Sample Paper – 6
Mathematics
Class XI Session 2022-23

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A

(Multiple Choice Questions)
Each question carries 1 mark.

1. If A and B are two sets such that $n(A) = 50$, $n(B) = 20$, $n(A \cap B) = 40$, then $n(A \cup B)$ is equal to:
(a) 20 (b) 30
(c) 40 (d) 50 1

2. Shweta write a complex number in her notebook $3 - 4i$, real and imaginary part of the complex number is:
(a) 3, 4 (b) 3, –4
(c) 4, 3 (d) 4, –3 1

3. If the 8^{th} term of G.P is 192 with a common ratio of 2, then the 12^{th} term is:
(a) 1640 (b) 2084
(c) 3072 (d) 3126 1

4. Chandan notices that a wheel in factory making 18 revolutions per second. If the radius of the wheel is 49 cm, then what linear distance does a point of its rim travel in three minutes? (Take $\pi = 22/7$)
(a) 9.97 km (b) 9.90 km
(c) 9.80 km (d) 9.85 km 1

5. The distance between (3, 2, –1) and (–1, –1, –1) is:
(a) 5 units (b) 6 units
(c) 7 units (d) 8 units 1

6. If $-3 \leq \frac{5 - 3x}{2} \leq 4$, then x:
(a) $\left[1, \frac{11}{3}\right]$ (b) $[-5, 5]$
(c) $\left[-\frac{11}{3}, \infty\right]$ (d) $[-\infty, \infty]$ 1

7. The value of $\lim_{x \to 3} (4x^3 - 2x^2 - x + 1)$ is equal to:
(a) 40 (b) 20
(c) 38 (d) 88 1

8. The tangent of angle between the lines whose intercepts on the axes are a, $-b$ and b, $-a$, respectively, is:
(a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{b^2 - a^2}{2}$
(c) $\frac{b^2 - a^2}{2ab}$ (d) None of these 1

9. An investigator interviewed 100 students to determine the performance of three drinks: milk, coffee, and tea. The investigator reported that 10 students take all three drinks milk, coffee, and tea; 20 students take milk and coffee; 25 students take milk and tea; 20 students take coffee and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of the three drinks is:
(a) 10 (b) 20
(c) 25 (d) 30 1

10. Equation of a circle with centre $(-a, -b)$ and radius = 100 is:
(a) $(x - a)^2 + (y - b)^2 = 100$
(b) $(x + a)^2 + (y + b)^2 = 10000$
(c) $x^2 + y^2 = 100$
(d) None of the above 1

11. If x, $2y$, $3z$ are in A.P., where the distinct numbers x, y, z are in G.P., then the common ratio of the G.P. is:

(a) 3 : 1 (b) 1 : 3

(c) 2 : 1 (d) 1 : 2 1

12. If $\frac{x+3}{x+5} > 3$, then $x \in$

(a) (–6, 5) (b) (–∞, –6)

(c) (–6, ∞) (d) (6, 12) 1

13. Mean deviation about median for 3, 4, 9, 5, 3, 12, 10, 18, 7, 19, 21 is:

(a) 4.27 (b) 5.24

(c) 5.27 (d) 4.24 1

14. A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. Then, the possible outcomes of this experiment is:

(a) {RR, BB} (b) {RR, B, B, RR}

(c) {BB, R} (d) {RR, RB, BR, BB} 1

15. The value of x such that $\frac{{}^{x}P_4}{{}^{x-1}P_4} = \frac{5}{3}$, $x > 4$ is:

(a) 11 (b) 15

(c) 12 (d) 10 1

16. How many elements will be there in the cartesian product of A and B, if number of elements in A and B are respectively 10 and 7?

(a) 3 (b) 17

(c) 70 (d) 10^7 1

17. The space is divided into .... parts by placing three axes perpendicular to each other.

(a) 3 (b) 4

(c) 6 (d) 8 1

18. The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is:

(a) 50 (b) 202

(c) 51 (d) None of these 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): Value of $\sin\frac{\pi}{3} = \sin 60° = \frac{\sqrt{3}}{2}$.

Reason (R): Value of π in degree is $\frac{22}{7}$. 1

20. Assertion (A): The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is 7/10.

Reason (R): The distance between the lines $ax + by = c_1$ and $ax + by = c_2$ is given by $\left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$. 1

SECTION - B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. In each of the following cases, find a and b.

(A) $(2a + b, a - b) = (8, 3)$

(B) $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

OR

In function $f = \{(1, 1), (0, -2), (3, 0), (2, 4)\}$ be a linear function defined by formula, $f(x) = ax + b$. Then find 'a' and 'b'. 2

22. If $z_1 = \sqrt{2}(\cos 30° + i \sin 60°)$

$z_2 = \sqrt{3}(\cos 60° + i \sin 30°)$, then show that $\text{Re}(z_1 z_2) = 0$ 2

23. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die. 2

24. In how many ways can a student choose a programme of 7 courses (3 optional and 4 compulosry) if 9 course (5 optional and 4 compulsory) are available for every student?

OR

A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions. 2

25. If $a \cos\theta + b \sin\theta = m$ and $a \sin\theta - b \cos\theta = n$ then show that $a^2 + b^2 = m^2 + n^2$. 2

SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Let A = {1, 2, 3, 4} B = {1, 2, 3} and C = {2, 4}. Find all sets X satisfying each pair of conditions:

(A) $X \subset B$ and $X \not\subset C$

(B) $X \subset B$, $X \neq B$ and $X \not\subset C$

(C) $X \subset A$, $X \subset B$ and $X \subset C$

OR

Which of the following pairs of sets are disjoint?

