

# Chapter 1. Language of Algebra

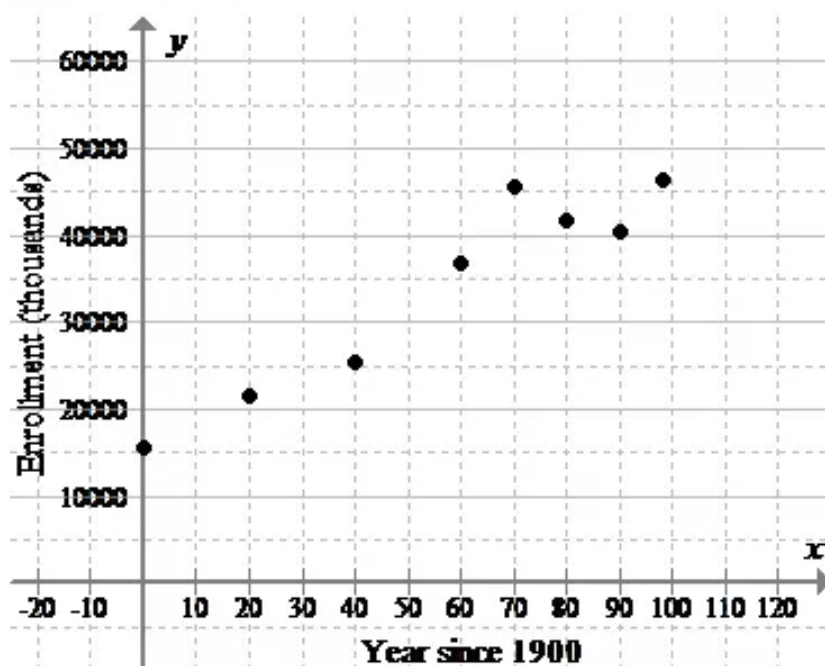
## Ex. 1.8

### Answer 1AA.

Consider the following table showing the number of students enrolled in elementary and secondary schools in the United States for the years:

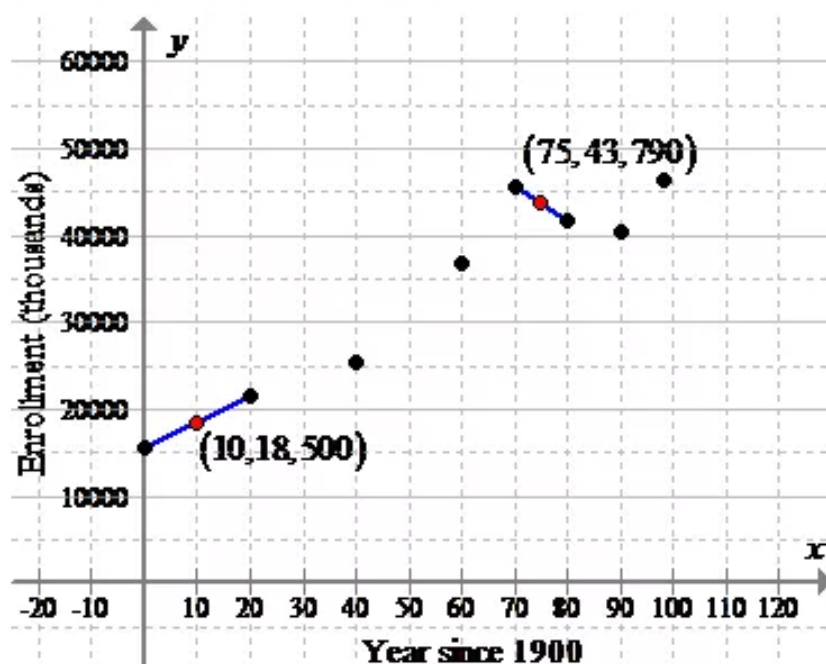
Year	Enrollment (thousands)	Year	Enrollment (thousands)
1900	15,503	1970	45,550
1920	21,578	1980	41,651
1940	25,434	1990	40,543
1960	36,807	1998	46,327

Draw the graph as shown below:



The objective is to estimate the number of students in elementary and secondary schools in 1910 and in 1975.

To estimate the number of students in elementary and secondary schools in 1910 and in 1975, draw a line between the points  $(0, 15503)$  and  $(20, 21578)$  and between the points  $(70, 45550)$  and  $(80, 41651)$ . Find the mid points of the two lines which give the coordinates for 1910 and 1975.



The mid points are  $(10, 18,500)$  and  $(75, 43,790)$ .

Therefore, the estimated number of students in elementary and secondary schools in 1910 is 18,500.

The estimated number of students in elementary and secondary schools in 1975 is 43,790.

### Answer 1CU.

The objective is to explain the importance of order of a number in an order pair.

An order pair is a set of numbers, or coordinates written in the form  $(x, y)$ . The  $x$  value called  $x$ -coordinate, corresponds to horizontal axis ( $x$ -axis) and the  $y$  value, or  $y$ -coordinate, corresponds to vertical axis ( $y$ -axis).

An order pair  $(x, y)$  is a rule which indicates, first move along the  $x$ -axis then move along the  $y$ -axis. That is why order of a number in a order pair is important.

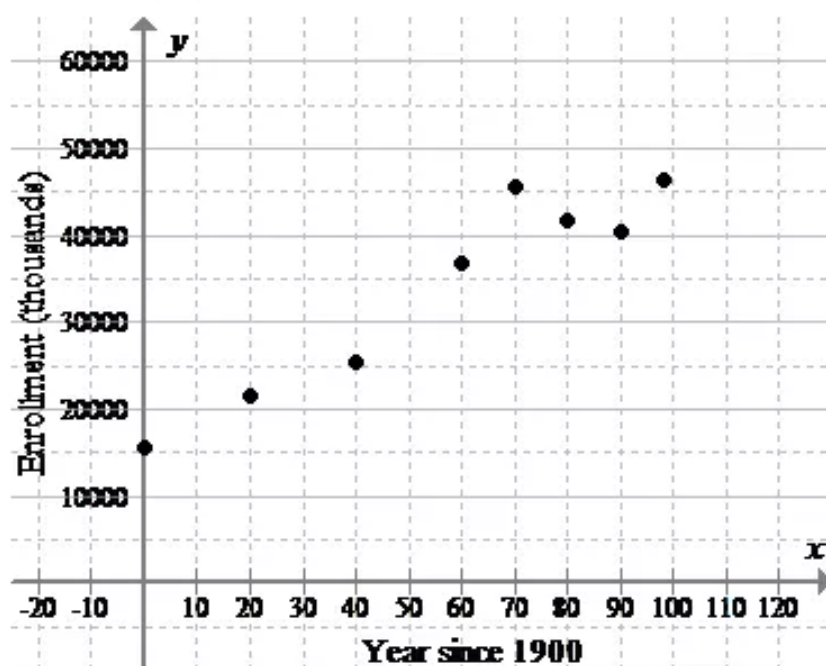
For example, to put the order pair  $(2, 3)$  in  $x$ - $y$  plane, first move 2 units along the  $x$ -axis then move 3 units along  $y$ -axis.

