Lecture - 14

Filters :-

W.r.t to components present in the N/W filters are classified as

(I) **Active** Filter (II) Passive Filter

→ Active filter consist of op-amp and capacitor

→ **In** the active filter it is possible to inc. the **gain** of the system

→ Generally inductor is not preferred in the active filter since its size is bulky and cost is high

→ **Passive** filter consist of LC series and parallel **sections** (Reactive N/W)

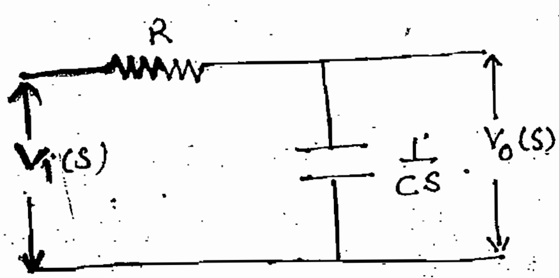
→ **Based** on the operating frequency filters are **classified** as .

(I) **LPF** (II) HPF (III) BPF (IV) BEF (Band stop)

(V) **All pass**

LPF:-

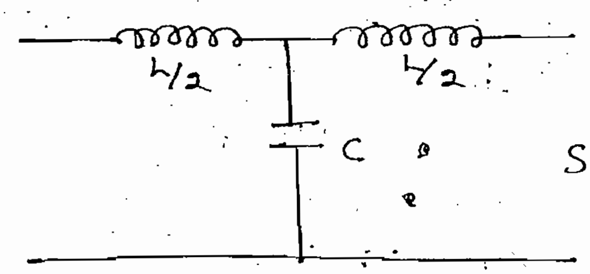
First order



$$V_o(S) = V_i(S)\frac{\frac{1}{CS}}{R+\frac{1}{CS}}$$

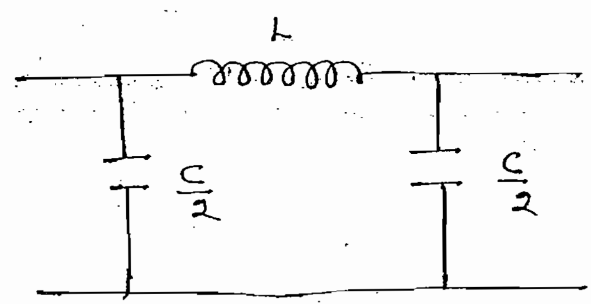
$$\Rightarrow \boxed{\frac{V_o(S)}{V_i(S)} = \frac{1}{1+RCS}}$$

Second order

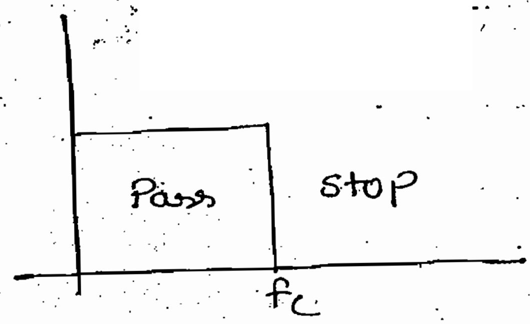


Sym-T

$X_L = 2\pi f L$, $X_C = \frac{1}{2\pi f C}$



Sym-π



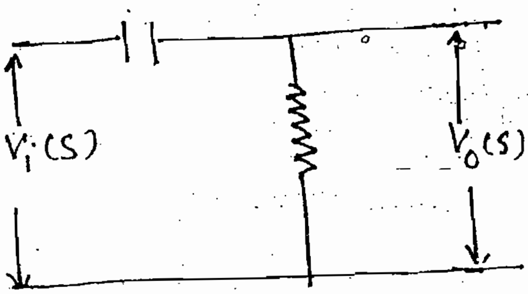
→ Inductor provide low resistance path and capacitor bypass higher frequency

L →

HPF :-

→ For lower freq. capacitor provides high impedance

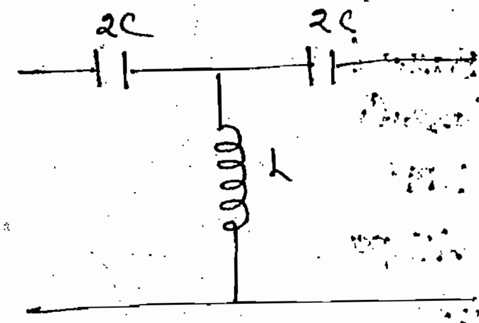
First Order

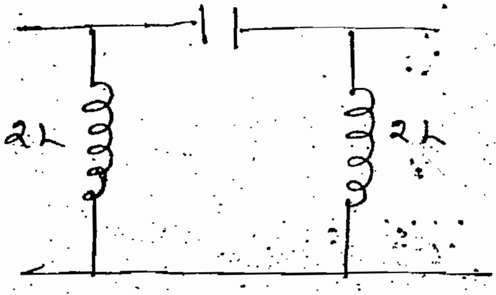


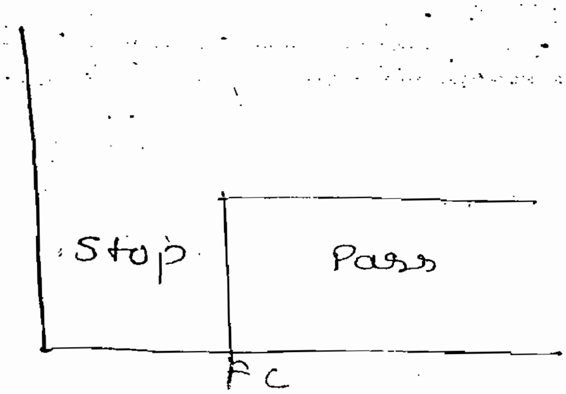
$$V_o(S) = V_i(S)\frac{R}{R+\frac{1}{CS}}$$