(A) {1, 2, 3, 4} and $\{x : x$ is a natural number and $4 \leq x \leq 6\}$

(B) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(C) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$ 3

27. If for a distribution of 18 observations, $\Sigma(x_i - 5) = 10$ and $\Sigma(x_i - 5)^2 = 50$, find the mean and standard deviation. 3

28. If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$. 3

29. Find the equation of the circle which touches x-axis and whose centre is (1, 2).

OR

Find the equation of the circle which touches the both axes in first quadrant and whose radius is a. 3

30. Let R be a relation from N to N defined by $R = \{(a, b) : a, b \in N$ and $a = b^3\}$. Are the following true? Justify your answer in each case.

(A) $(a, a) \in R$, For all $a \in N$

(B) $(a, b) \in R$ implies that $(b, a) \in R$

(C) $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$ 3

31. Find $\lim_{x \to 0} [f(x)]$, where $f(x) = \begin{cases} \frac{|x|}{x} \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$.

OR

Evaluate: $\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$. 3

SECTION - D

(This section comprises of long answer-type questions (LA) of 5 marks each.)

32. The length of three unequal edges of a rectangular solid block are in G.P. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. Find the length of the largest edge. 5

33. Find the range of each of the following functions:

(A) $f(x) = \frac{1}{\sqrt{x-3}}$ (B) $f(x) = \sqrt{36 - x^2}$ 5

34. If $x - iy = \frac{(a+7)^2}{2a+i}$, then find the value of $x^2 + y^2$.

OR

Show that $|z - 2/z - 3| = 2$ represents a circle. Find its center and radius. 5

35. Two dice are rolled, A is the event that the sum of the numbers on the two dice is 6 and B is the event that at least one of the dice shows 4. Are the two events A and B (A) mutually exclusive? (B) exhaustive?

OR

From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years? 5

SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (A), (B), (C) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. Case-Study 1:

Ms. Khushi and Mr. Daksh decide to construct a Pascal triangle with the help of binomial theorem. They use the formula for the expansion is $(x+y)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} y^r$

$$= {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + \ldots + {}^nC_{n-1} x^1 y^{n-1} + {}^nC_n x^0 y^n.$$

(A) Find the coefficient of x^k $(0 \le k \le n)$ in the expansion of $E = 1 + (1+x) + (1+x)^2 + \ldots (1+x)^n$.

OR

Find the coefficient of y is the expansion of $\left(y^2 + \frac{c}{y}\right)^5$. 2

(B) Find the number of terms in the expansion of $(1+\sqrt{5}x)^7 + (1-\sqrt{5}x)^7$. 1

(C) Find the sum of coefficients of even powers of x in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$. 1

37. Case-Study 2:

Pankaj and his father were walking in a large park. They saw a kite flying in the sky. The position of Kite, Pankaj and Pankaj's father are at (20, 30, 10), (4, 3, 7) and (5, 3, 7) respectively.

(A) Find the distance between Pankaj and Kite.

OR

Find the distance between Pankaj's father and kite. 2

(B) The co-ordinates of Pankaj lie in which quadrant? 1

(C) If co-ordinate of kite, Pankaj and Pankaj's father form a triangle, then find the centroid of it. 1

38. Case-Study 3:

Let f be a real valued function, the function defined by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

For a function f(x) = sin x + cos x, answer the following questions.

(A) Find $\frac{d}{dx}(f'(x))$ at $x = 90°$. 2

(B) If $f(x) = \cos^2 x - \sin^2 x$, then find the value of $f' = (30°)$. 2

SOLUTION

SECTION - A

1. *(b)* 30

Explanation: Given, A and B are two sets such that $n(A) = 50$, $n(B) = 20$, $n(A \cap B) = 40$,

We have,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Putting the values,

$$n(A \cup B) = 50 + 20 - 40$$

$$n(A \cup B) = 30$$

2. *(b)* 3, –4

Explanation: Here complex number

$$3 - 4i$$

So, a general complex number can be written as Re + i(*Im*).

So, Real part = 3

And *Imaginary part* = –4

3. *(c)* 3072

Explanation: Here,

$$a_8 = 192$$

and $r = 2$

So, $ar^7 = 192$

$\Rightarrow$ $a.2^7 = 192$

$\Rightarrow$ $a = \frac{192}{128}$

$\Rightarrow$ $a = \frac{3}{2}$

Now, $a_{12} = ar^{11}$

$$= \frac{3}{2} \times 2^{11}$$

$$= 3 \times 2^{10}$$

$$= 3 \times 1024 = 3072$$

4. *(a)* 9.97 km

Explanation: Radius of the wheel = 49 cm

$\therefore$ Circumference of the wheel

$= 2\pi \times 49$ cm = 308 cm

Hence, the linear distance travelled by a point of the rim in one revolution = 308 cm.

Number of revolutions made by the wheel in 3 minutes *i.e.*, 180 seconds = 18 × 180 = 3240

$\therefore$ The linear distance travelled by a point of the rim in 3 minutes

= 308 × 3240 = 997920 cm = 9.97 km.

