Chapter - 15

Nuclear Physics

In 1931, for explanation of experimental obersations of α ray scattering experiments Rutherford proposed that nearly all of its mass and the entire charge of an atom is concentrated into a very small region called the "nucleus" situated at the centre of atom. You have studied in detail about this experiment and the nuclear atom model in previous chapter. Now it is a natural question to ask whether the nucleus has also some internal structure like internal structure of an atom. In this chapter we will make efforts to answer this question by discussing about nuclear constitution, nuclear size and nuclear forces.

Prior to the discovery of the nucleus it was known that some heavy elements like uranium, thorium etc, decay spontaneously by emitting certain particles called α, β and γ radiations. This phenomenon discovered by Becqueral in 1936 is called the radioctity. We will see later that the radioctity too is a nuclear phenomenon. After discussing the laws of radioactive decay and related definitions we will discuss about nuclear fission and nuclear reactors which at present play an important role in fullfilling our energy needs. Towards the end of this chapter we will discuss about fusion which is the source of energy generation in sun and other stars, and promises to be a pollution free source of energy in future.

15.1 Nuclear Structure

With the exception of normal hydrogen the nuclei of all other atoms are formed of two components called neutrons and protons. The nucleus of ordinary hydrogen contains only a single proton and no neutron. Two other forms of hydrogen (istopes) known as deuterium and Tritieum contains respectively 1 and 2 neutrons in addition to 1 proton. The proton is positively charged with magnitude of charge equal to the electronic charge while the neutron is electrically neutral. The masses of protons and neutrons are respectively as

$$m_n = 1.67626231 \times 10^{-27}\ kg$$

$$m_p = 1.6749286 \times 10^{\ 27}\ kg$$

(Later on we will describe masses in terms of another unit (u) and equivalent energy units). Electron is a fundamental particle of nature but neutrons and protons are not fundamental particles in a true sense. These are supposed to be made up of other particles called 'quarks'. In this chapter our emphasis is primarily on those properties of the nucleus which are not related to the internal structure of proton or neutron thus we will not be discussing about quarks any further.

The number of protons present in a nucleus of an element is called its proton number and it is also called as atomic number and is denoted symoblically by Z. This number is equal to the number of electrons present in a neutral atom of the said element. The number of neutrons present in a nucleus is termed as the neutron number and is denoted by N. If we ignore the difference of charge (q = + e for proton and q = o for neutron) the neutrons and protons are very nearly identical particles. Their masses are very nearly the same and inside the nucleus these are subjected to identical nuclear force. For these reasons we often classify neutrons and protons taken together as 'nucleons'. The number of nucleons (= Z + N) is called as the mass number of nucleus and is denoted by A. By specifying Z and A (and hence N) we can identify a particular nuclear species or a nuclide. As per convention a nuclide is symbolically denoted by ${}_{Z}^{A}X$ or ${}^{A}X_{Z}$ where

X = chemical symbol of element

Z = atomic number of element which is also the number of protons in the nucleus

A = mass number of nuclide which is equal to the number of nucleons in the nucleus

Thus, ${}_{2}^{4}He$ represents a helium nucleus which contains 2 protons and 4 nucleons and therefore 2 neutrons. Similarly ${}_{46}^{107}Ag$ represents a silver nucleus containing 46 protons and 107 nucleons therefore 61 neutrons.

15.1.1 Some important Definitions

Isotopes : These are atoms having same number of protons Z in their nuclei: but having different mass number A i.e the nuclei of different isotopes of same element contains same number of protons but different number of neutrons. For example consider three isotopes of oxygen ${}^{16}_{8}O,\ {}^{17}_{8}O,\ {}^{18}_{8}O$. As the chemical properties of any elements is determined by Z, the number of electrons in it, so all the isotopes of a given elements show identical chemical properties and they occupy same place in periodic table. They cannot be separated by chemical analysis but can be done so by mass-spectrograph.

Isobars : These are nuclei having same mass (nucleon) number A but different atomic (proton) number Z, and neutron number N. For example 1 ${}^{14}_{6}C$ and ${}^{14}_{7}N$ are isobars for each A = 14 but Z and N are different. As the atomic numbers are different these belong to different chemical elements and occupy different positions in periodic table. They can be separated by chemical means but not by mass spectrograph.

Isotones : These are nuclei belonging to different elements having same neutron number N but different atomic number Z and different mass number A. e.g ${}^{13}_{6}C$ and ${}^{14}_{7}N$ are isotones for each of which N = 7. These belongs to different elements and can be separated by both the chemical means and mass spectrograph.

Mirror Nulcei : In such nuclei the mass number A is same but proton number and neutron number are interchanged i.e number of neutrons in one equals the number of protons in other and Vice-Versa. e.g ${}^{7}_{4}Be(Z=4,\ N=3)$ and ${}^{7}_{3}Li(Z=3,\ N=4)$

Isomers : For these nuclei each A and Z are same but their radioactive properties (like half lives, nuclear energy states) are different. Isomers are represented by same chemical symbol with a marked as superscript to differentiate it with nucleus in ground state.

15.2 Nuclear Size

In previous chapter while analysing the α particle scattering experiments we have calculated the distance of closest approach for on α particle for a given nucleus. For α scattering experiment involving 5.5 MeV energy α particles and gold nucleus, Rutherford found this distance to be nearly 4.0×10^{14} m . At such a distance α particle retraces its original path after stopping momentarily due to the Coloumb repulsion of the nucleus. From this Rutherford concluded that the size of gold nucleus must be smaller than 4.0×10^{14} m . Rutherford also found the distance of clost approach for silver nuclei to be nearly 2.0×10^{14} m . Thus it is obvious that if we assume nucleus to be spherical its radius must be of the order of 10^{14} m .

In modern experiments for determining the nuclear radius high energy electrons or neutrons are utilised. In electron scattering experiments electron beams having energy 200 MeV or more are used for their de-Bragile wavelength is short enough for them to act nuclear structure sensitive probe. In effect such experiments diffraction pattern of scattered electrons from which shape of target (nucleus) is determined. It is worth noting that electron scattering experiments provides information regarding the charge distribution in nucleus while neutrons scattering experiments determines the distribution of matter (mass) in nucleus. From these experiments although we found that the nucleus has no sharply defined surface but majority of the nuclei are very nearly spherical with some of them having ellipsoidal surfaces. However, there is a general agreements that one can define an average or a mean radius for a nucleus as follows

$$R = R_0 A^{1/3} \qquad \ldots(15.1)$$

where A is the mass number of the nucleus and R_0 is constant with a value about 1.2×10^{15} m .

A convenient unit for measuring nucleur radius and nuclear distances is femtometer also called fermi and abbreviated as fm, such that

1 femtometer = 1 fermi = 1 fm = 10^{15} m, thus in this unit $R_0 = 1.2\,fm$. As discussed in previous chapter it is to be noted that nuclear radius is smaller by a factor of 10^4 compared to the atomic radius.

15.2.1 Nuclear Volume

If we asume that the nucleus to be spherical with a radius R then its volume is

$$V = 4/3\ \pi R^3$$

or $$V = (4/3)\ \pi R_0^{\ 3} A \qquad \ldots15.2$$

Therefore the volume of a nucleus is proportional to its mass number. This in turns means that the density of the nuclear matter is independent of its mass number and is same for all nulcei.

Example 15.1 Determine the radius of $^{27}_{13}\text{Al}$ nucleus.

Solution : The nuclear radius is given by

$R = R_0 A^{1/3}$ where $R_0 = 1.2\, fm$ and as per question A = 27, thereofre

$$R = 1.2 \times (27)^{1/3}$$

$$= 3.6\, fm = 3.6 \times 10^{-15}\, m$$

Example 15.2 Determine the potential energy due to electrical repulsion between two $^{27}_{13}\text{Al}$ nuclei when they just touch each other at the surface.

Solution : As per solution obtained in example 15.1 above the radius of each $^{27}_{13}Al$ nucleus is $R = 3.6 \times 10^{-15}$ m. When they just touch other at the surface the separation between their centres is $d = 2R = 7.2 \times 10^{-15}$ m. So the potential energy associated with this pair will be

$$U = \frac{q_1 q_2}{4\pi \in_0 d}$$

Here each nucleus contains 13 proton, so

$$q_1 = q_2 = 13 \times 1.6 \times 10^{-19}\, C$$

$$\therefore \quad U = \frac{\left(9 \times 10^9\, N m^2 / C^2\right)\left(13 \times 1.6 \times 10^{-19} C\right)^2}{7.2 \times 10^{-15} m}$$

$$= 540.8 \times 10^{24} \times 10^{-38}\ \text{Nm}$$

$$= 540.8 \times 10^{-14}\ \text{J} = \frac{540.8 \times 10^{-14}}{1.6 \times 10^{-19}}\ \text{eV}$$

Example 15.3 Estimate the numerical value of nuclear density for a nucleus of mass number A.

Solution : The mass of proton and neutrons are very nearly equal say m, then the mass of a necleus of mass number M = mA. From equation 15.2 the nuclear volume $V = \frac{4}{3}\pi R_0^3 A$

the density of nuclear matter ρ =

$$= \frac{m}{4/3\pi R_0^3} = \frac{3}{4}\frac{m}{\pi R_0^3}$$

Which is independtent of the mass number A. Taking $R_0 = 1.2 \times 10^{-15}$ m we obtain

$$\rho = \frac{3}{4 \times 3.14} \times \frac{1.67 \times 10^{-27}}{\left(1.2 \times 10^{-15}\, m\right)^3}$$

$$= 2.3 \times 10^{17}\, kg / m^3$$

From the above example it is clear that the density of nuclear matter is indepentent of the mass number and is quite high of the order of $10^{17}\ \text{kg/m}^3$. This is expected as the nuclear matter is confined to a very small volume. If we compare the density of nuclear matter with the density of water $\left(\rho_w = 10^3\, kg / m^3\right)$ then we find it to be greater than by a factor of 2.3×10^4. Matter with such a high density is found in neutron stars.