$$\frac{V_o(S)}{V_i(S)} = \frac{RCS}{RCS+1}$$

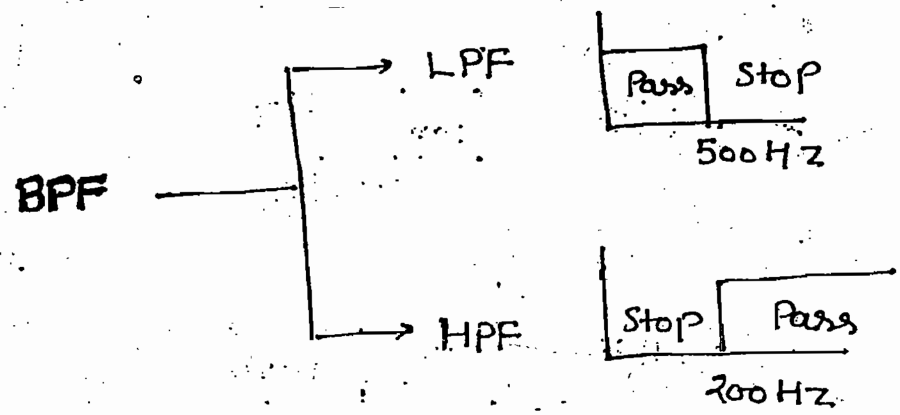
Second Order

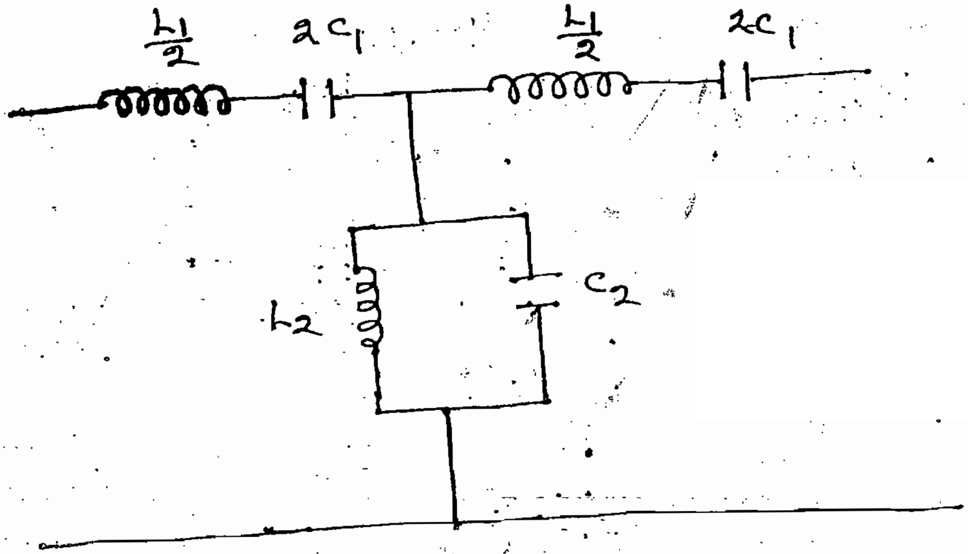


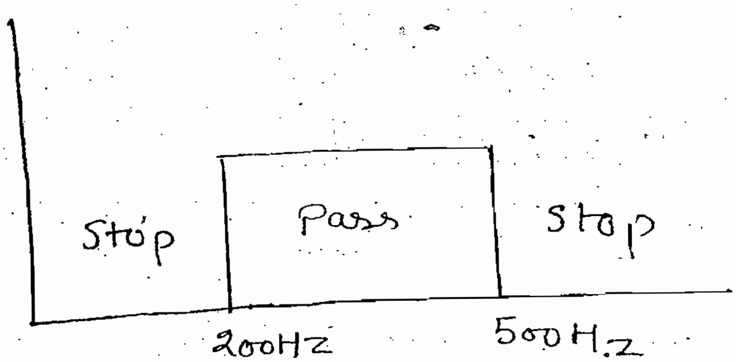




BPF:-



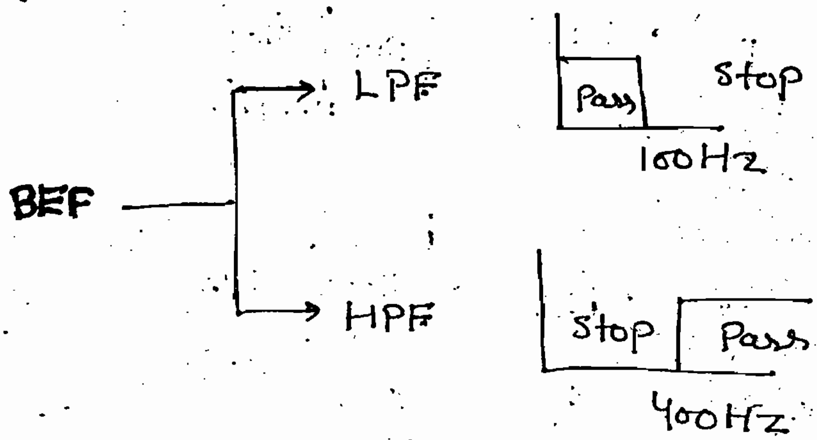


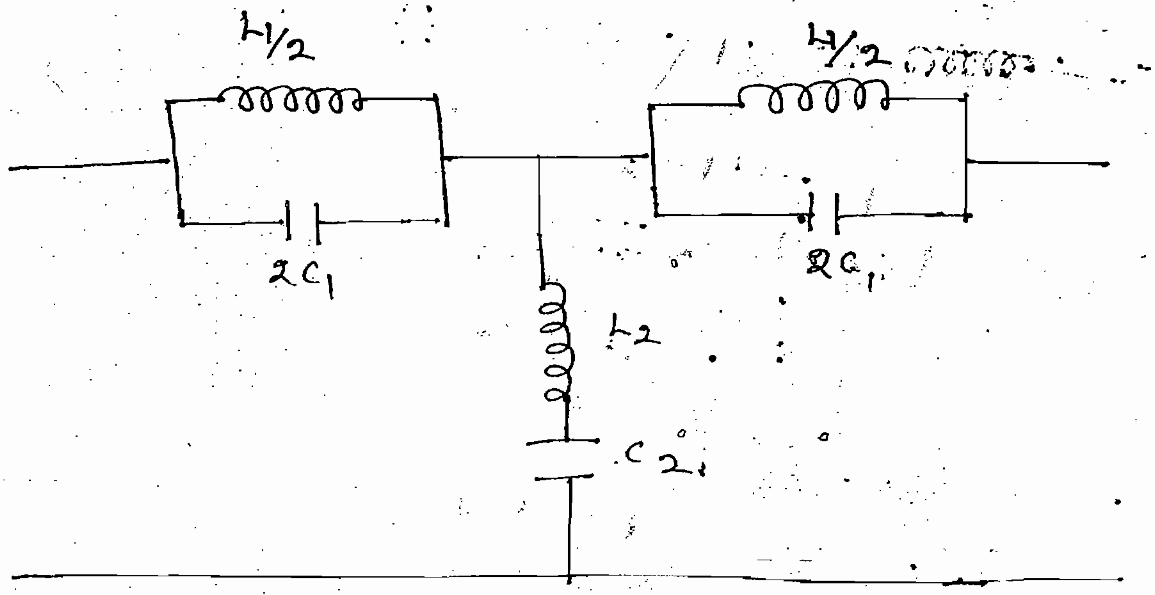


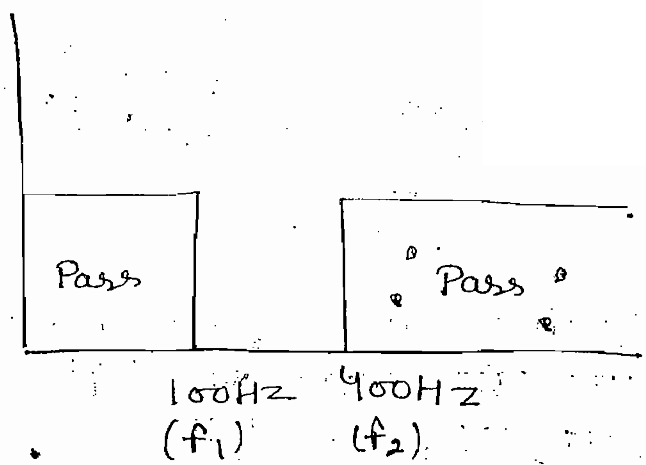
→ BPF is obtained with a combination of LPF & HPF and cut-off freq of LPF should be greate than cut-off freq of HPF

→ In the BPF first cut-off frequency is calculate w.r.t series resonance and second cut-off freq is calculated w.r.t parallel resonance

BEF :-







⇒ BEF is obtained with a combination of LPF and HPF and cut-off frequency of high pass filter should be greater than cut-off freq of low pass filter

Transfer Function of first order filter:-

$\frac{1}{1+Ts} \rightarrow$ LPF

$\frac{Ts}{1+Ts} \rightarrow$ HPF

$\frac{1-Ts}{1+Ts} \rightarrow$ All pass

Note:- In all pass filter zero's are present in the right half of plane and poles are present in left half of plane

Transfer Function of second order filter:-

$\frac{P}{s^2+as+b} \rightarrow$ LPF

$\frac{Ps^2}{s^2+as+b} \rightarrow$ HPF

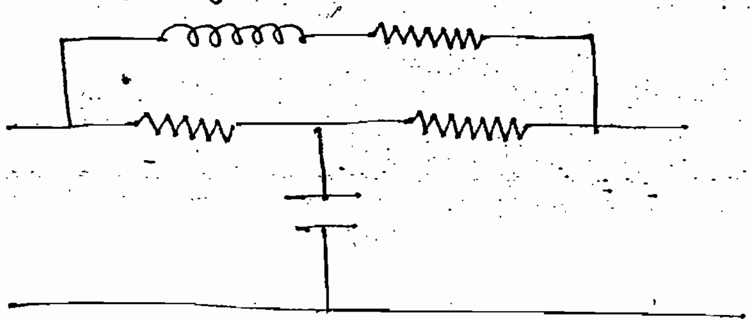
$\frac{Ps}{s^2+as+b} \rightarrow$ BPF

$\frac{Ps^2+q}{s^2+as+b} \rightarrow$ BEF

$\frac{s^2-Ps+q}{s^2+as+b} \rightarrow$ All Pass

Note:-
Poles and zeroes are symmetrical about jω-ax (All pass)