### Answer 2AA.

Consider the following table showing the number of students enrolled in elementary and secondary schools in the United States for the years:

Year	Enrollment (thousands)	Year	Enrollment (thousands)
1900	15,503	1970	45,550
1920	21,578	1980	41,651
1940	25,434	1990	40,543
1960	36,807	1998	46,327

Draw the graph as shown below:



The objective is to estimate the number of students in elementary and secondary schools in 2020.

As the scattered plot does not follow any pattern, the number of students in elementary and secondary schools in 2020 can not be estimated.

**Answer 2CU.**

The objective is to differentiate between dependent and independent variables.

Difference between dependent and independent variables are:

In a function dependent variables depend on independent variables, while independent variables are always independent.

In graphical representation of a function, dependent variables are plotted on vertical axis and independent variables are plotted on horizontal axis.

Example of dependent and independent variables:

In general, People become weaker as the age go on.

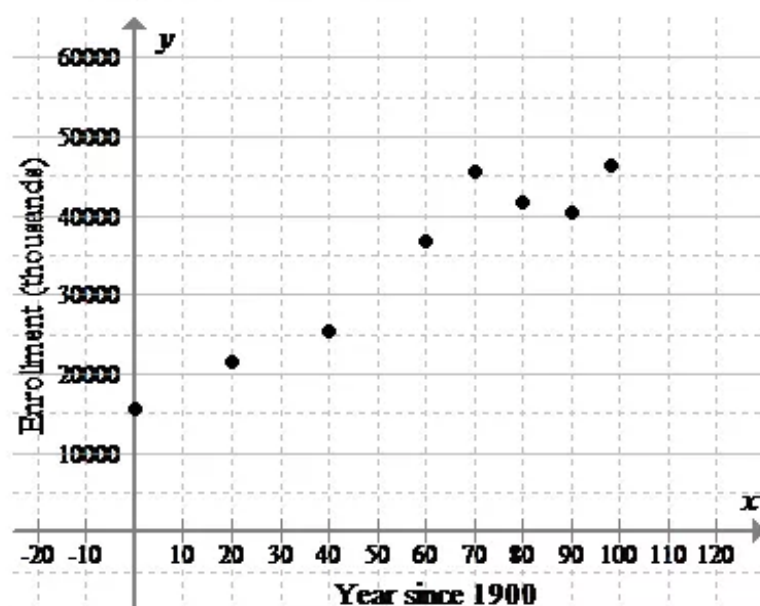
Here age is an independent variable as it is unaffected by the weakness, and weakness is dependent variable as it is affected by the age.

### Answer 3AA.

Consider the following table showing the number of students enrolled in elementary and secondary schools in the United States for the years:

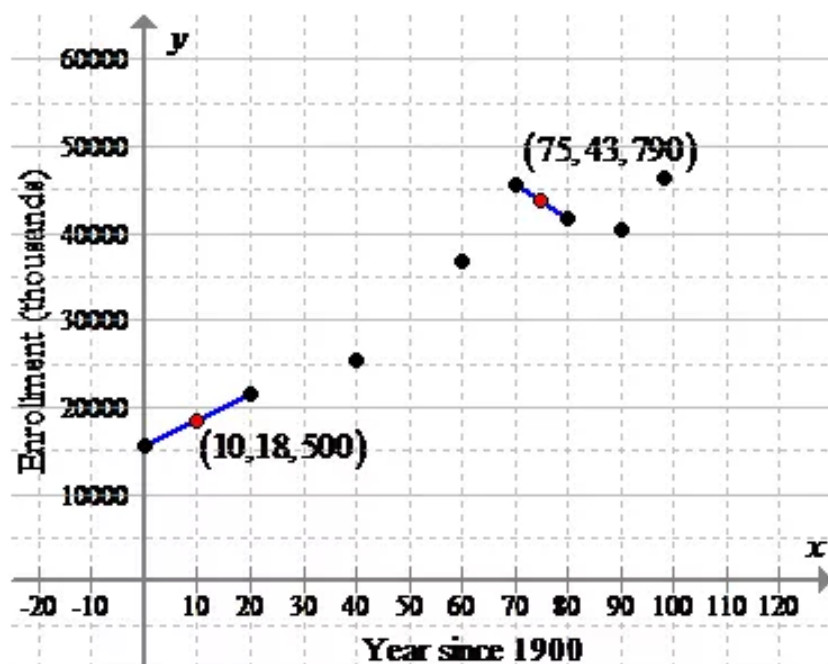
Year	Enrollment (thousands)	Year	Enrollment (thousands)
1900	15,503	1970	45,550
1920	21,578	1980	41,651
1940	25,434	1990	40,543
1960	36,807	1998	46,327

Draw the graph as shown below:



The objective is to describe the method to estimate the number of students in elementary and secondary schools in 1910 and in 1975.

To estimate the number of students in elementary and secondary schools in 1910 and in 1975, draw a line between the points  $(0, 15503)$  and  $(20, 21578)$  and between the points  $(70, 45550)$  and  $(80, 41651)$ . Find the mid points of the two lines which give the coordinates for 1910 and 1975.



The mid points are  $(10, 18,500)$  and  $(75, 43,790)$ .

Therefore, the estimated number of students in elementary and secondary schools in 1910 is 18,500.

The estimated number of students in elementary and secondary schools in 1975 is 43,790.

As the scattered plot does not follow any pattern, the number of students in elementary and secondary schools in 2020 can not be estimated.

### Answer 3CU.

The objective is to give an example of a relation and identify the domain and range.

Take an example of a relation:

A Xerox machine produces 4 true copy in a minute, working up to 10 minutes in a day. The machine production is a function of per minute working and number of true copy produced.

Identify the domain and range:

The domain contains number of minutes the machine works in a day. Since it works 10 minutes in a day. Therefore, reasonable domain values from 0 to 10 minutes.

The range contains daily production of true copy, which is  $4 \times 10$  copy or 40 copy.

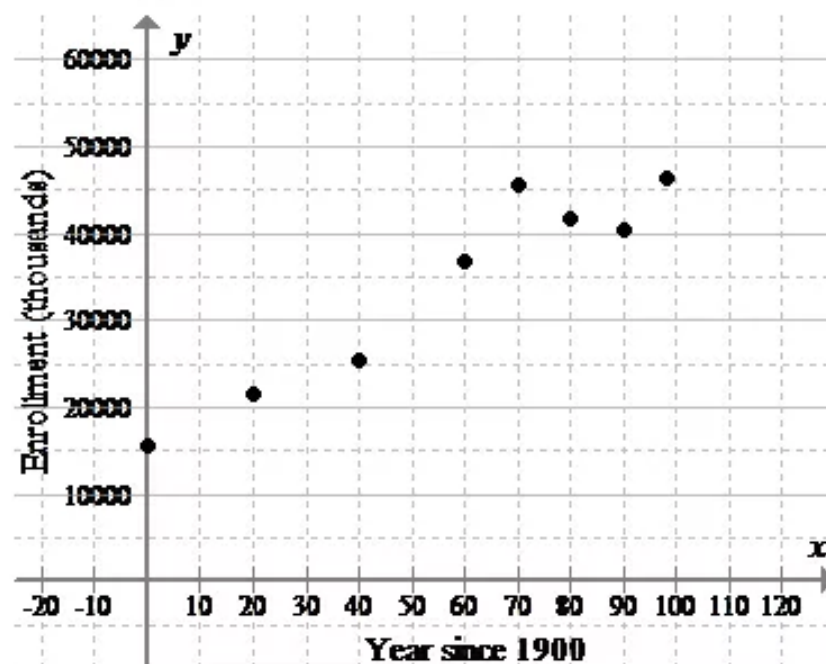
Therefore, the reasonable range values from 0 to 40 copy.