15.3 Atomic mass unit

The atomic and nuclear masses are of the order 10^{-25} kg to 10^{-27} kg. In practice it is not convenient to use such smaller quantities, therefore these are expressed in another smaller unit called unified atomic mass unit (u) [earlier this unit was called as atomic mass unit (amu)]. This unit is selected such that when expressed in this unit the mass of a 12 C atom (not the nucleus) is exactly 12 u.

$$\text{So} \quad 1u = \frac{{}^{12}_{6}\text{C (mass of carbon atom)}}{12}$$

$$= \frac{1.992647 \times 10^{-26}\, kg}{12}$$

$$= 1.66054 \times 10^{-27}\ \text{kg}$$

Note that the atomic masses refers to the masses of neutral atoms and note of bare nuclei. Thus an atomic mass always includes the masses of its Z electrons. Exact measurements of atomic masses is done by mass spectrograph. When expressed in u, atomic masses of many elements are found to be very nearly equal to integral multiples of the atomic mass of hydrogen atom. However, there are a few exceptions e.g the atomic mass of chlorine is 35.46 u.

On using the Einstein's famous mass-energy equivalence relation, $E = mc^2$ we can obtain energy equivalent to 1μ mass as follows

$$m = 1u = 1.66050 \times 10^{-27}\, kg$$

$\therefore$ Equivalent energy $E = (1u)c^2$

$$E = \left(1.6605 \times 10^{-27}\right)\left(2.9979 \times 10^{8}\right)^2 \text{ kg m}^2/\text{s}^2$$

$$= 1.4924 \times 10^{\ 10} \text{ J}$$

$$= \frac{1.4924 \times 10^{\ 10}}{1.602 \times 10^{-19}} \text{eV}$$

$$E = 931.5\, MeV$$

This suggests that one can write 1 μ = 931.5 MeV / c^2 or one can determine energy equivalent to a given mass difference expressed in μ or vice versa. In table 15.1 the masses of proton, neutrons electron and ordinary hydrogen atom are mentioned in different mass unit.

Table 15.1 Masses of proton, neutron, electron and hydrongen atom ($_1^1$H)in various mass units.

Particle	Mass		
	kg	**u**	**MeV / c^2**
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	$1.6750 \times 10^{\ 27}$	1.008665	939.29
Electron	9.1095×10^{-31}	0.0005486	0.511
$_1^1$H atom	$1.6736 \times 10^{\ 21}$	1.007825	938.79

Although to be exact 1 u = 931.5 MeV but for ease of numerial calculations in what follows we shall take 1 μ = 931 MeV .

15.4 Mass Defect and Nuclear Binding Energy

Except hydrogen $\left({}_1^1\text{H}\right)$ nucleus all other nuclei are composed of neutrons and protons. Thus it is quite natural to expect that the mass of a nucleus M must be equal to the sum of the masses of its constituent nucleons $\sum m$. However, the mass of the nucleus M as measured experimentally is alwyas found to be smaller than $\sum m$. This difference in mass is called the mass defect and is denoted by ΔM i.e

$$\Delta M = \sum m - M$$

If a nucleus of mass number A, consists of Z protons and N neutrons with m_p and m_n as mass of proton and neutrons respectively then

$$\sum M = Zm_p + Nm_n$$

then accordingly

$$\Delta M = Zm_p + Nm_n - M \qquad ...(15.3)$$

and as

$$N = A - Z$$

So one can also write

$$\Delta M = Zm_p + (A - Z)m_n - m \qquad ...(15.4)$$

The theoretical explanation of mass defect lies in the Eintein's mass-energy relationship. According to it the energy equivalent to mass defect $\Delta E_b = \Delta Mc^2$ is the binding energy of the nucleus. The necleons in a nucleus are bound together and to pull them apart from each other so that these are separated from each other by long distances the energy is to be given to the nucleus (Fig 15.10). This energy is called the binding energy of the nucleus. Alternatively if intially the nucleons are well separated from each other and are brought together to form a nucleus this much amount of energy is going to be released in the process. (Fig 15.1 (b).

One cannot make or break a nucleus in the manner suggested above but still the binding energy of the nucleus gives us an idea about how well the nucleons and bound together in a nucleus.

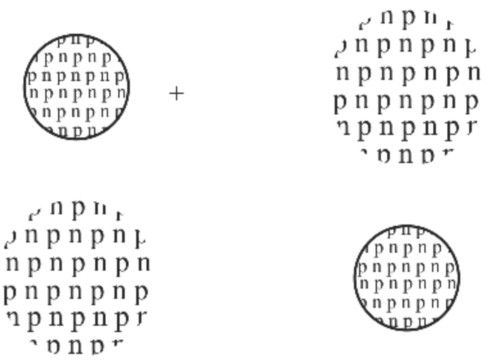

Fig 15.1 (a) The energy equal to the binding energy is to be given to the nucleus to break it into its constituents

nucleons. Each such nucleons is at rest and well separated from the other nucleons.

Fig 15.1 (b) Construction of a nucleus from its constituents nucleons, binding energy is released in the process

Now accroding to the mass energy relation

$$\Delta E_b = \Delta M c^2$$

So on substituting for from equation 15.3

$$\Delta E_b = (Zm_p + NM_n - M)c^2 \qquad \ldots(15.5)$$

So on substituting for M from equation if instead of nuclear masses we use atomic masses (as usually atomic masses are reported rather than the nuclear masses) then the above equation assumes the following from

$$\Delta E_b = \left(ZM_H + NM_n - {}_Z^A M\right)c^2 \qquad \ldots(15.5a)$$

Where M_H refers to the mass of ordinary hydrogen $\left({}_1^1\mathrm{H}\right)$ atom and ${}_Z^A M$ is the mass of the neutral atom of the nucles under consideration. Here it can be seen that Z hydrogen atoms contain Z electrons and atomic mass of atom ${}_Z^A M$ also includes the mass of Z electrons and hence the masses of electrons cancel out in above equation. (However, such cancellation may not take place in process of β decay discussed latter). Their is a slight difference between the binding energy calculated from equations 15.5 and 15.6 owing to the binding energy of electron in atomic masses, However as the atomic binding energy is of the order of a few eV while the nuclear binding energy is of MeV so the difference is quite small and is to be negleted.

Example 15.4 Calculate the binding energy for the following nuclei

(i) Deutron $\left({}_1^2\mathrm{H}\right)$ (ii) ${}_{50}^{120}Sn$ Given that $m_p = 1.007u$, $m_n = 1.008u$ mass of deutron nucleus $M_d = 2.013u$ and the mass of S_n nucleus $M_{Sn} = 119.902u$ $(1u = 931 MeV/c^2)$

Solution : The formula for binding energy is

$$\Delta E_b = \left[Zm_p + (A-Z)m_n - M\right]C^2$$

(i) For deutron $\because$ $Z = 1 \quad A = 2$

$$\text{So } \Delta E_b = \left[1m_p + 1m_n - M_d\right]c^2$$

$$= [1.007 + 1.008 - 2.013]uc^2$$

$$= [2.015 - 2.013] \times 931\ MeV$$

$$= 0.002 \times 931 = 1.862\ \mathrm{MeV}$$

(ii) For Sn nucleus Z = 50, A = 120 So A - Z = 70

$$\Delta E_b = [50 \times 1.007 + 70 \times 1.008 - 119.902] \times 931\ \mathrm{MeV}$$

$$= [50.35 + 70.56 - 119.902] \times 931\ \mathrm{MeV}$$

$$= [120.91 - 119.902] \times 931\ \mathrm{MeV}$$

$$= 1.008 \times 931 MeV = 938.448 MeV$$

In above example rather than taking the exact masses for m_p, m_n and nuclei for the sake of simplicity in calculation we have taken their approximate values, still we can note that the binding energy is in MeV range much higher than the atomic binding energy (a few eV). Also note that the binding energy of an intermediate mass nucleus like ${}_{50}^{120}S_n$ is quite large compared to a lighter mass nucleus like ${}_1^2\mathrm{H}$.

15.4.1 Binding Energy per Nucleon

The quantity obtained on dividing the binding energy ΔE_b of a nucleus by its mass number A is termed as the binding energy per nucleon. It is denoted by ΔE_{bn} or $\overline{\Delta E_b}$ i.e.

$$\Delta E_{bn} = \frac{\Delta E_b}{A} \qquad \ldots(15.6)$$

It is a very useful concept. Higher is the value of ΔE_{bn} more stable is the nucleus. If a graph is plotted between binding energy per nuleon for various nuclei and corresponding mass numbers then a curve as shwon in Fig 15.2 is obtained.

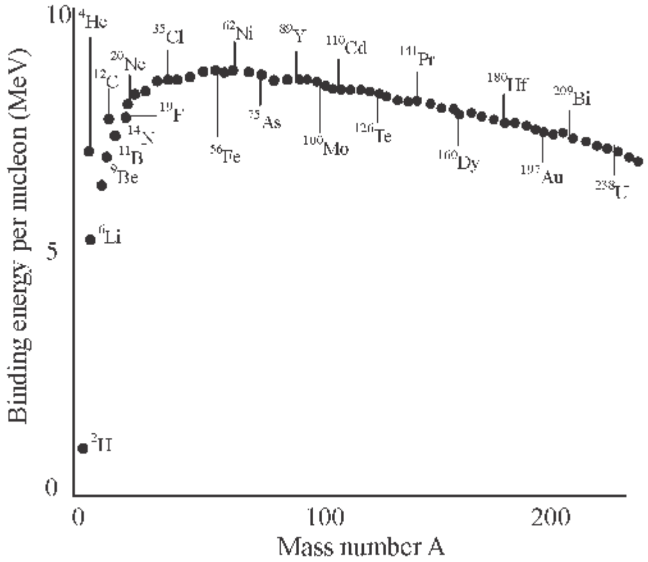

Fig 15.2 : Variation of binding energy per nucleon for some representative nuclides with corresponding mass number A.

