



conservation of energy, find the speed at perigee and the speed at apogee. It is necessary to have the spacecraft escape from the earth completely.

- a) 9840 m/sec(perigee),9760 m/sec(apogee)      b) 7840 m/sec(perigee),5760 m/sec(apogee)  
c) 10840 m/sec(perigee),8760 m/sec(apogee)      d) 8840 m/sec(perigee),6760 m/sec(apogee)
6. A body is moving forwards and backward. Change in frequency observed of source is 2%. What is velocity of the body? (Speed of sound is 300 m/s) [1]  
a) 3 m/s      b) 2.5 m/s  
c) 2 m/s      d) 6 m/s
7. A body A is thrown up vertically from the ground with a velocity  $v_0$  and another body B is simultaneously dropped from a height H. They meet at a height  $\frac{H}{2}$ , if  $v_0$  is equal to: [1]  
a)  $\sqrt{2gH}$       b)  $\sqrt{\frac{2g}{H}}$   
c)  $\sqrt{gH}$       d)  $\frac{1}{2}\sqrt{gH}$
8. The phase difference between two waves, represented by [1]  
 $y_1 = 10^{-6} \sin [100t + (\frac{x}{50}) + 0.5]$  m  
 $y_2 = 10^{-6} \cos [100 t + (\frac{x}{50})]$  m  
where x is expressed in metres and t is expressed in seconds, is approximately  
a) 1.07 rad      b) 1.5 rad  
c) 0.5 rad      d) 2.07 rad
9. An open tank filled with water (density  $\rho$ ) has a narrow hole at a depth of h below the water surface. The velocity of water flowing out is [1]  
a) 2gh      b)  $\sqrt{2gh}$   
c)  $h\rho g$       d) gh
10. Satellites orbiting the earth have a finite life and sometimes debris of satellites fall to the earth. This is because, [1]  
a) of viscous forces causing the speed of the satellite and hence height to gradually decrease.      b) the solar cells and batteries in satellites run out.  
c) of collisions with other satellites.      d) the laws of gravitation predict a trajectory spiralling inwards.
11. A thin uniform rod of length 2l and mass M is acted upon a constant torque. The angular velocity changes from zero to  $\omega$  in time t. The value of torque is: [1]  
a)  $\frac{Ml^2\omega}{3t}$       b)  $\frac{2Ml^2\omega}{3t}$   
c)  $\frac{Ml^2\omega}{12t}$       d)  $\frac{Ml^2\omega}{t}$
12. A spherical black body with a radius of 12 cm radiates 450 watt power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be [1]  
a) 225      b) 450  
c) 1800      d) 1000

13. **Assertion (A):** The Work-Energy theorem is applicable for non-inertial frames also. [1]  
**Reason (R):** The Work-Energy theorem is applicable to non-inertial frames provided we include the pseudo forces in the calculation of the net force acting on the body under consideration.
- a) Both A and R are true and R is the correct explanation of A.      b) Both A and R are true but R is not the correct explanation of A.  
c) A is true but R is false.      d) A is false but R is true.
14. **Assertion:** In an isochoric process, work done by the gas is zero. [1]  
**Reason:** In a process, if initial volume is equal to the final volume, work done by the gas is zero.
- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.      b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.  
c) Assertion is correct statement but reason is wrong statement.      d) Assertion is wrong statement but reason is correct statement.
15. **Assertion (A):** Even when orbit of a satellite is elliptical, its plane of rotation passes through the centre of earth. [1]  
**Reason (R):** According to law of conservation of angular momentum plane of rotation of satellite always remain same.
- a) Both A and R are true and R is the correct explanation of A.      b) Both A and R are true but R is not the correct explanation of A.  
c) A is true but R is false.      d) A is false but R is true.
16. **Assertion (A):** The division of a vector by another vector is not defined. [1]  
**Reason (R):** The division of a vector by a direction is not possible.
- a) Both A and R are true and R is the correct explanation of A.      b) Both A and R are true but R is not the correct explanation of A.  
c) A is true but R is false.      d) A is false but R is true.

### Section B

17. A stone is dropped into a well and its splash is heard at the mouth of the well after an interval of 1.45 s. Find the depth of the well. Given that velocity of sound in air at room temperature is equal to  $332 \text{ ms}^{-1}$ . [2]
18. Differentiate between dimensional and non-dimensional variables. [2]
19. The density of mercury is  $13.6 \text{ g cm}^{-3}$  in CGS system. Find its value in SI units. [2]
20. Is friction a non-conservative force? [2]
21. At a point above the surface of the earth, the gravitational potential is  $-5.12 \times 10^7 \text{ J/kg}$  and the acceleration due to gravity is  $6.4 \text{ m/s}^2$ . Assuming the mean radius of the earth to be 6400 km, calculate the height of the point above the earth's surface. [2]

OR

An astronaut, by mistake, drops his food packet from an artificial satellite orbiting around the earth. Will it reach the surface of the earth? Why?