5. *(a)* 5

Explanation: The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Two points are (3, 2, –1) and (–1, –1, –1), then

$$d = \sqrt{(3+1)^2 + (2+1)^2 + (-1+1)^2}$$

$$= \sqrt{16+9+0} = \sqrt{25}$$

= 5 units

6. *(a)* $\left[1, \frac{11}{3}\right]$

Explanation: We have, $-3 \le \frac{5-3x}{2} \le 4$

$\Rightarrow -6 \le 5 - 3x \le 8 \Rightarrow -11 \le -3x \le 3$

$\Rightarrow \frac{11}{3} \ge x \ge 1$, which can be written as

$$1 '' x '' \frac{11}{3}$$

$$\therefore x \in \left[1, \frac{11}{3}\right]$$

⚠ **Caution**

- *Always remember to change the inequality while changing the minus sign to positive*

7. *(d)* 88

Explanation:

We have, $\lim_{x \to 3} (4x^3 - 2x^2 - x + 1)$

Putting, $x = 3$, then

Limit (L) $= 4(3)^3 - 2(3)^2 - 3 + 1$

$= 4 \times 27 - 2 \times 9 - 3 + 1$

$= 108 - 18 - 2$

$= 90 - 2$

$= 88$

8. *(c)* $\frac{b^2 - a^2}{2ab}$

Explanation: Let the first equation of line having intercepts on the axes $a, -b$ is

$$\frac{x}{a} + \frac{y}{-b} = 1$$

$\Rightarrow$ $\frac{x}{a} - \frac{y}{b} = 1$

$\Rightarrow$ $bx - ay = ab$ ...(i)

Let the second equation of line having intercepts on the axes $b, -a$ is

$$\frac{x}{b} + \frac{y}{-a} = 1$$

$\Rightarrow$ $\frac{x}{b} - \frac{y}{a} = 1$

$\Rightarrow$ $ax - by = ab$ ...(ii)

Now, we find the slope of equation (i)

$$bx - ay = ab$$

$\Rightarrow$ $ay = bx - ab$

$\Rightarrow$ $y = \frac{b}{a}x - b$

Since, the above equation is in $y = mx + c$ form.

So, the slope of equation (ii) is

$$m_1 = \frac{b}{a}$$

Now, we find the slope of equation (ii),

$$ax - by = ab$$

$$\Rightarrow \quad by = ax - ab$$

$$\Rightarrow \quad y = \frac{a}{b}x - a$$

Since the above equation is in $y = mx + b$ form.

So, the slope of eq. (ii) is

$$m_2 = \frac{a}{b}$$

Let θ be the angle between the given two lines.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Putting the values of m_1 and m_2 in the above equation, we get

$$\tan\theta = \left|\frac{\frac{b}{a} - \frac{a}{b}}{1 + \left(\frac{b}{a}\right)\left(\frac{a}{b}\right)}\right|$$

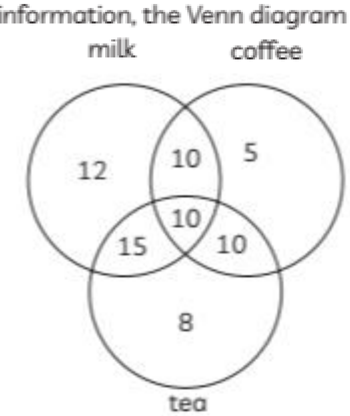
$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{b^2 - a^2}{ab}}{1+1}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{b^2 - a^2}{2ab}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{b^2 - a^2}{2ab}$$

9. *(d) 30*

Explanation: According to the given information, the Venn diagram is:



Now, $n(M \cup C \cup T) = 12 + 5 + 8 + 10 + 15 + 10 + 10 = 70$

Now, number of people who did not took any of the drinks is $n(M' \cap C' \cap T') = n(M \cup C \cup T)'$

$\Rightarrow n(M \cup C \cup T)' = N - n(M \cup C \cup T)$

$\Rightarrow n(M \cup C \cup T)' = 100 - 70 = 30$

Caution

➥ *Draw venn diagrams properly and always recheck it.*

10. *(b)* $(x + a)^2 + (y + b)^2 = 10000$

Explanation: Here $h = -a$, $k = -b$ and $r = 100$

So, equation will be $(x - h)^2 + (y - k)^2 = 100^2$

$(x + a)^2 + (y + b)^2 = 10000$

11. *(b) 1 : 3*

Explanation: Since x, $2y$, $3z$ are in A.P.

$$\therefore \quad 2y - x = 3z - 2y$$

$$\Rightarrow \quad 4y = x + 3z \quad \text{...(i)}$$

Now, x, y, z are in G.P.

$$\therefore \quad \text{Common ratio } r = \frac{y}{x} = \frac{z}{y} \quad \text{...(ii)}$$

$$\Rightarrow \quad y^2 = xz$$

Putting the value of x from eq. (i), we get

$$y^2 = (4y - 3z)z$$

$$\Rightarrow \quad y^2 = 4yz - 3z^2$$

$$\Rightarrow \quad 3z^2 - 4yz + y^2 = 0$$

$$\Rightarrow \quad 3z^2 - 3yz - yz + y^2 = 0$$

$$\Rightarrow \quad 3z(z - y) - y(z - y) = 0$$

$$\Rightarrow \quad (3z - y)(z - y) = 0$$

$$\Rightarrow \quad 3z - y = 0 \text{ and } z - y = 0$$

$$\Rightarrow \quad 3z = y \text{ and } z \neq y$$

[∵ z and y are distinct numbers]

