Identity

Exercise 71:

Solution 1:

1. $(a+2)(a+3)$
   $= a \times (a+3) + 2(a+3)$
   $= a^2 + 3a + 2a + 6$
   $= a^2 + 5a + 6$

2. $(m-1)(m-3)$
   $= m \times (m-3) - 1(m-3)$
   $= m^2 - 3m - m + 3$
   $= m^2 - 4m + 3$

3. $(b+2)(b-6)$
   $= b \times (b-6) + 2(b-6)$
   $= b^2 - 6b + 2b - 12$
   $= b^2 - 4b - 12$

4. $(a+b)(a+4b)$
   $= a \times (a+4b) + b(a+4b)$
   $= a^2 + 4ab + ab + 4b^2$
   $= a^2 + 5ab + 4b^2$

Exercise 72:

Solution 1:

1. $(p+q)^2$

 Here, first term = p, second term = q

 $\therefore (p+q)^2 = (p)^2 + 2 \times p \times q + (q)^2$

 $= p^2 + 2pq + q^2$

2. $(b+3)^2$

 Here, first term = b, second term = 3

 $\therefore (b+3)^2 = (b)^2 + 2 \times b \times 3 + (3)^2$

 $= b^2 + 6b + 9$

3. $(q+7)^2$

 Here, first term = q, second term = 7

 $\therefore (q+7)^2 = (q)^2 + 2 \times q \times 7 + (7)^2$

 $= q^2 + 14q + 49$

4. $(n+2)^2$

 Here, first term = n, second term = 2

 $\therefore (n+2)^2 = (n)^2 + 2 \times n \times 2 + (2)^2$

 $= n^2 + 4n + 4$

5. $(6+x)^2$

 Here, first term = 6, second term = x

 $\therefore (6+x)^2 = (6)^2 + 2 \times 6 \times x + (x)^2$

 $= 36 + 12x + x^2$

6. $(10+y)^2$

 Here, first term = 10, second term = y

 $\therefore (10+y)^2 = (10)^2 + 2 \times 10 \times y + (y)^2$

 $= 100 + 20y + y^2$

Solution 2:

1. $(x+4)^2 = (x)^2 + 2(x)(4) + (4)^2$
$= x^2 + 8x + 16$

2. $(3+m)^2 = (3)^2 + 2(3)(m) + (m)^2$
$= 9 + 6m + m^2$

3. $(3x+1)^2 = (3x)^2 + 2(3x)(1) + (1)^2$
$= 9x^2 + 6x + 1$

4. $(p+2q)^2 = (p)^2 + 2(p)(2q) + (2q)^2$
$= p^2 + 4pq + 4q^2$

5. $(x+2y)^2 = (x)^2 + 2(x)(2y) + (2y)^2$
$= x^2 + 4xy + 4y^2$

Solution 3:

1. $42^2 = (40+2)^2$
$= 40^2 + 2 \times 40 \times 2 + 2^2$
$= 1600 + 160 + 4$
$= 1764$

2. $105^2 = (100+5)^2$
$= 100^2 + 2 \times 100 \times 5 + 5^2$
$= 10000 + 1000 + 25$
$= 11025$

3. $51^2 = (50+1)^2$
$= 50^2 + 2 \times 50 \times 1 + 1^2$
$= 2500 + 100 + 1$
$= 2601$

4. $102^2 = (100+2)^2$
$= 100^2 + 2 \times 100 \times 2 + 2^2$
$= 10000 + 400 + 4$
$= 10404$

5. $53^2 = (50+3)^2$
$= 50^2 + 2 \times 50 \times 3 + 3^2$
$= 2500 + 300 + 9$
$= 2809$

Exercise 73:

Solution 1:

1. $(x-y)^2 = (x)^2 - 2 \times x \times y + (y)^2$
$= x^2 - 2xy + y^2$

2. $(x-4)^2 = (x)^2 - 2 \times x \times 4 + (4)^2$
$= x^2 - 8x + 16$

3. $(2n-5)^2 = (2n)^2 - 2 \times 2n \times 5 + (5)^2$
$= 4n^2 - 20n + 25$

4. $(7-4m)^2 = (7)^2 - 2 \times 7 \times 4m + (4m)^2$
$= 49 - 56m + 16m^2$

5. $(5y-9)^2 = (5y)^2 - 2 \times 5y \times 9 + (9)^2$
$= 25y^2 - 90y + 81$

6. $(2a-3b)^2 = (2a)^2 - 2 \times 2a \times 3b + (3b)^2$
$= 4a^2 - 12ab + 9b^2$

Solution 2:

