

Chapter 16. Area Theorems [Proof and Use]

Exercise 16(A)

Solution 1:

(i) $\triangle ADE$ and parallelogram ABED are on the same base AB and between the same parallels $DE \parallel AB$, so area of the triangle $\triangle ADE$ is half the area of parallelogram ABED.

$$\text{Area of ABED} = 2 (\text{Area of ADE}) = 120 \text{ cm}^2$$

(ii) Area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between the same parallels

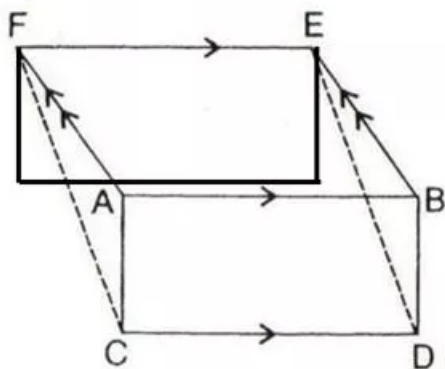
$$\text{Area of ABCF} = \text{Area of ABED} = 120 \text{ cm}^2$$

(iii) We know that area of triangles on the same base and between same parallel lines are equal

$$\text{Area of ABE} = \text{Area of ADE} = 60 \text{ cm}^2$$

Solution 2:

After drawing the opposite sides of AB, we get



Since from the figure, we get $CD \parallel FE$ therefore FC must be parallel to DE. Therefore it is proved that the quadrilateral CDEF is a parallelogram.

Area of parallelogram on same base and between same parallel lines is always equal and area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e, between same parallel lines.

So Area of CDEF = Area of ABDC + Area of ABEF

Hence Proved

Solution 3:

(i)

Since POS and parallelogram PMLS are on the same base PS and between the same parallels i.e. SP//LM.

As O is the center of LM and Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.

The area of the parallelogram is twice the area of the triangle if they lie on the same base and in between the same parallels.

So $2(\text{Area of } \triangle POS) = \text{Area of } PMLS$

Hence Proved.

(ii)

Consider the expression $\text{Area}(\triangle POS) + \text{Area}(\triangle QOR)$;

LM is parallel to PS and PS is parallel to RQ, therefore, LM is

Since triangle POS lie on the base PS and in between the parallels PS and LM, we have, $\text{Area}(\triangle POS) = \frac{1}{2} \text{Area}(\square PSLM)$,

Since triangle QOR lie on the base QR and in between the parallels LM and RQ, we have,

$$\text{Area}(\triangle QOR) = \frac{1}{2} \text{Area}(\square LMQR)$$

$$\begin{aligned} \text{Area}(\triangle POS) + \text{Area}(\triangle QOR) &= \frac{1}{2} \text{Area}(\square PSLM) + \frac{1}{2} \text{Area}(\square LMQR) \\ &= \frac{1}{2} [\text{Area}(\square PSLM) + \text{Area}(\square LMQR)] \\ &= \frac{1}{2} [\text{Area}(\square PQRS)] \end{aligned}$$

(iii)

In a parallelogram, the diagonals bisect each other.

Therefore, OS = OQ

Consider the triangle PQS, since OS = OQ, OP is the median of the triangle PQS.

We know that median of a triangle divides it into two triangles of equal area.

Therefore,

$$\text{Area}(\triangle POS) = \text{Area}(\triangle POQ) \dots (1)$$

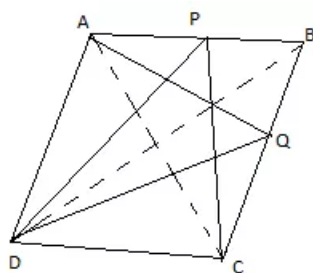
Similarly, since OR is the median of the triangle QRS, we have,

$$\text{Area}(\triangle QOR) = \text{Area}(\triangle SOR) \dots (2)$$

Adding equations (1) and (2), we have,

$$\text{Area}(\triangle POS) + \text{Area}(\triangle QOR) = \text{Area}(\triangle POQ) + \text{Area}(\triangle SOR)$$

Hence Proved.

Solution 4:

(i)

Given ABCD is a parallelogram. P and Q are any points on the sides AB and BC respectively, join diagonals AC and BD.

proof:

since triangles with same base and between same set of parallel lines have equal areas

$$\text{area}(\triangle CPD) = \text{area}(\triangle BCD) \dots (1)$$

again, diagonals of the parallelogram bisect area in two equal parts

$$\text{area}(\triangle BCD) = \left(\frac{1}{2}\right) \text{area of parallelogram } ABCD \dots (2)$$

from (1) and (2)

area(CPD)=1/2 area(ABCD)..... (3)

similarly area (AQD)=area(ABD)=1/2 area(ABCD)..... (4)

from (3) and (4)

area(CPD)=area(AQD),

hence proved.

(ii)

We know that area of triangles on the same base and between same parallel lines are equal

So Area of AQD= Area of ACD= Area of PDC = Area of BDC = Area of ABC=Area of APD + Area of BPC

Hence Proved

Solution 5:

(i)

Since triangle BEC and parallelogram ABCD are on the same base BC and between the same parallels i.e. BC//AD.

$$\text{So Area}(\triangle BEC) = \frac{1}{2} \times \text{Area}(\square ABCD) = \frac{1}{2} \times 48 = 24 \text{ cm}^2$$

(ii)

$$\begin{aligned}\text{Area}(\square ANMD) &= \text{Area}(\square BNMC) \\ &= \frac{1}{2} \text{Area}(\square ABCD) \\ &= \frac{1}{2} \times 2 \times \text{Area}(\triangle BEC) \\ &= \text{Area}(\triangle BEC)\end{aligned}$$

Therefore, Parallelograms ANMD and NBCM have areas equal to triangle BEC

Solution 6:

Since $\triangle DCB$ and $\triangle DEB$ are on the same base DB and between the same parallels i.e. DB//CE, therefore we get

$$\begin{aligned}\text{Ar.}(\triangle DCB) &= \text{Ar.}(\triangle DEB) \\ \text{Ar.}(\triangle DCB + \triangle ADB) &= \text{Ar.}(\triangle DEB + \triangle ADB) \\ \text{Ar.}(ABCD) &= \text{Ar.}(\triangle ADE)\end{aligned}$$

Hence proved

Solution 7:

$\triangle APB$ and parallelogram ABCD are on the same base AB and between the same parallel lines AB and CD.

$$\therefore \text{Ar.}(\triangle APB) = \frac{1}{2} \text{Ar.}(\text{parallelogram ABCD}) \dots\dots(i)$$