### Section C

22. A cylindrical vessel filled with water upto a height of 2 m stands on a horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. Find the minimum diameter of the hole so that the vessel [3]

begin to move on the floor, if the plug is removed. The coefficient of friction between the bottom of the vessel and the plane is 0.4 and total mass of water plus vessel is 100 kg.

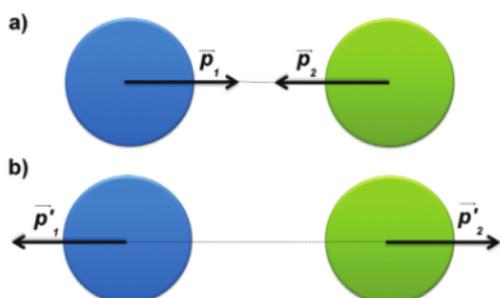
23. A certain substance has a mass of 50 g/mole. When 300 J of heat is added to 25g of sample of this material, its temperature rises from 25°C to 45°C. Calculate [3]
- the thermal capacity,
  - specific heat capacity, and
  - molar heat capacity of the sample.
24. Draw the following graphs for an object projected upward with a velocity  $v_0$ , which comes back to the same point after some time: [3]
- Acceleration versus time graph,
  - Speed versus time graph,
  - Velocity versus time graph.
25. Show that Newton's third law of motion is contained in the second law. [3]
26. If, at 50°C and 75 cm of mercury pressure, a definite mass of a gas is compressed [3]
- slowly
  - suddenly, then what will be the final pressure and temperature of the gas in each case if the final volume is one-fourth of the initial volume? ( $\gamma = 1.5$ )
27. How is centripetal force provided in case of the following? [3]
- Motion of planet around the sun,
  - Motion of moon around the earth.
  - Motion of an electron around the nucleus in an atom.
28. Mercury has an angle of contact equal to  $140^\circ$  with soda-lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is  $0.465 \text{ N m}^{-1}$ . Density of mercury =  $13.6 \times 10^3 \text{ kgm}^{-3}$ .

OR

What is venturi-meter? On which principle does it work? How is the principle of venturi-meter applied in automobiles?

#### Section D

29. **Read the text carefully and answer the questions:** [4]
- The kinetic energy of an object is the energy associated with the object which is under motion. It is defined as “the energy required by a body to accelerate from rest to stated velocity.” It is a vector quantity and the momentum of an object is the virtue of its mass. It is defined as the product of mass and velocity. It is a vector quantity. The relation between them is given by  $E = \frac{p^2}{2m}$ . In case of the elastic collision both of these quantities remain constant.







# Solution

## Section A

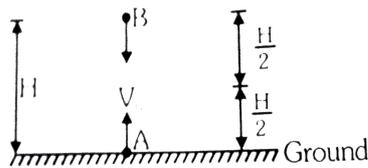
- (c)  $[\text{ML}^{-1}\text{T}^{-1}]$   
**Explanation:**  $[\eta] = \left[ \frac{F\Delta x}{A\Delta v} \right]$   
 $= \frac{[\text{MLT}^{-2}][\text{L}]}{[\text{L}^2][\text{LT}^{-1}]}$   
 $= [\text{ML}^{-1}\text{T}^{-1}]$
- (b) 94.9 cm  
**Explanation:**  $\nu = \frac{v}{4L} = \frac{300}{4 \times 1} = 75 \text{ Hz}$   
For shorter pipe,  
 $\nu + 4 = \frac{300}{4L'}$   
or  $75 + 4 = \frac{300}{4L'}$   
 $L' = \frac{300}{4 \times 79} \text{ m} = 94.9 \text{ cm}$
- (b) His moment of inertia decreases  
**Explanation:** When gymnast lowers his hand the distance of the mass from rotational axis decrease. Hence his moment of inertia decreases and angular velocity increases to conserve angular momentum.
- (a) is zero  
**Explanation:** At the critical temperature, the surface tension of a liquid becomes zero.
- (c) 10840 m/sec(perigee), 8760 m/sec(apogee)  
**Explanation:** To escape Earth, we need total energy of zero.  
( $E_{\text{final}} = 0$  because  $U \rightarrow 0$  as  $R \rightarrow \infty$  and  $K \rightarrow 0$  as  $v = 0$  at  $R \rightarrow \infty$ )  
So,  
 $K_p + U_p = 0$   
Looking for the new velocity at perigee;  
 $\frac{1}{2}mv_{p,\text{escape}}^2 = \frac{GMm}{R_p}$   
 $v_{p,\text{escape}} = \sqrt{\frac{2GM}{R_p}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.78 \times 10^6}}$   
 $= 1.084 \times 10^4 \text{ m/sec} = 10840 \text{ m/sec}$   
The similar calculation at apogee gives  
 $v_{a,\text{escape}} = \sqrt{\frac{2GM}{R_a}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{10.38 \times 10^6}}$   
 $= 8.76 \times 10^3 \text{ m/sec} = 8760 \text{ m/sec}$
- (a) 3 m/s  
**Explanation:** When the source is moving forward towards the observer, the apparent frequency is  $f_1 = \frac{v}{v-v_s} \times f$   
When source moves backwards  $f_2 = \frac{v}{v+v_s} \times f$   
 $f_2 - f_1 = f v \left[ \frac{1}{v+v_s} - \frac{1}{v-v_s} \right] = f v \left[ \frac{-2v_s}{v^2 - v_s^2} \right]$   
As  $v_s \ll v$ , so  
 $\frac{f_2 - f_1}{f} = \left| \frac{2v_s}{v} \right| = \frac{2}{100}$   
 $v_s = \frac{v}{100} = \frac{300}{100} = 3 \text{ m/s}$

7.

