Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

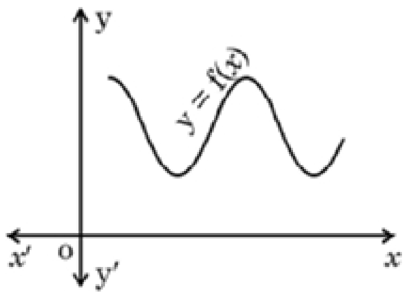
1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. The number $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ is **[1]**

a) an irrational number
b) an integer
c) not a real number
d) a rational number

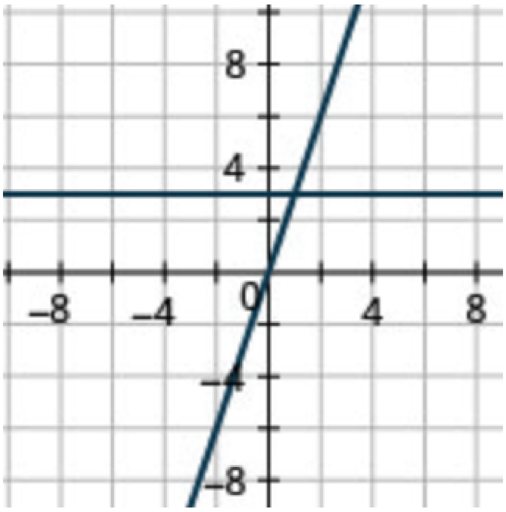
2. The graph of y = f(x) is shown in the figure for some polynomial f(x). **[1]**



The number of zeroes of f(x) is

a) 3
b) 4
c) 0
d) 2

3. The number of solutions of two linear equations representing intersecting lines is/are **[1]**



a) 1
b) 2
c) 0
d) ∞

4. The product of two consecutive even integers is 528. The quadratic equation corresponding to the above statement, is **[1]**

a) x (x + 2) = 528
b) 2x (2x + 1) = 528
c) (1 + x) 2x = 528
d) 2x (x + 4) = 528

5. If the sum of n terms of an A.P. is $3n^2 + 5n$ then which of its term is 164? **[1]**

a) 27th
b) none of these.
c) 28th
d) 26th

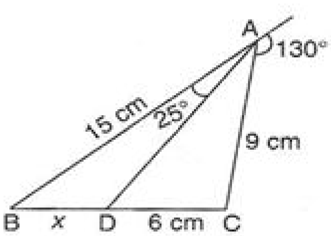
6. The distance of a point from the x-axis is called **[1]**

a) due point
b) origin
c) abscissa
d) ordinate

7. If the point C(k, 4) divides the join of the points .4(2, 6) and B(5,1) in the ratio 2:3 then the value of k is **[1]**

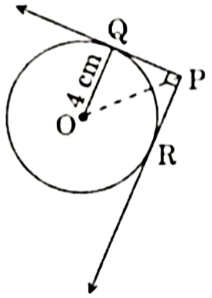
a) 16
b) $\frac{28}{5}$
c) $\frac{8}{5}$
d) $\frac{16}{5}$

8. Find the value of a given figure. **[1]**



a) 12 cm.
b) 6 cm.
c) 10 cm.
d) 15 cm.

9. In the given figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90^{o}$, then length of PQ is: **[1]**



a) 3 cm

b) 4 cm

c) 2 cm

d) $2\sqrt{2}$ cm

10. How many tangents can be drawn to a circle from a point on it? **[1]**

a) Two

b) Zero

c) Infinite

d) One

11. If x = a cosθ and y = b sinθ, then the value of $b^2x^2 + a^2y^2$ is **[1]**

a) a + b

b) a^2b^2

c) a - b

d) ab

12. $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta}$ is equal to **[1]**

a) sinθ + cosθ

b) sinθ - cosθ

c) 0

d) 1

13. The top of a broken tree has its top end touching the ground at a distance 15 m from the bottom, the angle made by the broken end with the ground is 30^o. Then length of broken part is **[1]**

a) 10 m

b) $10\sqrt{3}$ m

c) $\sqrt{3}$ m

d) $5\sqrt{3}$ m

14. Find the area of the sector of a circle having radius 6 cm and of angle 30^o. **[1]**

a) 8.42 cm^2

b) 9 cm^2

c) 9.42 cm^2

d) 9.52 cm^2

15. A piece of wire 20cm long is bent into the form of an arc of a circle subtending an angle of 60^o at its centre. The radius of the circle is **[1]**

a) $\frac{20}{6+\pi}$ cm

b) $\frac{30}{6+\pi}$ cm

c) $\frac{60}{\pi}$ cm

d) $\frac{15}{6+\pi}$ cm

16. In a family of two children, the probability of having at least one girl is: **[1]**

a) $\frac{1}{2}$

b) $\frac{2}{5}$

c) $\frac{3}{4}$

d) $\frac{1}{4}$

17. If a letter of English alphabet is chosen at random, then the probability of this letter to be a consonant is: **[1]**

a) $\frac{11}{13}$

b) $\frac{10}{13}$

c) $\frac{21}{26}$

d) $\frac{5}{26}$

18. The marks obtained by 9 students in Mathematics are 59, 46, 31, 23, 27, 40, 52, 35 and 29. The mean of the data **[1]**

is

a) 30

b) 41

c) 23

d) 38

19. **Assertion (A):** In a solid hemisphere of radius 10 cm, a right cone of same radius is removed out. The surface area of the remaining solid is 570.74 cm^2 [Take π = 3.14 and $\sqrt{2}$ = 1.4] **[1]**

Reason (R): **Reason (R):** Expression used here to calculate Surface area of remaining solid = Curved surface area of hemisphere + Curved surface area of cone

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its nth term $a_n = 6n - 7$ **[1]**

Reason (R): nth term of an AP, whose sum to n terms is S_n, is given by $a_n = S_n - S_{n-1}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

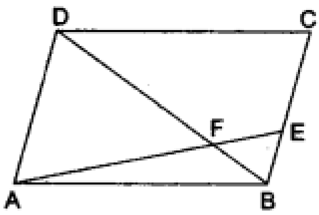
c) A is true but R is false.

d) A is false but R is true.