$\triangle ADQ$ and parallelogram ABCD are on the same base AD and between the same parallel lines AD and BQ.

$$\therefore \text{Ar.}(\triangle ADQ) = \frac{1}{2} \text{Ar.}(\text{parallelogram ABCD}) \dots\dots(ii)$$

Adding equation (i) and (ii), we get

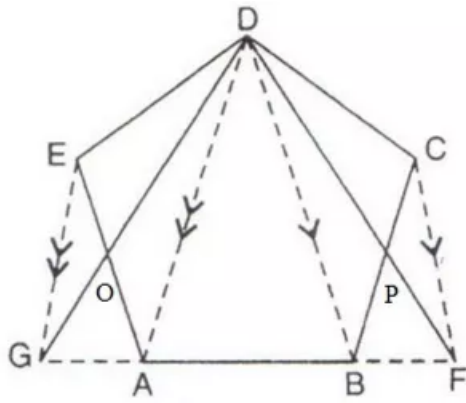
$$\begin{aligned}\therefore \text{Ar.}(\triangle APB) + \text{Ar.}(\triangle ADQ) &= \text{Ar.}(\text{parallelogram ABCD}) \\ \text{Ar.}(\text{quad.}ADQB) - \text{Ar.}(\triangle BPQ) &= \text{Ar.}(\text{parallelogram ABCD}) \\ \text{Ar.}(\text{quad.}ADQB) - \text{Ar.}(\triangle BPQ) &= \text{Ar.}(\text{quad.}ADQB) - \text{Ar.}(\triangle DCQ) \\ \text{Ar.}(\triangle BPQ) &= \text{Ar.}(\triangle DCQ)\end{aligned}$$

Subtracting Ar. $\triangle PCQ$ from both sides, we get

$$\begin{aligned}\text{Ar.}(\triangle BPQ) - \text{Ar.}(\triangle PCQ) &= \text{Ar.}(\triangle DCQ) - \text{Ar.}(\triangle PCQ) \\ \text{Ar.}(\triangle BCP) &= \text{Ar.}(\triangle DPQ)\end{aligned}$$

Hence proved.

Solution 8:



Since triangle EDG and EGA are on the same base EG and between the same parallel lines EG and DA, therefore

$$Ar.(\triangle EDG) = Ar.(\triangle EGA)$$

Subtracting $\triangle EOG$ from both sides, we have

$$Ar.(\triangle EOD) = Ar.(\triangle GOA) \quad (i)$$

Similarly

$$Ar.(\triangle DPC) = Ar.(\triangle BPF) \quad (ii)$$

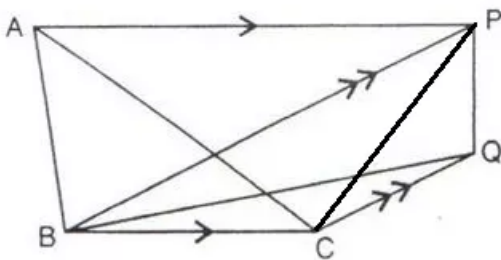
Now

$$\begin{aligned} Ar.(\triangle GDF) &= Ar.(\triangle GOA) + Ar.(\triangle BPF) + Ar.(\text{pen. } ABPDO) \\ &= Ar.(\triangle EOD) + Ar.(\triangle DPC) + Ar.(\text{pen. } ABPDO) \\ &= Ar.(\text{pen. } ABCDE) \end{aligned}$$

Hence proved

Solution 9:

Joining PC we get



$\triangle ABC$ and $\triangle BPC$ are on the same base BC and between the same parallel lines AP and BC.

$$\therefore Ar.(\triangle ABC) = Ar.(\triangle BPC) \quad \dots\dots(i)$$

$\triangle BPC$ and $\triangle BQP$ are on the same base BP and between the same parallel lines BP and CQ.

$$\therefore Ar.(\triangle BPC) = Ar.(\triangle BQP) \quad \dots\dots(ii)$$

From (i) and (ii), we get

$$\therefore Ar.(\triangle ABC) = Ar.(\triangle BQP)$$

Hence proved.

Solution 10:

(i)

$$\angle EAC = \angle EAB + \angle BAC$$

$$\angle EAC = 90^\circ + \angle BAC \quad \dots\dots(i)$$

$$\angle BAF = \angle FAC + \angle BAC$$

$$\angle BAF = 90^\circ + \angle BAC \quad \dots\dots(ii)$$

From (i) and (ii), we get

$$\angle EAC = \angle BAF$$

In $\triangle EAC$ and $\triangle BAF$, we have, $EA=AB$

$$\angle EAC = \angle BAF \text{ and } AC=AF$$

$$\therefore \triangle EAC \cong \triangle BAF \text{ (SAS axiom of congruency)}$$

(ii)

Since $\triangle ABC$ is a right triangle, we have,

$$AC^2 = AB^2 + BC^2 \quad [\text{Using Pythagoras Theorem in } \triangle ABC]$$

$$\Rightarrow AB^2 = AC^2 - BC^2$$

$$\Rightarrow AB^2 = (AR + RC)^2 - (BR^2 + RC^2) \quad [\text{Since } AC = AR + RC \text{ and Using Pythagoras Theorem in } \triangle BRC]$$

$$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (BR^2 + RC^2) \quad [\text{Using the identity}]$$

$$\Rightarrow AB^2 = AR^2 + 2AR \times RC + RC^2 - (AB^2 - AR^2 + RC^2) \quad [\text{Using Pythagoras Theorem in } \triangle ABR]$$

$$\Rightarrow 2AB^2 = 2AR^2 + 2AR \times RC$$

$$\Rightarrow AB^2 = AR(AR + RC)$$

$$\Rightarrow AB^2 = AR \times AC$$

$$\Rightarrow AB^2 = AR \times AF$$

$$\Rightarrow \text{Area}(\square ABDE) = \text{Area}(\text{rectangle } ARHF)$$

Solution 11:

(i)

In $\triangle ABC$, D is midpoint of AB and E is the midpoint of AC.

$$\frac{AD}{AB} = \frac{AE}{AC}$$

DE is parallel to BC.

$$\therefore \text{Ar.}(\triangle ADC) = \text{Ar.}(\triangle BDC) = \frac{1}{2} \text{Ar.}(\triangle ABC)$$

Again

$$\therefore \text{Ar.}(\triangle AEB) = \text{Ar.}(\triangle BEC) = \frac{1}{2} \text{Ar.}(\triangle ABC)$$