(c)  $\sqrt{gH}$

**Explanation:**

Let the two bodies A and B respectively meet at a time, at a height  $\frac{H}{2}$  ground.



Using  $S = ut + \frac{1}{2}at^2$

For a body A,  $u = V_0$ ,  $a = -g$ ,  $S = \frac{H}{2}$

$\therefore \frac{H}{2} = v_0t - \frac{1}{2}gt^2$  ... (i)

For a body B,  $u = 0$ ,  $a = +g$ ,  $S = \frac{H}{2}$

$\therefore \frac{H}{2} = \frac{1}{2}gt^2$  ... (ii)

Equating equation (i) and (ii) we get

$V_0t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

$V_0t = gt^2$  or  $t = \frac{V_0}{g}$

Substituting the value of t in equation (i), we get

$\frac{H}{2} = V_0 \times \left(\frac{V_0}{g}\right) - \frac{1}{2}g\left(\frac{V_0}{g}\right)^2 = \frac{V_0^2}{g} - \frac{1}{2}\frac{V_0^2}{g}$

$\frac{H}{2} = \frac{1}{2}\frac{V_0^2}{g}$  or  $v_0^2 = gH$

$V_0 = \sqrt{gH}$

8. (a) 1.07 rad

**Explanation:**  $y_1 = 10^{-6} \sin [100t + (\frac{x}{50}) + 0.5]$  m

$y_2 = 10^{-6} \cos [100t + (\frac{x}{50})]$  m

$= 10^{-6} \sin [100t + (\frac{x}{50}) + (\frac{\pi}{2})]$

$\Delta\phi = \frac{\pi}{2} - 0.05 = \frac{3.14}{2} - 0.5 = 1.57 - 0.5$

$= 1.07$  rad

9.

(b)  $\sqrt{2gh}$

**Explanation:** Velocity of efflux,  $v = \sqrt{2gh}$

10.

(d) the laws of gravitation predict a trajectory spiralling inwards.

**Explanation:** Due to the viscous atmosphere, friction force due to the atmosphere acts on the satellite which reduces its orbital speed and hence the energy of revolution around a planet. Due to the decrease in the energy of the satellite, its height gradually decreases.

11. (a)  $\frac{Ml^2\omega}{3t}$

**Explanation:**

As Torque ( $\tau$ ) is equal to the product of Moment of Inertia (I) and Angular acceleration ( $\alpha$ )

$\tau = I\alpha$

$\tau = I \frac{\Delta\omega}{\Delta t}$

$\tau = \left[ \frac{M(2l)^2}{12} \right] \left[ \frac{\omega}{t} \right]$

$\tau = \frac{Ml^2\omega}{3t}$

12.

(c) 1800

**Explanation:** Power radiated,  $P = \sigma AT^4$

$$\therefore P' = \sigma \left( \frac{A}{4} \right) (2T)^4 \quad [A \propto r^2]$$

$$= 4P = 4 \times 450 \text{ W} = 1800 \text{ W}$$

13. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** The Work-Energy theorem is applicable for non-inertial frames. The assertion is true.

If we include the pseudo forces in the calculation of the net force acting on the body under consideration then this theorem is applicable for the non-inertial frame also.

The assertion and reason both are true and the reason also explains the assertion.

14.

(c) Assertion is correct statement but reason is wrong statement.

**Explanation:** Assertion is correct statement but reason is wrong statement.

15. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** As no torque is acting on the planet, its angular momentum must stay constant in magnitude as well as direction. Therefore, plane of rotation must pass through the centre of earth.

16. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

### Section B

17. Let h be the depth of the well. Then time  $t_1$  taken by the stone to fall into well under gravity is given by

$$h = 0 + \frac{1}{2}gt_1^2 \text{ or } t_1 = \sqrt{\frac{2h}{g}}$$

Time taken for the splash to travel height h is given by  $t_2 = \frac{h}{v}$

where v = velocity of sound

But  $t_1 + t_2 = 1.45 \text{ s}$

$$\therefore \sqrt{\frac{2h}{g}} + \frac{h}{v} = 1.45$$

$$\text{or } \sqrt{\frac{2h}{9.8}} + \frac{h}{332} = 1.45$$

On solving, h = 9.9 m.

18. The quantities which have dimensions but do not possess a constant value are called dimensional variables e.g., velocity, force etc.

On the other hand, the quantities which have neither dimensions nor they have a constant value are called non-dimensional variables e.g., relative density, strain, etc.