Ques:- Identify type of the filter of the ckt shown



soln:-

Hp 11d

Soln:-

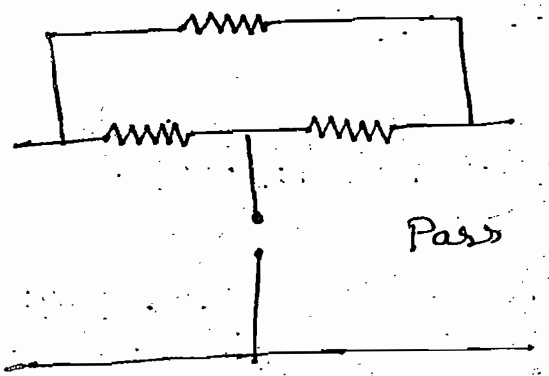
$f = 0$

$X_L = 2\pi f L = 0$

$L \rightarrow SC$

$X_C = \frac{1}{2\pi f c} = \infty$

$C \rightarrow OC$



Pass

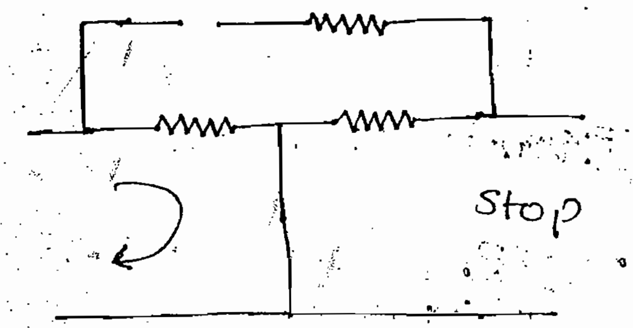
$f = \infty$

$X_L = 2\pi f L = \infty$

$L \rightarrow O \cdot C$

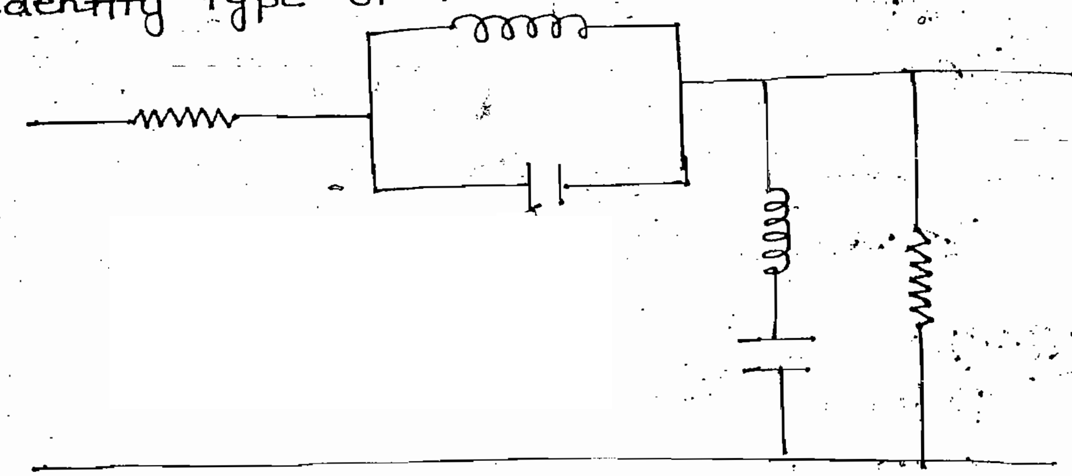
$X_C = \frac{1}{2\pi f C} = 0$

$C \rightarrow S \cdot C$



Stop

Ques:- Identify type of the filter of ckt shown



Soln:-

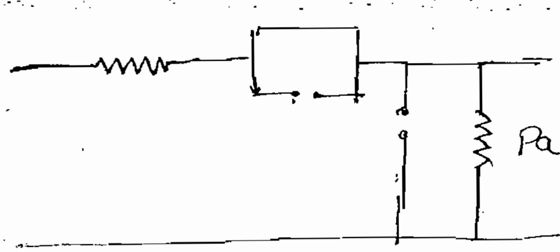
$f = 0$

$X_L = 2\pi f L = 0$

$L \rightarrow S \cdot C$

$X_C = \frac{1}{2\pi f C} = \infty$

$C \rightarrow O \cdot C$



Pass

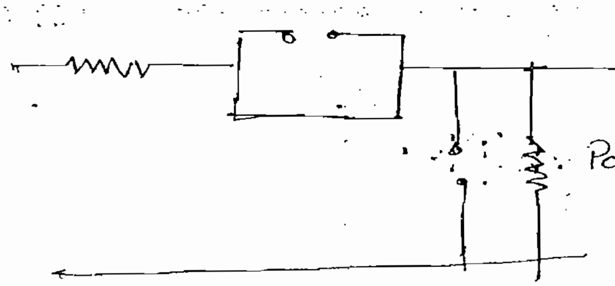
$f = \infty$