From the above two equations, we have

$$\text{Area}(\triangle ADC) = \text{Area}(\triangle AEB).$$

Hence Proved

(ii)

We know that area of triangles on the same base and between same parallel lines are equal

$$\text{Area}(\text{triangle } DBC) = \text{Area}(\text{triangle } BCE)$$

$$\text{Area}(\text{triangle } DOB) + \text{Area}(\text{triangle } BOC) = \text{Area}(\text{triangle } BOC) + \text{Area}(\text{triangle } COE)$$

$$\text{So Area}(\text{triangle } DOB) = \text{Area}(\text{triangle } COE)$$

Solution 12:

(i)

Since $\triangle EBC$ and parallelogram $ABCD$ are on the same base BC and between the same parallels i.e. $BC \parallel AD$.

$$\begin{aligned}\therefore \text{Ar.}(\triangle EBC) &= \frac{1}{2} \times \text{Ar.}(\text{parallelogram } ABCD) \\ (\text{parallelogram } ABCD) &= 2 \times \text{Ar.}(\triangle EBC) \\ &= 2 \times 480 \text{ cm}^2 \\ &= 960 \text{ cm}^2\end{aligned}$$

(ii)

Parallelograms on same base and between same parallels are equal in area

$$\text{Area of } BCFE = \text{Area of } ABCD = 960 \text{ cm}^2$$

(iii)

$$\text{Area of triangle } ACD = 480 = \left(\frac{1}{2}\right) \times 30 \times \text{Altitude}$$

$$\text{Altitude} = 32 \text{ cm}$$

(iv)

The area of a triangle is half that of a parallelogram on the same base and between the same parallels.

Therefore,

$$\text{Area}(\triangle ECF) = \frac{1}{2} \text{Area}(\square CBEF)$$

$$\text{Similarly, Area}(\triangle BCE) = \frac{1}{2} \text{Area}(\square CBEF)$$

$$\Rightarrow \text{Area}(\triangle ECF) = \text{Area}(\triangle BCE) = 480 \text{ cm}^2$$

Solution 13:

Here $AD = DB$ and $EC = DB$, therefore $EC = AD$

Again, $\angle EFC = \angle AFD$ (opposite angles)

Since ED and CB are parallel lines and AC cut this line, therefore

$$\angle ECF = \angle FAD$$

From the above conditions, we have

$$\triangle EFC = \triangle AFD$$

Adding quadrilateral $CBDF$ in both sides, we have

$$\text{Area of } \parallel \text{ gm } BDEC = \text{Area of } \triangle ABC$$

Solution 14:

In Parallelogram $PQRS$, $AC \parallel PS \parallel QR$ and $PQ \parallel DB \parallel SR$.

Similarly, $AQRC$ and $APSC$ are also parallelograms.

Since $\triangle ABC$ and parallelogram $AQRC$ are on the same base AC and between the same parallels, then

$$\text{Ar.}(\triangle ABC) = \frac{1}{2} \text{Ar.}(AQRC) \dots\dots (i)$$

Similarly,

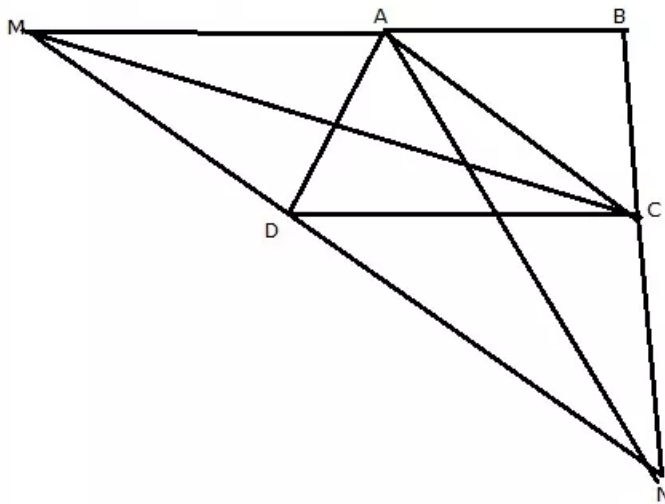
$$\text{Ar.}(\triangle ADC) = \frac{1}{2} \text{Ar.}(APSC) \dots\dots (ii)$$

Adding (i) and (ii), we get

$$\text{Area of quadrilateral } PQRS = 2 \times \text{Area of quad. } ABCD$$

Solution 15:

Given: ABCD is a trapezium



$AB \parallel CD, MN \parallel AC$

Join C and M

We know that area of triangles on the same base and between same parallel lines are equal.

So Area of $\triangle AMD$ = Area of $\triangle AMC$

Similarly, consider AMNC quadrilateral where $MN \parallel AC$.

$\triangle ACM$ and $\triangle ACN$ are on the same base and between the same parallel lines. So areas are equal.

So, Area of $\triangle ACM$ = Area of $\triangle CAN$

From the above two equations, we can say

Area of $\triangle ADM$ = Area of $\triangle CAN$

Hence Proved.

Solution 16:

We know that area of triangles on the same base and between same parallel lines are equal.

Consider ABED quadrilateral; $AD \parallel BE$

With common base, BE and between AD and BE parallel lines, we have

Area of $\triangle ABE$ = Area of $\triangle BDE$

Similarly, in BEFC quadrilateral, $BE \parallel CF$

With common base BC and between BE and CF parallel lines, we have

Area of $\triangle BEC$ = Area of $\triangle BEF$

Adding both equations, we have

Area of $\triangle ABE$ + Area of $\triangle BEC$ = Area of $\triangle BEF$ + Area of $\triangle BDE$

\Rightarrow Area of AEC = Area of DBF

Hence Proved

Solution 17:

Given: ABCD is a parallelogram.

We know that

Area of $\triangle ABC$ = Area of $\triangle ACD$

Consider $\triangle ABX$,

Area of $\triangle ABX$ = Area of $\triangle ABC$ + Area of $\triangle ACX$

We also know that area of triangles on the same base and between same parallel lines are equal.

Area of $\triangle ACX$ = Area of $\triangle CXD$

From above equations, we can conclude that

Area of $\triangle ABX$ = Area of $\triangle ABC$ + Area of $\triangle ACX$ = Area of $\triangle ACD$ + Area of $\triangle CXD$ = Area of ACXD Quadrilateral

Hence Proved

Solution 18:

Join B and R and P and R.