1. $(x-3)^2 = (x)^2 - 2(x)(3) + (3)^2$
$= x^2 - 6x + 9$

2. $(m-8)^2 = (m)^2 - 2(m)(8) + (8)^2$
$= m^2 - 16m + 64$

3. $(9-a)^2 = (9)^2 - 2(9)(a) + (a)^2$
$= 81 - 18a + a^2$

4. $(3x-7)^2 = (3x)^2 - 2(3x)(7) + (7)^2$
$= 9x^2 - 42x + 49$

5. $(10-3p)^2 = (10)^2 - 2(10)(3p) + (3p)^2$
$= 100 - 60p + 9p^2$

Solution 3:

1. $48^2 = (50-2)^2$
$= 50^2 - 2 \times 50 \times 2 + 2^2$
$= 2500 - 200 + 4$
$= 2304$

2. $199^2 = (200-1)^2$
$= 200^2 - 2 \times 200 \times 1 + 1^2$
$= 40000 - 400 + 1$
$= 39601$

3. $59^2 = (60-1)^2$
$= 60^2 - 2 \times 60 \times 1 + 1^2$
$= 3600 - 120 + 1$
$= 3481$

4. $78^2 = (80-2)^2$
$= 80^2 - 2 \times 80 \times 2 + 2^2$
$= 6400 - 320 + 4$
$= 6084$

5. $108^2 = (110-2)^2$
$= 110^2 - 2 \times 110 \times 2 + 2^2$
$= 12100 - 440 + 4$
$= 11664$

Exercise 74:

Solution 1:

1. $(x+1)(x-1)$

   First term = x, second term = 1

   $(x+1)(x-1) = (x)^2 - (1)^2$

   $= x^2 - 1$

2. $(p-8)(p+8)$

   First term = p, second term = 8

   $(p-8)(p+8) = (p)^2 - (8)^2$

   $= p^2 - 64$

3. $(6+n)(6-n)$

   First term = 6, second term = n

   $(6+n)(6-n) = (6)^2 - (n)^2$

   $= 36 - n^2$

4. $(3y+5)(3y-5)$

   First term = 3y, second term = 5

   $(3y+5)(3y-5) = (3y)^2 - (5)^2$

   $= 9y^2 - 25$

5. $(9+4p)(9-4p)$

   First term = 9, second term = 4p

   $(9+4p)(9-4p) = (9)^2 - (4p)^2$

   $= 81 - 16p^2$

6. $(2a+3b)(2a-3b)$

   First term = 2a, second term = 3b

   $(2a+3b)(2a-3b) = (2a)^2 - (3b)^2$

   $= 4a^2 - 9b^2$

7. $98 \times 102 = (100-2)(100+2)$

$= (100)^2 - (2)^2$

$= 10000 - 4$

$= 9996$

8. $203 \times 197 = (200+3)(200-3)$

$= (200)^2 - (3)^2$

$= 40000 - 9$

$= 39991$

9. $57 \times 63 = (60-3)(60+3)$

$= (60)^2 - (3)^2$

$= 3600 - 9$

$= 3591$

10. $54 \times 46 = (50+4)(50-4)$

$= (50)^2 - (4)^2$

$= 2500 - 16$

$= 2484$

Exercise 75:

Solution 1(1):

$x + 8 = 2$

Substituting $x = 0$, we have

L.H.S. $= x + 8 = 0 + 8 = 8$

R.H.S. $= 2$

Since, L.H.S. $\neq$ R.H.S.,

$\therefore$ $x + 8 = 2$ is not an identity.

Solution 1(2):

$y - 3 = 7$

Substituting $y = 0$, we have

L.H.S. $= y - 3 = 0 - 3 = -3$

R.H.S. $= 7$

Since, L.H.S. $\neq$ R.H.S.,

$\therefore$ $y - 3 = 7$ is not an identity.

Solution 1(3):

$x(x+2) = x^2 + 2x$

Substituting $x = 0$, we have

L.H.S. $= x(x+2) = 0(0+2) = 0$

R.H.S. $= x^2 + 2x = 0 + 2(0) = 0$

$\therefore$ L.H.S. = R.H.S.

Substituting $x = 1$, we have

L.H.S. $= x(x+2) = 1(1+2) = 1(3) = 3$

R.H.S. $= x^2 + 2x = 1^2 + 2(1) = 1 + 2 = 3$

$\therefore$ L.H.S. = R.H.S.

Substituting $x = 2$, we have

L.H.S. $= x(x+2) = 2(2+2) = 2(4) = 8$

R.H.S. $= x^2 + 2x = 2^2 + 2(2) = 4 + 4 = 8$

$\therefore$ L.H.S. = R.H.S.