$X_L = 2\pi f L = \infty$

$L \rightarrow O \cdot C$

$X_C = \frac{1}{2\pi f C} = 0$

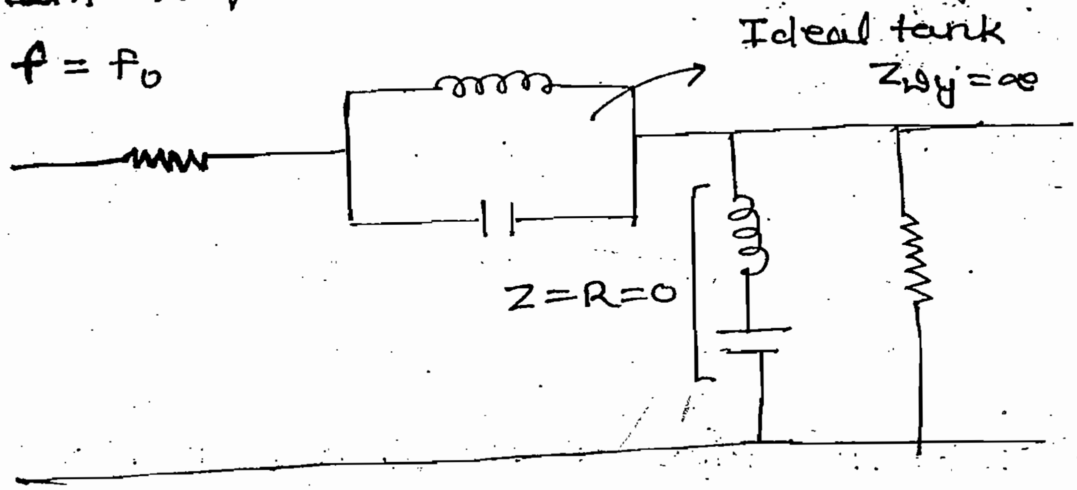
$C \rightarrow S \cdot C$

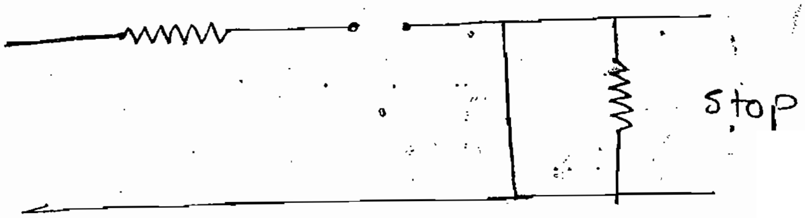


Pass

For BEF & All pass filter draw eq. ckt at resonant freq.

$f = f_0$

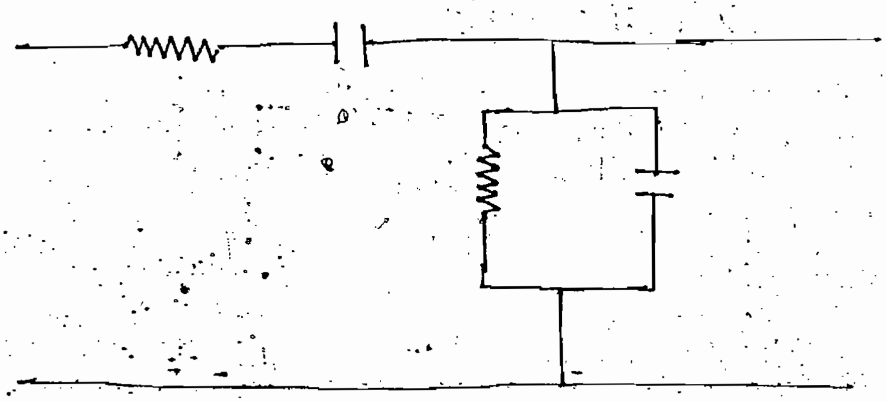




$\longrightarrow$ BEF , Ans

Note:-
When BEF eliminates only few frequency then it is also called as Notch filter

Ques:- Identify type of the filter of the ckt shown



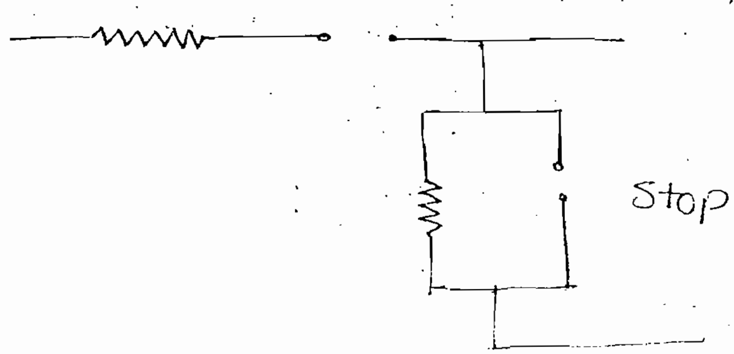
Soln:- $f = 0$

$X_L = 2\pi f L = 0$

$L \rightarrow S.C$

$X_C = \frac{1}{2\pi f C} = 0$

$C \rightarrow O.C$



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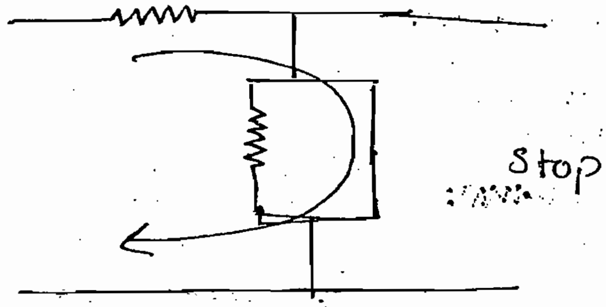
$f = \infty$