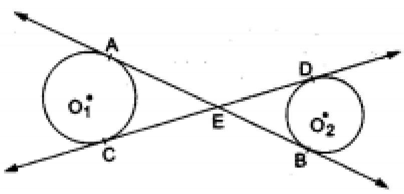
Section B

21. Prove that $5\sqrt{2}$ is irrational. **[2]**

22. ABCD is a parallelogram and E is a point on BC. If the diagonal BD intersects AE at F, prove that $AF \times FB = EF \times FD$. **[2]**



23. In the given figure, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD. **[2]**

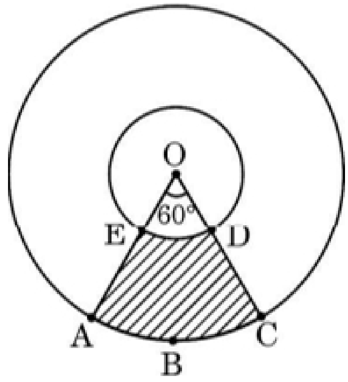


24. If sin (A + B) = 1 and cos (A – B) = 1, find A and B. **[2]**

OR

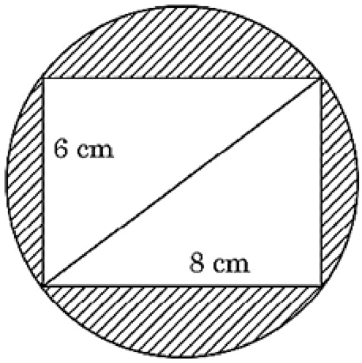
Prove the trigonometric identity: $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$.

25. In the given figure, two concentric circles with centre O are shown. Radii of the circles are 2 cm and 5 cm respectively. Find the area of the shaded region. **[2]**



OR

Reeti prepares a Rakhi for her brother Ronit. The Rakhi consists of a rectangle of length 8 cm and breadth 6 cm inscribed in a circle as shown in the figure. Find the area of the shaded region. (Use π = 3·14)



Section C

26. On morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? **[3]**

27. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane. **[3]**

28. Solve: $\frac{1}{2x-3}+\frac{1}{x-5}=1\frac{1}{9}, x\neq\frac{3}{2}, 5$ **[3]**

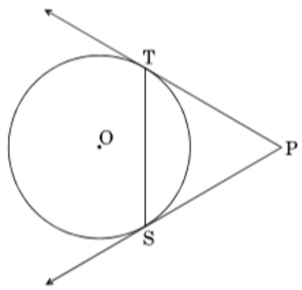
OR

At t minutes past 2 pm the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

29. If radii of the two circles are equal, prove that AB = CD where AB and CD are common tangents. **[3]**

OR

In the given figure, PT and PS are tangents to a circle with centre O, from a point P, such that PT = 4 cm and ∠TPS = 60^{o}. Find the length of the chord TS. Also, find the radius of the circle.



30. If secθ + tanθ = p, show that $\frac{p^2-1}{p^2+1}=\sin\theta$ **[3]**

31. Find the mean of the following frequency distribution: **[3]**

Class	Frequency

0-10	12
10-20	18
20-30	27
30-40	20
40-50	17
50-60	6

Section D

32. Points A and B are 70 km. apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars. **[5]**

OR

Ved travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and the rest by car. He takes 20 minutes longer if he travels 200 km by train and the rest by car. Find the speed of the train and the car.

33. In a trapezium ABCD, AB | | DC and DC = 2AB. EF | | AB, where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that, 7EF = 11AB. **[5]**

34. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the number. **[5]**

OR

If the ratio of the sum of the first n terms of two APs is (7n +1): (4n + 27) then find the ratio of their 9th terms.

35. The following table shows the ages of the patients admitted in a hospital during a month: **[5]**

Age(in years)	6 - 15	16 - 25	26 - 35	36 - 45	46 - 55	56 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Aashish and his family went for a vacation to Manali. There they had a stay in tent for a night. Aashish found that the tent in which they stayed is in the form of a cone surmounted on a cylinder. The total height of the tent is 42 m, diameter of the base is 42 m and height of the cylinder is 22 m.

i. What is curved surface area of cone? (1)

ii. If each person needs 126 m^2 of floor, then how many persons can be accommodated in the tent? (1)

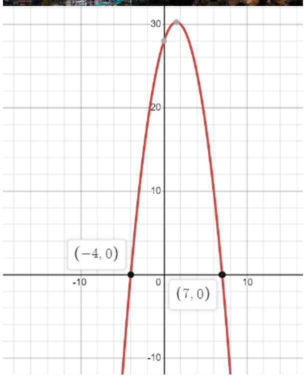
iii. What is the curved surface area of cylinder? (2)

OR

How much canvas required to make a tent? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Two friends Govind and Pawan decided to go for a trekking. During summer vacation, they went to Panchmarhi. While trekking they observed that the trekking path is in the shape of a parabola. The mathematical representation of the track is shown in the graph.