19. [Density] =  $M^1 L^{-3}$

$$n_2 = n_1 \left[ \frac{M_1}{M_2} \right]^1 \left[ \frac{L_1}{L_2} \right]^{-3}$$

$$= 13.6 \left[ \frac{1 \text{ g}}{1 \text{ kg}} \right]^1 \left[ \frac{1 \text{ cm}}{1 \text{ m}} \right]^{-3}$$

$$= 13.6 \left[ \frac{1 \text{ g}}{1000 \text{ g}} \right] \left[ \frac{1 \text{ cm}}{100 \text{ cm}} \right]^{-3} = 13.6 \times 10^3$$

$$\therefore 13.6 \text{ g cm}^{-3} = 13.6 \times 10^3 \text{ kgm}^{-3}$$

20. Yes. When the direction of motion of a body reverses, the direction of friction is also reversed. Work has to be done against friction both during the forward and return journey i.e., work done against friction along a closed path is not zero. So friction is a non-conservative force.

21. If r is the distance of the given point from the centre of the earth, the gravitational potential at the point,

$$v = -\frac{GM}{r} = -5.12 \times 10^7 \text{ J/kg}$$

Acceleration due to gravity at this point,

$$g = \frac{GM}{r^2} = 6.4 \text{ m/s}^2$$

$$\text{Clearly, } \frac{|V|}{g} = \frac{GM/r}{GM/r^2} = r$$

$$\text{thus, } r = \frac{5.12 \times 10^7 \text{ J/kg}}{6.4 \text{ m/s}^2} = 8 \times 10^6 \text{ m} = 8000 \text{ km}$$

Obviously, height of the point from the earth's surface =  $(r - R) = 8000 \text{ km} - 6400 \text{ km} = 1600 \text{ km}$

OR

The food packet will not fall on the earth. As the satellite, as well as an astronaut, were in a state of weightlessness hence, the food packet when dropped by mistake, will also start moving with the same velocity as that of satellite and will continue to move along with the satellite in the same orbit.

Section C

22. Velocity of efflux through the hole,  $v = \sqrt{2gh}$

$\therefore$  Distance moved by water in one second  $v = \sqrt{2gh}$

$\therefore$  Rate of the momentum =  $(\rho A \sqrt{2gh})(\sqrt{2gh}) = 2 ghA\rho$

According to Newton's second law of motion,

Force due to the velocity of efflux =  $2 gh A\rho$

Now, according to Newton's third law of motion,

Force on the vessel = Rate of the momentum Force on the vessel =  $2 gh A\rho$

The vessel will move, if force on the vessel = force of friction

or  $2gh A\rho = \mu Mg$

or  $A = \frac{\mu M}{2h\rho} = \frac{0.4 \times 100}{2 \times 2 \times 1000} = \frac{1}{100}$

Since, the hole is circular,

$A = \pi r^2 = \frac{\pi D^2}{4}$

$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 1}{100 \times 314}} = 0.113 \text{ m}$

So, the diameter of a hole  $D = 0.113 \text{ m}$

23. i. Total heat supplied to sample  $\Delta Q = 300 \text{ J}$  and rise in temperature  $\Delta T = T_2 - T_1 = 45 - 25 = 20^\circ\text{C}$

$\therefore$  Thermal capacity of substance is given as =  $\frac{\Delta Q}{\Delta T} = \frac{300}{20} = 15 \text{ J}^\circ\text{C}^{-1}$

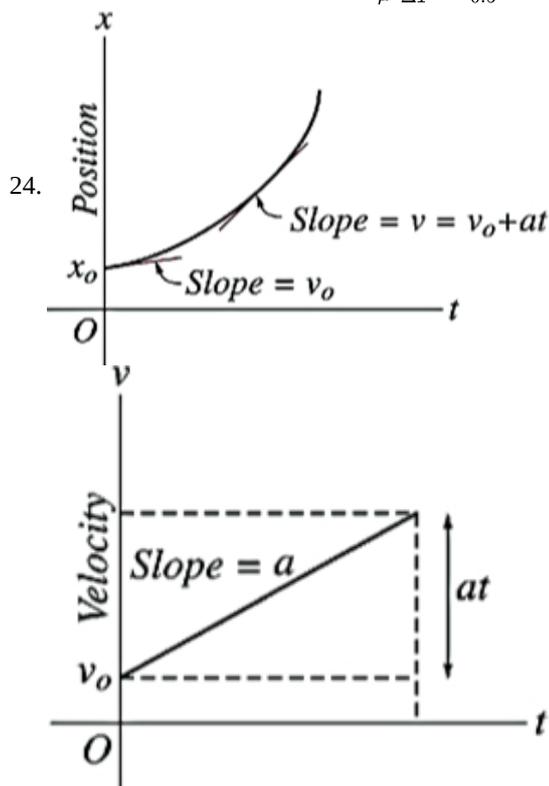
ii. mass,  $m = 25 \text{ g} = 0.025 \text{ kg}$