We know that the area of the parallelogram is equal to twice the area of the triangle, if the triangle and the parallelogram

are on the same base and between the parallels

Consider ABCD parallelogram:

Since the parallelogram ABCD and the triangle ABR lie on AB and between the parallels AB and DC, we have

$$\text{Area}(\square ABCD) = 2 \times \text{Area}(\triangle ABR) \dots (1)$$

We know that the area of triangles with same base and between the same parallel lines are equal.

Since the triangles ABR and APR lie on the same base AR and between the parallels AR and QP, we have,

$$\text{Area}(\triangle ABR) = \text{Area}(\triangle APR) \dots (2)$$

From equations (1) and (2), we have,

$$\text{Area}(\square ABCD) = 2 \times \text{Area}(\triangle APR) \dots (3)$$

Also, the triangle APR and the parallelogram ARQP

lie on the same base AR and between the parallels, AR and QP,

$$\text{Area}(\triangle APR) = \frac{1}{2} \times \text{Area}(\square ARQP) \dots (4)$$

Using (4) in equation (3), we have,

$$\text{Area}(\square ABCD) = 2 \times \frac{1}{2} \times \text{Area}(\square ARQP)$$

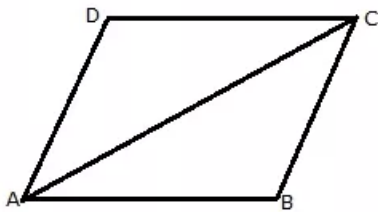
$$\text{Area}(\square ABCD) = \text{Area}(\square ARQP)$$

Hence proved.

Exercise 16(B)

Solution 1:

(i) Suppose ABCD is a parallelogram (given)



Consider the triangles ABC and ADC:

$$AB = CD \quad [\text{ABCD is a parallelogram}]$$

$$AD = BC \quad [\text{ABCD is a parallelogram}]$$

$$AC = AC \quad [\text{common}]$$

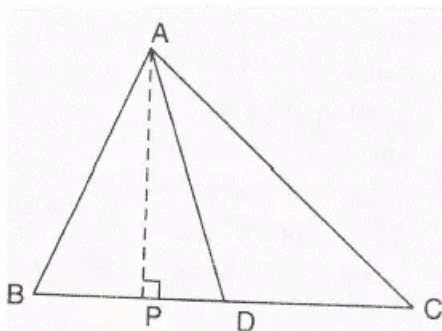
By Side – Side – Side criterion of congruence, we have,

$$\triangle ABC \cong \triangle ADC$$

Area of congruent triangles are equal.

Therefore, Area of ABC = Area of ADC

(ii) Consider the following figure:



Here $AP \perp BC$

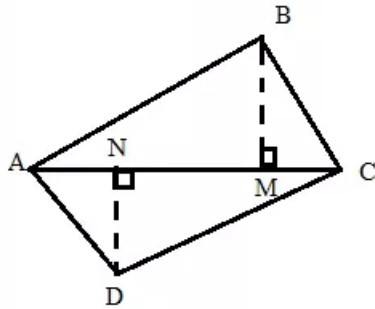
$$\text{Since } \text{Ar.}(\triangle ABD) = \frac{1}{2} BD \times AP$$

$$\text{And, } \text{Ar.}(\triangle ADC) = \frac{1}{2} DC \times AP$$

$$\therefore \frac{\text{Area}(\triangle ABD)}{\text{Area}(\triangle ADC)} = \frac{\frac{1}{2} BD \times AP}{\frac{1}{2} DC \times AP} = \frac{BD}{DC},$$

hence proved

(iii) Consider the following figure:



Here

$$\text{Ar.}(\triangle ABC) = \frac{1}{2} BM \times AC$$

$$\text{And, } \text{Ar.}(\triangle ADC) = \frac{1}{2} DN \times AC$$

$$\therefore \frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle ADC)} = \frac{\frac{1}{2} BM \times AC}{\frac{1}{2} DN \times AC} = \frac{BM}{DN},$$

hence proved

Solution 2:

AD is the median of $\triangle ABC$. Therefore it will divide $\triangle ABC$ into two triangles of equal areas.

$$\therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle ACD) \quad (i)$$

ED is the median of $\triangle EBC$

$$\therefore \text{Area}(\triangle EBD) = \text{Area}(\triangle ECD) \quad (ii)$$

Subtracting equation (ii) from (i), we obtain

$$\text{Area}(\triangle ABD) - \text{Area}(\triangle EBD) = \text{Area}(\triangle ACD) - \text{Area}(\triangle ECD)$$

$$\text{Area}(\triangle ABE) = \text{Area}(\triangle ACE). \text{ Hence proved}$$

Solution 3:

AD is the median of $\triangle ABC$. Therefore it will divide $\triangle ABC$ into two triangles of equal areas.

$$\therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle ACD)$$

$$\text{Area}(\triangle ABD) = \frac{1}{2} \text{Area}(\triangle ABC) \text{ (i)}$$

In $\triangle ABD$, E is the mid-point of AD. Therefore BE is the median.

$$\therefore \text{Area}(\triangle BED) = \text{Area}(\triangle ABE)$$

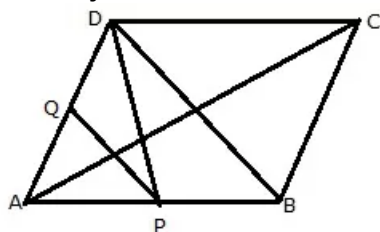
$$\text{Area}(\triangle BED) = \frac{1}{2} \text{Area}(\triangle ABD)$$

$$\text{Area}(\triangle BED) = \frac{1}{2} \times \frac{1}{2} \text{Area}(\triangle ABC) \text{ [from equation (i)]}$$

$$\text{Area}(\triangle BED) = \frac{1}{4} \text{Area}(\triangle ABC)$$

Solution 4:

We have to join PD and BD.



