Chapter 11

Algebra

Matchstick Pattern and Idea of Variable

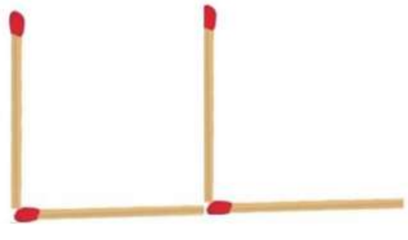
Matchstick Pattern

We see many patterns in our day to day life.
Ria and Sonia were playing with the matchsticks.
They both started to make patterns with the help of matchsticks.

Ria made one 'L' with the help of matchsticks.

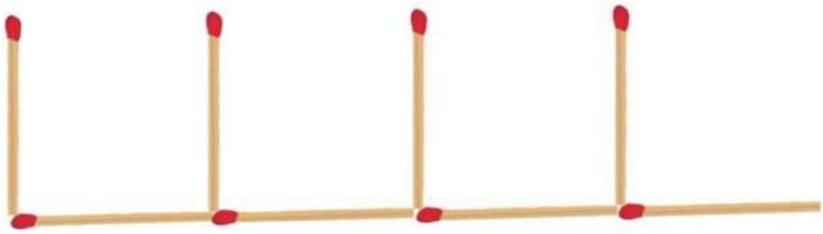
Ria made 'L' with two matchsticks

Sonia extended the pattern and made another 'L' with matchsticks.



Number of matchsticks = 4

They repeated the process again and again.



Let us sum up these patterns:-

Number of L's	1	2	3	4
Number of matchsticks	2	4	6	8

Here, we see a pattern

- If there is 1 'L' then the number of matchsticks is '2'.
- If there are 2 L' then the number of matchsticks is '4'.
- If there are 3 'L' then the number of matchsticks is '6'.
- If there are 4 'L' then the number of matchsticks is '8'

So, If there are 5 'L' then the number of matchsticks is '10' and so on.

<u>Idea of variable</u>

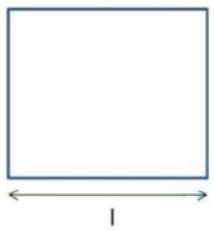
In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls.
The rule was:
Number of matchsticks required = 2n
Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3,4….
Let us look at the table once again. In the table, the value of n goes on changing (increasing).
As a result, the number of matchsticks required also goes on changing (increasing).

'n' is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4..... We wrote the rule for the number of matchsticks required using the variable n.

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values

Use of Variables in Geometry

Perimeter of square
Consider a square having side 'l' units.



We know that the perimeter of any figure is the sum of the length of the sides.

Here each side is 'l' units.

Perimeter of square = l+l+l+l = 4l.

So, here use of variable makes it easy to remember and use.

If side of square is

- 2 units, Perimeter = 4(l) = 4(2) = 8 units.
- 3 units, Perimeter = 4(l) = 4(3) = 12 units.
- 4 units, Perimeter = 4(l) = 4(4) = 16 units.
- 5 units, Perimeter = 4(l) = 4(5) = 20 units and so on.

Perimeter of rectangle

Consider a rectangle having length 'l' units and breadth 'b' units.

We know that the perimeter of any figure is the sum of the length of the sides.

Here, length 'l' units and breadth 'b' units.

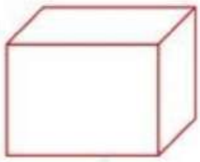
Perimeter of square = l+b+l+b = 2l+2b = 2(l+b).

So, here use of variable makes it easy to remember and use.

If length and breadth of rectangle is

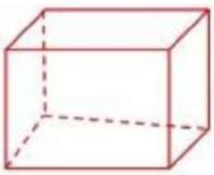
- 2 and 1 units respectively, Perimeter = 2(l+b) = 2(2+1) = 6 units.
- 3 and 1 units respectively, Perimeter = 2(l+b) = 2(3+1) = 8 units.
- 2 and 3 units respectively, Perimeter = 2(l+b) = 2(2+3) = 10 units.
- 1 and 2 units respectively, Perimeter = 2(l+b) = 2(1+2) = 6 units and so on.

Example: A cube is a three-dimensional figure as shown in Figure. It has six faces and all of them are identical squares. The length of an edge of the cube is given by l. Find the formula for the total length of the edges of a cube. (REFERENCE: NCERT)



Sol.

A cube has 6 faces.
Let us find the total number of edges of the cube.



Here, we drew all the edges.
So, the total count of edges = 12
Each edge measures 'l' units.
So total length = 12 (l) units

Use of Variables in Arithmetic

Commutativity of addition

$1+2 = 3$ and $2+1 = 3$.

As we know for whole numbers this is true for any two numbers.
This property is known as commutative property of addition of numbers.

It means changing the order of numbers will not affect the sum.
Let 'a' and 'b' be two numbers, then $a+b = b+a$.

Commutativity of multiplication

$1\times2 = 2$ and $2\times1 = 2$

As we know for whole numbers this is true for any two numbers.
This property is known as commutative property of multiplication of numbers.

It means changing the order of numbers will not affect the product.

Let 'a' and 'b' be two numbers, then $a\times b = b\times a$.