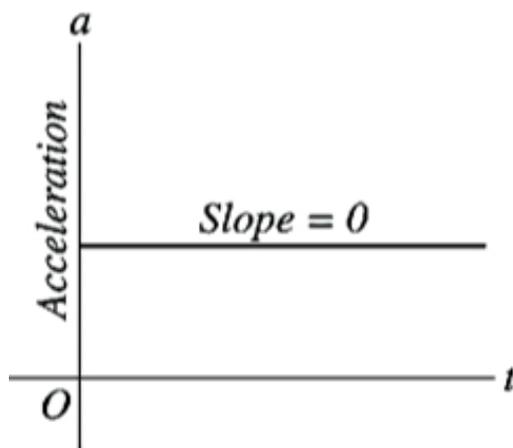
$\therefore$  Specific heat capacity,  $c = \frac{1}{m} \cdot \frac{\Delta Q}{\Delta T} = \frac{1}{0.025} \times 15 = 600 \text{ Jkg}^{-1} \text{ }^\circ\text{C}^{-1}$

iii. Number of moles in 25 g sample of 50 g/mol is

$\mu = \frac{25}{50} = 0.5 \text{ moles}$

$\therefore$  Molar heat capacity  $C = \frac{1}{\mu} \frac{\Delta Q}{\Delta T} = \frac{1}{0.5} \times 15 = 30 \text{ Jmol}^{-1} \text{ }^\circ\text{C}^{-1}$ .





25. Let  $F_{ba}$  be the force (action) exerted by A on B and  $\frac{d\mathbf{p}_B}{dt}$  be the resulting change of the momentum of B.

Let  $F_{AB}$  be the force (reaction) exerted by B on A and  $\frac{d\mathbf{p}_A}{dt}$  be the resulting change of momentum of A.

According to Newton's second law,  $F = \frac{d\mathbf{p}}{dt}$

Then,  $F_{BA} = \frac{d\mathbf{p}_B}{dt}$  and  $F_{AB} = \frac{d\mathbf{p}_A}{dt}$

$$\therefore F_{BA} + F_{AB} = \frac{d\mathbf{p}_B}{dt} + \frac{d\mathbf{p}_A}{dt} = \frac{d}{dt}(\mathbf{p}_B + \mathbf{p}_A) \dots(i)$$

In the absence of any external force, the rate of change of momentum of the whole system zero.

$$\text{i.e. } \frac{d}{dt}(\mathbf{p}_B + \mathbf{p}_A) = 0$$

$$\text{So, } F_{BA} + F_{AB} = 0 \text{ or } F_{BA} = -F_{AB}$$

or Action = - reaction

and it is a Newton's third law of motion.

Hence, proved.

26. Here  $V_2 = \frac{1}{4} V_1$ ,  $P_1 = 75$  cm of Hg,

$$T_1 = 50 + 273 = 323 \text{ K}$$

i. When the gas is compressed slowly, the process is isothermal.

$$\therefore P_1 V_1 = P_2 V_2 \text{ or } 75 \times V_1 = P_2 \times \frac{1}{4} V_1$$

$$\text{or } P_2 = 75 \times 4 = 300 \text{ cm of Hg}$$

As the process is isothermal, so  $T_2 = 50^\circ\text{C}$

ii. When the gas is compressed suddenly, the process is adiabatic.

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\text{or } P_2 = P_1 \left( \frac{V_1}{V_2} \right)^\gamma = 75 \left( \frac{V_1}{\frac{1}{4} V_1} \right)^{1.5}$$

$$= 75 \times 4 \times 4^{\frac{1}{2}} = 75 \times 4 \times 2 = 600 \text{ cm of Hg}$$

$$\text{Also, } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\text{or } T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

$$= 323 \times (4)^{0.5} = 323 \times 2$$

$$= 646 \text{ K} = 373^\circ\text{C}$$

27. i. The earth revolves round the sun. The earth is also acted upon by the centripetal force which is provided by the gravitational force of attraction between the sun and the earth.
- ii. The motion of moon around the earth is also in circular path. The necessary centripetal force is provided by the gravitational attraction of the earth on the moon.
- iii. In an atom, electrons revolve around the nucleus in various circular orbits. The necessary centripetal force for circular motion, is exerted by the electrostatic force of attraction between the positively-charged nucleus and the negatively charged electrons.

28. Angle of contact between mercury and soda lime glass,  $\theta = 140^\circ$

$$\text{Radius of the narrow tube, } r = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$\text{Surface tension of mercury at the given temperature, } s = 0.465 \text{ N m}^{-1}$$

$$\text{Density of mercury, } \rho = 13.6 \times 10^3 \frac{\text{kg}}{\text{m}^3}$$

Dip in the height of mercury = h

Acceleration due to gravity,  $g = 9.8 \frac{m}{s^2}$

Surface tension is related with the angle of contact and the dip in the height as:

$$s = \frac{h\rho gr}{2 \cos \theta}$$
$$\therefore h = \frac{2s \cos \theta}{\rho g}$$
$$= \frac{2 \times 0.465 \times \cos 140}{1 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$
$$= -0.00534 \text{ m}$$
$$= -5.31 \text{ mm}$$

Here, the negative sign shows the decreasing level of mercury. Hence, the mercury level dips by 5.34 mm.