The binding energy is maximum (8.8 MeV) for ^{62}Ni nucleus amongest all known stable nuclides. Also note that nuclei like $^{4}_{2}He$, $^{16}_{18}O$, etc are more stable than their neighbours becasue of their higher binding energy per nucleon.

From the study of this curve following conculsions are drawn.

(i) Initially the value of ΔE_{bn} increases, attains a maximum and then decreases slowly.

The nuclei for which the nucleon number is a multiple of 4 i.e A = 4, 8, 12, 16 ..... have higher values of ΔE_{bn} compared to their immediate neighbours. So these are relatively more stable (this suggests a shell structure for nucleons in a nucleus, like the shell structure for electrons in atoms. Inert gases having a completely filled outermost shell are more stable than other elements, like wise nuclei with nucleon numbers suggested above also have completely filled nuclear shells. You will learn more about this in higher classes).

(ii) The elements with A ~ 50 to A ~ 80 are most stable. For them average ΔE_{bn} ~ 8.7 MeV per nucleon. For both A < 50 and A > 80, ΔE_{bn} decreases. The binding energy per nuclear is maximum near A~60, (therefore having A nearing this value) like F_e, N_i and C_o are very stable E_{bn} is maximum = 8.8 MeV for ^{62}Ni nucleus.It is a bit smaller for $^{60}_{26}Fe$ nucleus. This is the reason why molten Ni and Fe are most abundent in earth core.

(iii) For intermediate mass numbers (30 < A < 170) the binding energy per nucleon can be assumed to be practically constant at about 8 MeV. In this range ΔE_{bn} does not vary significantly with A, indicating short range and saturation property of nuclear forces about which we shall learn in brief in following section.

(iv) As mentioned above, nuclei having intermediate mass numbers are relatively more stable compared to those having higher mass numbers. Therefore if a heavy nucleus breaks into two nuclei of inermediate masses, the total binding energy increases while the rest mass energy decreases. Thus energy is released in the process in the form of kinetic energies of the fragments and (or) in some other forms. This process called nuclear fission will be discussed in a latter section of this chapter.

(v) Likewise it can be imagined that two light nuclei (A≤10) can be combined to form a relatively heavier nucleus. As the binding energy per nucleon for the lighter nuclei is smaller compared to the middle mass nuclei there is a possibility of release of energy in the process. This process called fusion will be discussed in detail towards the end of this chapter.

Some others conclusions which we can not infer directly from the curve shown in Fig 15.2 are as follows. Nuclei with even A, even Z are usually stable and mass abundant, Nuclei with odd A, even odd Z are unstable, in general. Nuclei with odd Z and even A are also unstable with exceptions like ^{2}H, ^{6}Li, ^{10}B, ^{14}N which are stable.

15.5 Nuclear Forces

Two protons (positive charges) which are so near in a nucleus repels each other with such a large electrostatic force, then what keeps a nucleus from breaking? Clearly there should be some strong attractive force operating with in the nucleus which helds the nulceons bound together as nuclei of many of the elements available in nature are stable. This force can not be gravitational as due to very small masses of nucleons and small value of G, even for inter nuclear distance gravitational forces between two nucleons is so week to counteract the repulsive electrostatic force. In fact the gravitational forces are completly ignored in the domain of nuclear physics. Therefore, there must be some other kind of force operating between the nucleons inside a nucleus to held the nucleons bound together. This force is called as the nuclear force. The overall effect of the nuclear force is that it is much stronger than the repulsive Coloumb force operating between two protons and thus the nucleus stays bound.

Unlike electrostatic or gravitational force there is no simple single mathematical expression to determine the nuclear force operating between two nucleons. In fact, many details of the nuclear force are yet to be understood. Some of the qualitative features of the nuclear forces are as follows.

(i) Nuclear forces are independent of charge. For a given separation the nuclear force between two protons is the same as that between two neutrons or between a neutron and a proton. [electrons are not affected by the nuclear forces that is the reason why in electron scattering experiments electrons are scattered by nuclear charges and therefore electron scattering experiments provides information about the distribution of nuclear charges]. Likewise in neutron scattering experiment there is no role of nuclear charge but of nuclear force. Thus neutron scattering experiments provides information about distribution of mass in a nucleus].

(ii) Nuclear forces are short range. The range upto which the nuclear force acts is called nuclear range and it is of the order of a few femto meter, however with in this range the nuclear force is much larger than the electrostatic force (50 ~ 60 times larger). Outside the nuclear range nuclear forces are not effective.

(iii) Nuclear forces are non central in nature. In addition to separation between the nuleons the force between a pair of nucleons also depends on relative orientations of spins of the neuleons.

(iv) Along with the short range of nuclear forces the fact that the density of nuclear matter is constant and the binding energy per nucleon for middle mass nuclei is roughly constant indicates that each nucleon in a nucleus does not interact with every other nulceons in the nuleus. It interacts only with a few neighbouring nucleons. (Consider a nucleon in a nueleus of mass number A. If it would interact with all other nucleons then we would be having $A(A-1)/2$ such an interactions. In such a case the binding energy would be proportional to $A(A-1)$ and for $A >> 1$ this would mean A^2 i.e. not a constant). This property of nuclear force is called saturation of the nuclear force. This is different from electrostatic force. (A proton in a nucleus interacts with all other electrons and number of such interactions $\frac{Z(Z-1)}{2} \sim Z^2$

(v) Nuclear forces are attractive in general. However for separations less than 1 fm the nuclear force between a pair of nucleons tends to be repulsive. A detail discussion of this property is beyond the level of the present study.

15.6 Radioactivity

At the beginning of this chapter we mentioned the discovery of radioactivity by Becquerel in 1896 which indicated spontaneous disintegration of heavy elements like uranium, thorium etc by emission of particles or radiation. During the process new atoms (elements) are formed which may themselves be radioactive and the process continues till a stable element is formed.

Becquerel discovered radioactivity accidentally when he found that uranyl potassium sulphat crystals emitted an invisible radiation that could darken a photographic plate when the plate was covered to exculde light. From a series of experiments he also found these radiations capable of ionisation of gases. The most significant investigations of the phenomenon were conducted by polish scientists Marie curie and Piere Curie. After several years of laborious chemical separation on tons of pitch belnde, a radio active ore, the Curies discovered two previously unknown elements both of which were radio active. These were named Polonium and radium. Experimental work by Rutherford showed that radio active radiations was of three types which he called alpha, beta and gamma rays. Later experiments showed that alpha rays are helium nuclei, β rays are electrons or positrons and gamma rays are high energy photons. Experinments also predicted that the radioactivity is a nuclear phenomenon which involves decay or disintegration of an unstable nucleus. Some important facts regarding radioactivity are as follows.

(i) Radioactivity is not influenced by external parameters like pressure, temperature, phase of radioactive material (solid, liquid, or gas). Radioactivity is not affected by chemical reactions or chemical combination (e.g both uranium or its salts (compounds) are radioactive). As outer atomic electrons are involved in chemical reaction, therefore electronic configuration of an atom plays no role in the phenomenon of radioactivity. Also the emission α of particle, energetic β particles or high energy γ ray photons is not possible from external part of an atom thus radioactivity is purely a nuclear phenomenon.

(ii) In radioactive decay of any nucleus, conservation laws like mass-energy conservation, linear and angular mommontum conservation along with conservation of nucleon number must be obeyed.

(iii) A nucleus X shall be unstable for α or β decay in principle if its mass is more than the sum of the masses of decay products.

(iv) The energy realeased per atom in radioactive decay is few MeV while that in chemical reactions is few eV only.

15.6.1 Rutherford-Soddy Law of Radioactive Decy

Radioactive decay is a random process. It is a statistical phenomenon that obeys the laws of probability (In fact it is the phenomenon of radioactivity which provided the first evidence that the laws governing the subatomic world are statistical in natural). In reference to the decay of atoms present in a sample of radioactive material each decay is an independent event. There is absolutely no way to predict whether any given atom in a radioactive sample will be among the small number of nuclei that decays during the next second. All have the same probability. The radioactive decays law was given by Rutherford and Soddy. According to this law the rate of decay of nuclei (-dN/dt) at some given instant is proportional to the number of nuclei N present at that instant i.e.

$$-\frac{dN}{dt} \propto N$$

or $$\frac{dN}{dt} = -\lambda N \qquad \ldots (15.7)$$

The negative sign indicates that N decreases as t increasses. λ is a constant called decay ot disintegration constant. It has a characteristics value for every radioative nuclide and its SI unit is inverse second (s^{-1}). To understand equation 15.7 the logic is as follows. Assume that at certain instant the number of radioactive nuclei is N. How many of them are going to decay in the next small interval dt? This number will be proportional to both N and dt. For each nucleus there is a chance of decay in interval dt. So more the number of nuclei present at instant t more will decay in next time interval dt. Likewise, if dt is made slightly longer more nuclei will decay because each nuclei will hare more chance of decaying. Hence

$$-dN \propto N dt \quad \text{or} \quad dN = -\lambda N\, dt$$

which is same as equation 15.7. On rearranging equation 15.7

$$\frac{dN}{N} = -\lambda dt$$

and than integrating both sides, obtaining

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

or $$\ell nN - \ell nN_0 = -\lambda t$$

or $$\frac{N}{N_0} = e^{-\lambda t}$$

or $$N = N_0 e^{-\lambda t} \qquad \ldots (15.8)$$

Here N_0 is the number of active nuclei at $t = 0$. From the equation 15.8 it is obvious that the number of active nuclei decreases exponentially with time. This has been shown graphically in Fig 15.3.

