frac{10}{5} = 0$</p> <p>Similarly, $5 \left(\frac{3 - \sqrt{19}}{5} \right)^2 - 6 \left(\frac{3 - \sqrt{19}}{5} \right) - 2 = 0$</p>	<p>[1]</p> <p>[1/2]</p> <p>[1½]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1/2]</p> <p>[1/2]</p>
24.	<p>Let the three middle most terms of the AP be $a - d$, a, $a + d$.</p> <p>We have, $(a - d) + a + (a + d) = 225$</p> <p>$\Rightarrow 3a = 225 \Rightarrow a = 75$</p> <p>Now, the AP is</p> <p>$a - 18d, \dots, a - 2d, a - d, a, a + d, a + 2d, \dots, a + 18d$</p> <p>Sum of last three terms:</p> <p>$(a + 18d) + (a + 17d) + (a + 16d) = 429$</p> <p>$\Rightarrow 3a + 51d = 429 \Rightarrow a + 17d = 143$</p> <p>$\Rightarrow 75 + 17d = 143$</p> <p>$\Rightarrow d = 4$</p> <p>Now, first term = $a - 18d = 75 - 18(4) = 3$</p> <p>\therefore The AP is 3, 7, 11, ..., 147.</p>	<p>[1]</p> <p>[1/2]</p> <p>[1]</p> <p>[1/2]</p> <p>[1]</p>

25.	<p>Given: A right triangle ABC right angled at B. To prove: $AC^2 = AB^2 + BC^2$ Construction: Draw $BD \perp AC$ Proof: In $\triangle ADB$ and $\triangle ABC$ $\angle ADB = \angle ABC$ (each 90°) $\angle BAD = \angle CAB$ (common)</p>  <p>$\triangle ADB \sim \triangle ABC$ (By AA similarity criterion)</p> <p>Now, $\frac{AD}{AB} = \frac{AB}{AC}$ (corresponding sides are proportional)</p> <p>$\Rightarrow AB^2 = AD \times AC$... (i)</p> <p>Similarly $\triangle BDC \sim \triangle ABC$</p> <p>$\Rightarrow BC^2 = CD \times AC$... (ii)</p> <p>Adding (1) and (2)</p> <p>$AB^2 + BC^2 = AD \times AC + CD \times AC$ $\Rightarrow AB^2 + BC^2 = AC \times (AD + CD)$ $\Rightarrow AB^2 + BC^2 = AC^2$, Hence Proved.</p> <p style="text-align: center;">OR</p> <p>Given: $\triangle ABC \sim \triangle PQR$</p> <p>To prove: $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$</p> <p>Construction: Draw $AM \perp BC$, $PN \perp QR$</p>  <p>$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC}{QR} \times \frac{AM}{PN}$... (i)</p> <p>In $\triangle ABM$ and $\triangle PQN$ $\angle B = \angle Q$ ($\because \triangle ABC \sim \triangle PQR$) $\angle M = \angle N$ (each 90°) $\triangle ABM \sim \triangle PQN$ (AA similarity criterion)</p> <p>Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$... (ii)</p> <p>But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ($\triangle ABC \sim \triangle PQR$) ... (iii)</p>	<p>[1/2]</p> <p>[1]</p> <p>[1]</p> <p>[1/2]</p> <p>[1]</p> <p>[1/2]</p> <p>[1]</p> <p>[1/2]</p>
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	<p>Hence, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{BC}{QR} \times \frac{AM}{PN}$ from (i)</p> <p>$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [from (ii) and (iii)]</p> <p>$= \left(\frac{AB}{PQ}\right)^2$</p> <p>$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$ Using (iii)</p>	<p>[1/2]</p> <p>[1/2]</p>
26.	<p>Draw $\triangle ABC$ in which $BC = 7$ cm, $\angle B = 45^\circ$, $\angle A = 105^\circ$ and hence $\angle C = 30^\circ$. Construction of similar triangle $A'BC'$ as shown below:</p> 	<p>[1]</p> <p>[3]</p>
27.	$\begin{aligned} \text{LHS} &= \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} \\ &= \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} \times \frac{\cos \theta + \sin \theta + 1}{\cos \theta + \sin \theta + 1} \\ &= \frac{(\cos \theta + 1)^2 - \sin^2 \theta}{(\cos \theta + \sin \theta)^2 - 1^2} \\ &= \frac{\cos^2 \theta + 1 + 2 \cos \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta - 1} \\ &= \frac{2 \cos^2 \theta + 2 \cos \theta}{2 \sin \theta \cos \theta} \\ &= \frac{2 \cos \theta (\cos \theta + 1)}{2 \sin \theta \cos \theta} \\ &= \frac{\cos \theta + 1}{\sin \theta} = \operatorname{cosec} \theta + \cot \theta = \text{RHS} \end{aligned}$	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>