BD is the diagonal of the parallelogram ABCD. Therefore it divides the parallelogram into two equal parts.

$$\therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle DBC)$$

$$= \frac{1}{2} \text{Area}(\text{parallelogram ABCD}) \text{ (i)}$$

DP is the median of $\triangle ABD$. Therefore it will divide $\triangle ABD$ into two triangles of equal areas.

$$\therefore \text{Area}(\triangle APD) = \text{Area}(\triangle DPB)$$

$$= \frac{1}{2} \text{Area}(\triangle ABD)$$

$$= \frac{1}{2} \times \frac{1}{2} \text{Area}(\text{parallelogram ABCD}) \text{ [from equation (i)]}$$

$$= \frac{1}{4} \text{Area}(\text{parallelogram ABCD}) \text{ (ii)}$$

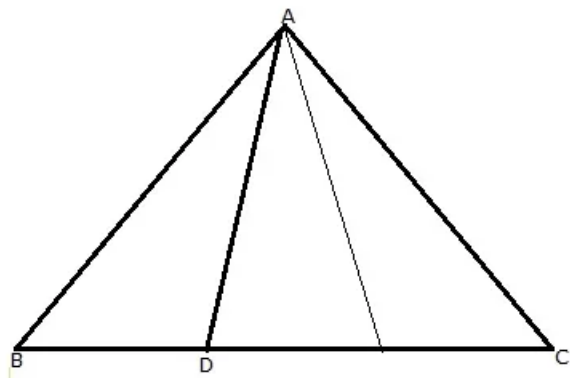
In $\triangle APD$, Q is the mid-point of AD. Therefore PQ is the median.

$$\therefore \text{Area}(\triangle APQ) = \text{Area}(\triangle DPQ)$$

$$= \frac{1}{2} \text{Area}(\triangle APD)$$

$$= \frac{1}{2} \times \frac{1}{4} \text{Area}(\text{parallelogram ABCD}) \text{ [from equation (ii)]}$$

$$\text{Area}(\triangle APQ) = \frac{1}{8} \text{Area}(\text{parallelogram ABCD}), \text{ hence proved}$$

Solution 5:

$$\text{In } \triangle ABC, \because BD = \frac{1}{2} DC \Rightarrow \frac{BD}{DC} = \frac{1}{2}$$

$$\therefore \text{Ar.}(\triangle ABD) : \text{Ar.}(\triangle ADC) = 1:2$$

$$\text{But Ar.}(\triangle ABD) + \text{Ar.}(\triangle ADC) = \text{Ar.}(\triangle ABC)$$

$$\text{Ar.}(\triangle ABD) + 2\text{Ar.}(\triangle ABD) = \text{Ar.}(\triangle ABC)$$

$$3 \text{Ar.}(\triangle ABD) = \text{Ar.}(\triangle ABC)$$

$$\text{Ar.}(\triangle ABD) = \frac{1}{3} \text{Ar.}(\triangle ABC)$$

Solution 6:

Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases. So, we have

$$\frac{\text{Area of } \triangle DPB}{\text{Area of } \triangle PCB} = \frac{DP}{PC} = \frac{3}{2}$$

Given: Area of $\triangle DPB = 30$ sq. cm

Let 'x' be the area of the triangle PCB

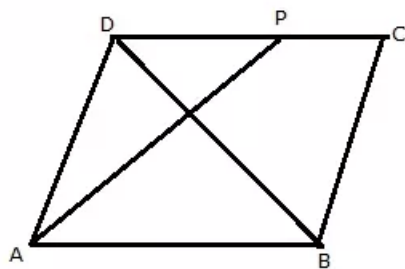
Therefore, we have,

$$\frac{30}{x} = \frac{3}{2}$$

$$\Rightarrow x = \frac{30}{3} \times 2 = 20 \text{ sq. cm.}$$

So area of $\triangle PCB = 20$ sq. cm

Consider the following figure.



From the diagram, it is clear that,

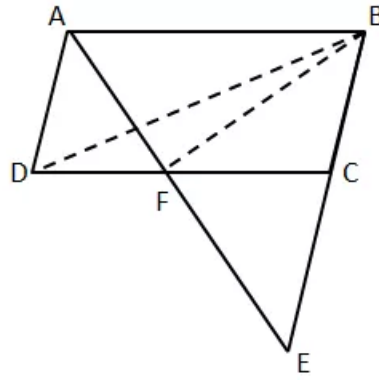
$$\begin{aligned} \text{Area}(\triangle CDB) &= \text{Area}(\triangle DPB) + \text{Area}(\triangle CPB) \\ &= 30 + 20 \\ &= 50 \text{ sq. cm} \end{aligned}$$

Diagonal of the parallelogram divides it into two triangles $\triangle ADB$ and $\triangle CDB$ of equal area.

Therefore,

$$\begin{aligned} \text{Area}(\text{||gm } ABCD) &= 2 \times \triangle CDB \\ &= 2 \times 50 = 100 \text{ sq. cm} \end{aligned}$$

Solution 7:



$BC = CE$ (given)

Also, in parallelogram ABCD, $BC = AD$

$\Rightarrow AD = CE$

Now, in $\triangle ADF$ and $\triangle ECF$, we have

$AD = CE$

$\angle ADF = \angle ECF$ (Alternate angles)

$\angle DAF = \angle CEF$ (Alternate angles)

$\therefore \triangle ADF \cong \triangle ECF$ (ASA Criterion)

$\Rightarrow \text{Area}(\triangle ADF) = \text{Area}(\triangle ECF)$ (1)

Also, in $\triangle FBE$, FC is the median (Since $BC = CE$)

$\Rightarrow \text{Area}(\triangle BCF) = \text{Area}(\triangle ECF)$ (2)

From (1) and (2),

$\text{Area}(\triangle ADF) = \text{Area}(\triangle BCF)$ (3)

Again, $\triangle ADF$ and $\triangle BDF$ are on the base DF and between parallels DF and AB.

$\Rightarrow \text{Area}(\triangle BDF) = \text{Area}(\triangle ADF)$ (4)

From (3) and (4),

$\text{Area}(\triangle BDF) = \text{Area}(\triangle BCF) = 30 \text{ cm}^2$

$\therefore \text{Area}(\triangle BCD) = \text{Area}(\triangle BDF) + \text{Area}(\triangle BCF) = 30 + 30 = 60 \text{ cm}^2$

Hence, Area of parallelogram ABCD $= 2 \times \text{Area}(\triangle BCD) = 2 \times 60 = 120 \text{ cm}^2$

Solution 8:

In $\triangle ABC$,

R and Q are the mid - points of AC and BC respectively.

$$\Rightarrow RQ \parallel AB$$

that is $RQ \parallel PB$

So, $\text{area}(\triangle PBQ) = \text{area}(\triangle APR) \dots (i) \dots$ (Since $AP = PB$ and triangles on the same base and between the same parallels are equal in area)

Since P and R are the mid - points of AB and AC respectively.

$$\Rightarrow PR \parallel BC$$

that is $PR \parallel BQ$

So, quadrilateral PMQR is a parallelogram.

Also, $\text{area}(\triangle PBQ) = \text{area}(\triangle PQR) \dots (ii) \dots$ (diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

from (i) and (ii),

$$\text{area}(\triangle PQR) = \text{area}(\triangle PBQ) = \text{area}(\triangle APR) \dots (iii)$$

Similarly, P and Q are the mid - points of AB and BC respectively.

$$\Rightarrow PQ \parallel AC$$

that is $PQ \parallel RC$

So, quadrilateral PQCR is a parallelogram.