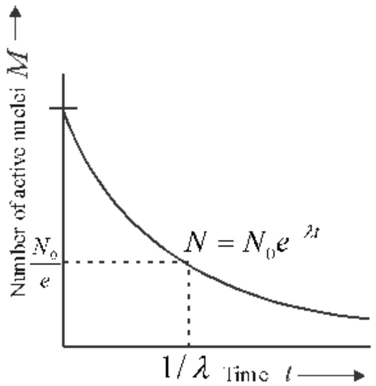

Fig 15.3 : The exponential decay of radioactive atoms

The number of nuclei that have decayed in time t is

$$N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0\left(1 - e^{-\lambda t}\right) \quad \ldots (15.9)$$

Thus, the fraction of nuclei that have decayed in time *t* is

$$\frac{N_0 - N}{N_0} = 1 - e^{-\lambda t}$$

Decay constant

From equation 15.7

$$\lambda = \frac{\left|(dN/dt)\right|}{N}$$

hence, the decay constant is the rate of decay of radioactive atoms per atom

$$\text{Also } \lambda = \left(-\frac{dN}{N}\right)\frac{1}{dt}$$

Thus, the decay constant is the probability of decay per unit time. In addition to this if in equation 15.7 we take $t = 1/\lambda$, then

$$N = N_0 e^{-1} = \frac{N_0}{e} = 0.368\, N_0$$

Thus, the decay constant is the reciprocal of that time in which the fraction of active atoms reduces to 1/e or 0.368 i.e 36.8% atoms remains active or about 63.2% of the atoms decays. This can be calculated from fig 15.3.

Activity : We are often interested in determining the number of nuclei decaying per second than N (as it is more convinient to measure than N). This is called the activity of the sample. It is also called decay rate of sample and by defination it is a positive quantity, denoted by R.

$$\text{Activity } R = \left|\frac{dN}{dt}\right| \quad \ldots (15.10)$$

From equation 15.11

$$\left|\frac{dN}{dt}\right| = \lambda N$$

$$\therefore \quad R = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$= R_0 e^{-\lambda t} \quad \ldots (15.11)$$

$$\text{Where} \quad R_0 = \lambda N_0 \quad \ldots (15.12)$$

is initial activity of the sample. The SI unit for activity is the becquerel (Bq) and

1 Bq = 1 disintegration/second

However, a traditional unit of activity, the curie (Ci) still in common use is defined as the activity of 1 g of radium (^{226}Ra) with a value.

$$1\,C_i = 3.7 \times 10^{10} \text{ disintegration/s}$$

$$= 3.7 \times 10^{10} \text{ Bq}$$

The graphical representation of equation 15.12 is similar to that shown in Fig 15.3 however, we have to take R in place of N on y axis.

15.6.2 Half Life

The time in which the number of active nuclei present in a sample of a radioactive element raduces to half of its initial value is called as the half life of that radioactive element. If we denote it by T then from equation 15.8 for t=T we have $N = N_0/2$ i.e

$$\frac{N_0}{2} = N_0 e^{-\lambda T}$$

$$\text{or } e^{\lambda T} = 2$$

$$\text{or } \lambda T = \ell n 2$$

$$\therefore \quad T = \frac{\ell n 2}{\lambda} = \frac{0.693}{\lambda} \quad \ldots (15.13)$$

Therefore the half life of a radioactive material is inversely proportional to its decay constant. It is contant for a given material and is not affected by external parameters like pressure, temperature etc. As the activity also decays exponentially with time with a decay constant λ thus the activity of a sample decays to half of its initial value in on half life period. Some radioactive nuclides have half-lives which are only a millionth of a second, while for others half lives are billions of years. Thus half life varies in a wide span. e.g an isotope of polonium $^{214}_{84}P_0$ half-life is only 10^{-15} s while for Uranium $^{238}_{92}U$ half life is 4.5×10^9 years. In fig. 15.4 the radioactive decay is depicted in terms of number of half-lifes.

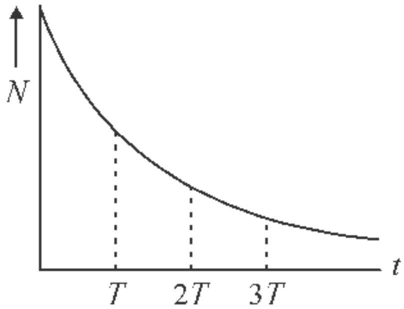

Fig 15.4 Exponenetial decay of a radioactive material, after each half life the number of active atoms reduces to half the value present at the begining of proceeding half life.

On using equation (15.13) equation 15.8 can be rewritten at

$$N = N_0 e^{-\lambda t} = N_0 e^{-(\ell n 2)t/T}$$

$$= \frac{N_0}{\left[e^{\ell n 2}\right]^{t/T}} = \frac{N_0}{2^{t/T}} \qquad \left[\because\ e^{\ell n 2} = 2\right]$$

$$\text{or } \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \qquad \ldots (15.14)$$

Likewise, we can show that

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T} \qquad \ldots (15.15)$$

The above relations are very useful in calculations of T and λ as we shall see shortly in numerical examples to follow.

15.6.3 Average life

According to the radioactive decay law, number of active atoms decreases exponentially with time however complete disintegration of a sample is possible only in infinite time i.e the life of individual active atoms can have any value between O and? The average life of a radioactive substance is defined as the ratio of the total life time of all the radioactive atoms to the total number of such atoms in it. It is denoted by τ then

$$\tau = \frac{\text{Sum of the ages of all atoms}}{\text{Number of atoms}} \qquad \ldots (15.16)$$

Consider a sample containing N_0 radioactive atoms initially $(t = 0)$ and after time t the number of active atoms is N. Then the number of atoms undergoing decay in a very small next time interval dt is dN. Since dt is very small therefore we are safe in assuming that the each of these dN atoms has a life time t. Then the sum of lives of these dN atoms is tdN. As stated earlier the life time span of atoms in a sample is in between O and therefore the sum of lives of all N_0 atoms (say S) present in the sample will be

$$S = \int_0^\infty t\, dN$$

From equation 15.17

$$S = \int_0^\infty t\, dN = \int_0^\infty \lambda N t\, dt = \int_0^\infty \lambda N_0 e^{-\lambda t} \cdot t\, dt$$

$$\tau = \frac{\int_0^\infty t\, dN}{N_0} = \int_0^\infty \frac{\lambda N_0 e^{-\lambda t} \cdot t\, dt}{N_0}$$

$$= \lambda \int_0^\infty t\, e^{-\lambda t} dt$$

On solving above integral we obtain

$$\tau = \frac{1}{\lambda} \qquad \ldots (15.17)$$

Thus, the average life is reciprocal of decay constant. Recall that in a time $t = 1/\lambda$ the number of active reduces to $1/e$ of its initial value. Therefore the average life time can also be defined as that time in which number of active atoms reduced to $1/e$ of its initial number.

From equation (15.13) and (15.17) it can be noted that

$$T = \frac{\ell n2}{\lambda} = \tau \ell n2 = 0.693\,\tau \qquad \ldots (15.18)$$

Note that all the equations derived above are of statistical nature. They do not predict the exact behaviour for each individual atom. In one half life-half the initially active atoms will decay but which of the atom will decay in this half life period can never be predicted. Also note that these equations will work out well only if N is sufficiently large.

Example 15.5 Consider a radoactive sample of 1000 atoms of half life T. Then how many atoms remain active after time T/2.

Solution : Use $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

given $t = T/2$ so $\frac{t}{T} = \frac{1}{2}$

$$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}$$

or $\quad N = \frac{1}{\sqrt{2}} N_0 = (0.707) \times 1000$

$= 707$ atoms

Example 15.6 The activity of a radioactive sample drops to 1/32 of its initial value in 7.5 h. Find the half life of atoms of the sample.

Solution : Given $\frac{R}{R_0} = \frac{1}{32}, t = 7.5h$

Therefore, using $\quad \frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T}$

We have,

$$\frac{1}{32} = \left(\frac{1}{2}\right)^{7.5/T}$$

or $\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{7.5/T}$

or $5 = \frac{7.5}{T}$

$$\therefore \quad T = \frac{7.5}{5} = 1.5\,h$$

Example 15.7 What is the activity of a 10 kg sample of ^{235}U if the half life of uranium ^{235}U is 7.04×10^8 years. [Take 1 year $= 3.15 \times 10^7$ s and atomic mass of $^{235}U = 252$ g/mol]

Solution : For a sample of mass M containg N atoms each of atomic mass M, N is given by

$N = \frac{m}{M} N_A$ where N_A is Avogadro number

on substituting relevent values

$$N = \frac{10 \times 10^3}{235}\left[6.02 \times 10^{23}\right] = 2.56 \times 10^{25}$$

therefore activity $R = \lambda N = \frac{(\ell n2) N}{T}$

$$= \frac{0.693 \times \left(2.56 \times 10^{25}\right)}{7.04 \times 10^8 \text{ year}}$$

$$= 2.52 \times 10^{16} \text{ disintegration/year}$$

$$= \frac{2.52 \times 10^{16}}{3.15 \times 10^7}$$

$$= 8.0 \times 10^8 \text{ Bq}$$

Example 15.8 ^{214}Bi nucleus can decay by two channels. In one of decay channels it decays by α emission with a decay constant λ_1, according to

$$^{214}Bi \xrightarrow{\lambda_1} {}^{210}T\ell + {}^4_2He$$

or it decays via β emission with a decay constant λ_2 according to

$$^{214}Bi \xrightarrow{\lambda_2} {}^{214}Po + {}^{0}_{-1}e$$

If the life time corresponding to the two channels are T_1 and T_2 and in a sample of ^{214}Bi some atoms decays via first and other via second channel. Then obtain an expression for effective half life for such a sample.