### Answer 4AA.

Consider the following table showing the number of students enrolled in elementary and secondary schools in the United States for the years:

Year	Enrollment (thousands)	Year	Enrollment (thousands)
1900	15,503	1970	45,550
1920	21,578	1980	41,651
1940	25,434	1990	40,543
1960	36,807	1998	46,327

Draw the graph as shown below:

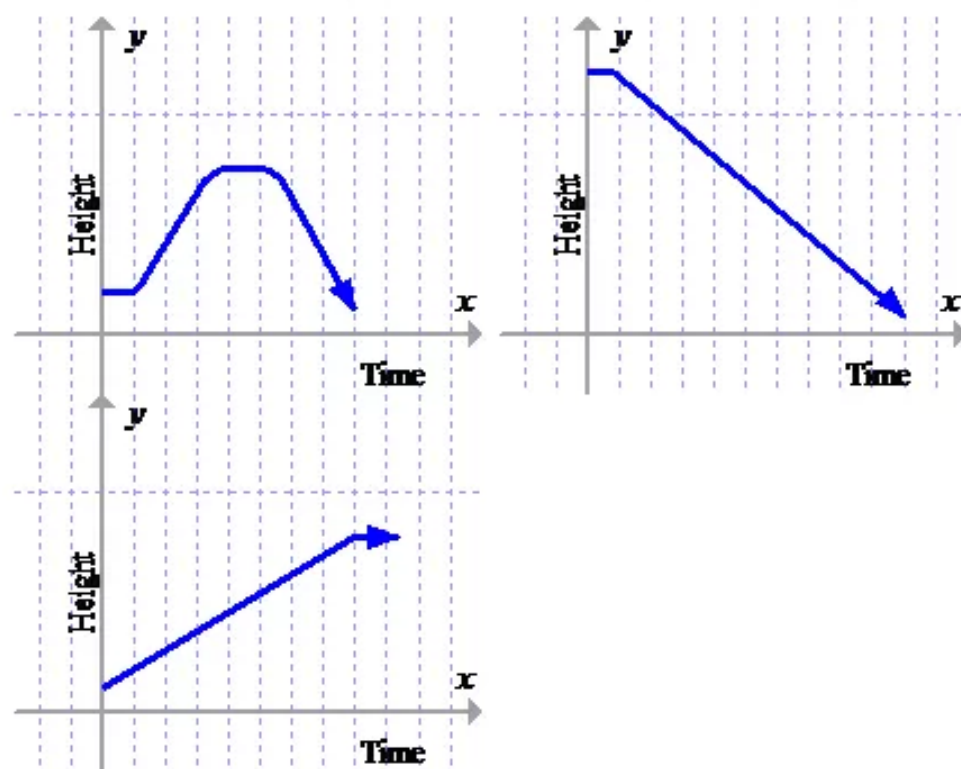


As the scattered plot does not follow any pattern, the number of students in elementary and secondary schools in 2020 can not be estimated accurately.

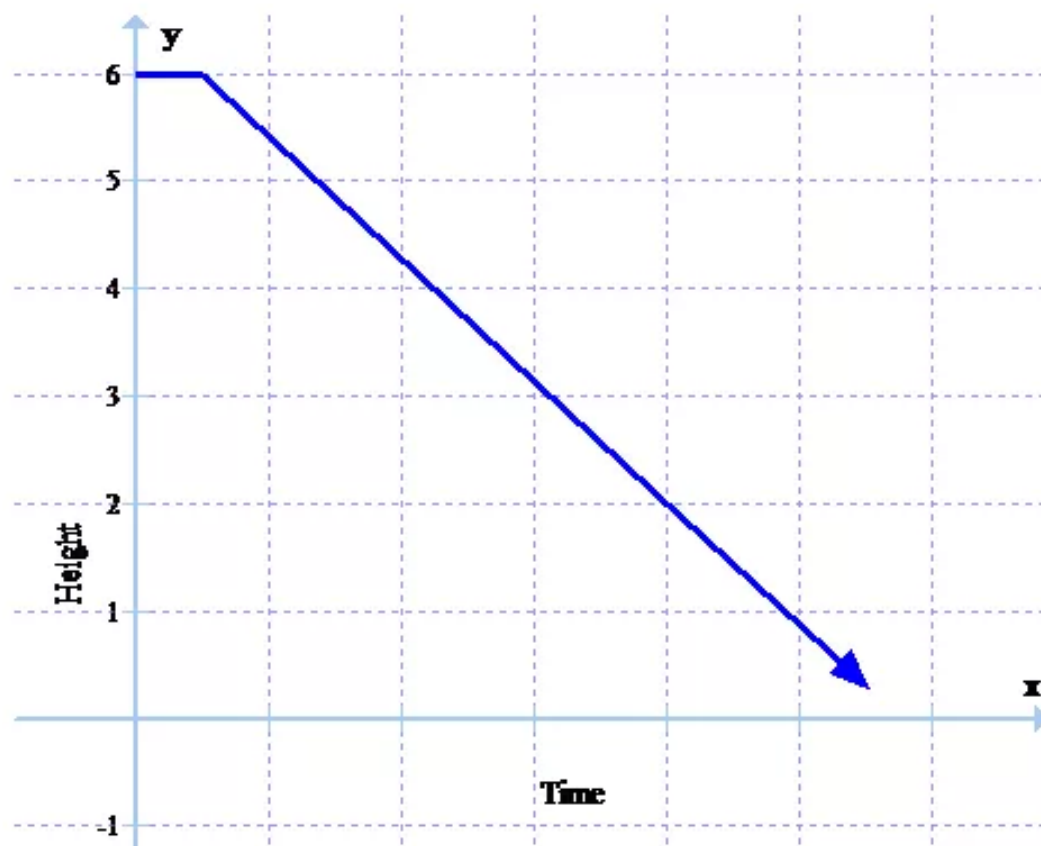
### Answer 5CU.



Consider the following graphs that show height of skydiver just before she jumps from a plane:



When a skydiver jumps from a plane, her falling path resemble with graph B because she will gain two motions, one is downward motion due to gravity and another is horizontal motion due to motion of the plane. Thus, the correct option is **Graph B**. The correct graph is shown below:



**Answer 6CU.**

The objective is to identify dependent and independent variable from the given data.

In the above table, time is the independent variable because it is unaffected by the change in height. And height is the dependent variable which is affected by the time (In a relation independent variable is that variable which is not dependent on other variables and dependent variables are affected by other variables).

### Answer 7CU.

Consider the table that represents height of an object above the ground at several intervals after it was dropped of a height of 5 meter:

Time(S)	0	0.2	0.4	0.6	0.8	1
Height(cm)	500	480	422	324	186	10

The objective is to write the order pairs from the given data.

As the Time is an independent variable, therefore time represents x-coordinate and height is a dependent variable therefore, Height represents y-coordinate.