$$\Rightarrow \quad \frac{z}{y} = \frac{1}{3}$$

$$\Rightarrow \quad r = \frac{1}{3} \quad \text{(from eq. (ii))}$$

12. *(b)* $(-\infty, -6)$

Explanation: We have, $x + 3 > 3x + 15$

$\Rightarrow -12 > 2x \Rightarrow -6 > x$

$\Rightarrow x < -6$

13. *(c) 5.27*

Explanation: The data is

3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21

$$\text{Now median} = \left(\frac{11+1}{2}\right)^{\text{th}} \text{ term} = 9$$

Now $|x_i - M|$ are

6, 6, 5, 4, 2, 0, 1, 3, 9, 10, 12

$$\text{Therefore, MD(M)} = \frac{1}{11}\sum |x_i - M|$$

$$= \frac{1}{11} \times 58$$

$$= 5.27$$

14. *(d) {RR, RB, BR, BB}*

Explanation: The sample space for this experiment is

S = {RR, RB, BR, BB}, where R denotes the red balls and B denotes the black balls.

15. *(d) 10*

Explanation: Given that $\frac{{}^{x}P_4}{{}^{x-1}P_4} = \frac{5}{3}$

$\Rightarrow \quad \frac{x!}{(x-4)!} \times \frac{(x-5)!}{(x-1)!} = \frac{5}{3}$

$\Rightarrow \quad 3x = 5(x-4)$

$x = 10$

16. *(c) 70*

Explanation: $n(A) = 10$

$n(B) = 7$

So number of elements in A × B

$= n(A) \times n(B)$

$= 10 \times 7$

$= 70$

17. *(d) 8*

18. *(c) 51*

Explanation: Given,

$(x + a)^{100} + (x - a)^{100}$

$= ({}^{100}C_0 x^{100} + {}^{100}C_1 x^{99} a + {}^{100}C_2 x^{98} a^2 + \ldots) + ({}^{100}C_0 x^{100} - {}^{100}C_1 x^{99} a + {}^{100}C_2 x^{98} a^2 + \ldots)$

$= 2({}^{100}C_0 x^{100} + {}^{100}C_2 x^{98} a^2 + \ldots + {}^{100}C_{100} a^{100})$

So, there are 51 terms.

19. *(c) A is true but R is false.*

Explanation: Here $\sin \frac{\pi}{3} = \sin 60° = \frac{\sqrt{3}}{2}$

Value of π in degree is 180°.

20. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Given lines are

and $4x + 3y = \frac{15}{2}$

Distance between them is

$$d = \left|\frac{11 - 15/2}{\sqrt{16+9}}\right|$$

$$= \left|\frac{7}{10}\right| = \frac{7}{10}$$

SECTION - B

21. (A) Given that: $(2a + b, a - b) = (8, 3)$

Comparing both sides, we get

$2a + b = 8$ ...(i)

$a - b = 3$ ...(ii)

Solving (i) and (ii) we get $a = \frac{11}{3}$ and $b = \frac{2}{3}$

(B) Given that: $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

Comparing both sides, we get

$\frac{a}{4} = 0 \Rightarrow a = 0, a - 2b = 6 + b$

$\Rightarrow \quad a - 3, b = 6$

$\Rightarrow \quad 0 - 3b = 6$

$\Rightarrow \quad b = -2$

So, $\quad a = 0, b = -2.$

OR

$f = \{(1, 1)\} (0, -2), (3, 0), (2, 4)\}$

and $\quad f(x) = ax + b$

then, $\quad f(0) = -2$

$\Rightarrow \quad -2 = a \times 0 + b$

$\Rightarrow \quad -2 = b$

and $\quad f(1) = 1$

$\Rightarrow \quad 1 = a \times 1 + b$

$\Rightarrow \quad 1 = a + b$

$\Rightarrow \quad 1 = a - 2$

$\Rightarrow \quad 3 = a$

22. Given,

$z_1 = \sqrt{2}(\cos 30° + i \sin 60°)$ and

$z_2 = \sqrt{3}(\cos 60° + i \sin 30°)$

$\therefore \quad z_1 z_2 = \left[\sqrt{2}(\cos 30° + i \sin 60°)\right] \times \left[\sqrt{3}(\cos 60° + i \sin 30°)\right]$

$= \sqrt{6}\,[(\cos 30° \cos 60° - \sin 30° \sin 60°) + i(\cos 30° \sin 30° + \sin 60° \cos 60°)]$

$= \sqrt{6}\left[\cos(60° + 30°) + i\left(\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2}\right)\right]$

[∵ cos(A + B) = cos A cos B − sin A sin B]

$= \sqrt{6}\left[\cos 90° + i\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \sqrt{6}\left[0 + i\left(\frac{\sqrt{3}}{2}\right)\right]$

$= 0 + i\left(\frac{\sqrt{6}(\sqrt{3})}{2}\right) = \frac{0 + 3\sqrt{2}i}{2}$

$\therefore$ Re$(z_1 z_2) = 0$

Hence proved.

23. It is given that, 2 × Probability of even number = Probability of odd number

$\Rightarrow$ P(A) = 2 P(B)

Let (A odd number, B even number)

$\Rightarrow$ P(A) : P(B) = 2 : 1

$\therefore$ Probability of occuring odd number,

$$P(A) = \frac{2}{2+1} = \frac{2}{3}$$

And probability of occurring even number,

$$P(B) = \frac{1}{2+1} = \frac{1}{3}$$

Now, G be the even that a number greater than 3 occur in a single roll of die.