i. What are the zeroes of the polynomial whose graph is given? (1)

ii. What will be the expression of the given polynomial p(x)? (1)

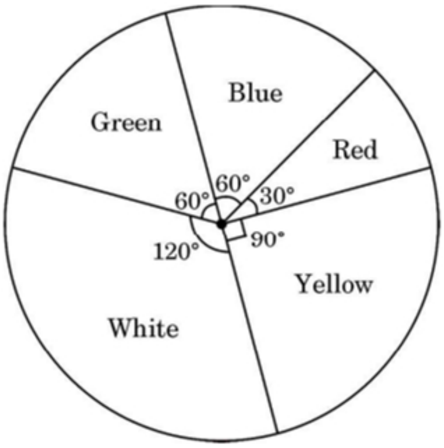
iii. What is the product of the zeroes of the polynomial which represents the parabola? (2)

OR

In the standard form of quadratic polynomial, $ax^2 + bx + c$, whar are a, b, and c? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Some students were asked to list their favourite colour. The measure of each colour is shown by the central angle of a pie chart given below:



i. If a student is chosen at random, then find the probability of his/her favourite colour being white? (1)

ii. What is the probability of his/her favourite colour being blue or green? (1)

iii. If 15 students liked the colour yellow, how many students participated in the survey? (2)

OR

What is the probability of the favourite colour being red or blue? (2)

Solution

Section A

1. **(a)** an irrational number

   Explanation: $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

   $= \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

   $= \frac{(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$

   $= \frac{(\sqrt{5})^2+(\sqrt{2})^2+2\times\sqrt{5}\times\sqrt{2}}{5-2}$

   $= \frac{5+2+2\sqrt{10}}{3}$

   $= \frac{7+2\sqrt{10}}{3}$

   Here $\sqrt{10} = \sqrt{2} \times \sqrt{5}$

   Since $\sqrt{2}$ and $\sqrt{5}$ both are an irrational number

   Therefore, $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ is an irrational number.

2. **(c)** 0

   Explanation: Here y = f(x) is not intersecting or touching the X-axis.

   $\therefore$ Number of zeroes of f(x) = 0

3. **(a)** 1

   Explanation: The number of solutions of two linear equations representing intersecting lines is 1 because two linear equations representing intersecting lines has a unique solution.

4. **(a)** x (x + 2) = 528

   Explanation: Let the first number = x

   Second number = x + 2

   According to question

   x(x + 2) = 528

5. **(a)** 27th

   Explanation: Sum of n terms of an A.P = $3n^2+5n$

   Let a be the first term and d be the common difference

   $S_n = 3n^2 + 5n$

   $\therefore S_1 = 3(1)^2 + 5 \times 1 = 3 + 5 = 8$

   $S_2 = 3(2)^2 + 5 \times 2 = 12 + 10 = 22$

   $\therefore$ First term (a)=8

   $a_2 = \mathrm{S}_2 - \mathrm{S}_1 = 22 - 8 = 14$

   $\therefore d = a_2 - a_1 = 14 - 8 = 6$

   Now a_n = a + (n - 1)d

   $\Rightarrow 164 = 8 + (n - 1) \times 6$

   $\Rightarrow$ 6n - 6 = 164 - 8 = 156

   $\Rightarrow$ 6n = 156 + 6 = 162

   $\Rightarrow n = \frac{162}{6} = 27$

   $\therefore$ 164 is 27^{th} term

6. **(d)** ordinate

   Explanation: The distance of a point from the x-axis is the y (vertical) coordinate of the point and is called ordinate.

7. **(d)** $\frac{16}{5}$

Explanation: By Section Formula,

The X-coordinate of C = $\frac{2(5)+3(2)}{2+3}$

$\Rightarrow$ K = $\frac{16}{5}$

8.

(c) 10 cm.

Explanation: Here, $\angle$CAD = $180^\circ - (130^\circ + 25^\circ)$ = 25^o

Now, since $\angle$CAD = $\angle$DAB, therefore, the AD is the bisector of $\angle$BAC.

Since the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$\therefore \frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{x}{6} = \frac{15}{9}$

$\Rightarrow x = \frac{15 \times 6}{9}$ = 10 cm

9.

(b) 4 cm

Explanation: Join OR & OQ

Now PQOR becomes a quadrilateral

$\angle$QPR = 90^o (given)

PQ = PR ($\because$ tangent from external point)

OQ = OR = (radius of same circle)

$\angle$OQP = $\angle$ORP = 90^o ($\because$ tangents and radius are perpendicular)

$\therefore \angle$QOR = 360^o - $\angle$OQP - $\angle$QPR - $\angle$ORP

$\angle$QOR = 360^o - 90 - 90 - 90

$\angle$QOR = 90^o

$\therefore$ PQOR becomes a square

sides of a square are same

$\therefore$ PQ = 4 cm

10. **(a)** Two

Explanation: Two

11.