Substituting $x = 3$, we have

L.H.S. $= x(x+2) = 3(3+2) = 3(5) = 15$

R.H.S. $= x^2 + 2x = 3^2 + 2(3) = 9 + 6 = 15$

$\therefore$ L.H.S. = R.H.S.

Hence, $x(x+2) = x^2 + 2x$ is an identity.

Solution 1(4):

$p(p-4) = p^2 - 4p$

Substituting $p = 0$, we have

L.H.S. $= p(p-4) = 0(0-4) = 0$

R.H.S. $= p^2 - 4p = 0 - 4(0) = 0$

$\therefore$ L.H.S. = R.H.S.

Substituting $p = 1$, we have

L.H.S. $= p(p-4) = 1(1-4) = -3$

R.H.S. $= p^2 - 4p = 1^2 - 4(1) = 1 - 4 = -3$

$\therefore$ L.H.S. = R.H.S.

Substituting $p = 2$, we have

L.H.S. $= p(p-4) = 2(2-4) = 2(-2) = -4$

R.H.S. $= p^2 - 4p = 2^2 - 4(2) = 4 - 8 = -4$

$\therefore$ L.H.S. = R.H.S.

Substituting $p = 3$, we have

L.H.S. $= p(p-4) = 3(3-4) = 3(-1) = -3$

R.H.S. $= p^2 - 4p = 3^2 - 4(3) = 9 - 12 = -3$

$\therefore$ L.H.S. = R.H.S.

Hence, $p(p-4) = p^2 - 4p$ is an identity.

Solution 1(5):

$3m = 9 - m$

Substituting $m = 2$, we have

L.H.S. $= 3m = 3(2) = 6$

R.H.S. $= 9 - m = 9 - 2 = 7$

Since, L.H.S. $\neq$ R.H.S.,

$\therefore 3m = 9 - m$ is not an identity.

Solution 1(6):

$n + 5 = 2(n + 2) - n + 1$

Substituting $n = 0$, we have

L.H.S. $= n + 5 = 0 + 5 = 5$

R.H.S. $= 2(n + 2) - n + 1 = 2(0 + 2) - 0 + 1 = 2(2) + 1 = 4 + 1 = 5$

$\therefore$ L.H.S. = R.H.S.

Substituting $n = 1$, we have

L.H.S. $= n + 5 = 1 + 5 = 6$

R.H.S. $= 2(n + 2) - n + 1 = 2(1 + 2) - 1 + 1 = 2(3) = 6$

$\therefore$ L.H.S. = R.H.S.

Substituting $n = 2$, we have

L.H.S. $= n + 5 = 2 + 5 = 7$

R.H.S. $= 2(n + 2) - n + 1 = 2(2 + 2) - 2 + 1 = 2(4) - 1 = 8 - 1 = 7$

$\therefore$ L.H.S. = R.H.S.

Substituting $n = 3$, we have

L.H.S. $= n + 5 = 3 + 5 = 8$

R.H.S. $= 2(n + 2) - n + 1 = 2(3 + 2) - 3 + 1 = 2(5) - 2 = 10 - 2 = 8$

$\therefore$ L.H.S. = R.H.S.

Hence, $n + 5 = 2(n + 2) - n + 1$ is an identity.

Solution 1(7):

$7\left(m^2+5\right)=35+7m^2$

Substituting m = 0, we have

L.H.S. = $7\left(m^2+5\right)=7\left(0+5\right)=7\left(5\right)=35$

R.H.S. = $35+7m^2=35+7\left(0\right)=35$

$\therefore$ L.H.S. = R.H.S.

Substituting m = 1, we have

L.H.S. = $7\left(m^2+5\right)=7\left(1^2+5\right)=7\left(6\right)=42$

R.H.S. = $35+7m^2=35+7\left(1^2\right)=35+7=42$

$\therefore$ L.H.S. = R.H.S.

Substituting m = 2, we have

L.H.S. = $7\left(m^2+5\right)=7\left(2^2+5\right)=7\left(9\right)=63$

R.H.S. = $35+7m^2=35+7\left(2^2\right)=35+7\left(4\right)=35+28=63$

$\therefore$ L.H.S. = R.H.S.

Substituting m = 3, we have

L.H.S. = $7\left(m^2+5\right)=7\left(3^2+5\right)=7\left(14\right)=98$

R.H.S. = $35+7m^2=35+7\left(3^2\right)=35+7\left(9\right)=35+63=98$

$\therefore$ L.H.S. = R.H.S.

Hence, $7\left(m^2+5\right)=35+7m^2$ is an identity.