The order pairs are :

$(0, 500), (0.2, 480), (0.4, 422), (0.6, 324), (0.8, 186), (1, 10)$

### Answer 8CU.

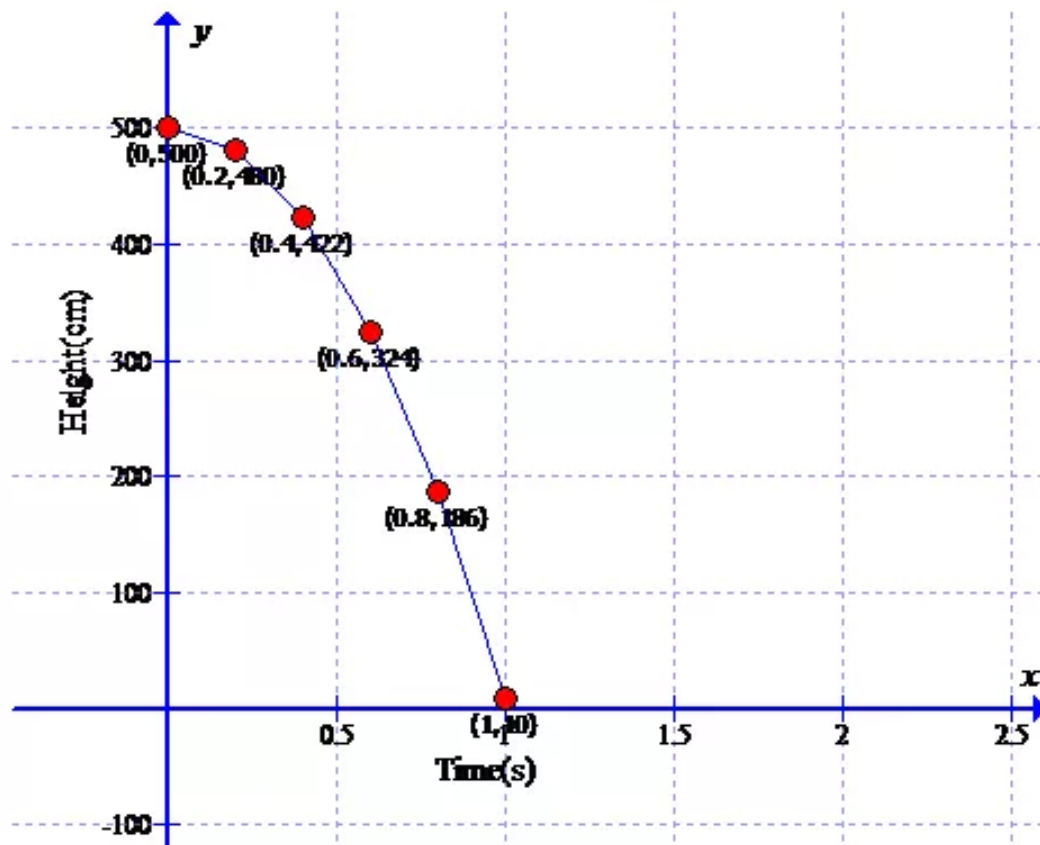
Consider the table that represents height of an object above the ground at several intervals after it was dropped of a height of 5 meter:

Time(S)	0	0.2	0.4	0.6	0.8	1
Height(cm)	500	480	422	324	186	10

The objective is to plot the graph of the given data.



The graph showing the relation between height and time is plotted below.

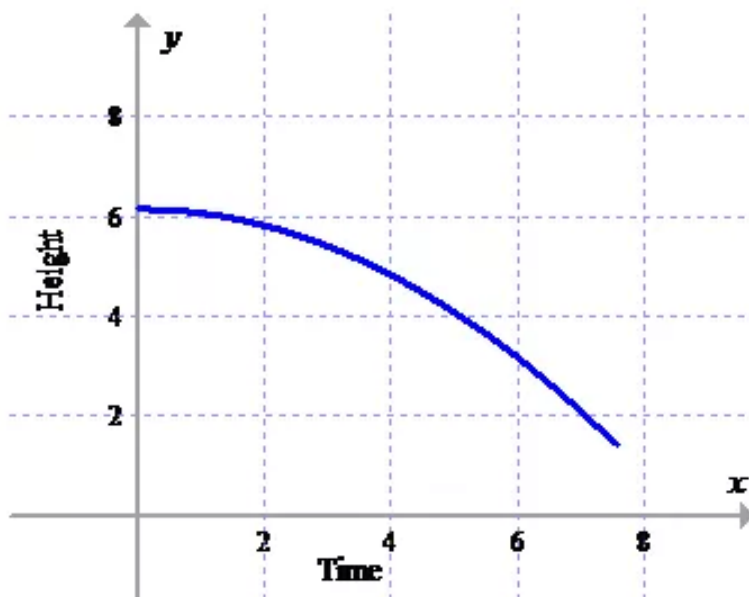


**Answer 9CU.**

Consider that Paul is a pitcher for his school baseball team. And he releases the ball to catcher.

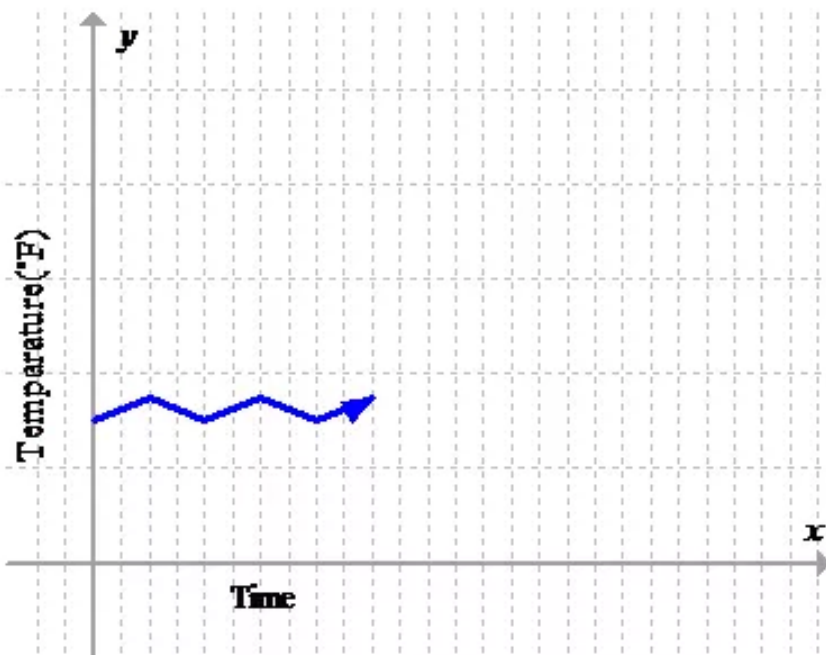
The objective is to plot the reasonable graph that shows the height of the baseball from the ground, from the time he release the ball until the time the catcher catches the ball.

At the point 0, Paul holds the ball and throws the ball to the catcher. The ball will follow a curve path as it will first gain a horizontal movement then a vertical movement. The valid relation is shown in the graph below.