OR

Venturi-meter is a device used to measure the flow speed of a liquid. It is basically based on Bernoulli's principle and works on the principle that when a liquid flows in the tube from wide neck to a narrow constriction, the speed of flow increases and the pressure falls.

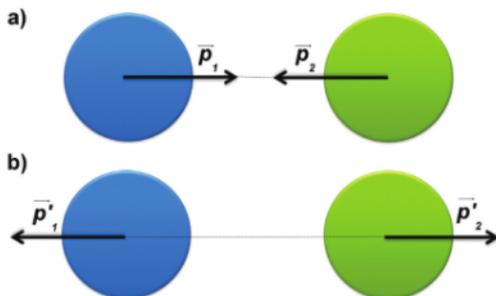
Bernoulli's principle states that with the increase in the velocity of the fluid its pressure decreases (or) there is a decrease in the fluid pressure energy. This decrease in the fluid pressure in the areas where the flow velocity is increased is called the Bernoulli effect.

It is utilised in the carburettor of automobiles. The carburettor has a venturi channel (fine nozzle) through which air flows with a large speed. The pressure is then lowered at the narrow neck as a result, the valve of petrol chambers opens and the petrol is sucked up in the chamber to provide the correct mixture of air and petrol necessary for combustion.

#### Section D

#### 29. Read the text carefully and answer the questions:

The kinetic energy of an object is the energy associated with the object which is under motion. It is defined as "the energy required by a body to accelerate from rest to stated velocity." It is a vector quantity and the momentum of an object is the virtue of its mass. It is defined as the product of mass and velocity. It is a vector quantity. The relation between them is given by  $E = \frac{P^2}{2m}$ . In case of the elastic collision both of these quantities remain constant.



- (i) (b) 4:1

**Explanation:** 4:1

- (ii) (c) 125%

**Explanation:** 125%

- (iii) (d) light object

**Explanation:** light object

OR

- (d) positive

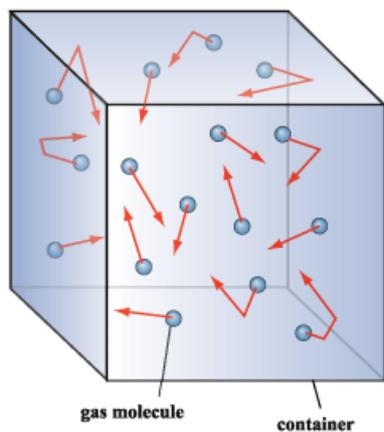
**Explanation:** positive

- (iv) (b) no work is done on it

**Explanation:** no work is done on it

#### 30. Read the text carefully and answer the questions:

Gas molecules move in random motion inside the container. The **pressure exerted** by the gas is due to the continuous collision of the molecules against the walls of the container. Due to this continuous collision, the walls experience a continuous force which is equal to the total momentum imparted to the walls per second.



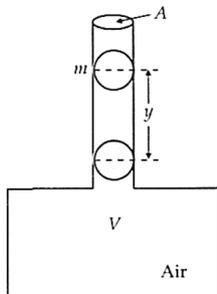
- (i) **(d)** 1:2  
**Explanation:** 1:2
- (ii) **(c)** directly proportional to the density  
**Explanation:** directly proportional to the density
- (iii) **(a)** pressure increase  
**Explanation:** pressure increase
- (iv) **(b)** becomes double  
**Explanation:** becomes double

OR

- (d)**  $M^1L^2T^{-2}K^{-1}$   
**Explanation:**  $M^1L^2T^{-2}K^{-1}$

### Section E

31. Oscillations of a ball in the neck of an air chamber. The figure shows an air chamber of volume  $V$ , having a neck of area of cross-section  $A$  and a ball of mass  $m$  fitting smoothly in the neck. If the ball be pressed down a little and released, it starts oscillating up and down about the equilibrium position.



If the ball be depressed by distance  $y$ , then the decrease in volume of air in the chamber is  $\Delta V = Ay$ .

$$\therefore \text{Volume strain} = \frac{\Delta V}{V} = \frac{Ay}{V}$$

If pressure  $P$  is applied to the ball, then hydrostatic stress =  $P$

$\therefore$  Bulk modulus of elasticity of air,

$$E = -\frac{P}{\Delta V/V} = -\frac{P}{Ay/V} \text{ or } P = -\frac{EA}{V}y$$

$$\text{Restoring force, } F = PA = -\frac{EAy}{V}A = -\frac{EA^2}{V}y$$

Thus  $F$  is proportional to  $y$  and acts in its opposite direction. Hence the ball executes SHM with force constant,

$$k = \frac{EA^2}{V}$$

The period of oscillation of the ball is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{EA^2/V}} = 2\pi\sqrt{\frac{mV}{EA^2}}$$

i. If the P-V variations are isothermal, then  $E = P$ ,

$$\therefore T = 2\pi\sqrt{\frac{mV}{PA^2}}$$

ii. If the P-V variations are adiabatic, then  $E = \gamma P$ ,

$$\therefore T = 2\pi\sqrt{\frac{mV}{\gamma PA^2}}$$

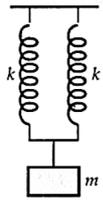
OR

For each spring,

$$F = -ky \dots(i)$$

where  $F$  = restoring force,  $k$  = spring factor and  $y$  = displacement of the spring.