Also, $\text{area}(\triangle RQC) = \text{area}(\triangle PQR) \dots (iv) \dots$ (diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

From (iii) and (iv),

$$\text{area}(\triangle PQR) = \text{area}(\triangle PBQ) = \text{area}(\triangle RQC) = \text{area}(\triangle APR)$$

$$\text{So, } \text{area}(\triangle PBQ) = \frac{1}{4} \text{area}(\triangle ABC) \dots (v)$$

Also, since S is the mid - point of PQ,

BS is the median of $\triangle PBQ$

$$\text{So, } \text{area}(\triangle QSB) = \frac{1}{2} \text{area}(\triangle PBQ)$$

from (v),

$$\text{area}(\triangle QSB) = \frac{1}{2} \times \frac{1}{4} \text{area}(\triangle ABC)$$

$$\Rightarrow \text{area}(\triangle ABC) = 8 \text{area}(\triangle QSB)$$

Exercise 16(C)

Solution 1:

(i)
Ratio of area of triangles with same vertex and bases along the same line is equal to the ratio of their respective bases. So, we have:

$$\frac{\text{Area of } \triangle DOC}{\text{Area of } \triangle BOC} = \frac{DO}{BO} = 1 \text{ ----1}$$

Similarly

$$\frac{\text{Area of } \triangle DOA}{\text{Area of } \triangle BOA} = \frac{DO}{BO} = 1 \text{ -----2}$$

We know that area of triangles on the same base and between same parallel lines are equal.

$$\text{Area of } \triangle ACD = \text{Area of } \triangle BCD$$

$$\text{Area of } \triangle AOD + \text{Area of } \triangle DOC = \text{Area of } \triangle DOC + \text{Area of } \triangle BOC$$

$$\Rightarrow \text{Area of } \triangle AOD = \text{Area of } \triangle BOC \text{ -----3}$$

From 1, 2 and 3 we have

$$\text{Area}(\triangle DOC) = \text{Area}(\triangle AOB)$$

Hence Proved.

(ii)

Similarly, from 1, 2 and 3, we also have

$$\text{Area of } \triangle DCB = \text{Area of } \triangle DOC + \text{Area of } \triangle BOC = \text{Area of } \triangle AOB + \text{Area of } \triangle BOC = \text{Area of } \triangle ABC$$

$$\text{So Area of } \triangle DCB = \text{Area of } \triangle ABC$$

Hence Proved.

(iii)

We know that area of triangles on the same base and between same parallel lines are equal.

Given: triangles are equal in area on the common base, so it indicates $AD \parallel BC$.

So, ABCD is a parallelogram.

Hence Proved

Solution 2:

Ratio of area of triangles with the same vertex and bases along the same line is equal to the ratio of their respective bases.

So, we have

$$\frac{\text{Area of } \triangle APD}{\text{Area of } \triangle BPD} = \frac{AP}{BP} = \frac{1}{2}$$

Area of parallelogram ABCD = 324 sq.cm

Area of the triangles with the same base and between the same parallels are equal.

We know that area of the triangle is half the area of the parallelogram if they lie on the same base and between the parallels.

Therefore, we have,

$$\begin{aligned}\text{Area}(\triangle ABD) &= \frac{1}{2} \times \text{Area}(\text{||gm } ABCD) \\ &= \frac{324}{2} \\ &= 162 \text{ sq. cm}\end{aligned}$$

From the diagram it is clear that,

$$\text{Area}(\triangle ABD) = \text{Area}(\triangle APD) + \text{Area}(\triangle BPD)$$

$$\Rightarrow 162 = \text{Area}(\triangle APD) + 2\text{Area}(\triangle APD)$$

$$\Rightarrow 162 = 3\text{Area}(\triangle APD)$$

$$\Rightarrow \text{Area}(\triangle APD) = \frac{162}{3}$$

$$\Rightarrow \text{Area}(\triangle APD) = 54 \text{ sq. cm}$$

(ii)

Consider the triangles $\triangle AOP$ and $\triangle COD$

$$\angle AOP = \angle COD \text{ [vertically opposite angles]}$$

$$\angle CDO = \angle APD \text{ [AB and DC are parallel and DP is the transversal, alternate interior angles are equal]}$$

Thus, by Angle – Angle similarity, $\triangle AOP \sim \triangle COD$.

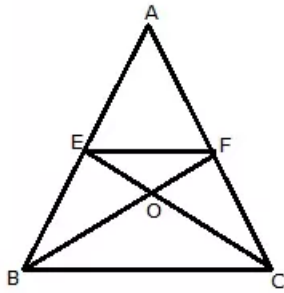
Hence the corresponding sides are proportional.

$$\begin{aligned}\frac{AP}{CD} &= \frac{OP}{OD} = \frac{AP}{AB} \\ &= \frac{AP}{AP + PB} \\ &= \frac{AP}{3AP} \\ &= \frac{1}{3}\end{aligned}$$

Solution 3:

E and F are the midpoints of the sides AB and AC.

Consider the following figure.



Therefore, by midpoint theorem, we have, $EF \parallel BC$

Triangles BEF and CEF lie on the common base EF and between the parallels, EF and BC

Therefore, $Ar.(\triangle BEF) = Ar.(\triangle CEF)$

$\Rightarrow Ar.(\triangle BOE) + Ar.(\triangle EOF) = Ar.(\triangle EOF) + Ar.(\triangle COF)$

$\Rightarrow Ar.(\triangle BOE) = Ar.(\triangle COF)$

Now BF and CE are the medians of the triangle ABC

Medians of the triangle divides it into two equal areas of triangles.