So, the possible outcomes are 4, 5 and 6 out of which two are even and one odd.

$\therefore$ Required probability = P(G) = 2 × P(A) × P(B)

$$= 2 \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$$

24. To choose a programme of 7 courses (3 optional and 4 compulsory) from 9 courses (including 5 optional and 4 compulsory).

Here, the order is not important.

So, each selection is a combination.

Number of ways of selecting 3 optional from 5 optional = $^5C_3 = \frac{5!}{3!2!} = 10$

Number of ways of selecting 4 compulsory from 4 compulsory = $^4C_4 = 1$

Hence, required number of ways

$= 10 \times 1$

$= 10$

OR

We know that,

$$^nC_r = \frac{n!}{r!(n-r)!}$$

No. of questions in group A = 6

No. of questions in group B = 6

According to the question,

The different ways in which the question can be attempted are,

Group A	2	3	4	5
Group B	5	4	3	2

Hence, the number of different ways of doing questions,

$$= (^6C_2 \times {}^6C_5) + (^6C_3 \times {}^6C_4) + (^6C_4 \times {}^6C_3) + (^6C_5 \times {}^6C_2)$$

$= (15 \times 6) + (20 \times 15) + (15 \times 20) + (6 \times 15)$

$= 780$

25. Given, $a \cos\theta + b \sin\theta = m$ ...(i)

and $a \sin\theta - b \cos\theta = n$ ...(ii)

On squaring and adding of eqs (i) and (ii), we get

$m^2 + n^2 = (a \cos\theta + b \sin\theta)^2 + (a \sin\theta - b \cos\theta)^2$

$\Rightarrow m^2 + n^2 = a^2 \cos^2\theta + b^2 \sin^2\theta + 2ab \cos\theta \sin\theta + a^2 \sin^2\theta + b^2 \cos^2\theta - 2ab \sin\theta \cos\theta$

$\Rightarrow m^2 + n^2 = a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\sin^2\theta + \cos^2\theta)$

$\Rightarrow m^2 + n^2 = a^2 + b^2$

Hence, proved.

SECTION - C

26. (A) We have,

$X \subset B$ and $X \not\subset C$

$\Rightarrow$ X is a subset of B but X is not a subset of C.

$\Rightarrow$ $X \in P(B)$ but $X \notin P(C)$

$\Rightarrow$ X = {1}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}.

(B) We have,

$X \subset B$, $X \neq B$ and $X \not\subset C$

$\Rightarrow$ X is a subset of B other than B itself, and X is not a subset of C.

$X \in P(B)$

$\Rightarrow$ $X \in P(B)$, $X \notin P(C)$ but $X \neq B$

$\Rightarrow$ X = {1}, {3}, {1, 2}, {1, 3}, {2, 3}

(C) We have,

$X \subset A$, $X \subset B$ and $X \subset C$

$\Rightarrow$ $X \in P(A)$, $X \in P(B)$ and $X \in P(C)$

$\Rightarrow$ X is a subset of A, B, and C.

$\Rightarrow$ X = ϕ, {2}.

OR

(A) Let $A = \{1, 2, 3, 4\}$ and $B = \{x : x$ is a natural number and $4 \le x \le 6\} = \{4, 5, 6\}$

We know that two sets are disjoint if they have no common element.

Here, $A \cap B = \{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\} \neq \phi$

Since, there is a common element in both sets.

Hence, the given pair of sets is not disjoint.

(B) Let $A = \{a, e, i, o, u\}$ and $B = \{c, d, e, f\}$

We know that have two sets that are disjoint if they have no common element.

Here, $A \cap B = \{a, e, i, o, u\} \cap \{c, d, e, f\}$

$= \{e\} \neq \phi$

Since there is a common element in both sets.

Hence, the given sets are not disjoint.

(C) Let $A = \{x : x \text{ is an even integer}\} = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$ and $B = \{x : x \text{ is an odd integer}\} = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$

We know that have two sets that are disjoint if they have no common element.

Here, $A \cap B = \phi$

Hence, the given pair of sets are disjoint.

27. We have $\sum_{i=1}^{18}(x_i - 5) = 10$ and $\sum_{i=1}^{18}(x_i - 5)^2 = 50$

$$\Rightarrow \sum_{i=1}^{18} x_i - \sum_{i=1}^{18} 5 = 10 \text{ and } \sum_{i=1}^{18} x_i^2 - 10\sum_{i=1}^{18} x_i + \sum_{i=1}^{18} 25 = 50$$

$$\Rightarrow \sum_{i=1}^{18} x_i - 18 \times 5 = 10 \text{ and, } \sum_{i=1}^{18} x_i^2 - 10\sum_{i=1}^{18} x_i + 18 \times 25 = 50$$

$$\Rightarrow \sum_{i=1}^{18} x_i = 100 \text{ and, } \sum_{i=1}^{18} x_i^2 - 10 \times 100 + 18 \times 25 = 50$$

$$\Rightarrow \sum_{i=1}^{18} x_i = 100 \text{ and, } \sum_{i=1}^{18} x_i^2 = 600$$

$$\therefore \text{ Mean} = \frac{1}{18}\sum_{i=1}^{18} x_i = \frac{100}{18} = 5.55$$

$$\text{S.D.} = \sqrt{\frac{1}{18}\sum_{i=1}^{18} x_i^2 - \left(\frac{1}{18}\sum_{i=1}^{18} x_i\right)^2} = \sqrt{\frac{600}{18} - \left(\frac{100}{18}\right)^2}$$