Distributivity of numbers

If we have to calculate, 5×42 instead of calculating it directly we can do it like,

$5 \times 42 = 5 \times (40 + 2) = 5\times 40 + 5\times 2 = 200+10 = 210$.

As we know for whole numbers this is true for any three numbers.
This property is known as the distributive property of multiplication over addition.

Let 'a', 'b', and 'c' be three numbers, then $a\times(b+c) = a\times b + b\times c$

Expressions with Variables

Expressions with variables

We know that variables can take different values, they have no fixed value. But they are numbers. That is why as in the case of numbers, operations of addition, subtraction, multiplication, and
division can be done on them.

One important point must be noted regarding the expressions containing variables.

A number expression like $(4 \times 3) + 5$ can be immediately evaluated as $(4 \times 3) + 5 = 12 + 5 = 17$

But an expression like $(4x + 5)$, which contains the variable x, cannot be evaluated.

Only if x is given some value, an expression like $(4x + 5)$ can be evaluated. For example,
when $x = 3$, $4x + 5 = (4 \times 3) + 5 = 17$ as found above.

Let us see some expressions and how they are formed?

(a) $y + 7$
  7 added to y.
(b) $t - 1$
  1 subtracted from t.
(c) 8b
  b multiplied by 8.
(d) $3x + 5$
  First x multiplied by 3, then 5 is added to the product

Give expressions for the following:

(a) 14 subtracted from x
  $x - 14$
(b) 25 added to s
  $s + 25$
(c) q multiplied by 16
  16 q
Example: The length of a rectangular cinema hall is 3 meters less than 3 times the breadth of the hall. What is the length, if the breadth is b meters?

Sol.
Breadth of hall = b meters
Now, let the length of the hall be 'l'.

It is given that,
The length of a rectangular cinema hall is 3 meters less than 3 times the breadth of the hall.
3 times the breadth of the hall = 3b
3 meters less than 3 times the breadth of the hall = 3b – 3
So, length, l = 3b-3.

Equations

Equation
Any equation is a condition on a variable. It is satisfied only for a definite value of the variable.

For example:
The equation 2n = 8 is satisfied only by the value 4 of the variable n.

Similarly, the equation x – 3 = 10 is satisfied only by the value 13 of the variable x.

An equation has an equal sign (=) between its two sides. The equation says that the value of the left-hand side (LHS) is equal to the value of the right-hand side (RHS).

The statement 2n is greater than 8, i.e. $2n > 8$ is not an equation. Similarly, the statement 2n is smaller than 8 i.e. $2n < 8$ is not an equation.
Also, the statements
$(x - 3) > 10$ or $(x - 3) < 10$ are not equations.

Now, let us consider 7 – 4 = 3
There is an equal sign between the LHS and RHS. Neither of the two sides contains a variable. Both contain numbers. We may call this a numerical equation.

Example:

State which of the following are equations with a variable. In the case of equations with a variable, identify the variable.

(a) x + 20 = 70
  Yes, x is a variable.
(b) 8 × 3 = 24
  No, this a numerical equation.

(c) $2p > 30$
No, this is not an equation.
(d) $n - 4 = 100$
Yes, n is a variable.
(e) $20b = 80$
Yes, b is a variable.

Solution of Equation

The value of the variable in an equation which satisfies the equation is called a solution to the equation.
Thus, $n = 4$ is a solution to the equation $2n = 8$.
Note, $n = 6$ is not a solution to the equation $2n = 8$.

Example: Pick out the solution from the values given in the bracket next to each equation.
Show that the other values do not satisfy the equation.
(REFERENCE: NCERT)
(a) $5m = 60$ (10, 5, 12, 15)
Sol.
Equation is $5m = 60$.
(10, 5, 12, 15) is the given set of values.
Let us try each value one by one.

- If $m = 10$
  $LHS = 5m = 5(10) = 50$
  and $RHS = 60$
  $LHS \neq RHS$

So this is not a solution to the given equation.

- If $m = 5$
  $LHS = 5m = 5(5) = 25$
  and $RHS = 60$
  $LHS \neq RHS$

So this is not a solution to the given equation.

- If $m = 12$
  $LHS = 5m = 5(12) = 60$
  and $RHS = 60$
  $LHS = RHS$

So this is a solution to the given equation.

- If $m = 15$

  $LHS = 5m = 5(15) = 75$
  and $RHS = 60$
  $LHS \neq RHS$

So this is not a solution to the given equation.

(b) $n + 12 = 20$ (12, 8, 20, 0)

Sol.

Equation is $n+12 = 20$.

(12, 8, 20, 0) is the given set of values.

Let us try each value one by one.

- If $n = 12$
  $LHS = n+12 = 12 + 12 = 24$
  And $RHS = 20$
  $LHS \neq RHS$

So this is not a solution to the given equation.

- If $n = 8$
  $LHS = n+12 = 8 + 12 = 20$
  and $RHS = 20$
  $LHS = RHS$

So this is a solution to the given equation.

- If $n = 20$
  $LHS = n+12 = 20 + 12 = 32$
  and $RHS = 20$

  $LHS \neq RHS$

So this is not a solution to the given equation.

- If $n = 0$
  $LHS = n+12 = 0 + 12 = 12$
  and $RHS = 20$
  LHS ≠ RHS