28.	<p>In $\triangle BTP \Rightarrow \tan 30^\circ = \frac{TP}{BP}$</p>  <p style="text-align: right;">Correct Figure</p> $\Rightarrow \frac{1}{\sqrt{3}} = \frac{TP}{BP}$ $BP = TP\sqrt{3} \quad \dots(i)$ <p>In $\triangle GTR$,</p> $\tan 60^\circ = \frac{TR}{GR} \Rightarrow \sqrt{3} = \frac{TR}{GR} \Rightarrow GR = \frac{TR}{\sqrt{3}} \quad \dots(ii)$ <p>Now, $TP\sqrt{3} = \frac{TR}{\sqrt{3}}$ (as $BP = GR$)</p> $\Rightarrow 3TP = TP + PR$ $\Rightarrow 2TP = BG \Rightarrow TP = \frac{50}{2} \text{ m} = 25 \text{ m}$ <p>Now, $TR = TP + PR = (25 + 50) \text{ m}$.</p> <p>Height of tower $= TR = 75 \text{ m}$.</p> <p>Distance between building and tower $= GR = \frac{TR}{\sqrt{3}}$</p> $\Rightarrow GR = \frac{75}{\sqrt{3}} \text{ m} = 25\sqrt{3} \text{ m}$	<p>[1]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1]</p> <p>[1/2]</p> <p>[1/2]</p>
29.	<p>Capacity of mug (actual quantity of milk) $= \pi r^2 h - \frac{2}{3} \pi r^3$</p> $= \pi r^2 \left(h - \frac{2}{3} r \right)$ $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(14 - \frac{2}{3} \times \frac{7}{2} \right)$ $= \frac{2695}{6} \text{ cm}^3$ <p>Amount dairy owner B should charge for one mug of milk</p> $= \frac{2695}{6} \times \frac{80}{1000} = ₹ 35.93$ <p>Value exhibited by dairy owner B: honesty (or any similar value)</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>

30.

Daily pocket allowance (in ₹)	Number of children (f_i)	Mid-point (x_i)	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11–13	3	12	–3	–9
13–15	6	14	–2	–12
15–17	9	16	–1	–9
17–19	13	18	0	0
19–21	k	20	1	k
21–23	5	22	2	10
23–25	4	24	3	12

$$\Sigma f_i = 40 + k$$

$$\Sigma f_i u_i = k - 8$$

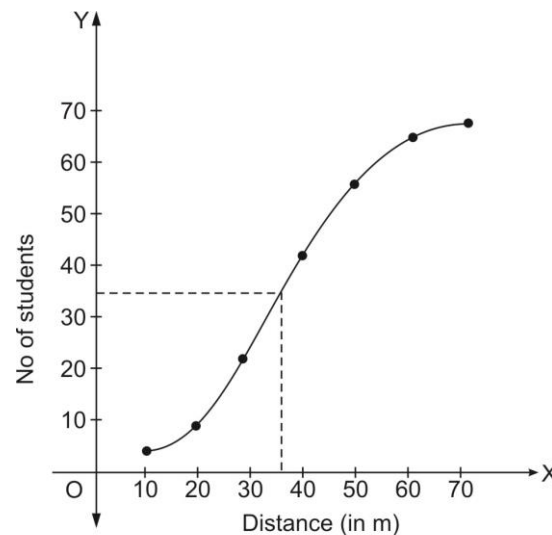
$$\text{Mean} = \bar{x} = a + h \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right)$$

$$18 = 18 + 2 \left(\frac{k - 8}{40 + k} \right)$$

$$\Rightarrow k = 8$$

OR

Less than	Number of Students
10	4
20	9
30	22
40	42
50	56
60	64
70	68



Less than Ogive

Median distance is value of x that corresponds to

$$\text{Cumulative frequency } \frac{N}{2} = \frac{68}{2} = 34$$

Therefore, Median distance = 36 m

[2]

[1]

[1]

[1]

[2]

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