**Answer 10PA.**

Consider the following graph that represents Michelle's temperature when he was sick:

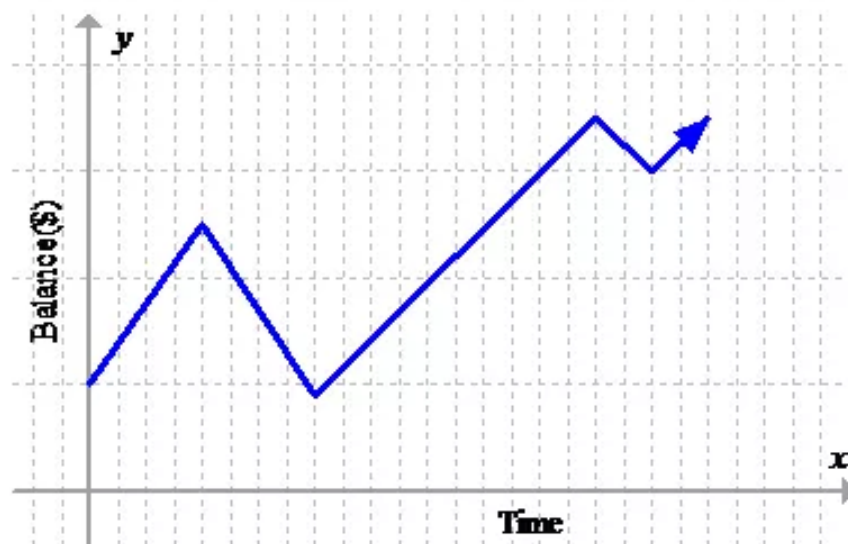


The objective is to explain the graph that represents Michelle's temperature when he was sick.

At the point 0, Michelle's temperature is normal. Gradually his temperature goes high and again his temperature gradually goes down, and the process is repeated. That means, his temperature is a dependent variable and time is an independent variable.

#### Answer 11PA.

Consider the following graph that represents Rashaad's checking account balance:

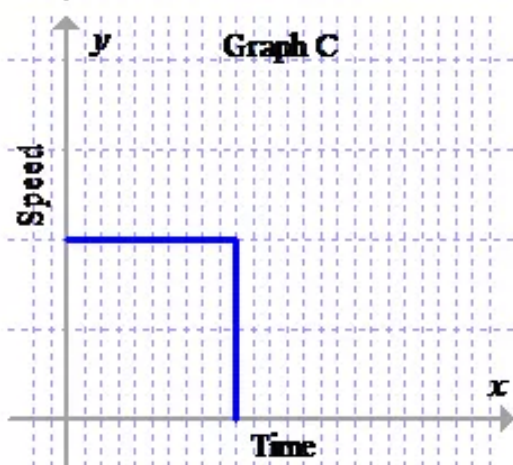
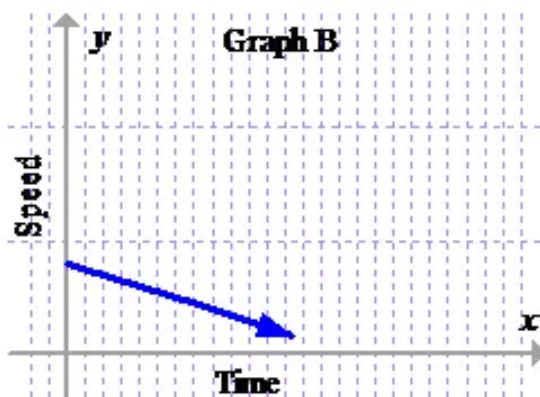
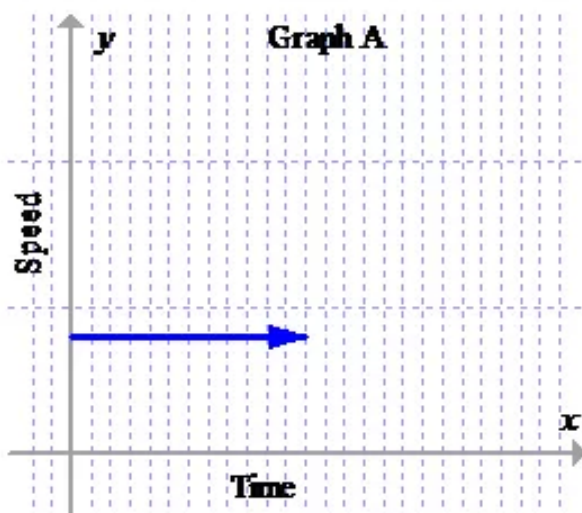


The objective is to explain the graph.

The graph shows that Rashaad's account is increasing as he makes deposits and earns interest. Then he pays some bills, He then makes some deposits and earns interest and so on.

#### Answer 12PA.

Consider the following graphs that represent speed of a radio-controlled car:

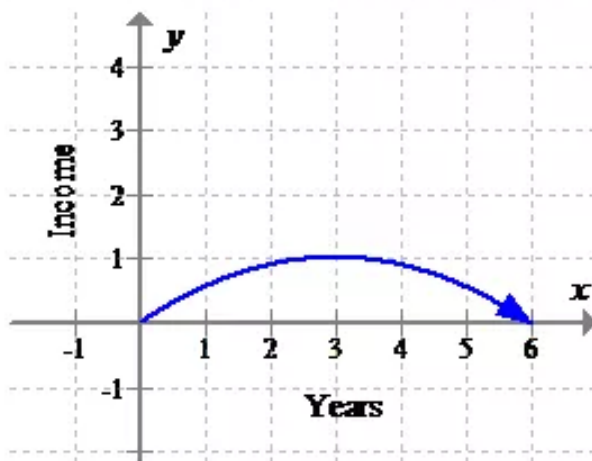
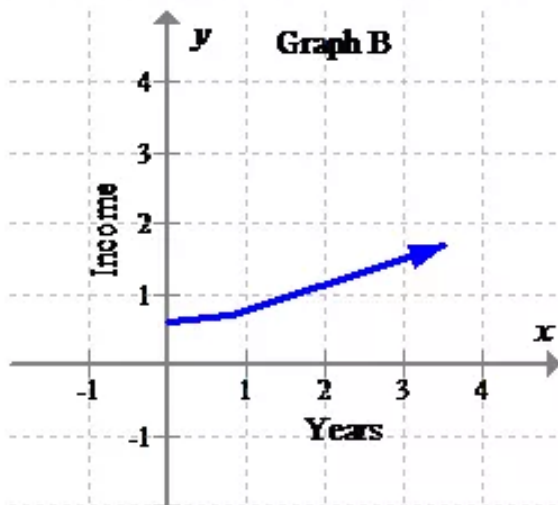
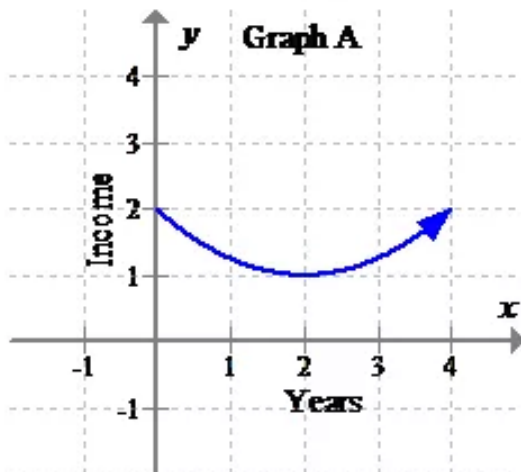


The objective is to choose the valid graph.