Solution : According to question decay constants for first and second processes are λ_1 and λ_2. The probability that an active nuclei decays by the first process in time dt is $\lambda_1 dt$. Similarly the probability that ir decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ this probability is also equal to λdt. Therefore

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or effective decay constant, $\lambda = \lambda_1 + \lambda_2$ . . . (i)

and as $\lambda_1 = \frac{0.693}{T_1}$, $\lambda_2 = \frac{0.693}{T_2}$

and if the effective half life is T then $\lambda = \frac{0.693}{T_2}$ on substituting for λ_1, λ_2 and λ in equation (i), we obtain

$$\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$

or $$T = \frac{T_1 T_2}{T_1 + T_2}$$

Example 15.9 In some radioactive process assume that a nucleus A is transforming into a nucleus B with a decay constant λ_A. The nucleus B so formed is itself radioactive and is decaying in to another nucleus C with a decay constant λ_B. Let N_A and N_B be the number of nuclei of A and B at time *t*. Find the condition for which number of nuclei of B becomes constant.

Solution : The number of nuclei of A decaying in a small time interval *t* and *t+dt* is $\lambda_A N_A dt$. This is also the number of nuclei B produced in this interval. For decay of B into C the number of nuclei of B decaying in the same time interval is $\lambda_B N_B dt$. The number of nuclei of B wil be constant if their rate of production is equal to their decay rate. i.e

$$\lambda_A N_A dt = \lambda_B N_B dt$$

or $\lambda_A N_A = \lambda_B N_B$

Example 15.10 ^{238}U, decays into ^{206}Pb with a half life of 4.47×10^8 y. In a smaple of rock 1.19 mg of ^{238}U and 3.09 mg of ^{206}Pb are found. Assuming all lead to be formed from uranium, estimate the age of rock.

Solution : Let, here N_F = number of product ^{206}Pb nuclei at time t.

N_I = number of ^{238}U nuclei at time t.

m_u = mass of U in sample

m_{pb} = mass of Pb in sample

M_{pb} = atomic mass of Pb

M_u = atomic mass of U

and N_A = Avogadro number, then

$$N_F = \frac{m_{pb}}{M_{Pb}} N_A, \quad N_I = \frac{m_U}{M_U} N_A$$

$$\therefore \quad \frac{N_F}{N_I} = \frac{m_{Pb}}{m_U} \frac{M_u}{M_{Pb}}$$

$$= \frac{3.09(mg)}{1.19(mg)} \frac{238g/mol}{206g\,mol} = 3$$

If N_0 = initial number of ^{238}U nuclei, then

$$N_F + N_I = N_0$$

$$3N_I + N_I = N_0$$

or $N_I = \frac{N_0}{4}$

Thus, at time t the number of uranium nuclei reduces to 1/4 of its initial value (value at the time of rock formation). Therefore

$$t = 2T = 2 \times 4.47 \times 10^3 = 8.94 \times 10^8 y.$$

Example 15.11 How much time it will take to reduce a radioactive sample to reduce to 10% due to decay. The half life of materials is 22 years.

Solution : Use $\frac{N}{N_0}=\left(\frac{1}{2}\right)^{t/T}$

$\frac{10}{100}=\left(\frac{1}{2}\right)^{t/22}$ or $\frac{1}{10}=\left(\frac{1}{2}\right)^{t/22}$

or $(2)^{t/22}=10$

On taking log of both sides

$\frac{t}{22}\log 2=\log 10$

$\frac{t}{22}\times 0.301=1$

$t=\frac{22}{0.301}=73y$

15.7 α, β and γ rays and their properties

α, β and γ rays emitted in the process of radioactive decay process are collectively known as the nuclear radiations. In this section we will study main properties of these radiations.

15.7.1 Properties of α particles

The important properties of α rays are as follows.

(i) α particles are positively charged particles. In fact these are doubly ionsied helium atoms i.e helium nuclei ($_2^4He$) having mass four times the mass of a proton and change twice the protonic charge.

(ii) As these are charged particles so these are deflected by both electric and magnetic fields. Owing to their relatively higher mass deflection of alpha particles is comparatively smaller than β particles for a given electric or magnetic field.

(iii) The velocity of α particles is in the range $1.4\times10^7 \sim 1.7\times10^7$ m/s i.e $v_\alpha \sim 0.05c$ (where c is the velocity of light in free space).

(iv) On passing through gases alpha particles collide with gas atoms to knockout electron from them so gases are ionised. Their ionisation power is 100 times more than that of β rays and 10,000 times more than γ rays. This is due to relatively higher charge and relatively smaller velocity of α particles.

(v) The distance covered by α particles in air at N.T.P is called range of α particles i.e distance after covering which the penetration power of α particles is no more is called range. The range in air is small 2.7 cm to 8.6 cm. According to Gieger and Nuttal law the decay constant λ and energy E of α particles are related as $\ell n\ \lambda = A_1 + B_1 \ell n\ E$. The graph between $\ell n\ \lambda$ and ℓn E is a straight line. Future the range depends on energy $R_\alpha \propto E^{3/2}$ or $R_\alpha \propto v^3$ ($V\alpha$ = Velocity of α particle). Using it, the Giger- Nuttal relation becomes $\ell n\ \lambda = A + B \ell n\ R$ so graph between $\ell n\ \lambda$ and $\ell n\ R$ is a straight line. The value of B is same for various radioactive series (mentioned later in this chapter).

(vi) The penetration power of α particles is much smaller comapred to β and γ rays. These are stopped by piece of a card board or 0.1 mm thick aluminium sheet. The reasons lies in their relatively large mass and ionisation power so while passing through a material the energy of α particle decreases rapidly (in comparision to β or γ particles). On stopping by materials α particles produce heating effect.

(vi) They effect photographic plates and produces fluorescence in Zns or barium platinocynoide.

(viii) After emission of an α particle the atomic number Z of nucleus decreases by 2 while mass number decreases by 4 and size of nucleus reduces.

(ix) The energy spectrum of α particles is a discrete line spectrum which is indicative of presence of discrete energy states for a nucleus.

15.7.2 Properties of β rays

β radiations consists of charged particles. For β^- decay, these radiations are electrons coming out of nucleus (this happens when neutrons are converted into protons inside the nucleus). For β^+ decays these are positrons. β decay usually implies β^- decays. Main properties of β particles are as follows

(i) β^- rays being electrons have charge

$= +e = -1.6 \times 10^{-19}$ C while

β^- rays being positrons have charge

$= -e = -1.6 \times 10^{19}$ C

positron is the antiparticle of electron

(ii) As β rays are charged these are deflected by both electric and magnetic fields. Deflection is larger compared to that for α particles. For β^- particles direction is opposite to that for α particles while for β^+ particles direction is same as to that for α particles.

(iii) Velocity of β particle range from 1% to 99% of the velocity of light. Also the velocity of β particles emitted from the same source differs very much so is the kinetic energy.

(iv) The kinetic energy of β particles emitted from a radioactive materials is distributed continuously from zero to a maximum value, hence energy spectrum of β particles is continuous. For this reason range of β particles varies but is much large than of α particles in air it is tens of centimeter.

(v) β rays produces medium ionisation in gas through which β radiation pass. Their ionisation power 1/100th of α particles but 100 times larger compared to γ particles.

(vi) Their penetration power is 100 times larger than α particles but smaller by the same factor compared to γ rays. They can penetrate an aluminium sheet of thickness 10 cm.

(vii) These also affects the photographic plate produce fluroscence in ZnS barium plantinocyanide, calcium tungustate, willemite etc.

(viii) In β decay atomic number Z changes by unity while the mass number A and size of the nucleus are unaffected.

15.7.3 Properties of γ rays

The main properties of γ rays are as follows -

(i) γ rays are electromagnetic radiations (photons) of very high frequency or very small wave lengths (wavelength range is from 10^{-4} Å to 1 Å). These are uncharged and the rest mass of γ ray photons is zero.

(ii) Being uncharged these are not deflected by electric or magnetic fields.

(iii) The velocity of γ ray photons is same as that of velocity of light.

(iv) These produce a very weak ionisation in gases as these are uncharged and move with very high velocity.

(v) The range of γ ray photons is very large in air it is several hundred meters.

(vi) Their penetration power is much larger compared to α and β rays. γ rays may penetrate a 30 cm thick iron plate. If γ rays of intensity I_0 enters some material then after passing through x thickness of material the intensity is given by

$$I = I_0 e^{-\mu x}$$

where μ is called absorption coefficient, μ depends on nature of material and wavelenght of γ radiation $(\mu \propto \lambda^3)$.

(vii) These affects photographic plates and produce flourescence in ZnS, barium platinocyahide etc.

(viii) Like X rays, γ rays are also diffracted by crystals. However their sources of origin are different. Emission of X rays is an atomic property due to electron transition between atomic energy levels, while γ rays are emitted when an excited nucleus makes a transition to a lower energy state or ground state thus emission of γ ray is as nuclear property.

(ix) Depending on energy of γ rays, their interaction with matter results in phenomenon of photo electric effect, compton effect and pair production.

All the above radiation produces hecating effect when absorbed in a medium. The human body when exposed to α, β and γ radiation suffers incurable burns. Excess exposure may lead to cancer.

15.8 α, β and γ Decay

So far more than 1000 nuclides are known, however a majority of them are unstable. An unstable nucleus changes its composition by emitting some particle while a stable nucleus does not. The two main processes by which a nucleus decays are α decay and β decay.

Often after α or β decay the product nucleus is formed in its excited states and emittes γ ray photons while returning to in the ground state. In all these decay processes conservation laws like mass-energy conservation, linear and angular momentum conservation, charge conservation must be obeyed, and nucleon number must also be conserved. It is also ncessary that mass of the nuleus under going decay must be greater than the sum of the masses of decay products. In this section we will discuss α, β and γ decay.