(b) a^2b^2

Explanation: Given: x = a cosθ and y = b sinθ

$\therefore b^2x^2 + a^2y^2$

$= b^2(a \cos\theta)^2 + a^2(b \sin\theta)^2$

$= b^2a^2 \cos^2\theta + a^2b^2\sin^2\theta$

$\Rightarrow b^2x^2 + a^2y^2$

$= a^2b^2(\cos^2\theta + \sin^2\theta)$

$= a^2b^2$

[$\because \sin^2\theta + \cos^2\theta = 1$]

12. **(a)** sinθ + cosθ

Explanation: We have, $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin\theta \times \sin\theta}{\sin\theta - \cos\theta} + \frac{\cos\theta \times \cos\theta}{\cos\theta - \sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} - \frac{\cos^2\theta}{\sin\theta - \cos\theta}$

$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$

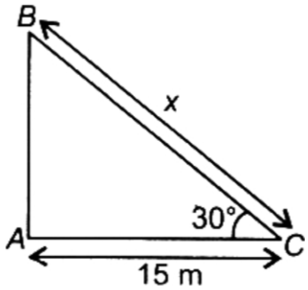
$= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta - \cos\theta}$

$= \sin\theta + \cos\theta$

13.

(b) $10\sqrt{3}$ m

Explanation: Let the length of broken part (BC) = x



In $\triangle$ABC,

cos 30^o = $\frac{15}{x}$

$\Rightarrow$ x $\times$ $\frac{\sqrt{3}}{2}$ = 15

$\Rightarrow$ x = $5\sqrt{3} \times 2$ = $10\sqrt{3}$ m

14.

(c) 9.42 cm^2

Explanation: Radius of a circle = r = 6 cm

Central angle $= \theta$ = 30^o

$\therefore$ Area of the sector $= \frac{\pi r^2 \theta}{360}$

$= \left(\frac{3.14 \times 6 \times 6 \times 30^{\circ}}{360^{\circ}}\right)$ cm^2

= 9.42 cm^2

15.

(c) $\frac{60}{\pi}$ cm

Explanation: Given: Length of arc = 20 cm

$\Rightarrow \frac{\theta}{360^{\circ}} \times 2\pi$r= 20

$\Rightarrow \frac{60^{\circ}}{360^{\circ}} \times 2\pi$r= 20

$\Rightarrow \frac{\pi r}{3}$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right)$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right)$ = 20

$\Rightarrow$ r = $\frac{60}{\pi}$ cm

16.

(c) $\frac{3}{4}$

Explanation: $\frac{3}{4}$

17.

(c) $\frac{21}{26}$

Explanation: We have,

Number of vowels = 5 (a, e, i, o, u)

Number of consonants = 21 (26 - 5 = 21)

Number of possible outcomes = 21

Number of total outcomes = 26

$\therefore$ Required Probability = $\frac{21}{26}$

18.

(d) 38

Explanation: Given: 59, 46, 31, 23, 27, 40, 52, 35 and 29

Mean = $\frac{\text{Sum of all observations}}{\text{Number of observations}}$

= $\frac{59+46+31+23+27+40+52+35+29}{9}$

= $\frac{342}{9}$

= 38

19.

(d) A is false but R is true.

Explanation: A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: nth term of an AP be $a_n = S_n - S_{n-1}$

$a_n = 3n^2 - 4n - 3(n-1)^2 + 4(n-1)$

$a_n = 6n - 7$

So, both A and R are true and R is the correct explanation of A.

Section B

21. Let us assume that $5\sqrt{2}$ is rational. Then, there exist positive co-primes a and b such that

$5\sqrt{2} = \frac{a}{b}$

$\sqrt{2} = \frac{a}{b} - 5$

$\sqrt{2} = \frac{a-5b}{b}$

As a-5b and b are integers .

So, $\frac{a-5b}{b}$ is rational number .

But $\sqrt{2}$ is not rational number .

Since a rational number cannot be equal to an irrational number. Our assumption that $5\sqrt{2}$ is rational wrong.

Hence, $5\sqrt{2}$ is irrational.

22. Given: ABCD is a parallelogram and E is a point on BC. The diagonal BD intersects AE at F.

To prove: $AF \times FB = EF \times FD$

Proof: Since ABCD is a parallelogram, then its opposite sides must be parallel.

$\therefore$In $\triangle ADF$ and $\triangle EBF$

$\angle \mathrm{FDA} = \angle \mathrm{EBF}$ and $\angle FAD = \angle FEB$ [Alternate interior angles]

$\angle AFD = \angle BFE$ [vertically opposite angles]

Therefore,by AAA criteria of similar triangles,we have,

$\triangle \mathrm{ADF} = \triangle \mathrm{EBF}$

Since the corresponding sides of similar triangles are proportional. Therefore,we have,

$\frac{AF}{FD} = \frac{EF}{FB}$

$\Rightarrow AF \times FB = EF \times FD$

23. We know that tangent segments to a circle from the same external point are congruent.

So, $EA = EC$ for the circle having centre O_1

And, $ED = EB$ for the circle having centre O_2

Now, Adding ED on both sides in $EA = EC$, we get

$EA + ED = EC + ED$

$\Rightarrow EA + EB = EC + ED$

$\Rightarrow AB = CD$

24. $\sin(A+B) = 1 = \sin 90^\circ$

$A + B = 90^\circ \quad (1)$

$\cos(A-B) = 1 = \cos 0^\circ$

$\Rightarrow A - B = 0^\circ (2)$

Solving (1) and (2) we get

$A = 45^\circ, \quad B = 45^\circ$

OR

L.H.S = $\sec^4\theta - \sec^2\theta$

= $\sec^2\theta (\sec^2\theta - 1)$

= $\sec^2\theta (\tan^2\theta)$ [$\because 1 + tan^2\theta = sec^2\theta$ or $tan^2\theta = sec^2\theta - 1$]

= $(1 + \tan^2\theta) \tan^2\theta = \tan^2\theta + \tan^4\theta$ = R.H.S

Hence proved.