Initially the radio-controlled car achieves a certain speed and moves along with that constant speed. When the car hits on the wall its speed instantly becomes zero. Graph C shows the relation.

**Answer 13PA.**

Consider the following graph that represents the income of an older person:



The objective is to choose the valid graph.

When a person gets older his income increases until his retirement. Here income is a dependent variable and year is an independent variable. Graph B represent valid relation.

**Answer 14PA.**

Consider the following table that represents cost of parking:

Time(h)	0- 1:59	2- 3:59	4- 5:59	6- 11:59	12- 24
Cost(\$)	1	2	4	5	30
After 24 hours: an additional \$15 per day or portion thereof					

The objective is to write the order pairs up to 36 hours parking.

Rewrite the table for 36 hours.

Time(h)	Cost(\$)
0	1
1	1
2	2
3	2
4	4
5	4
6	5
7	5
8	5
9	5
10	5
11	5
12	30
13	30



14	30
15	30
16	30
17	30
18	30
19	30
20	30
21	30
22	30
23	30
24	30
25	45
26	$30+15=45$
27	45
28	$30+15=45$
29	45

30	$30+15=45$
31	45
32	45
33	45
34	45
35	45
36	45

### Answer 15PA.

Consider the following table which represents cost of parking:

Time(h)	0- 1:59	2- 3:59	4- 5:59	6- 11:59	12- 24
Cost(\$)	1	2	4	5	30
After 24 hours: an additional \$15 per day or portion thereof					

The objective is to plot the graph from the given table which represents cost of parking.

Rewrite the table for 36 hours parking.

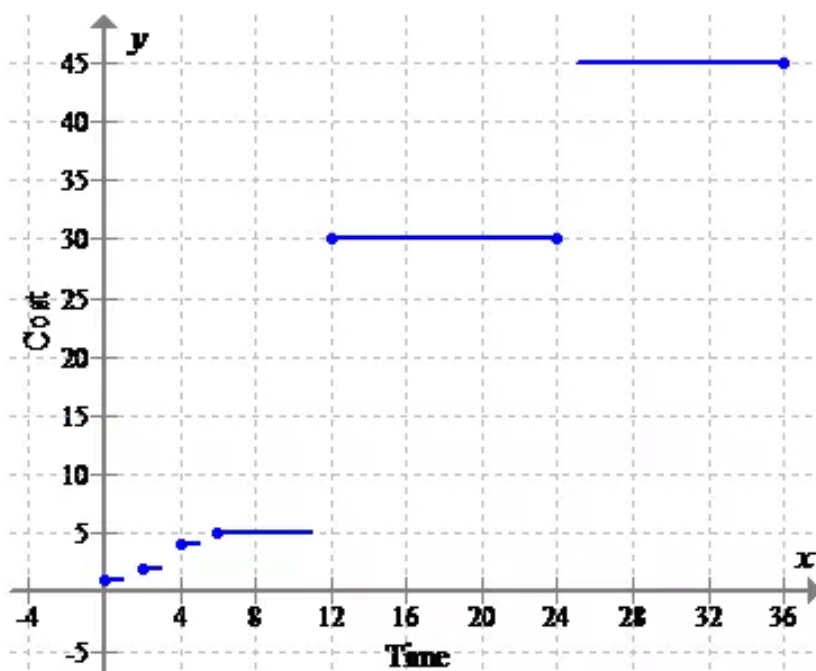
Time(h)	Cost(\$)
0	1
1	1

2	2
3	2
4	4
5	4
6	5
7	5
8	5
9	5
10	5
11	5
12	30
13	30
14	30

15	30
16	30
17	30
18	30
19	30
20	30
21	30
22	30
23	30
24	30
25	45
26	$30+15=45$
27	45
28	$30+15=45$

29	45
30	$30+15=45$
31	45
32	45
33	45
34	45
35	45
36	45

The graph of the above table is shown below.



**Answer 16PA.**



Consider the following table which represents cost of parking:

Time(h)	0- 1:59	2- 3:59	4- 5:59	6- 11:59	12- 24
Cost(\$)	1	2	4	5	30
After 24 hours: an additional \$15 per day or portion thereof					

The objective is to calculate parking cost from Monday 7:00 am to Tuesday 9:00 pm.

If you arrive on Monday at 7:00 am and depart on Tuesday at 9:00 pm, the total duration of parking is 38 hours.

The cost of parking is

$$\begin{aligned} & \$ (30 + 15) \\ & = \$45 \end{aligned}$$

The solution is \$45.

### Answer 17PA.

Consider the following table that represents the relation between sides and interior angle of a polygon:

The objective is to identify the independent and dependent variables.

Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	heptagon
Sides	3	4	5	6	7
Interior angle Sum	180	360	540	720	900

Sides are the independent variables as they are not affected by sum of interior angle and sum of the interior angles are dependent variables as they are affected by the sides.

### Answer 18PA.

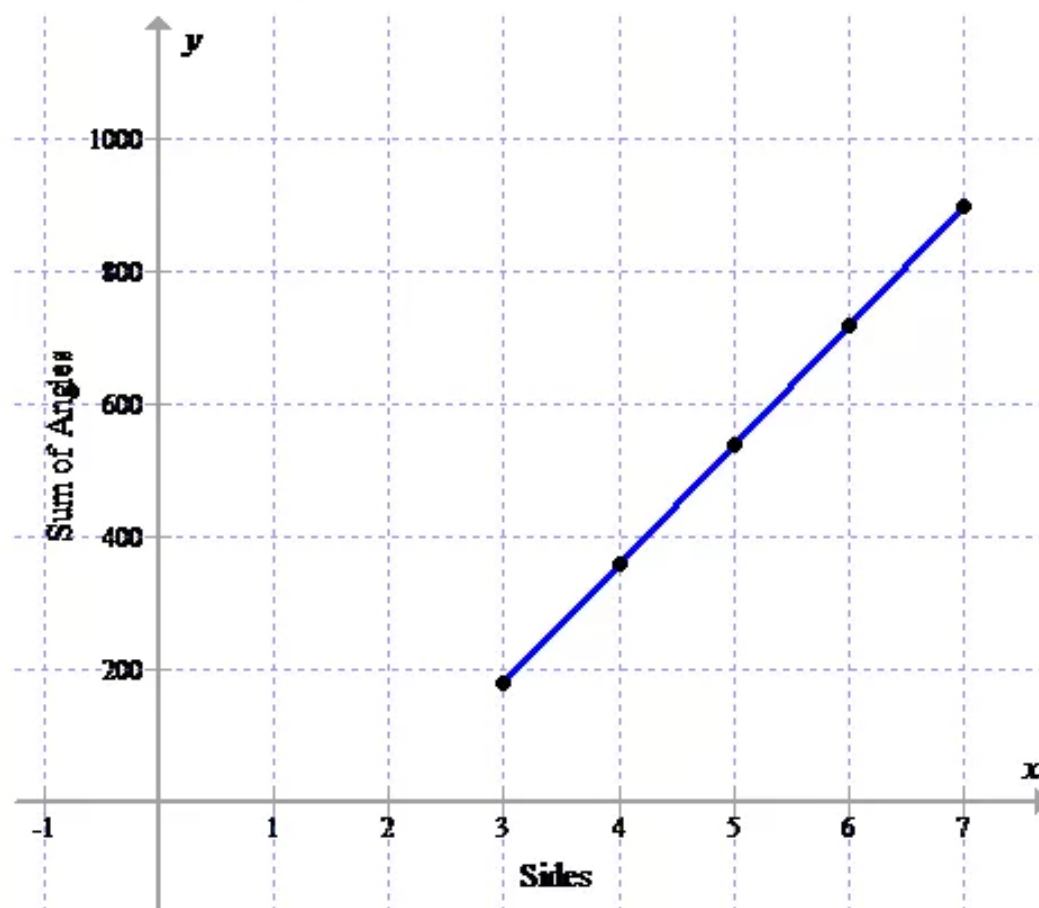
Consider the following table that represents the relation between sides and sum of interior angles of a convex polygon.

Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	heptagon
Sides	3	4	5	6	7
Interior angle Sum	180	360	540	720	900

The objective is to plot the graph from the given data that represents the relation between sides and sum of interior angle of a polygon.

Plot the ordered pairs  $(3,180), (4,360), (5,540), (6,720), (7,900)$  on the coordinate system and join them.

The graph of the given table is shown below.



**Answer 19PA.**

The objective is to predict relation between sides and sum of interior angles for polygon.

Consider the following table that represents the relation between sides and sum of interior angle of a polygon:

The objective is to predict relation between sides and sum of interior angles for polygon.

Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	heptagon
Sides	3	4	5	6	7
Interior angle Sum	180	360	540	720	900

Relation between sides and sum of interior angles can be predicted from the above table.

$$\text{Sum of interior angles} = (\text{Number of sides} - 2) \times 180^\circ$$

Now, verify the above relation for quadrilateral.

Number of sides of a quadrilateral is 4, so

$$(4 - 2) \times 180^\circ$$

$$= 2 \times 180^\circ$$

$$= 360^\circ$$

$$\text{Sum of interior angles of octagon} = (\text{Number of side} - 2) \times 180^\circ$$

$$= (8 - 2)180^\circ = 1440^\circ$$

$$= (6)180^\circ$$

$$= \boxed{1080^\circ}$$

$$\text{Sum of interior angles of nonagon} = (9 - 2) \times 180^\circ$$

$$= (7)180^\circ$$

$$= \boxed{1260^\circ}$$

$$\text{Sum of interior angles of octagon} = (10 - 2) \times 180^\circ$$

$$= (8)180^\circ$$

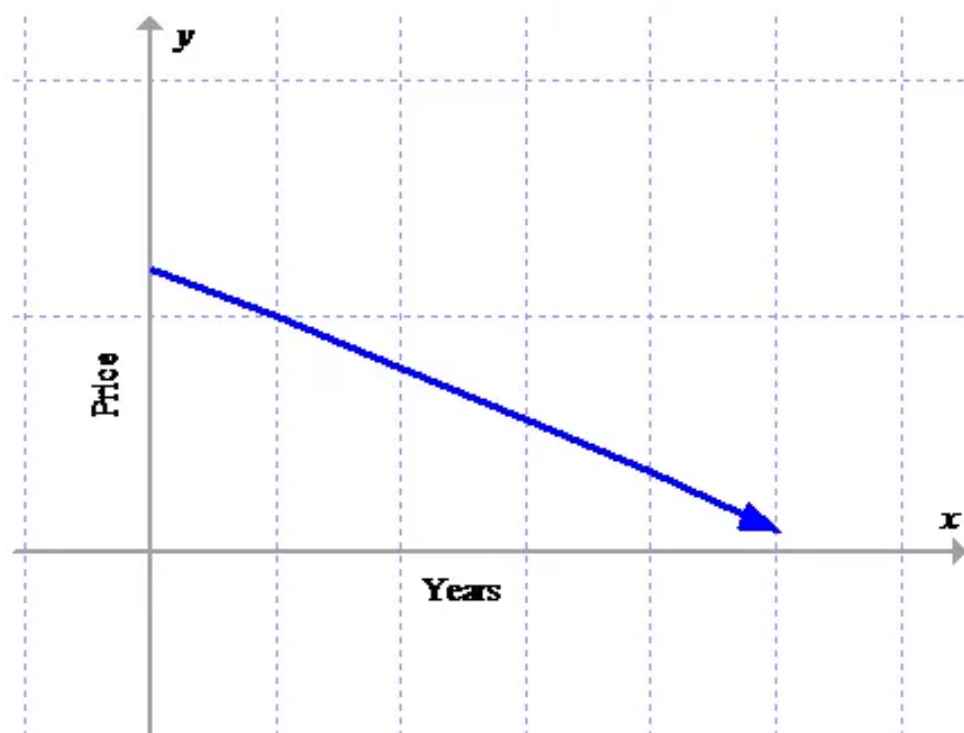
$$= 1440^\circ$$

**Answer 20PA.**

Consider that a car was purchased new in 1970. The owner has taken excellent care of the car, and it has relatively low millage.

The objective is to plot the reasonable graph.

As the car was purchased in 1970 and it has relatively low millage, so the value of the car will decrease as it gets older. The reasonable graph is shown below.

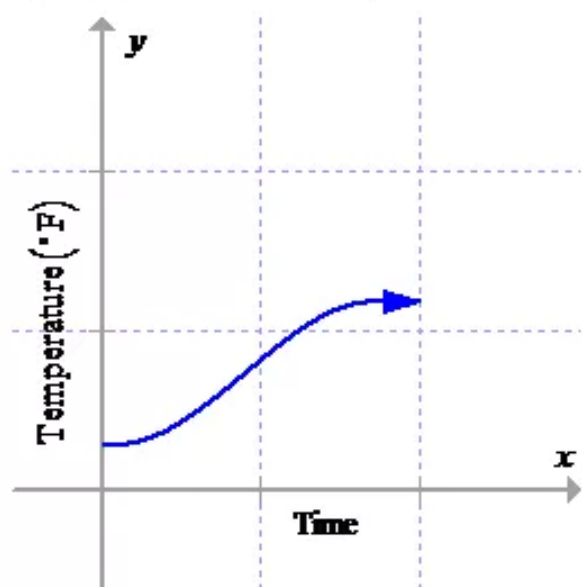


### Answer 21PA.

Consider that when ice is exposed to  $32^{\circ}\text{F}$ , it begins to melt. The ice is removed from freezer and placed on a counter room.

The objective is to plot the reasonable graph that represents relation between temperature and time.

When ice is exposed to temperature above  $32^{\circ}\text{F}$ , it begins to melt and gradually it comes into equilibrium with room temperature. The relation is shown below.



### Answer 22PA.

(a)

Consider that Mallory is 23 years older than Lisa.