- i. In Figure, let the mass  $m$  produce a displacement  $y$  in each spring and  $F$  be the restoring force in each spring. If  $k_1$  be the spring factor of the combined system, then



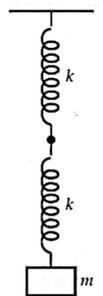
$$2F = -ky$$

$$\text{or } F = -\frac{k_1}{2}y \dots(ii)$$

Comparing (i) and (ii), we get

$$\frac{k_1}{2} = k \text{ or } k_1 = 2k$$

- ii. In Figure, as the length of the spring is doubled, the mass  $m$  will produce double the displacement ( $2y$ ). If  $k_2$  be the spring factor of the combined system, then

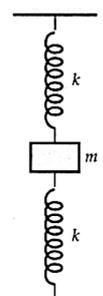


$$F = -k_2(2y) = -2k_2y \dots(iii)$$

Comparing (i) and (iii),

$$2k_2 = k \text{ or } k_2 = \frac{k}{2}$$

- iii. In figure, the mass  $m$  stretches the upper spring and compresses the lower spring, each giving rise to a restoring force  $F$  in the same direction. If  $k_3$  be the spring factor of the combined system, then



$$2F = -k_3 y$$

$$\text{or } F = -\frac{k_3}{2}y \dots(iv)$$

Comparing (i) and (iv),

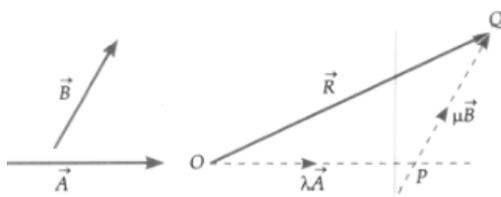
$$\frac{k_3}{2} = k \text{ or } k_3 = 2k$$

32. **Resolution of a vector.** It is the process of splitting a vector into two or more vectors in such a way that their combined effect is same as that of the given vector. The vectors into which the given vector is splitted are called component vectors.

A component of a vector in any direction gives a measure of the effect of the given vector in that direction. The resolution of a vector is just opposite to the process of vector addition.

**Resolution of a vector along two given directions.**

Suppose we wish to resolve a vector  $\vec{R}$  in the direction of two coplanar and non-parallel vectors  $\vec{A}$  and  $\vec{B}$ , as shown in Figure.



Resolving a vector  $\vec{R}$  in the directions of  $\vec{A}$  and  $\vec{B}$ .

Suppose  $\vec{OQ}$  represent vector  $\vec{R}$  in the directions of  $\vec{A}$  and  $\vec{B}$ .

Q draw lines parallel to vectors  $\vec{A}$  and  $\vec{B}$  respectively to meet at point P. From triangle law of vector addition.

$$\vec{OQ} = \vec{OP} + \vec{PQ}$$

As  $\vec{OP} \parallel \vec{A}$  therefore,  $\vec{OP} = \lambda \vec{A}$

As  $\vec{PQ} \parallel \vec{B}$  therefore,  $\vec{PQ} = \mu \vec{B}$

Here  $\lambda$  and  $\mu$  are scalar. Hence

$$\vec{R} = \lambda \vec{A} + \mu \vec{B} \dots (i)$$

Thus the vector  $\vec{R}$  has been resolved in the direction of  $\vec{A}$  and  $\vec{B}$ . Here  $\lambda \vec{A}$  is the component of  $\vec{R}$  in the direction  $\vec{A}$  and  $\mu \vec{B}$  is the component in the direction of  $\vec{B}$ .

**Uniqueness of resolution.** Let us assume that  $\vec{R}$  can be resolved in the directions of  $\vec{A}$  and  $\vec{B}$  in another way.

Then  $\vec{R} = \lambda' \vec{A} + \mu' \vec{B} \dots (ii)$

From equation (i) and (ii), we have

$$\lambda \vec{A} + \mu \vec{B} = \lambda' \vec{A} + \mu' \vec{B}$$

$$\text{or } (\lambda - \lambda') \vec{A} = (\mu' - \mu) \vec{B}$$

But  $\vec{A}$  and  $\vec{B}$  are non-zero vectors acting along different directions. The above equation is possible only if

$$\lambda - \lambda' = 0 \text{ and } \mu' - \mu = 0$$

$$\text{or } \lambda' = \lambda \text{ and } \mu' = \mu$$

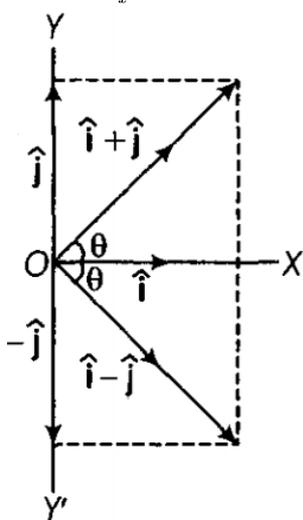
Hence there is one and only one way in which a vector  $\vec{R}$  can be resolved in the directions of vectors  $\vec{A}$  and  $\vec{B}$ .