25. Area of sector OABC = $\frac{\pi \times 5^2 \times 60^\circ}{360^\circ} = \frac{25\pi}{6}$ cm^2

Area of sector OED = $\frac{\pi \times 2^2 \times 60^\circ}{360^\circ} = \frac{4\pi}{6}$ cm^2

Area of shaded region = $\frac{25\pi}{6} - \frac{4\pi}{6} = \frac{21}{6} \times \frac{22}{7}$ = 11 cm^2

OR

Diagonal of rectangle = $\sqrt{6^2+8^2}$ = 10

$\therefore$ Radius of circle r = $\frac{10}{2}$ = 5

Area of circle = 3.14 $\times$ 5 $\times$ 5

= 78.5

Area of rectangle = 6 $\times$ 8 = 48

Area of shaded region = 78.5 - 48

= 30.5 cm^2

$\therefore$ Area of shaded region is 30.5 cm^2

Section C

26. Since, the three persons start walking together.

$\therefore$ The minimum distance covered by each of them in complete steps = LCM of the measures of their steps

$40 = 8 \times 5 = 2^3 \times 5$

$42 = 6 \times 7 = 2 \times 3 \times 7$

$45 = 9 \times 5 = 3^2 \times 5$

Hence LCM (40, 42, 45)

$= 2^3 \times 3^2 \times 5 \times 7 = 8 \times 9 \times 5 \times 7 = 2520$

$\therefore$ The minimum distance each should walk so that each can cover the same distance

= 2520 cm = 25.20 meters.

27. Let P and Q be the two positions of the plane and let A be the point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from point A are 60° and 30° respectively.

$\therefore \angle PAB = 60^o, \angle QAB = 30^o$. It is also given that PB = 3600 $\sqrt{3}$ metres

In $\triangle$ ABP, we have

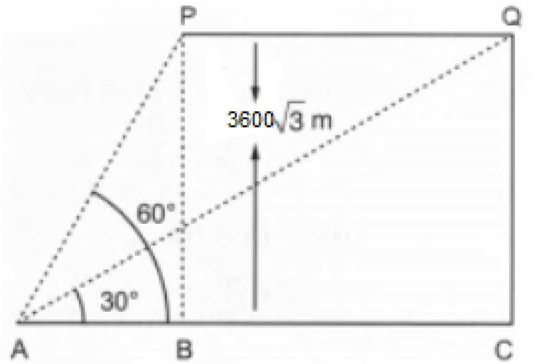
tan 60^o = $\frac{BP}{AB}$

$\Rightarrow \sqrt{3} = \frac{3600\sqrt{3}}{AB}$

$\Rightarrow$ AB = 3600 m

In $\triangle$ ACQ, we have

tan 30^o = $\frac{CQ}{AC}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC}$

$\Rightarrow$ AC = 3600 $\times$ 3 = 10800 m

$\therefore$ PQ = BC = AC - AB = 10800 - 3600 = 7200 m

Thus, the plane travels 7200 m in 30 seconds.

Hence, Speed of plane = $\frac{7200}{30}$ = 240 m / sec = $\frac{240}{1000} \times 60 \times 60$ = 864 km / hr

28. The given equation is:

$\frac{1}{2x-3} + \frac{1}{x-5} = \frac{10}{9}$

$\Rightarrow \frac{x-5+2x-3}{(2x-3)(x-5)} = \frac{10}{9}$

$\Rightarrow \frac{3x-8}{(2x-3)(x-5)} = \frac{10}{9}$

$\Rightarrow$ 27x - 72 = 10[(2x - 3) (x - 5)] (By cross multiplication method)

$\Rightarrow$ 27x - 72 = 10[$2x^2$ - 10x - 3x + 15]

$\Rightarrow$ 27x - 72 = 10[$2x^2$ - 13x + 15]

$\Rightarrow$ 27x - 72 = $20x^2$ - 130x + 150

$\Rightarrow$ $20x^2$ - 157x + 222 = 0

Here , a = 20, b = -157, c = 222

Therefore, by quadratic formula we have:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{157 \pm \sqrt{(-157)^2 - 4(20)(222)}}{40}$

$\Rightarrow x = \frac{157 \pm \sqrt{24649 - 17760}}{40}$

$\Rightarrow x = \frac{157 \pm \sqrt{6889}}{40}$

$\Rightarrow x = \frac{157 \pm 83}{40}$

$\Rightarrow x = \frac{157 + 83}{40}$ or $x = \frac{157 - 83}{40}$

$\Rightarrow$ x = 6 or x = $\frac{37}{20}$

OR

Since, there are 60 minutes gap between 2 PM & 3 PM.

Time needed by minutes hand after t minutes past 2 PM to show 3 PM = (60 - t) minutes

According to the question ;

60 - t = $\frac{t^2}{4}$ - 3

$\Rightarrow$ 63 = $\frac{t^2}{4}$ + t

$\Rightarrow$ 63 = $\frac{t^2 + 4t}{4}$

$\Rightarrow$ 252 = t^2 + 4t

$\Rightarrow$ t^2 + 4t - 252 = 0

$\Rightarrow$ t^2 + 18t - 14t - 252 = 0

$\Rightarrow$ t(t + 18) - 14(t + 8) = 0

$\Rightarrow$ (t + 18)(t - 14) = 0

$\Rightarrow$ t + 18 = 0 or t - 14 = 0

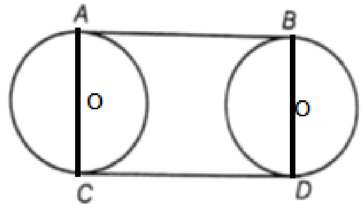
$\Rightarrow$ t = -18 or t = 14

Since time cannot be negative, t $\neq$ -18

Hence, t = 14 minutes.