$X_L = 2\pi f L = \infty$

$L \rightarrow O.C$

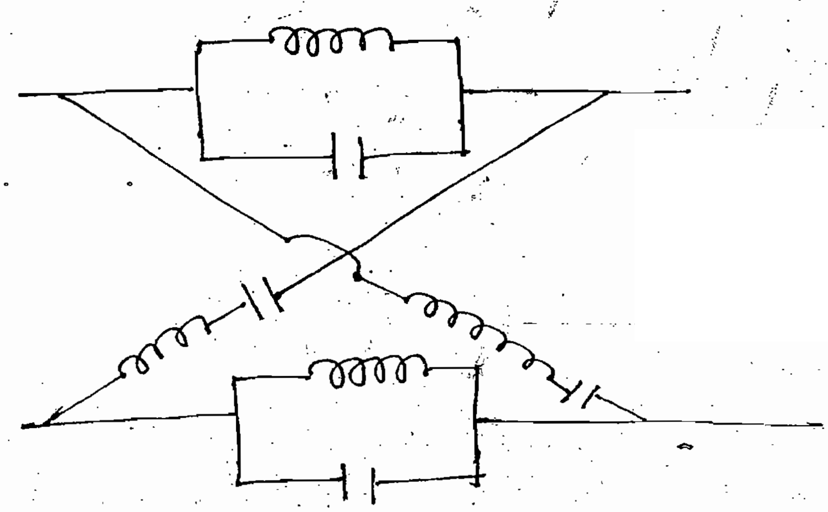
$X_C = \dfrac{1}{2\pi f C} = 0$

$C \rightarrow S.C$



$\longrightarrow$ BPF

Ques:- Identify type of the filter of the ckt shown



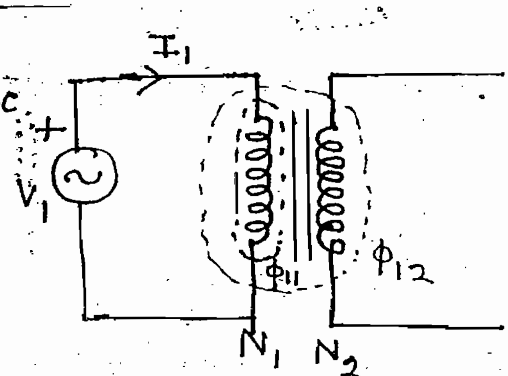
Ans:- All pass ($f = 0, f = \infty, f = f_0$)

Magnetic Coupled Circuits:-

Case-(1):-

$I_1 \longrightarrow \phi_1$; $\phi_1 \rightarrow \phi_{11} \rightarrow$ leakage flux ; $\phi_1 \rightarrow \phi_{12}$

ϕ_{12} $\downarrow$ Useful flux (or) Mutual flux



$$e_1 \propto \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{di_1} \cdot \frac{di}{dt} \quad \left(L = \frac{N\phi}{i} \right)$$

$$\boxed{e_1 = -L_1 \frac{di_1}{dt}}$$

Self induced emf

$$e_2 \propto \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} \quad \left(M_{21} = \frac{N_2 \phi_{12}}{i_1} \right)$$

Mutual inductance of second inductor w.r.t. 1

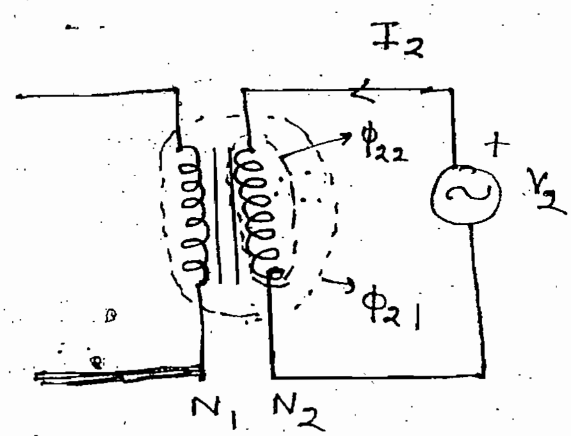
$$\boxed{e_2 = -M_{21} \frac{di_1}{dt}}$$

Mutual Induced emf

Case-(II):-

$I_2 \rightarrow \phi_2$

- $\phi_{22} \rightarrow$ leakage flux
- $\phi_{21} \rightarrow$ Useful flux or Mutual flux



$$e_2 \propto \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

$$e_2 = -N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} \quad \left(L = \frac{N\phi}{i} \right)$$

$$\boxed{e_2 = -L_2 \frac{di_2}{dt}}$$

self induced emf

$$e_1 \propto \frac{d\phi_{21}}{dt}$$

$$e_1 = -N_1 \frac{d\phi_{21}}{dt}$$

$$e_1 = -N_1 \frac{d\phi_{21}}{di_2} \cdot \frac{di_2}{dt}$$

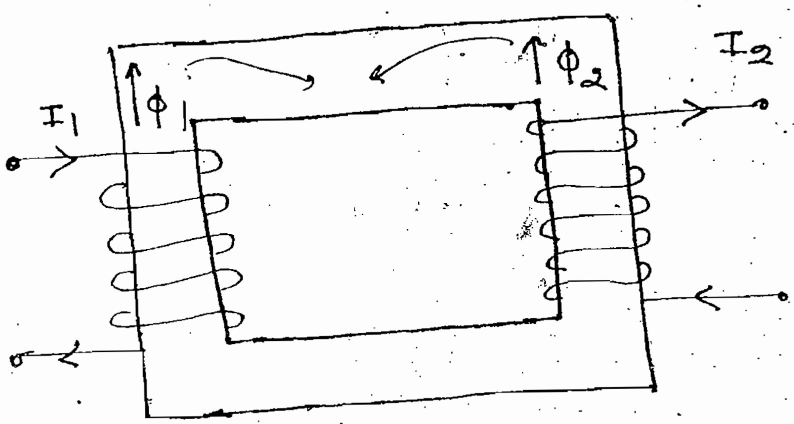
$$\left(M_{12} = \frac{N_1 \phi_{21}}{i_2} \right)$$

$$\boxed{e_1 = -M_{12} \frac{di_2}{dt}}$$

Mutual induced emf

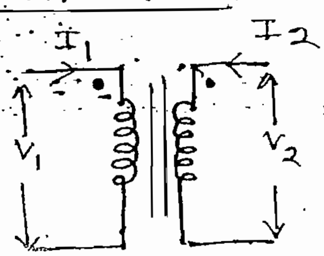
If current is flowing in any one of inductor then the sign of self & mutually induced voltage is same