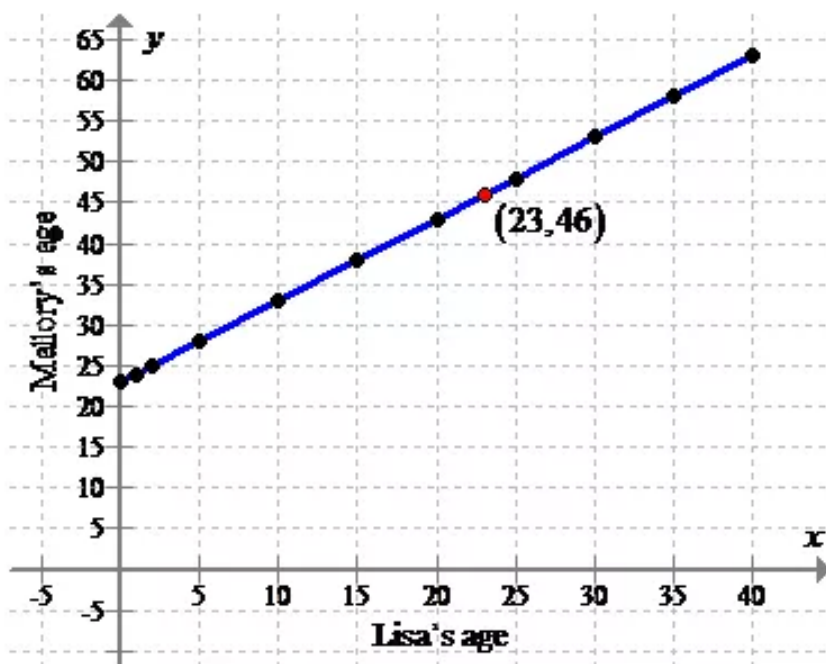
The objective is to draw a graph showing Mallory's age as a function of Lisa's age for the first 40 years of Lisa's life.

To draw the graph, make a table of some points of Lisa's age and Mallory's age:

Lisa's age	Mallory's age
0	23
1	24
2	25
5	28
10	33
15	38
20	43
23	46
25	48
30	53
35	58
40	63



Now, plot the ordered pairs in the coordinate system and join them as shown below:



(b)

The point on the graph when Mallory is twice as old as Lisa is  $(23, 46)$ .

### Answer 23PA.

Real-world data can be recorded and visualized in a graph and by expressing an event as a function of another event.

A graph gives a visual representation of the situation which is easier to analyze and evaluate.

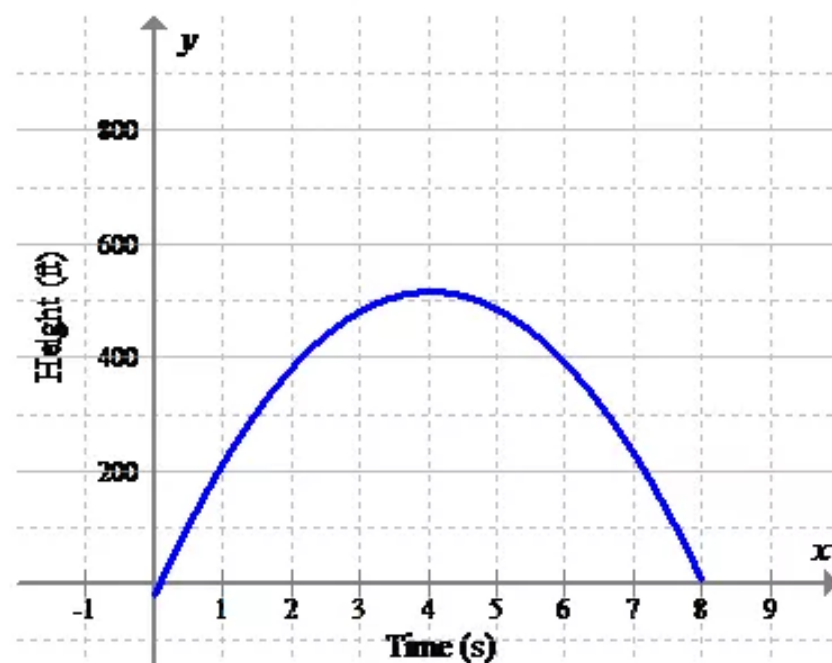
It gives a summary of what happens during the first 24 hours from the time of a concussion.

For example, during the first 24 hours, blood flow to the brain decreases to 50% at the moment of the injury and gradually increases to about 60%.

The graph also gives an explanation of the time in which significant improvement occurs.

### Answer 24PA.

Consider the following figure showing the height of a model rocket shot straight up.



The objective is to find the number of seconds it took to reach the maximum height.

#### Answer 25PA.

Consider that the charges for each back up CD is \$2.50.

His expenses include \$875 for the CD recording equipment and \$0.35 for each blank CD.

The objective is to find the equation to calculate the profit for the recording of  $n$  CDs.

The profit for  $n$  CDs is

$$\begin{aligned}
 & n(\text{charge for one CD back up} - \text{cost of one blank CD}) - \$875 \\
 &= n(\$2.50 - \$0.35) - \$875 \\
 &= n(\$2.15) - \$875 \\
 &= \$ (2.15n - 875)
 \end{aligned}$$

Thus, the correct option for the equation to calculate the profit for the recording of  $n$  CDs is

$$(A) \ 2.15n - 875$$

#### Answer 26MYS.

The above conditional statement is written without using the words 'if' and 'then'.

Thus, the hypothesis for the above statement is you have a computer.

The conclusion for the above statement is you can send e-mail.

The statement in if-then form is "If you have a computer, then you can send e-mail."

#### Answer 27MYS.

The above conditional statement is written without using the words 'if' and 'then'.

Thus, the hypothesis for the above statement is a shopper has 9 or fewer items.

The conclusion for the above statement is the shopper can use the express line.

The statement in if-then form is "If a shopper has 9 or fewer items, then the shopper can use the express line."

### Answer 28MYS.

The properties used in each step of the solution are shown below:

$$\begin{aligned}ab(a+b) &= (ab)a + (ab)b \\&= a(ab) + (ab)b \text{ [Commutative property: } ab = ba\text{]} \\&= (a \cdot a)b + a(b \cdot b) \text{ [Associative property: } a(bc) = (ab)c\text{]} \\&= a^2b + ab^2 \text{ [Substitution property: } a \cdot a = a^2; b \cdot b = b^2\text{]}\end{aligned}$$

### Answer 29MYS.

To find the value of  $n$ , solve the equation.

$$\begin{aligned}(12-9)4 &= n(4) \\(3)4 &= 4n \text{ [Substitution property: } 12-9=3\text{]} \\12 &= 4n \text{ [Substitution property: } 3 \cdot 4 = 12\text{]} \\3 &= n \text{ [Divide each side by 3]}\end{aligned}$$

Therefore,  $n = \boxed{3}$ .

### Answer 30MYS.

Consider the following statement:

$$7(n) = 0$$

The objective is to name the property used in the statement and to find the value of  $n$ .

The Multiplicative property of zero says that for any real number  $a$ ,

$$a \cdot 0 = 0$$

Thus, the property used in the above equation is multiplicative property of zero.

And from the above equation,

$$7(0) = 0$$

Therefore,  $n = \boxed{0}$ .

**Answer 31MYS.**

Consider the following statement:

$$n(87) = 87$$

The objective is to name the property used in the statement and to find the value of  $n$ .

The Multiplicative identity property says that for any real number  $a$ ,

$$\begin{aligned} a \cdot 1 &= 1 \cdot a \\ &= a \end{aligned}$$

where 1 is called the multiplicative identity.

Thus, the property used in the above equation is multiplicative identity.

And from the above equation,

$$1(87) = 87$$

Therefore,  $n = \boxed{1}$ .