Thus, we have, $Ar. \triangle ABF = Ar. \triangle CBF$

Subtracting $Ar. \triangle BOE$ on the both the sides, we have

$Ar. \triangle ABF - Ar. \triangle BOE = Ar. \triangle CBF - Ar. \triangle BOE$

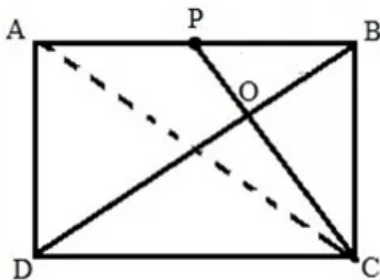
Since, $Ar.(\triangle BOE) = Ar.(\triangle COF)$,

$Ar. \triangle ABF - Ar. \triangle BOE = Ar. \triangle CBF - Ar. \triangle COF$

$Ar. (quad. AEOF) = Ar.(\triangle OBC)$, hence proved

Solution 4:

(i) Joining AC we have the following figure



Consider the triangles $\triangle POB$ and $\triangle COD$

$\angle POB = \angle DOC$ [vertically opposite angles]

$\angle OPB = \angle ODC$ [AB and DC are parallel, CP and BD are the transversals, alternate interior angles are equal]

Therefore, by Angle – Angle similarity criterion of congruence,

$\triangle POB \sim \triangle COD$

Since P is the midpoint $AP = BP$, and $AB = CD$, we have $CD = 2BP$

Therefore, we have,

$$\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2}$$

$\Rightarrow OP:OC = 1:2$

(ii)

Since from part (i), we have

$$\frac{BP}{CD} = \frac{OP}{OC} = \frac{OB}{OD} = \frac{1}{2},$$

Ratio between the areas of two similar triangles is equal to the ratio between the squares of the corresponding sides.

Here, $\triangle DOC$ and $\triangle POB$ are similar triangles.

Thus, we have,

$$\frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = \frac{DC^2}{PB^2}$$

$$\Rightarrow \frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = \frac{(2PB)^2}{PB^2}$$

$$\Rightarrow \frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = \frac{4PB^2}{PB^2}$$

$$\Rightarrow \frac{\text{Ar.}(\triangle DOC)}{\text{Ar.}(\triangle POB)} = 4$$

$$\begin{aligned}\Rightarrow \text{Ar.}(\triangle DOC) &= 4\text{Ar.}(\triangle POB) \\ &= 4 \times 40 \\ &= 160 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Now consider } \text{Ar.}(\triangle DBC) &= \text{Ar.}(\triangle DOC) + \text{Ar.}(\triangle BOC) \\ &= 160 + 80 \\ &= 240 \text{ cm}^2\end{aligned}$$

Two triangles are equal in area if they are on the equal bases and between the same parallels.

$$\text{Therefore, } \text{Ar.}(\triangle DBC) = \text{Ar.}(\triangle ABC) = 240 \text{ cm}^2$$

Median divides the triangle into areas of two equal triangles.

Thus, CP is the median of the triangle ABC .

$$\text{Hence, } \text{Ar.}(\triangle ABC) = 2\text{Ar.}(\triangle PBC)$$

$$\Rightarrow \text{Ar.}(\triangle PBC) = \frac{\text{Ar.}(\triangle ABC)}{2}$$

$$\Rightarrow \text{Ar.}(\triangle PBC) = 120 \text{ cm}^2$$

(iii)

From part(ii) we have,

$$\text{Ar.}(\triangle ABC) = 2\text{Ar.}(\triangle PBC) = 240 \text{ cm}^2$$

Area of a triangle is half the area of the Parallelogram if both are on equal bases and between the same parallels.

$$\text{Thus, } \text{Ar.}(\triangle ABC) = \frac{1}{2} \text{Ar.}(\text{||gm } ABCD)$$

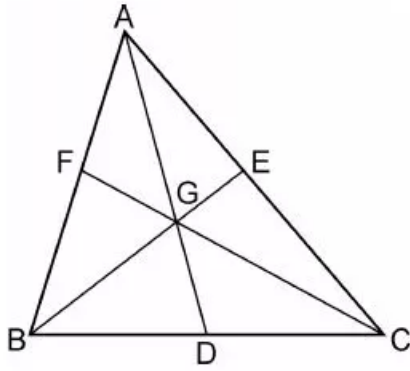
$$\Rightarrow \text{Ar.}(\text{||gm } ABCD) = 2 \text{Ar.}(\triangle ABC)$$

$$\Rightarrow \text{Ar.}(\text{||gm } ABCD) = 2 \times 240$$

$$\Rightarrow \text{Ar.}(\text{||gm } ABCD) = 480 \text{ cm}^2$$

Solution 5:

(i) The figure is shown below



Medians intersect at centroid.

Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.

Centroid divides the medians in the ratio 2:1

That is $AG:GD = 2:1$

Since BG divides AD in the ratio 2:1, we have,

$$\frac{\text{Area}(\triangle AGB)}{\text{Area}(\triangle BGD)} = \frac{2}{1}$$

$$\Rightarrow \text{Area}(\triangle AGB) = 2\text{Area}(\triangle BGD)$$

From the figure, it is clear that,

$$\text{Area}(\triangle ABD) = \text{Area}(\triangle AGB) + \text{Area}(\triangle BGD)$$

$$\Rightarrow \text{Area}(\triangle ABD) = 2\text{Area}(\triangle BGD) + \text{Area}(\triangle BGD)$$

$$\Rightarrow \text{Area}(\triangle ABD) = 3\text{Area}(\triangle BGD) \dots (1)$$

(ii)

Medians intersect at centroid.

Given that G is the point of intersection of medians and hence G is the centroid of the triangle ABC.

Centroid divides the medians in the ratio 2:1

That is $AG:GD = 2:1$

Similarly CG divides AD in the ratio 2:1, we have,

$$\frac{\text{Area}(\triangle AGC)}{\text{Area}(\triangle CGD)} = \frac{2}{1}$$

$$\Rightarrow \text{Area}(\triangle AGC) = 2\text{Area}(\triangle CGD)$$

From the figure, it is clear that,

$$\text{Area}(\triangle ACD) = \text{Area}(\triangle AGC) + \text{Area}(\triangle CGD)$$

$$\Rightarrow \text{Area}(\triangle ACD) = 2\text{Area}(\triangle CGD) + \text{Area}(\triangle CGD)$$

$$\Rightarrow \text{Area}(\triangle ACD) = 3\text{Area}(\triangle CGD) \dots (2)$$

(iii)

Adding equations (1) and (2), we have,

$$\text{Area}(\triangle ABD) + \text{Area}(\triangle ACD) = 3\text{Area}(\triangle BGD) + 3\text{Area}(\triangle CGD)$$

$$\Rightarrow \text{Area}(\triangle ABC) = 3[\text{Area}(\triangle BGD) + \text{Area}(\triangle CGD)]$$

$$\Rightarrow \text{Area}(\triangle ABC) = 3[\text{Area}(\triangle BGC)]$$

$$\Rightarrow \frac{\text{Area}(\triangle ABC)}{3} = [\text{Area}(\triangle BGC)]$$

$$\Rightarrow \text{Area}(\triangle BGC) = \frac{1}{3} \text{Area}(\triangle ABC)$$

Solution 6:

Consider that the sides be x cm, y cm and $(37-x-y)$ cm. also, consider that the lengths of altitudes be $6a$ cm, $5a$ cm and $4a$ cm.

$$\therefore \text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\therefore \frac{1}{2} \times x \times 6a = \frac{1}{2} \times y \times 5a = \frac{1}{2} \times (37-x-y) \times 4a$$

$$6x = 5y = 148 - 4x - 4y$$

$$6x = 5y \text{ and } 6x = 148 - 4x - 4y$$

$$6x - 5y = 0 \text{ and } 10x + 4y = 148$$

Solving both the equations, we have

$$x = 10 \text{ cm, } y = 12 \text{ cm and } (37-x-y) \text{ cm} = 15 \text{ cm}$$

Solution 7:

(i)

Consider the triangles $\triangle AFE$ and $\triangle DFC$.