Note :-

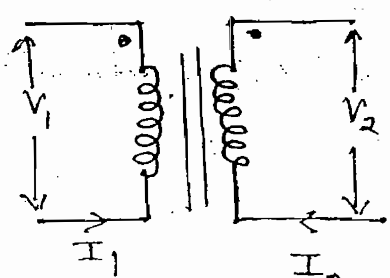


In above figure flux of the two inductors are completely closed path in opposite direction. Hence sign of mutually induced voltage is opposite to sign of self induced voltage

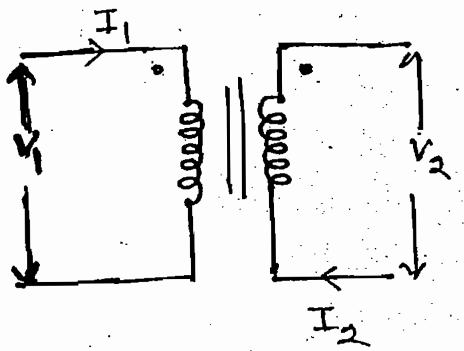
Dot convention :-



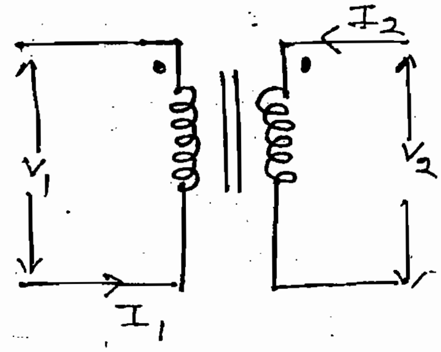
(I)



(II)



(III)



(IV)

$$M_{12} = M_{21} = M$$

$$\left.\begin{aligned} V_1 &= L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ V_2 &= L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{aligned}\right\} \text{Valid for (I) \& (II)}$$

$$\left.\begin{aligned} V_1 &= L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ V_2 &= L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \end{aligned}\right\} \text{Valid for (III) \& (IV)}$$

Note:-

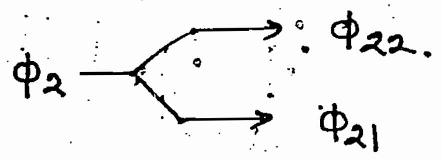
→ When either both currents are entering or both are leaving at dotted terminal sign of mutually induced voltage is same as the sign of self induced voltage.

→ When one current is entering and other current is leaving at dotted terminal sign of the mutually induced voltage is opposite to sign of self induced voltage

Coefficient of Coupling / Coupling Factor:-

$$K = \frac{\text{Useful flux}}{\text{Total flux}} \longrightarrow K_1 = K_2 \longrightarrow \text{conditio}$$

$$\phi_1 \begin{cases} \rightarrow \phi_{11} \\ \rightarrow \phi_{12} \end{cases} \qquad K_1 = \frac{\phi_{12}}{\phi_1}$$



$$k_2 = \frac{\phi_{21}}{\phi_2}$$

$$K = \sqrt{k_1 k_2}$$

For <u>ideal system</u> <u>$k = 1$</u> and for <u>practical system</u> the range of <u>K is 0 to 1</u> .

$$M_{21} = \frac{N_2 \phi_{12}}{i_1}$$

$$M_{12} = \frac{N_1 \phi_{21}}{i_2}$$

$$M_{12} = M_{21} = M$$

$$M^2 = M_{12} M_{21}$$

$$\Rightarrow M^2 = \frac{N_1 \phi_{12}}{i_1} \cdot \frac{N_2 \phi_{12}}{i_1} \quad \text{--- (I)}$$

$$\phi_{12} = k_1 \phi_1 \quad \text{--- (II)}$$

$$\phi_{21} = k_2 \phi_2 \quad \text{--- (III)}$$

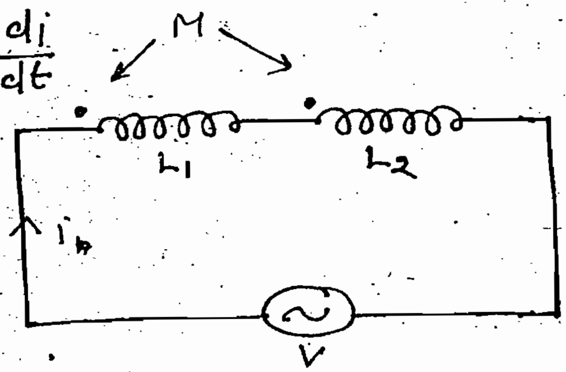
subsitute eq-(II) & (III) in eq-(I)

$$M^2 = k_1 k_2 \frac{N_1 \phi_1}{i_1} \cdot \frac{N_2 \phi_2}{i_2}$$

$$M^2 = K^2 L_1 L_2$$

$$\Rightarrow \boxed{M = K\sqrt{L_1 L_2}}$$

$$V = L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$



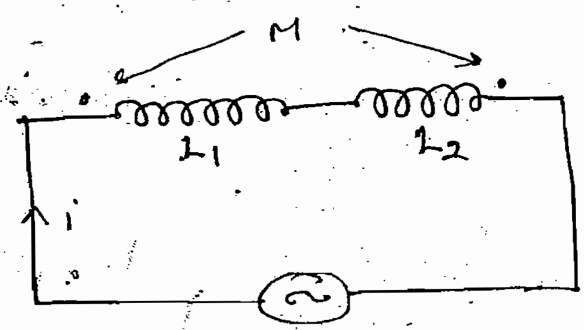
$$L_{eq}\frac{di}{dt} = (L_1+L_2+2M)\frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