15.8.1 α – Decay

When a nucleus undergoes α decay, it transforms into a different nuclide by emitting an α particle (a helium nucleus $^{4}_{2}He$). Therefore the α emission reduces the mass number by four and atomic (proton) number by two. Since the atomic number is changed the product nuclide belongs to a different element. Nucleus that undergoes decay is called parent nucleus and the product nucleus is called daughter nucleus. Thus the α decay of parent nucleus $^{A}_{Z}X$ can be written symbollicaly as

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He \quad \ldots (15.20)$$

From above equation note that both the nucleon number and charge are conversed. An example for the α decay is

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He \quad \ldots 15.21$$

α decay occurs for all the nulcei with mass number $A > 210$. We can see that leavy nuclei are unsatble because of large Coulomb repulsion force between their constituent protons. α emission reduces mass number and size of the parent nucleus so it tends towards attaining stability. As required, for decay process the total mass-energy of decay products must be less than mass-energy of the parent nucleus. The difference in mass-energy of the parent and sum of mass-energy of decay product is called distintegration energy or Q value of the process. If the masses of parent atom X, daughter nucleus γ and α particle are denoted by M_x M_y and M_x respectively then

$$Q = \left(M_x - M_y - M_\alpha\right)c^2 \ldots (15.22)$$

The distintegration energy Q represents the decrease in the binding energy of system and appears in the form of kinetic energies of daughter nucleus and α particle. If initally the parent nucleus is at rest then from conservation of linear momentum the daughter nucleus and α particle must have equal and opposite momentum. However as $M_y >> M_\alpha$ therefore, speed of a α particle is much more than the of daughter nucleus. Therefore, Q is mostly associated with the kinetic energy of α particle. From conservation of momentum and energy it can be shown that the kinetic energy of α particle K_α is related to the mass number A of parent nucleus and Q value, according to

$$K_\alpha \simeq \frac{A-4}{A}Q \quad \ldots (15.23)$$

Since $A > 210$, So K_a is only slightly less than Q.

According to equations (15.22) and (15.23) α particles must be emitted with a discrete energy K_a. However, experiments suggest that α particles are emitted with a set of discrete energies, with the maximum value given by equation 15.20. This occurs because the energy of nucleus is quantized, like quantized energies in an atom. In equation (15.20) we assume that the daughter nucleus is formed in its ground state. If the daughter nucleus is formed in one of its excited states, however, less energy is available for the decay and α particle is emitted with less than the maximum energy. The fact that the alpha particles have a discrete set of energies is a direct evidence of energy quantization in nucleus.

It is a natural question to ask that why a nucleus can not decay by neutron or proton emission. This does not happen because in such a case sum of masses of decay product $Y+n$ or $Y+p$ exceeds the mass of parent X. In such a case $Q<0$ and process is not energetically favourable to proceed spontaneously. In fact, in α decay the binding energy per nucleon for α particles is high enough (~ 7.1 MeV) so as to reduces masses of product $Y + \alpha$ that much for α decay to be possible.

If the nucleus that reults from a radioactive decay is itself radioactive then it will also decay and so on. The sequence of decays is known as a radioactive decay series. As in alpha decay the mass number decreases by 4 so if the mass number of parent nucleus is 4n (n is integer) than the mass number of daughter and other successive nuclei in the decay series will also have mass numbers equals to 4 times an integer. Similarly, it the mass number of the original nucleus is 4n+1, where n is an integer all the nuclei in the decay chain will have mass number given by 4n+1 with n decreasing by unity at each decay. We can see, therefore that there are four possible α decay series depending upon wherer A equals to 4n, 4n+1, 4n+2 or 4n+3 where n is an integer. All but one of these decay series are found in nature. The 4n+1 series is not found because it longest member (other than the stable end product 209 Bi) is ^{237}Np which has a half life of 2×10^6 y. Because this is much less thant the age of the earth, this series has disapeared. These four series are shown in table 15.2 below.

Table 15.2 : Different Radioactive series

Mass number	Series	Parent nucleus	Stable end products
4n	Thorium	$^{232}_{90}Th$	$^{208}_{82}Pb$
4n+1	Neptunium	$^{237}_{93}Np$	$^{209}_{83}Bi$
4n+2	Uranium	$^{238}_{82}U$	$^{206}_{82}Pb$
4n+3	Actinium	$^{235}_{82}U$	$^{207}_{82}Pb$

The half live values are different for different emitters. e.g for ^{238}U for α decay the half 4.47×10^9 y life is while it is 550s only for ^{228}U. It is a natural question to ask that through energy is released in each α decay but why is such a huge variation in half life of various α emittes. Newtonian mechanics does not provide answer to such questions, from the point of view of newtonian mechanics even it is not possible for the α decay process to take place at all. Such questions can be answered only with the help of quantum mechanics details of which cannot be possible to discuss here. For the sake of knowledge we try to give a brief account of Gammow theory for α decay in very simple terms.

According to this theory α particles assumed to exist in nucleus prior to α decay. In Fig 15.2 the potential energy function for an α particle and the residual nucleus is shown as a function of separation r between them. This energy is a combination of

(i) The energy associated with attractive nuclear forces inside the nucleus ($r<R_1$) and

(ii) The energy associated Coulomb repulsion between residual nucleus and α particles (outside the nucleus after decay) ($r>R_1$).

From figure it is apparent that a Coulomb energy barrier is present at the surface of nucleus.

The line marked by Q_α depicts the distintegration energy for α decay (which is nearly equal to the kinetic energy of α particle). From figure it is clear that in region $R_1 < r < R_2$ the energy of α particle E is less than potential energy suggesting its kinetic energyto be negative which is impossible. Thus in realm of newtonian mechanics α decay is not possible.

However, if we think of α particle as a matter wave then according to quantum machanical consideration there is a smal but finite probability for the matter wave to tunnel through this barrier. This is knwon as the tunnel effect meaning that α decay is possible. The tunnelling probability is a very sensititve function of barrier dimensions (barrier height and width). α decay processes for which Q_α is small and barrier height is quite high the tunneling probability is quite small. For nuclei like ^{238}U it is so. Calculation shows that if we considered the α particle to be repeatedly colliding with the nuclear barrier then for such nuclei the α particle has to collide 10^{38} times before coming out of the nucleus. Therefore such a nucleus will have large half life for α emission.

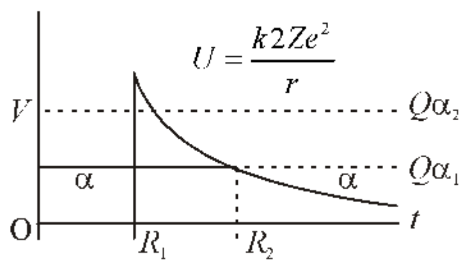

Fig 15.5 The potential energy function for α decay from the nucleus. The shaded region depicts the Coulomb potential barrier that opposes the decay process.

Next we consider an α decay for which disintegration energy is $Q'_\alpha\left(>Q_\alpha\right)$ from figure it is clear that for such α particles both the barrier height and width are comparatively small. Since tunelling probability increases rapidly with reduction in barrier height and width such α particles are emitted easily and consequently for corresponding nucleus the half life is much small.

15.8.2 β decay

When a nucleus decays by emitting an electron or a positron the decay process is termed as β decay. In β minus (β) decay the nucleus emits an electron and an antineutrino and daughter nucleus has the same mass number as that of parent but atomic number increases by unity. Symbolic form for β decay process is

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{-1}e + \overline{v} \quad \ldots(15.24)$$

and as an example we have

$$^{234}_{90}\text{Th} \longrightarrow {}^{234}_{91}\text{Pa} + {}^{o}_{1}e + \overline{v}$$

Note that both the nucleon number and charge are conserved in the process. Antineutrino $\overline{v}$ is a neutral particle with practically no rest mass. It interacts so weakly with matter to make its detection very difficult. In above example the total charge before decay is + 90 e and after decay it is 91 e + (–e) + 0 = 90 e. Since both electron and antineutrino are not nucleon so nucleon number stays conserved at 234.

It may seen strange that a nucleus emits electron (positron) and antineutrino (neutrino) as nucleus contains only proton and neutrons. (There is ample evidence to suggest non existence of electrons in a nucleus). However in previous chapter we have seen that atom emits photons but we have never said that atom contains photons. What we say actually is that photons are formed at the time of emission (during transition of atom from excited state to ground state). Same is true for electron (positron) and anti neutrino (neutrino) in case of β decay. These are formed at the time of emission process. In negative beta (β^-) decay a neutron inside nucleus transforms itself into a proton according to the following equation

$$n \longrightarrow p + e^- + \overline{v} \qquad \ldots(15.25)$$

Such a transformations takes place under the influence of special type of weak nuclear forces (known in general as weak interaction) proton remains in nucleus. Since both protons and neutrons are nucleons so in β decay one nucleus is changing into other so nucleon number is uncharged.

For β decay, the disintegation energy can be calculated as follows. Let m_x and m_y are nulcear masses of X and Y and m_e is the mass of electron, then mass defect

$$\Delta m = m_x - \left[m_y + m_e\right] \qquad \ldots(15.26)$$

Where antineurino is assumed to have zero rest mass. Now if we add and substract Zm_e to to the right hand side of above equation, we obatin

$$\Delta m = \left(m_x + Zm_e\right) - \left[m_y + \left(Z+1\right)m_e\right]$$

$$\text{or } \Delta m = \left(M_x\right) - \left(M_y\right) \qquad \ldots(15.27)$$

Where M_x and M_y are atomic masses of elements X and Y respectively then disintegration energy is given by

$$Q = \Delta mc^2 = \left(M_x - M_y\right)c^2 \qquad \ldots(15.28)$$

In above equations we have neglected the contributions due to binding energies of electrons in atoms X and Y which small enough to be ignored.

Disintegration energy Q appears in the form of kinetic energy of decay products. Because of relatively higher mass of residual nucleus it can be assumed that the energy Q is shared- in varying proportions between the emitted electron and the antineutrino. Sometimes the electron gets nearly all the emitted energy and sometimes the antineutrino does. In every case, however the sum of energies of electron and antineutrino gives the same value Q. Thus emitted electrons can have any value of energy between O and Q. Therefore the energy spectrum of electrons emitted in β^- decay is continuous between zero and Q (figure 15.6). Recall that the energy spectrum for α paricles is discrete.