$$= \sqrt{\frac{10800 - 10000}{324}} = \sqrt{\frac{800}{324}}$$

$$= \sqrt{2.46} = 1.56$$

28. Given, $\tan x = \frac{b}{a}$

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{\sqrt{(a+b)^2} + \sqrt{(a-b)^2}}{\sqrt{(a-b)(a+b)}}$$

$$= \frac{(a+b)+(a-b)}{\sqrt{a^2 - b^2}}$$

$$= \frac{2a}{\sqrt{a^2 - b^2}}$$

$$= \frac{2a}{a\sqrt{1 - \left(\frac{b}{a}\right)^2}}$$

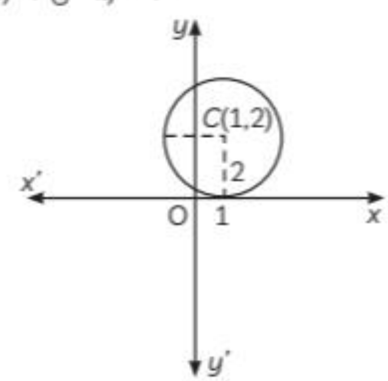
$$= \frac{2}{\sqrt{1 - \tan^2 x}} \qquad \left[\because \frac{b}{a} = \tan x\right]$$

$$= \frac{2\cos x}{\sqrt{\cos^2 x - \sin^2 x}}$$

$$= \frac{2\cos x}{\sqrt{\cos 2x}} \qquad [\therefore \cos 2x = \cos^2 x - \sin^2 x]$$

29. Given that, centre of the circle is (1, 2).

$(x - h)^2 + (y - k)^2 = r^2$



Important

➥ *When centre of the circle is given and circle touches x or y-axis then its radius = ordinate of centre or radius = abscissa of centre respectively.*

Put, $h = 1, k = 2$

$\therefore \quad r = 2$

So, the equation of circle is

$$(x - 1)^2 + (y - 2)^2 = 2^2$$

$$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 4y + 4 = 4$$

$$\Rightarrow \quad x^2 - 2x + y^2 - 4y + 1 = 0$$

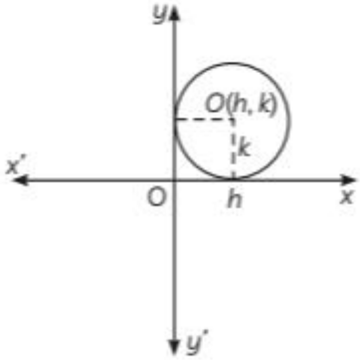
$$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0$$

OR

Given that radius of the circle is a and let centre be (h, k)

So, equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

Put, $h = k = r = a$



So, the equation of required circle is

$$(x-a)^2+(y-a)^2=a^2$$

$$\Rightarrow \quad x^2-2ax+a^2+y^2-2ay+a^2=a^2$$

$$\Rightarrow \quad x^2+y^2-2ax-2ay+a^2=0$$

30. Given $R=\{(a,b): a, b \in N \text{ and } a=b^3\}$.

(A) Since, $a=a^3$, is not true, for $a \in N$

$(a, a) \notin R$

(B) Let $(a, b) \in R$, where $a, b \in N$

$\Rightarrow a=b^3$.

$\Rightarrow b \neq a^3$, for some $a, b \in N$

For $a=8, b=2$, we have $(a, b) \in R$ but $(b, a) \notin R$.

(C) Let $(a, b) \in R$ and $(b, c) \in R$, where $a, b, c \in N$.

$\Rightarrow a=b^3$ and $b=C^3$

$\Rightarrow a \neq c^3$, for some $a, c \in N$

$\Rightarrow (a, c) \notin R$.

31. $$f(x)=\begin{cases}\frac{|x|}{x} \text{ if } x \neq 0 \\ 0 \text{ if } x=0\end{cases}=\begin{cases}\frac{x}{x} \text{ if } x>0 \\ -\frac{x}{x} \text{ if } x<0 \\ 0 \text{ if } x=0\end{cases}=\begin{cases}1 \text{ if } x>0 \\ -1 \text{ if } x<0 \\ 0 \text{ if } x=0\end{cases}$$

Given R.H.L $=\lim_{x \to 0^+}[f(x)]$

$$=\lim_{x \to 0}[1]=1$$

L.H.L $=\lim_{x \to 0^-}[f(x)]$

$$=\lim_{x \to 0}[-1]=-1$$

Since, $\lim_{x \to 0^+}[f(x)] \neq \lim_{x \to 0^-}[f(x)]$

Hence, $\lim_{x \to 0}[f(x)]$ does not exist.

OR

$$\lim_{x \to a} \frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}} \times \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$$

$$=\lim_{x \to a} \frac{(\sin x-\sin a)(\sqrt{x}+\sqrt{a})}{x-a}$$

$$=\lim_{x \to a} \frac{\left(2\cos\frac{x+a}{2} \cdot \sin\frac{x-a}{2}\right)(\sqrt{x}+\sqrt{a})}{x-a}$$

$$=\lim_{x \to a}\left(2\cos\frac{x+a}{2} \cdot \frac{\sin\frac{x-a}{2}}{2 \times \frac{x-a}{2}}\right)(\sqrt{x}+\sqrt{a})$$

$$=\lim_{x \to a} \cos\left(\frac{x+a}{2}\right)(\sqrt{x}+\sqrt{a})$$

$$\left[\because \lim_{\frac{x-a}{2} \to 0} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}=1\right]$$

Taking limit we have

$$=\cos\left(\frac{a+a}{2}\right)(\sqrt{a}+\sqrt{a})=\cos a \times 2\sqrt{a}=2\sqrt{a} \cdot \cos a$$

Hence, required answer is $2\sqrt{a} \cdot \cos a$.