OR

i. As we know  $\hat{i}$  and  $\hat{j}$  are unit vectors, Magnitude of  $(\hat{i} + \hat{j}) = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$  units

If vector  $(\hat{i} + \hat{j})$  makes an angle of  $\theta$  with the x - axis, then

$$\tan \theta = \frac{A_y}{A_x} = \frac{1}{1} = 1 = \tan 45^\circ \text{ or } \theta = 45^\circ$$



ii. Similarly, magnitude of

$$(\hat{i} - \hat{j}) = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

If vector  $(\hat{i} - \hat{j})$  makes an angle  $\theta$ , with x - axis, then

$$\tan \theta = \frac{A_y}{A_x} = \frac{(-1)}{1} = -1$$

$$= -\tan 45^\circ \Rightarrow \theta = -45^\circ \text{ with } \hat{i}$$

Hence, resultant vector  $(\hat{i} - \hat{j})$  makes an angle of  $45^\circ$  from x-axis in negative direction.

iii. To determine the component of  $A = 2\hat{i} + 3\hat{j}$  in the direction of  $(\hat{i} + \hat{j})$

Let us assume  $B = (\hat{i} + \hat{j})$ , then

$$A \cdot B = AB \cos\theta = (A \cos\theta) \cdot B$$

$$\text{or } A \cos\theta = \frac{A \cdot B}{B}$$

$$\Rightarrow A \cos\theta = \frac{A \cdot B}{B} = \frac{(2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{(1)^2 + (1)^2}}$$

$$= \frac{2\hat{i} \cdot \hat{i} + 3\hat{j} \cdot \hat{j}}{\sqrt{2}}$$

$$= \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}. \text{ This is the component of vector A in the direction of } (\hat{i} + \hat{j})$$

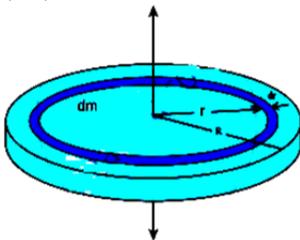
iv. Unit vector along  $(\hat{i} + \hat{j})$ ,  $\hat{n} = \frac{(\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|} = \frac{(\hat{i} + \hat{j})}{\sqrt{2}}$

Component of A along  $(\hat{i} - \hat{j})$

The magnitude of the component of A in the direction of

$$(\hat{i} - \hat{j}) = \frac{(2\hat{i} + 3\hat{j}) \cdot (\hat{i} - \hat{j})}{|\hat{i} - \hat{j}|} = \frac{2\hat{i} \cdot \hat{i} - 3\hat{j} \cdot \hat{j}}{\sqrt{(1)^2 + (-1)^2}} = \frac{2-3}{\sqrt{2}} = \frac{-1}{\sqrt{2}}. \text{ This is the component of vector A in the direction of } (\hat{i} - \hat{j}).$$

33. Consider a disc of mass M and radius R. This disc is made up of many infinitesimally small rings, as shown in the figure. Consider one such ring of mass (dm) and thickness (dr) and radius (r). The moment of inertia (dI) of this small ring is,  $dI = (dm)r^2$



$$I = \int dI$$

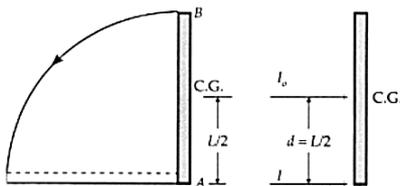
$$I = \int_0^R \frac{2M}{R^2} r^3 dr = \frac{2M}{R^2} \int_0^R r^3 dr$$

$$I = \frac{2M}{R^2} \left[ \frac{r^4}{4} \right]_0^R = \frac{2M}{R^2} \left[ \frac{R^4}{4} - 0 \right]$$

$$I = \frac{1}{2} MR^2$$

OR

Let M be the mass and L be the length of the metre scale. When the upper end of the rod strikes the floor, its centre of gravity falls through height  $\frac{L}{2}$ .



M.I. of the scale about the lower end A,  $I =$  M.I. of the scale about the parallel axis through CG +  $Md^2$

$$= I_0 + Md^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3} \quad \left[ \because d = \frac{L}{2} \right]$$

$$\text{Also, } \omega = \frac{v}{r} = \frac{v}{L}$$

Gain in rotational K.E.

$$= \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{ML^2}{3} \cdot \frac{v^2}{L^2} = \frac{Mv^2}{6}$$

Now, Gain in rotational K.E. = Loss in P.E.

$$\frac{Mv^2}{6} = Mg \cdot \frac{L}{2} \quad \text{or} \quad v^2 = 3gl$$

$$\text{or } v = \sqrt{3gL} = \sqrt{3 \times 9.8 \times 1} = 5.4 \text{ ms}^{-1}$$