$\angle AFE = \angle DEC$ [Vertically opposite angles]

$\angle FAE = \angle DCF$ [AB and DC are parallel lines, AC is a transversal,
alternate interior angles are equal]

Thus, by Angle – Angle similarity, we have,

$$\triangle AFE \sim \triangle DFC$$

Therefore, we have,

$$\frac{DF}{FE} = \frac{DC}{AE} = \frac{CF}{AF} = \frac{2}{1}$$

$$\Rightarrow DF:FE = 2:1$$

(ii)

Since from part(i) we have $DF:FE = 2:1$, therefore,

$$\text{Area}(\triangle DCF) = 4\text{Area}(\triangle AFE) \dots (1)$$

Also we know that,

$$\text{Area}(\triangle ADF) + \text{Area}(\triangle AFE) = \text{Area}(\triangle ADE)$$

$$\Rightarrow 60 + \text{Area}(\triangle AFE) = \text{Area}(\triangle ADE) \quad [\text{Area}(\triangle ADF) = 60 \text{ cm}^2]$$

$$\Rightarrow 2\text{Area}(\triangle ADE) = 2[60 + \text{Area}(\triangle AFE)]$$

Median divides the triangle into two equal areas of triangle.

$$\text{Therefore, } 2\text{Area}(\triangle ADE) = \text{Area}(\triangle ABD)$$

$$\Rightarrow \text{Area}(\triangle ABD) = 2[60 + \text{Area}(\triangle AFE)]$$

$$\Rightarrow \text{Area}(\triangle ABD) = 120 + 2\text{Area}(\triangle AFE) \dots (2)$$

Triangles with equal bases and between the parallels are of equal area.

$$\text{Area}(\triangle ABD) = \text{Area}(\triangle ACD)$$

Thus, Equation (2), becomes,

$$\text{Area}(\triangle ACD) = 120 + 2\text{Area}(\triangle AFE) \dots (3)$$

From the figure, it is clear that,

$$\text{Area}(\triangle ACD) = \text{Area}(\triangle DCF) + \text{Area}(\triangle ADF)$$

$$\Rightarrow \text{Area}(\triangle ACD) = \text{Area}(\triangle DCF) + 60$$

$$\Rightarrow \text{Area}(\triangle ACD) = 4\text{Area}(\triangle AFE) + 60 \dots (4)$$

Equating equations (3) and (4), we have,

$$120 + 2\text{Area}(\triangle AFE) = 4\text{Area}(\triangle AFE) + 60$$

$$\Rightarrow 2\text{Area}(\triangle AFE) = 60$$

$$\Rightarrow \text{Area}(\triangle AFE) = \frac{60}{2}$$

$$\Rightarrow \text{Area}(\triangle AFE) = 30$$

$$\Rightarrow \text{Area}(\triangle ADE) = \text{Area}(\triangle ADF) + \text{Area}(\triangle AFE)$$

$$\Rightarrow \text{Area}(\triangle ADE) = 60 + 30$$

$$\Rightarrow \text{Area}(\triangle ADE) = 90 \text{ cm}^2$$

(iii)

Median of a triangle divides it into two equal areas of triangle.

$$\text{Area}(\triangle ADB) = 2\text{Area}(\triangle ADE)$$

$$\Rightarrow \text{Area}(\triangle ADB) = 2\text{Area}(\triangle ADE)$$

$$\Rightarrow \text{Area}(\triangle ADB) = 2 \times 90 \text{ cm}^2$$

$$\Rightarrow \text{Area}(\triangle ADB) = 180 \text{ cm}^2$$

(iv)

Since DB divides the parallelogram ABCD into two equal triangles, therefore Area of $\triangle DBC = \text{Area of } \triangle ADB$

$$= 180 \text{ cm}^2$$

Thus the area of the parallelogram ABCD = Area of $\triangle ADB + \text{Area of } \triangle DBC$

$$\text{or } \triangle DBC$$

$$= 180 \text{ cm}^2 + 180 \text{ cm}^2$$

$$= 360 \text{ cm}^2$$

Solution 8:

Here BCED is a parallelogram, since $BD = CE$ and $BD \parallel CE$.

$\text{ar.}(\triangle DBC) = \text{ar.}(\triangle EBC)$... (Since they have the same base and are between the same parallels)

In $\triangle ABC$,

BE is the median,

$$\text{So, } \text{ar.}(\triangle EBC) = \frac{1}{2} \text{ar.}(\triangle ABC)$$

$$\text{Now, } \text{ar.}(\triangle ABC) = \text{ar.}(\triangle EBC) + \text{ar.}(\triangle ABE)$$

$$\text{Also, } \text{ar.}(\triangle ABC) = 2\text{ar.}(\triangle EBC)$$

$$\Rightarrow \text{ar.}(\triangle ABC) = 2\text{ar.}(\triangle DBC)$$

Solution 9:

Given :

$$\triangle CAD = 140 \text{ cm}^2$$

$$\triangle ODC = 172 \text{ cm}^2$$

$$AB \parallel CD$$

As Triangle DBC and $\triangle CAD$ have same base CD and between the same parallel lines Hence,

$$\text{Area of } \triangle DBC = \text{Area of } \triangle CAD = 140 \text{ cm}^2$$

$$\text{Area of } \triangle OAC = \text{Area of } \triangle CAD + \text{Area of } \triangle ODC = 140 \text{ cm}^2 + 172 \text{ cm}^2 = 312 \text{ cm}^2$$

$$\text{Area of } \triangle ODB = \text{Area of } \triangle DBC + \text{Area of } \triangle ODC = 140 \text{ cm}^2 + 172 \text{ cm}^2 = 312 \text{ cm}^2$$