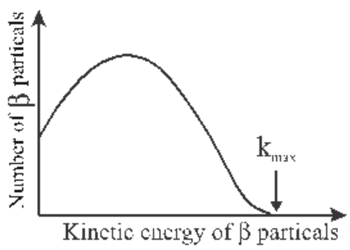

Fig. 15.6 Enrgy spectrum of β particles

In positive beta decay a positron (e) and a neutrino (υ) are emitted from the nucleus. The symbolic representation of such a decay is

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}e + v \quad \ldots (15.29) $$

and an example of such a decay is

$$ {}^{12}_{11}Na \longrightarrow {}^{12}_{10}Ne + {}^{0}_{1}e^{-} + v $$

Here, the mass number A is unchanged but atomic number Z is decreased by one. Like antineutrino a neutrino is uncharged and of negligible rest mass. Positron and neutron are formed at the time of decay as a proton inside nucleus changes into a neutron, positron and neutrino.

$$ p \longrightarrow n + e^{+} + v \qquad \ldots (15.30) $$

Like β^{-} decay, in β^{-} decay, nucleon number and charge is conserved. For β^{+} decay the disintegration energy is $Q = (M_x - M_Y - 2\,m_e)\,C^2$ given by and energy of positron can have any value between zero and Q.

In some nuclei another from of β decay, called electron capture is observed. In such nuclides β decay is not energetically favoured, however, nucleus may capture an orbital electron (usually electron from K shell) which combines with a proton in nucleus to form a neutron. The neutron remains in nucleus and a neutrino is emitted in the process. The symbolic representation of such a process is

$$ {}^{A}_{Z}X + {}^{o}_{-1}e \longrightarrow {}^{A}_{Z-1}Y + v \qquad \ldots (15.32) $$

and an example is

$$ {}^{64}_{29}Cu + {}^{0}_{-1}e \rightarrow {}^{64}_{28}Ni + v $$

The Q value for the process is given by

$$ Q = \left(M_x - M_y\right)c^2 $$

As the nucleus captures an atomic electron there exists a vacancy in corresponding orbit. To fill this vacancy electrons from higher energy states make transition to this orbit as a result X rays are produced.

From β decay we found that neutrons and protons are not fundamental particles in nature. It can also be noted that the process $n \rightarrow p + e^{-} + \overline{v}$ is possible both inside and outside the nucleus i.e an isolated neutron can decay into a proton. However, the process $p \rightarrow n + e + v$ is not possible out side the nucleus. As mass of neutron is more than that of proton, an isolated proton can not decay in a proton.

15.8.3 The Neutrino Hypothesis

Prior to 1930, for explanation of β decay it was assumed that in decay process the decaying nucleus decays into a residual nucleus and an electrons (positron) only i.e β decay processes were assumed to be like

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{o}_{1}e \qquad \ldots (15.34) $$

or

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{o}_{-1}e \qquad \ldots (15.34a) $$

However, there were several difficulties associated with such an assumption. If only an electron and residual nuleus are available as a result of the decay process then owing to the higher mass of the residual nucleus, all the disintegration energy must be available to electrons only making this energy to be unique. This is against the observed experimental fact that the β energy spectrum is a continuous one. It was apparent as if the principle of energy conservation was violated in the process. The smae was the difficulty with the conservation of linear momentum. If we assume the parent nucleus X to be at rest before the decay, then after decay electron and recoiled nucleus Y must move in opposite directions. However, experiments suggested that electrons could move at various angles to the direction of recoiled nucleus? Likewise angular momentum conservation seems to be violeted in the said process. Scientists were started thiking as β decay to be an exceptional process in which conservation laws like energy, momentum, angular momentum etc could not be obeyed.

To explain the apparent non conservation of energy and momentum Pauli in 1930 suggested that a third particle is also emitted in process. Later on Fermi named this particle as neutrino (neutrino means little neutral one). We have already seen how the neutrino and β particle share energy such that energy is always conserved. Although the rest mass of neutrino is zero it has a momentum due to energy. Thus the momentum conservation can be explained as the vector sum of the linear momenta of electron and neutrino must be equal and opposite to the that of recoiling residual nucleus.

(The neutrino is assigned a spin 1/2 to hold the conservation of angular momentum in β decay the details regarding this are not discussed here because of the level of study. Antineutrino is antiparticle of neutrino and is same as neutrino in every respect except for a property called helicity).

Neutrino interaeacts so weakly with matter to make their detection difficult. Neotrinos were first detected by Reines and Cowar in 1956.

15.8.4 γ Decay

A nucleus can exists in states having energies more than in ground state. This is similar to what we have seen in case of atoms with a difference that atomic energy states are in range of a few eV and keV while nucleus energy states range in MeV an excited nucleus is represented by putting a superscipt * on its symbol. When an excited nucleus returns to a lower energy state or ground state photons having energy equal to the difference in initial and final states are emitted. Such photos are called γ ray photons. Their energy is in MeV range. Often a nucleus is formed in its excited state after α or β decay therefore a γ decay follows. The symbolic representation of γ decay is as follows

$$ {}_Z^A X^* \longrightarrow {}_Z^A X + \gamma \qquad \ldots(15.35)$$

In γ decay as there is no change in A and Z so there is no transformation of one element into the other. In γ decay all known conservation laws are obeyed and γ ray energy spectrum is discrete. In Figure 15.7 decay of ${}_{12}^{27}Mg$ by successive β and γ emission into ${}_{13}^{27}\mathrm{Al}$ is shown.

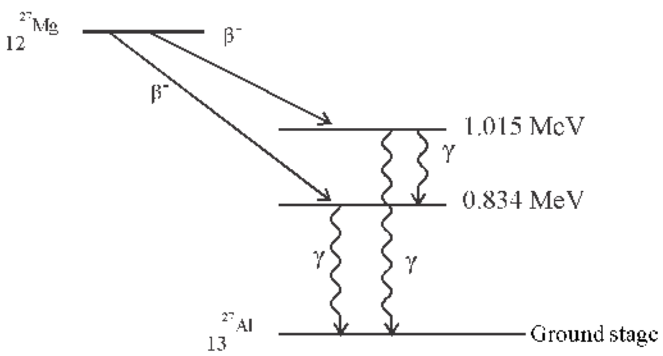

Fig. 15.7 Decay of ${}_{12}^{27}\mathrm{Mg}$ after β decay to ${}_{13}^{27}\mathrm{Al}$ emtting γ rays

Example : Radioactive nuclide ${}_{90}^{228}\mathrm{Th}$ by successive decays is ultimately converted into ${}_{83}^{213}\mathrm{Bi}$. How many α and β particles are emitted in this process.

Solution : For the process

$$ {}_{90}^{228}\mathrm{Th} \longrightarrow {}_{83}^{212}\mathrm{Bi} $$

the net decrease in mass number and atomic number is as follows

$$\Delta A = 228 - 212 = 16$$

and $\qquad \Delta Z = 90 - 83 = 7$

Since in β decay mass number is not changed, the change in ΔA must correspond to α decay. As in each α decay mass number changes by 4 so number of α particles emitted must be 16/4 = 4. However expected decrease in Z due to emission of 4 α particle = 4 x 2 = 8 therefore the final value of Z must be 82 but according to question the final value of Z is 83. This is only possible if $1\beta^-$ particle is also emitted in the process so as to obatin a final value of 83 instead of 82. Therefore 4 α and $1\beta^-$ particles are to be emitted in the given process.

Example 15.13 ${}_{97}^{238}\mathrm{U}$ nucleus undergoes α decay with a half life of 4.5×10^9 y. Write the decay equation and from the data given below estimate the kinetic energy of emitted α particles.

$$M\left({}_{92}^{238}\mathrm{U}\right) = 238.0507\,\mathrm{u}$$

$$M\left({}_{2}^{4}\mathrm{He}\right) = 4.0026\,\mathrm{u}$$

$$M\left({}_{90}^{234}\mathrm{Th}\right) = 234.0435\,\mathrm{u}$$

Take $u = 931\,\mathrm{MeV}/c^2$ and assume the nucleus to be at rest initially.

Solution : The required decay equation is

$$ {}_{92}^{238}\mathrm{U} \rightarrow {}_{90}^{234}\mathrm{Th} + {}_{2}^{4}\mathrm{He} $$

and for this process the Q value is given by

$$Q = \left[M\left({}_{92}^{238}\mathrm{U}\right) - M\left({}_{90}^{234}\mathrm{Th}\right) + M\left({}_{2}^{4}\mathrm{He}\right)\right]c^2$$

on substituting values of various quantities

$$Q = [238.0507 - 234.0435 - 4.0026]c^2$$

$$= [0.0046] \times 931 = 4.28 \text{MeV}$$

Assuming initially $^{238}_{92}U$ to be at rest, from conservation of linear momentum.

$$0 = \mathbf{p}_\alpha + \mathbf{p}_{Th}$$

$$\therefore \quad p_\alpha = p_{Th}$$

or $$\frac{K_\alpha}{K_{Th}} = \frac{p_\alpha^2/2m_\alpha}{p_{Th}^2/2m_{Th}} = \frac{m_{Th}}{m_\alpha} = \frac{A-4}{4}$$