SECTION - D

32. Let the length, breadth and height of a rectangular block be

$\frac{a}{r}$, a and ar. [Since they are in G.P.]

$\therefore$ Volume $= l \times b \times h$

$$\Rightarrow \quad 216=\frac{a}{r} \times a \times ar$$

$$\Rightarrow \quad a^3=216$$

$$\Rightarrow \quad a=6$$

Now total surface area $=2[lb+bh+lh]$

$$\Rightarrow \quad 252=2\left[\frac{a}{r} \cdot a+a \cdot ar+\frac{a}{r} \cdot ar\right]$$

$$\Rightarrow \quad 252=2\left[\frac{a^2}{r}+a^2r+a^2\right]$$

$$\Rightarrow \quad 252=2a^2\left[\frac{1}{r}+r+1\right]$$

$$\Rightarrow \quad 252=2 \times (6)^2\left[\frac{1+r^2+r}{r}\right]$$

$$\Rightarrow \quad 252=72\left[\frac{1+r^2+r}{r}\right]$$

$$\Rightarrow \quad \frac{252}{72} = \frac{1+r+r^2}{r}$$

$$\Rightarrow \quad \frac{7}{2} = \frac{1+r+r^2}{r}$$

$2 + 2r + 2r^2 = 7r$

$\Rightarrow \quad 2r^2 - 5r + 2 = 0$

$\Rightarrow \quad 2r^2 - 4r - r + 2 = 0$

$\Rightarrow \quad 2r(r - 2) - 1(r - 2) = 0$

$\Rightarrow \quad (r - 2)(2r - 1) = 0$

$\Rightarrow r - 2 = 0$ and $2r - 1 = 0$

$\therefore r = 2, \frac{1}{2}$

Therefore, the three edges are:

If $r = 2$ then edges are 3, 6, 12.

If $r = \frac{1}{2}$ then edges are 12, 6, 3.

So, the length of the longest edge = 12

33. (A) We have, $f(x) = \frac{1}{\sqrt{x-3}}$

Clearly, $f(x)$ takes real values for all x satisfying $x - 3 > 0 \Rightarrow x > 3 \Rightarrow x \in (3, \infty)$.

$\therefore$ Domain $(f) = (3, \infty)$

For any $x > 3$ we have

$x - 3 > 0 \Rightarrow \sqrt{x-3} > 0 \Rightarrow \frac{1}{\sqrt{x-3}} > 0$

$\Rightarrow \quad f(x) > 0$

Thus, $f(x)$ takes all real values greater than zero. Hence, Range $(f) = (0, \infty)$.

(B) We have, $f(x) = \sqrt{36 - x^2}$

We observe that $f(x)$ is defined for all x satisfying

$36 - x^2 \geq 0 \Rightarrow x^2 - 36 \leq 0$

$\Rightarrow \quad (x - 6)(x + 6) \leq 0$

$\Rightarrow \quad -6 \leq x \leq 6 \Rightarrow x \in [-6, 6]$.

$\therefore$ Domain $(f) = [-6, 6]$.

Let $y = f(x)$. Then,

$y = \sqrt{36 - x^2}$

$\Rightarrow y^2 = 36 - x^2$

$\Rightarrow x^2 = 36 - y^2$

$\Rightarrow x = \sqrt{36 - y^2}$

Clearly, x will take real values, if

$36 - y^2 \geq 0 \Rightarrow y^2 - 36 \leq 0$

$\Rightarrow \quad (y - 6)(y + 6) \leq 0 \Rightarrow -6 \leq y \leq 6$

$\Rightarrow \quad y \in [-6, 6]$

Also,

$y = \sqrt{36 - x^2} \geq 0$ for all $x \in [-6, 6]$.

Therefore, $y \in [0, 6]$ for all $x \in [-6, 6]$.

Hence, Range $(f) = [0, 6]$

34. Given, $x - iy = \frac{(a+7)^2}{2a+i}$

We know that if two complex numbers are equal, then their conjugates are also.

Taking conjugate of both sides, we get

$$x + iy = \frac{(a+7)^2}{2a-i}$$

Multiplying corresponding sides of both equations (1) and (2), we get

$(x - iy)(x + iy)$

$$= \frac{(a+7)^2}{2a+i} \times \frac{(a+7)^2}{2a-i}$$

$$\Rightarrow \quad x^2 - i^2y^2 = \frac{(a+7)^4}{(2a)^2 - i^2}$$

$$\Rightarrow \quad x^2 + y^2 = \frac{(a+7)^4}{4a^2 + 1}$$

OR

We have $|z - 2)/(z - 3)| = 2$

Putting $= x + iy$, we get

$$\left|\frac{x+iy-2}{x+iy-3}\right| = 2$$

$\Rightarrow |x - 2 + iy| = 2\,|x - 3 + iy|$

$\Rightarrow \sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$\Rightarrow x^2 - 4x + 4 + y^2 = 4(x^2 - 6x + 9 + y^2)$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$$

$$\Rightarrow \left(x - \frac{10}{3}\right)^2 + y^2 + \frac{32}{3} - \frac{100}{9} = 0$$

$$\Rightarrow \left(x - \frac{10}{3}\right)^2 + (y-0)^2 = \frac{4}{9}$$

Hence, centre of the circle is $\left(\frac{10}{3}, 0\right)$ and radius is $\frac{2}{3}$.