29. Given: AB and CD are two common tangents to two circles of equal radii.

To prove



Construction: OA, OC, O'B and O'D proof

Now, $\angle$OAB = 90° and $\angle$OCD = 90° as OA $\perp$ AB and OC $\perp$ CD

A tangent at any point of a circle is perpendicular to radius through the point of contact

Thus, AC is a straight line.

Also, $\angle O'BA = \angle O'DC = 90^\circ$

A tangent at a point on the circle is perpendicular to the radius through point of contact

so ABCD is a quadrilateral with four sides as AB, BC, CD and AD

But as $\angle A = \angle B = \angle C = \angle D = 90^\circ$

so, ABCD is a rectangle.

Hence, AB = CD opposite sides of the rectangle are equal.

OR

$\because$ PT = PS (tangents from an external point P)

$\therefore$ $\angle$PTS = $\angle$PST

Using Angle Sum Property in $\triangle$PTS

$\angle$PTS + $\angle$PST + $\angle$TPS = 180^o

2$\angle$PTS = 180 - 60 = 120^o

$\angle$PTS = 60^o

$\Rightarrow$ PTS Is a equilateral triangle

So, TS = 4 cm

Now, In $\triangle$PTO

As PO is angle bisector of $\angle$TPS, $\angle$OTP = 90^o

tan 30^o = $\frac{OT}{TP}$

$\frac{1}{\sqrt{3}}$ = $\frac{OT}{4}$

OT = $\frac{4}{\sqrt{3}}$

OT = $\frac{4\sqrt{3}}{3}$ cm

$\therefore$ radius of circle = $\frac{4\sqrt{3}}{3}$ cm

30. We have,

$\text{LHS} = \frac{p^2-1}{p^2+1} = \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$

$\Rightarrow \quad \text{LHS} = \frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}$

$\Rightarrow \quad \text{LHS} = \frac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+2\sec\theta\tan\theta+(1+\tan^2\theta)}$

$\Rightarrow \quad \text{LHS} = \frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+2\sec\theta\tan\theta+\sec^2\theta}$

$\Rightarrow \quad \text{LHS} = \frac{2\tan^2\theta+2\tan\theta\sec\theta}{2\sec^2\theta+2\sec\theta\tan\theta} = \frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)} = \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta\cdot\sec\theta} = \sin\theta$ =RHS

31.

Class interval	**Frequency**	**Midpoint**	**$f_i x_i$**
0-10	12	5	60
10-20	18	15	270
20-30	27	25	675
30-40	20	35	700
40-50	17	45	765
50-60	6	55	330
Total	$\sum f_i$=100		$\sum f_i x_i =$ 2800

Mean= $\frac{\sum f_i x_i}{\sum f_i} = \frac{2800}{100}$ = 28

Section D

32. Suppose, P and Q be the cars starting from A and B respectively and let their speeds be x km/hr and y km/hr respectively.

Case I:

When the cars P and Q move in the same direction.

Distance covered by the car P in 7 hours = 7x km

Distance covered by the car Q in 7 hours = 7y km

Let the cars meet at point M.

$\therefore AM = 7x\ km\ \ and\ \ BM = 7y\ km$

$\therefore AM - BM = AB$

$\Rightarrow 7x - 7y = 70$

$\Rightarrow 7(x-y) = 70$

$\Rightarrow x - y = 10$ ....(i)

Case II:

When the cars P and Q move in the opposite directions.
Distance covered by the car P in 1 hour = x km
Distance covered by the car Q in 1 hour = y km
In this case, let the cars meet at the point N

$\therefore AN = x\ km\ \ and\ \ BN = y\ km$
$\therefore AN + BN = AB$
$\Rightarrow x + y = 70$ ....(ii)
Adding (i) and (ii), we get
$2x = 80$
$\Rightarrow x = 40$
Putting $x = 40$ in (i), we get
$40 - y = 10$
$\Rightarrow y = (40 - 10) = 30$
$\therefore x = 40, y = 30$
Speed of car starting from point A = $40 km/hr.$
Speed of car Starting from point B = $30 km/hr.$

OR

Suppose the speed of the train be x km/hr and the speed of the car be y km/hr.

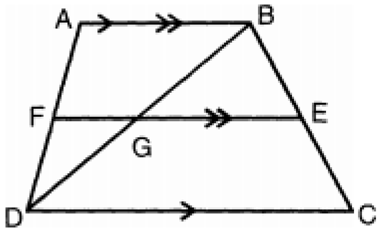
CASE I

Distance covered by car is $(600 - 120)km$ = $480km.$
Now, Time taken to cover 480 km by train $\frac{120}{x}\text{hrs}$ $[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}]$
Time taken to cover 480 km by car =$\frac{480}{y}$ hrs
$\therefore \frac{120}{x} + \frac{480}{y} = 8$
$\Rightarrow 8\left(\frac{15}{x} + \frac{60}{y}\right) = 8$
$\Rightarrow \frac{15}{x} + \frac{60}{y} = 1$
$\Rightarrow \frac{15}{x} + \frac{60}{y} - 1 = 0$ ....(i)