→ Series Aiding

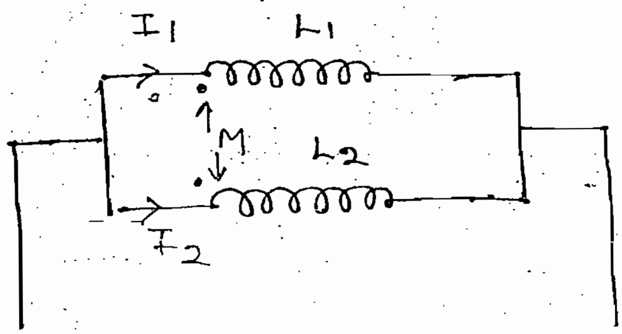
→ <u>series opposing</u>:-

$$\boxed{L_{eq} = L_1 + L_2 - 2M}$$



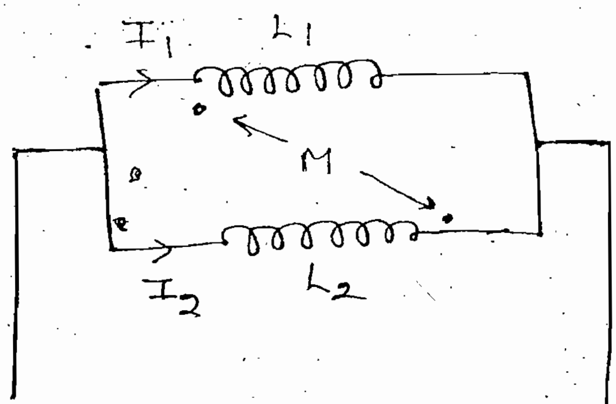
→ <u>Parallel Aiding</u>:-

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1+L_2-2M}}$$

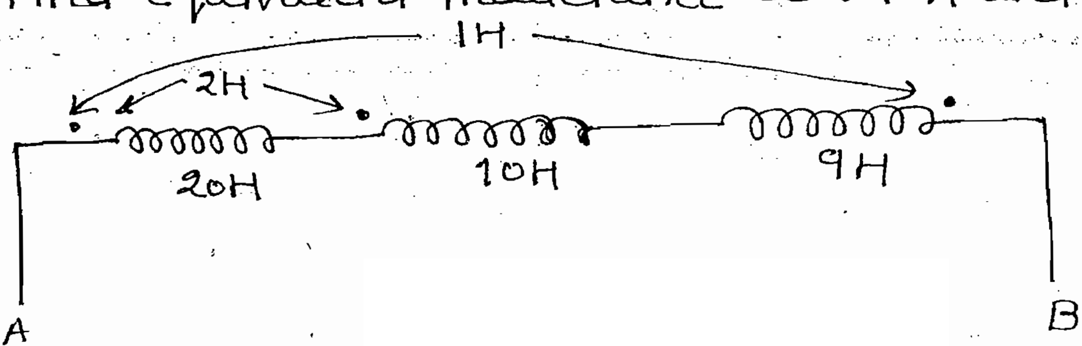


→ <u>Parallel Opposing</u>:-

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1+L_2+2M}}$$



<u>Ques</u>:- Find equivalent inductance w.r.t A and B:



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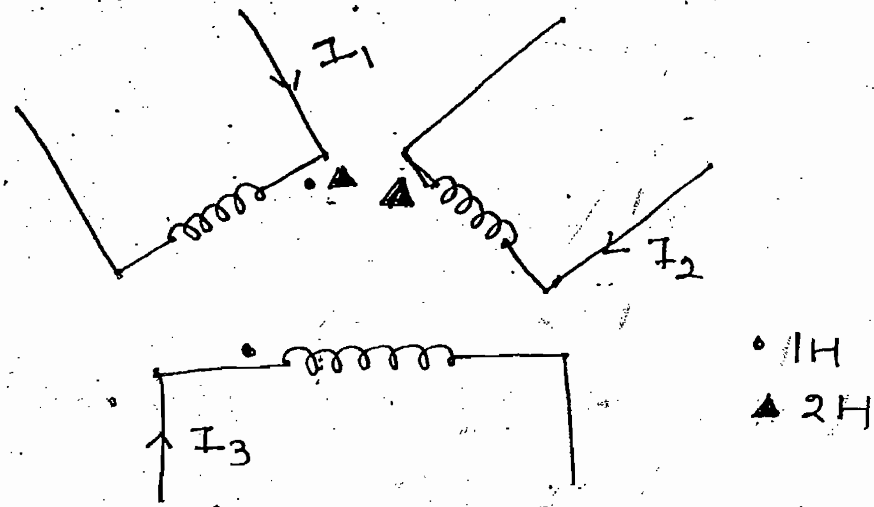
Soln:- $L_{eq} = L_1 + L_2 + L_3 \pm 2M_1 \pm 2M_2 \pm 2M_3$