35. When two dice are rolled, sample space is

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

A = {(2,4), (3,3), (4,2), (5,1), (1,5)}

B = {(1,4), (2,4), (4,1), (4,2), (4,4), (3,4), (4,5), (4,6), (4,3), (5,4), (6,4)}

$A \cap B = \{(2,4), (4,2)\}$ and $A \cup B \neq S$.

(A) A and B are not mutually exclusive.

(B) A and B are not exhaustive.

OR

Total number of persons =5

So, $n(S) = 5$

Probability spokesperson is male

There are 3 males

So, $n(A) = 5$

Probability spokesperson is male = P(A)

$$= \frac{n(A)}{n(S)}$$

$$= \frac{3}{5}$$

Probability spokesperson is over 35 years old

Let B be the event that person selected is over 35

There are 2 person over 35 years old.

So, $n(B) = 2$

Probability that the spokesperson is over 35 years old = P(B)

$$= \frac{n(B)}{n(S)}$$

$$= \frac{2}{5}$$

We need to find probability that the spokesperson will be either male or over 35 years = $P(A \cup B)$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

To find $P(A \cup B)$,

we must find $P(A \cap B)$ first.

Probability that spokesperson is male and over 35 years old

Here, 1 person is both male and over 35 years old.

So, $n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$= \frac{1}{5}$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$$

$$= \frac{3+2-1}{5} = \frac{4}{5}$$

Hence, probability that the spokesperson will be either male or over 35 years = $\frac{4}{5}$

SECTION - E

36. We have,

(A) $E = \frac{(1+x)^{n+1}-1}{(1+x)-1}$

$$= \frac{{}^{n+1}C_0 + {}^{n+1}C_1 x + {}^{n+1}C_2 x^2 + \ldots - 1}{x}$$

$$= {}^{n+1}C_1 + {}^{n+1}C_2 x + {}^{n+1}C_3 x^2 + \ldots$$

Coeffcient of $x^k = {}^{n+1}C_{k+1}$

OR

$$(y^2 + c/y)^5 = {}^5C_0\left(\frac{c}{y}\right)^0 (y^2)^{5-0} + {}^5C_1\left(\frac{c}{y}\right)^1 (y^2)^{5-1} + \ldots + {}^5C_5\left(\frac{c}{y}\right)^5 (y^2)^{5-5}$$

$$= \sum_{r=0}^{5} {}^5C_r\left(\frac{c}{y}\right)^r (y^2)^{5-r}$$

We need coeffcient of $y \Rightarrow 2(5-r) - r = 1$

$\Rightarrow \quad 10 - 3r = 1$

$\Rightarrow \quad r = 3$

So, coeffcient of $y = {}^5C_3 . c^3$

$= 10c^3$

(B) Given expansion is

$$\left(1+\sqrt{5}x\right)^7 + \left(1-\sqrt{5}x\right)^7.$$

Here, $n = 7$, which is odd.

Total number of terms

$$= \frac{n+1}{2}$$

$$= \frac{7+1}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

(C) $(r+1)^{th}$ term $= {}^{11}C_r (x)^{11-r} . x^{-r}$

$= {}^{11}C_r . x^{11-2r}$

Even power of x exists only if $11 - 2r$ is an even number which is not possible

Thus, sum of coefficients = 0

37. (A) Required distance

$$= \sqrt{(20-4)^2+(30-3)^2+(10-7)^2}$$

$$= \sqrt{16^2+27^2+3^2}$$

$$= \sqrt{256+729+9}$$

$$= \sqrt{994}$$

= 31.52 units

OR

Required distance

$$= \sqrt{(20-5)^2+(30-3)^2+(10-7)^2}$$

$$= \sqrt{15^2+27^2+3^2}$$

$$= \sqrt{255+729+9}$$

$$= \sqrt{963}$$

= 31.03 units

(B) Because in (4, 3, 7); all are positive.

Thus, the coordinate lies in the I quadrant.

(C) Centroid

$$= \left(\frac{20+4+5}{3}, \frac{30+3+3}{3}, \frac{10+7+7}{3}\right)$$

= (9.67, 12, 8)

38. (A) We have, $f'(x) = \cos x - \sin x$

$$\therefore \frac{d}{dx}\left(f'(x)\right) = \frac{d}{dx}\left(\cos x - \sin x\right)$$

$$= \frac{d}{dx}\left(\cos x\right) - \frac{d}{dx}\left(\sin x\right)$$

$= -\sin x - \cos x$

$\therefore \frac{d}{dx}\left(f'(x)\right)$ at x = 90°

$-\quad \sin 90° - \cos 90°$

$= -1 - 0$

$= -1$

(B) We have

$f'(x) = \cos^2 - \sin^2 x$

$\Rightarrow \quad f'(x) = -2\cos x.\sin x - 2 \sin x \,.\, \cos x$

$= -4 \cos x \sin x$

$\Rightarrow \quad f'(30) = -4 \cos (30°) \sin (30°)$

$$= -4 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = -\sqrt{3}$$