CASE II

Distance travelled by car is $(600 - 200)km$ = $400km$
Now, Time taken to cover $200km$ by train = $\frac{200}{x}$hrs
Time taken to cover $400km$ by train =$\frac{400}{y}$hrs
In this case the total time of journey is 8 hour 20 minutes
$\therefore \frac{200}{x} + \frac{400}{y} = 8\text{ hrs }20\ minutes$
$\Rightarrow \frac{200}{x} + \frac{400}{y} = 8\frac{1}{3}$ $\left[\because 8\text{ hrs }20\text{ minutes } = 8\frac{20}{60}\text{hrs} = 8\frac{1}{3}\text{hrs}\right]$
$\Rightarrow \frac{200}{x} + \frac{400}{y} = \frac{25}{3}$
$\Rightarrow \quad 25\left(\frac{8}{x} + \frac{16}{y}\right) = \frac{25}{3}$
$\Rightarrow \quad \frac{8}{x} + \frac{16}{y} = \frac{1}{3}$
$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} = 1$
$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} - 1 = 0$ ....(ii)
Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in equations (i) and (ii), we get
15 u+ 60v -1=0 ....(iii)
24u+ 48v -1=0 ....(iv)
By using cross-multiplication, we have
$\frac{u}{60\times-1-48\times-1} = \frac{-v}{15\times-1-24\times-1} = \frac{1}{15\times48-24\times60}$
$\Rightarrow \frac{u}{-60+48} = \frac{-v}{-15+24} = \frac{1}{720-1440}$
$\Rightarrow \quad \frac{u}{-12} = \frac{v}{-9} = \frac{1}{-720}$
$\Rightarrow \quad u = \frac{-12}{-720} = \frac{1}{60}$ and $v = \frac{-9}{-720} = \frac{1}{80}$
Now, $u = \frac{1}{x} \Rightarrow \frac{1}{60} = \frac{1}{x} \Rightarrow x = 60$
and, $v = \frac{1}{y} \Rightarrow \frac{1}{80} = \frac{1}{y} \Rightarrow y = 80$

Speed of train = $60 km/hr$
Speed of car=$80 km/hr$.

33. 

In a trapezium ABCD, AB|| DC ,. EF || AB and CD=2AB

and also $\frac{BE}{EC}=\frac{4}{3}$ -------------(1)

AB || CD and AB || EF

$\therefore \frac{AF}{FD}=\frac{BE}{EC}=\frac{4}{3}$

In ΔBGE and ΔBDC

$\angle BEG=\angle BCD$ (∵ corresponding angles)

$\angle GBE=\angle DBC$ (Common)

$\therefore \Delta BGE \sim \Delta BDC$ [By AA similarity]

$\Rightarrow \frac{EG}{CD}=\frac{BE}{BC}$ ....(2)

Now, from (1) $\frac{BE}{EC}=\frac{4}{3}$

$\Rightarrow \frac{EC}{BE}=\frac{3}{4}$

$\Rightarrow \frac{EC}{BE}+1=\frac{3}{4}+1$

$\Rightarrow \frac{EC+BE}{BE}=\frac{7}{4}$

$\Rightarrow \frac{BC}{BE}=\frac{7}{4}$ or $\frac{BE}{BC}=\frac{4}{7}$

from equation (2), $\frac{EG}{CD}=\frac{4}{7}$

So $EG=\frac{4}{7}CD$ ......(3)

Similarly, $\Delta DGF \sim \Delta DBA$ (by AA similarity)

$\Rightarrow \frac{DF}{DA}=\frac{FG}{AB}$

$\Rightarrow \frac{FG}{AB}=\frac{3}{7}$

$\Rightarrow FG=\frac{3}{7}AB$ ...(4)

$\left[\begin{array}{l} \because \frac{AF}{AD}=\frac{4}{7}=\frac{BE}{BC} \\ \Rightarrow \frac{EC}{BC}=\frac{3}{7}=\frac{DE}{DA} \end{array}\right]$

Adding equations (3) and (4),we get,

$EG+FG=\frac{4}{7}CD+\frac{3}{7}AB$

$\Rightarrow EF=\frac{4}{7}\times(2AB)+\frac{3}{7}AB$

$=\frac{8}{7}AB+\frac{3}{7}AB=\frac{11}{7}AB$

$\therefore \quad 7EF=11AB$

34. Let the four parts be (a-3d), (a-d), (a+d) and (a+3d).

Then, (a-3d)+(a-d)+(a+d)+(a+3d)=32

⇒4a=32

⇒a=8

It is given that

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{7}{15}$

$\Rightarrow \frac{a^2-9d^2}{a^2-d^2}=\frac{7}{15}$

$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$

⇒960-135d^2=448-7d^2

⇒128d^2=512

⇒d^2=4

⇒d=±2

When d=2,

a-3d=8-3(2)=2

a-d=8-2=6
a+d=8+2=10
a+3d=8+3(2)=14
when d=-2,
a-3d=8-3(-2)=14
a-d=8-(-2)=10
a+d=8+(-2)=6
a+3d=8+3(-2)=2
Thus, the four parts are 2,6,10,14 or 14,10,6,2

OR

Let a_1 and a_2 be the first terms and d_1 and d_2 be the common difference of the two APs respectively.

Let S_n and S'_n be the sums of the first n terms of the two APs and T_n and T'_n be their nth terms respectively.