$\Rightarrow L_{eq} = 20 + 10 + 9 + 2(2) + 0 - 2(1)$

Ques:- Develop inductance matrix of the figure shown

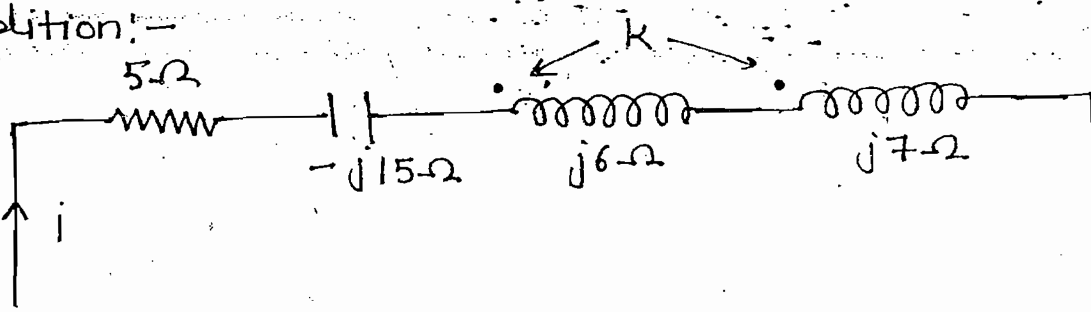


Soln:- Diagonal points denotes self inductance

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

$$L = \begin{bmatrix} 15 & -2 & 1 \\ -2 & 20 & 0 \\ 1 & 0 & 10 \end{bmatrix}$$

Ques:- Find the value of K under resonance condition:-



Soln:-

$2\pi f \, (L_{eq} = L_1 + L_2 + 2M)$

$$M = K\sqrt{L_1 L_2}$$

$$\Rightarrow 2\pi f M = 2\pi f \, K\sqrt{L_1 L_2}$$

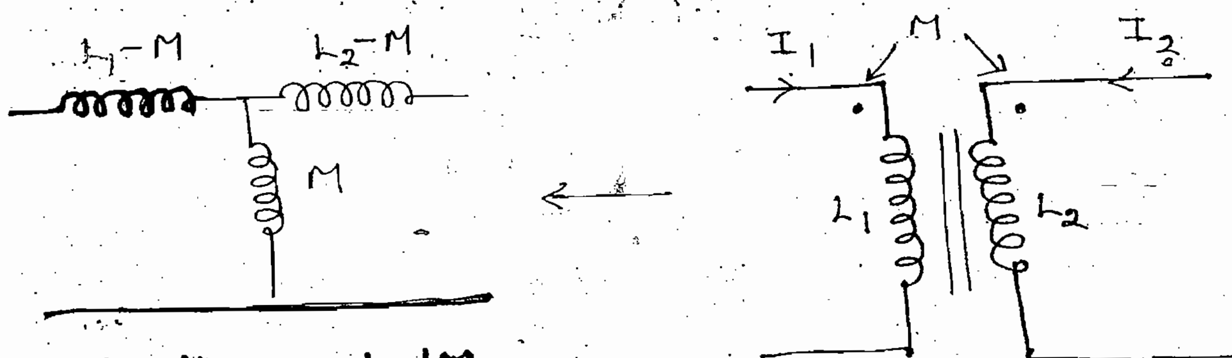
$$X_M = K\sqrt{X_1 X_2}$$

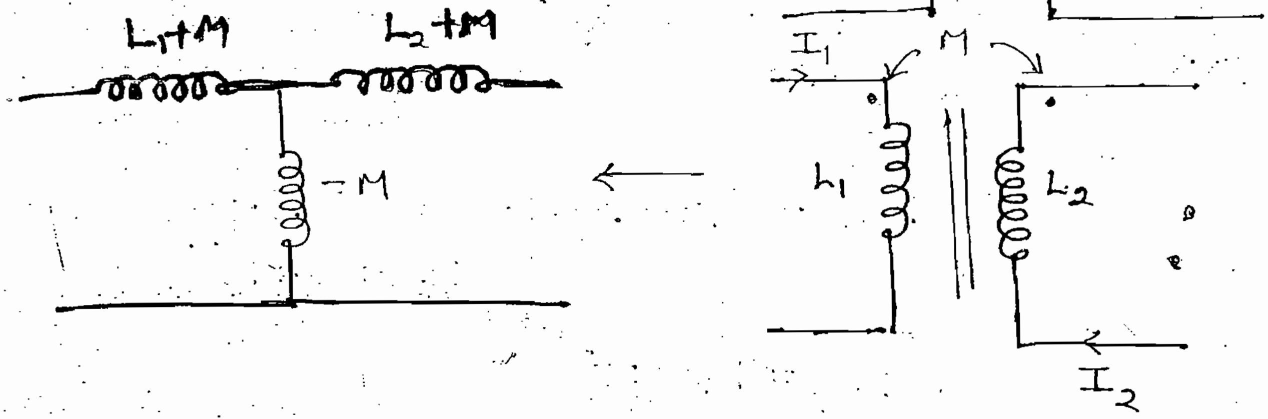
$$X_{eq} = X_1 + X_2 + 2X_M$$

$$\Rightarrow X_{eq} = X_1 + X_2 + 2K\sqrt{X_1 X_2}$$

$$\Rightarrow 15 = 6 + 7 + 2K\sqrt{6 \times 7}$$

$$\Rightarrow \boxed{K = \frac{1}{\sqrt{42}}} \quad \text{Ans}$$



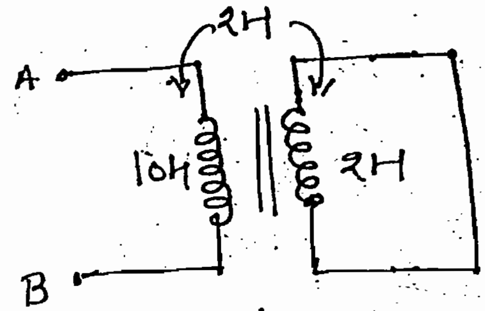


$$\boxed{W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2}$$

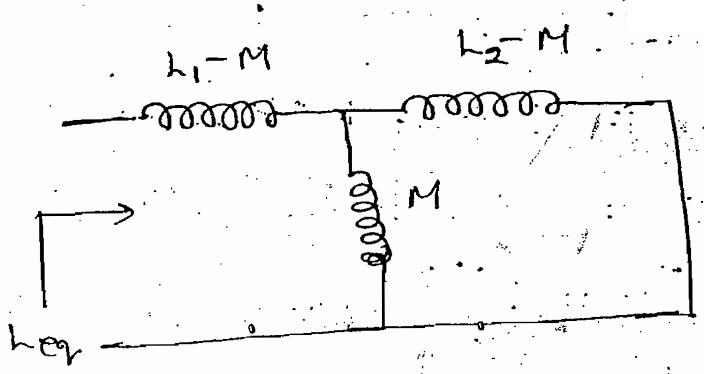
$+ \rightarrow$ fig-(I) $\quad$ $-ve \rightarrow$ fig(II)

Energy stored in the coupled coils.

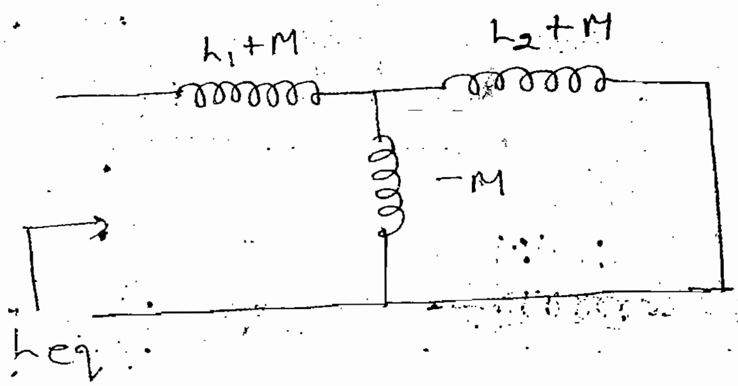
Ques:- Find L_{eq} w.r.t A and B



Soln:-



$$L_{eq} = L_1 - M + \frac{M(L_2 - M)}{L_2 - M + M} = L_1 - \frac{M^2}{L_2}$$



$$L_{eq} = (L_1 + M) + \frac{(-M)(L_2 + M)}{L_2 + M - M}$$

$$\boxed{L_{eq} = L_1 - \frac{M^2}{L_2}}$$

$$\Rightarrow \quad L_{eq} = 10 - \frac{2^2}{2} = 8H, \text{ Ans}$$