Then, $\frac{S_n}{S'_n} = \frac{7n+1}{4n+27} \Rightarrow \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{7n+1}{4n+27}$
$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{7n+1}{4n+27}$ ....(i)
To find the ratio of mth terms, we replace n by (2m -1) in the above expression.
Replacing n by (2 $\times$ 9 -1), i.e., 17 on both sides in (i), we get
$\frac{2a_1+(17-1)d_1}{2a_2+(17-1)d_2} = \frac{7\times17+1}{4\times17+27} \Rightarrow \frac{2a_1+16d_1}{2a_2+16d_2} = \frac{120}{95}$
$\Rightarrow \frac{a_1+8d_1}{a_2+8d_2} = \frac{24}{19}$
$\Rightarrow \frac{a_1+(9-1)d_1}{a_2+(9-1)d_2} = \frac{24}{19}$
$\Rightarrow \frac{a_1+(9-1)d_1}{a_2+(9-1)d_2} = \frac{24}{19}$
$\Rightarrow \frac{T_9}{T'_9} = \frac{24}{19}$
$\therefore$ required ratio = 24:19.

35.

Class Interval	**Frequency f_i**	**Class marks x_i**	$u_i = \frac{(x_i-A)}{h} = \frac{(x_i-40.5)}{10}$	**f_iu_i**
5.5 - 15.5	6	10.5	-3	-18
15.5 - 25.5	11	20.5	-2	-22
25.5 - 35.5	21	30.5	-1	-21
35.5 - 45.5	23	40.5 = A	0	0
45.5 - 55.5	14	50.5	1	14
55.5 - 65.5	5	60.5	2	10
Total	$\Sigma f_i = 80$			$\Sigma f_iu_i = -37$

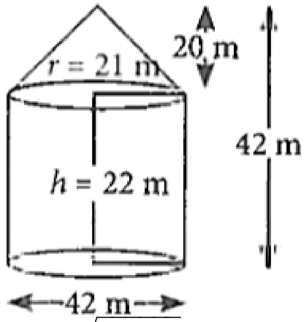
$A = 40.5, h = 10$
we know that, Mean, $\bar{x} = A + \left\{h \times \frac{\Sigma f_iu_i}{\Sigma f_i}\right\}$
$= 40.5 + \left\{10 \times \frac{(-37)}{80}\right\}$
$= 40.5 - \frac{37}{8}$
$= 40.5 - 4.63 = 35.87$
The modal class of the given data is 35.5 - 45.5.
$\therefore x_k = 35.5$, h = 10, $f_k = 23$, $f_{k-1} = 21$, $f_{k+1} = 14$
we know that, Mode, $M_o = x_k + \left\{h \times \frac{(f_k-f_{k-1})}{(2f_k-f_{k-1}-f_{k+1})}\right\}$
$= 35.5 + \left\{10 \times \frac{(23-21)}{(2\times23-21-14)}\right\}$
$= 35.5 + \left(\frac{10\times2}{11}\right)$
$= 35.5 + 1.82 = 37.32$
Clearly, mode > mean.

Section E

36. i. Curved surface area of cone

$= \pi rl$

$= \frac{22}{7} \times 21 \times 29 = 1914\ m^2$



[$\because l = \sqrt{r^2 + h_1^2} = \sqrt{(21)^2 + (20)^2} = \sqrt{841} = 29$ m]

ii. Area of floor = πr^2

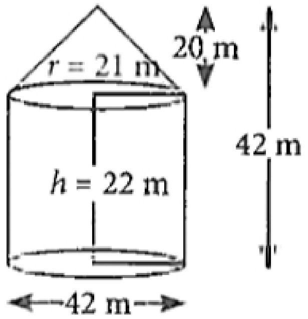
$= \frac{22}{7} \times 21 \times 21 = 1386\ m^2$

Number of persons that can be accommodated in the tent = $\frac{1386}{126} = 11$

iii. Curved surface area of cylinder

$= 2\pi rh$

$= \frac{22}{7} \times 21 \times 44 = 2904\ m^2$

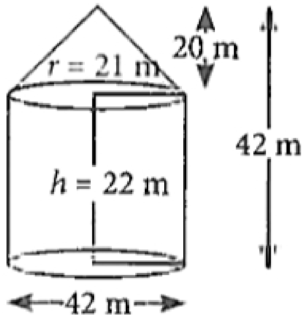


OR

Required area of canvas = Curved surface area of cone + Curved surface area of cylinder

$= \pi rl + 2\pi rh = \pi r\ (l + 2h)$

$= \frac{22}{7} \times 21\ (29 + 44) = 4818\ m^2$



37. i. Point of intersection of graph of polynomial, gives the zeroes of the polynomial.

$\therefore$ zeroes = -4 and 7

ii. Since, zero's are $\alpha = -4$, $\beta = 7$

$\alpha + \beta = -4 + 7 = 3$

$\alpha\beta = -4 \times 7 = -28$

$P(x) = x^2$ - (Sum of zeroes)x + product of zeroes

$P(x) = x^2 - 3x + (-28)$

$P(x) = x^2 - 3x - 28$

iii. Product of zeroes = -4×7

$= -28$

OR

a is a non-zero real number, b and c are any real numbers c.

38. i. Since, Total angle in a pie chart is 360^o

So, Total no. of Sample Space = 360

Let 'E' be the event of having 'White' as favourite colour.

P(E) = $\frac{favourable\ outcome}{Total\ Outcome}$

= $\frac{120}{360}$

= $\frac{1}{3}$

ii. P(Blue or Green) = $\frac{60+60}{360}$

= $\frac{120}{360}$ = $\frac{1}{3}$

iii. Since, Yellow represent 90^o in the Pie Chart

90^o = 15 Students

360^o = $\frac{15}{90} \times 360$ = 60 students

Hence, 60 Students participated in the survey.

OR

P(Red or Blue) = $\frac{30+60}{360}$

= $\frac{90}{360}$ = $\frac{1}{4}$