(A is mass number of parent nucleus)

or $$K_{Th} = \frac{4}{A-4}K_\alpha$$

But $K_\alpha + K_{Th} = Q$

$$K_\alpha + \frac{4K_\alpha}{A-4} = Q$$

or $$K_\alpha = \frac{A-4}{A}Q$$

$$= \frac{238-4}{238} \times 4.28 = 4.20 \text{MeV}$$

Example 15.14 For the decay scheme shown in the adjoining diagram calculate the maximum kinetic energy of emitted β particles and radiation frequencies in γ decay. Given

$$M\left(^{198}_{79}Au\right) = 197.9682\,u,$$

$$M\left(^{198}_{80}Hg\right) = 197.9667\,u$$

and / assume $\quad 1u = 931 \text{MeV}/c^2$

Solution : In β^- decay if the daughter nucleus is formed in its ground state then the maximum kinetic energy available to β^- particle is equal to the Q value, which in this case is

$$Q = \left[M\left(^{198}_{79}Au\right) - M\left(^{198}_{80}Hg\right)\right]c^2$$

$$= [197.9682u - 197.9667u]c^2 \times 931 \text{MeV}/c^2$$

$$= 1.396 \text{ MeV}$$

However, in question the nucleus being formed by emission of β particles indicated by β_1^- is in its second excited state which is 1.008 MeV above its ground state so the maximum kinetic energy available to such β particles will be

$$k(\beta_1) = 1.396 - 1.008 = 0.288 \text{MeV}$$

Like wise for β particles indicated by β_2 daughter nucleus is being formed in an excited state at energy 0.412 MeV above the ground state, so for such β particles the maximum kinetic energy will be

$$k(\beta_2) = 1.396 - 0.412 = 0.984 \text{ MeV}$$

For various transitions shown in figure, the frequencies can be obtained using $v = \frac{\Delta E}{h}$ as follows

$$v(\gamma_1) = \frac{1.008 \times 10^6 \times 1.6 \times 10^{-19} J}{6.63 \times 10^{-34} J \cdot s} = 2.62 \times 10^{20} \text{Hz}$$

$$v(\gamma_2) = \frac{(1.008 - 0.412) \times 10^6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.63 \times 10^{20} \text{Hz}$$

$$v(\gamma_3) = \frac{0.412 \times 10^6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 0.99 \times 10^{20} \text{Hz}$$

Example 15.15 For β^- decay,

$^{25}A\ell \to {}^{25}Mg + e^- + v$ calculate Q value. Given, $M\left(^{25}A\ell\right) = 24.990u \quad M\left(^{25}Mg\right) = 24.9858u$

Solution : For given β^- decay, the Q value is

$$Q = \left[M\left(^{25}A\ell\right) - M\left(^{25}Mg\right) - 2m_e\right]c^2$$

where m_e is mass of electron

$$\therefore \quad Q = [24.9904u - 24.9858u]c^2 - 2[0.511 MeV]$$

[in last term of above expression, energy equivalent of rest mass of electron is used]

$$=[0.0046]\times 931 MeV - 1.022 MeV$$

$$= 4.282 - 1.022 = 3.26 MeV$$

15.9 Nuclear Energy

You are well aware of various forms of energy. In view of Einstein mass-energy relation you also know that matter itself is a concentrate of energy. But the energy that we need to perform different task in our day to day life is required in specific forms. For example, heat is required to cook food, boiling water while operation of appliances like fans, cooler, bulbs etc. requires electrical energy. Fuels like coal, natural gas, wood all contain internal energy but this internal energy can not be converted directly into heat. To obtain heat from fuel it is essential to burn them which involves a chemical reaction. In such chemical reactions we are tinkering with atoms of fuels rearranging their outer electrons in a more stable configuration. Likewise, we can also obtain energy from a nuclear system via different nuclear reactions. In our discussion about binding energy per nucleon we have seen that this quantity is more for intermediate mass nuclei to make them relatively more stable. To achieve stability heavy nuclei have a tendecny to fission into middle mass nuclei along with a release of energy. Also, lighter nuclei have a tendecny to fuse to form a middle mass nuclei and again energy is released. In both fission and fusion nucleons are beining rearranged to obtain a more stable configuration. However in chemical reaction energy released is of eV to keV order. Although in both combustion of fuel or fission of nuclear fuel like uranium the decrease in rest mass energy of fuel appears as energy. However in case of nuclear fuel a much larger fraction of rest mass energy is converted into the other forms of energy. For example, theoretically burning of 1 kg coal gives energy to operate 100 W bulb for 8hours. While fission of 1 kg ^{235}U gives energy to operate the same bulb for 3×10^4 y years. Like fission, fusion is also a promising source of energy. In future, about which we shall be discussing at the end of this chapter.

15.10 Nuclear Fission

A few years after the discovery of neutron in 1932, Fermi found that when various elements are bomarded with neutrons, new radioactive elements are produced. Neutron is a useful nuclear projectile, as it is electrically neutral it experieneces no Coulomb repulsive force when it reaches a nuclear surface. Even slowly moving thermal neutrons can enter a nucleus to interact with its nucleons. Thermal neutrons are neutrons in thermal equilibrium with matter at room temperature. At a temperature of T = 300K the average kinetic energy of thermal neutrons is

$$K_{av} = \frac{3}{2}kT = \frac{3}{2}\left(8.62\times10^{-5} eV/K\right)\left(300K\right)$$

$$= 0.04 eV$$

These neutrons are useful for nuclear reactions.

In 1939 German Scientists Otto Hahn and Fritz Strassmann on bombarding uranium with thermal neutrons found that many new radioactive elements were produced one of them was similar to barium in chemical properties. Later on this element was positively identified as barium (Z=56). It was difficult for Hahn and Strassmann to explain that how could a middle mass element like barium is produced by bomardment of uranium (Z=92) with neutrons?

This difficulty was resolved by physicists Lee Meitner and Otto Frisch. They put forward a mechanism by which a uranium nucleus after absorbing a thermal neutron could spilt into two nuclei of intermediate masses (one of which might well be barium) along with the release of energy. They termed the process fission. In addition to fission fragments and energy, neutrons are also released in the fission process. It is worth noting here that fission products are not unique, which can be seen from few fission process illustrated below for the fission of ^{235}U

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \longrightarrow {}^{236}_{92}\text{U} \rightarrow {}^{144}_{56}\text{Ba} + {}^{90}_{36}\text{Kr} + 2\,{}^{1}_{0}\text{n}$$

$$\rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2\,{}^{1}_{0}\text{n}$$

$$\rightarrow {}^{148}_{57}\text{La} + {}^{85}_{35}\text{Br} + 3\,{}^{1}_{0}\text{n}$$

$$\rightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}$$

$\rightarrow$ and many more

Fission products are two middle mass nuclei of different mass number. Usually the fission products themselves are radioactive and undergo a series of β decays till a stable end product is formed. An example is shown below

$${}^{140}_{54}\text{Xe} \xrightarrow{\beta} {}^{140}_{55}\text{Ca} \xrightarrow{\beta} {}^{140}_{56}\text{Ba} \xrightarrow{\beta} {}^{140}_{57}\text{La} \xrightarrow{\beta} {}^{140}_{58}\text{Ce}$$

For fission of $^{235}_{92}U$ on an average 2.5 neutrons are obtained per fission event. These neutrons are called fast neutrons having nearly 2 MeV energy each. These neutrons are not capable of further fission of ^{235}U unless these are moderated to thermal speeds. In Fig 15.8 a graph is plotted between percentage yields of of different fission products and respective mass numbers for the case of fission of ^{235}U. More than 100 nuclides which belongs to 20 different elements are shown. For most fission product mass number lies between 90~ 100 and 135 ~100. The most propbable mass numbers are A = 95 and A = 140. The probability of having nearly equal mass numbers us small.

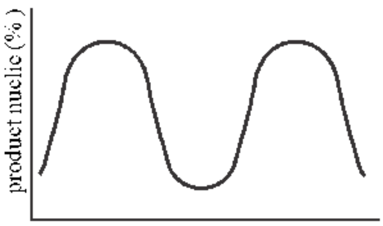

Fig. : 15.8 Graph between percentage yield of fission products and mass number

For fission energy released Q is much higher than in chemical reactions. To estimate the amount of energy released in fission we take the help of binding energy per nucleon curve. From this curve we can see that for heavy nuclides binding energy per nucleon E_{bn} is approximately 7.6 MeV while for middle mass nuclides it is approximately 8.5 MeV.

Next, assume that a high mass nuclide A = 240 undergoes a fission to yield two middle mass nuclei with A = 120 each. Then total binding energy for nucleus A = 240 is $\Delta E_{bi} = \Delta E_{bni} A$

$$\Delta E_{bi} = (7.6) \times 240 \text{ MeV}$$

and for two (A = 120) nuclei the total binding energy

$$\Delta E_{bf} = 2(\Delta E_{bnf}) A/2 = 2(8.5) \times 120$$

$$= (8.5) \times 240 \text{ MeV}$$

Therefore, the energy released in the process

$$Q = \Delta E_{bf} - \Delta E_{bi}$$

$$= (8.5 - 7.6) \times 240 = 216 \text{ MeV}$$

Therefore the energy per fission of ^{235}U is of the order of $Q \sim 200$ MeV. The most of this energy appears in the form of kinetic energies of fission products and partly in kinetic energy of neutrons and subsequent decay products.

The fission of a nucleus can be explained by a the liquid drop model developed by Bohr and wheeler. Here, we are giving a brief account of this model.

In liquid drop model a nucleus is treated like a spherical charged liquid drop which is in equilibrium under the effects of internal attractive forces and Coulomb repulsive forces. Fig 15.9 shows the process of fission for a ^{235}U nucleus. When such a nucleus absorbs a thermal neutron, the potential energy of the nucleus associated with nucleons gets converted into internal excitation energy. The excitation energy for this process is nearly 6.5 MeV (see example 15.12) and due to this excitation energy the nucleus starts vibrating violently [Fig 15.9 (b)]. Figure 15.9 (c) shows that the oscillating nucleus sooner or later develops a short neck and assumes a dumble like shape. If conditions are right then two globs part of dumble separates apart (fission) due to mutual electrostatic repulsion other wise the nucleus emits a γ ray to assume its original shape. According to calculations done by Bohr and Wheeler based on quantum mechanics the critical energy required to break the ^{235}U nucleus comes out to be nearly 5.3 MeV which is less than the excitation energy so fission of ^{235}U is possible by thermal neutrons.

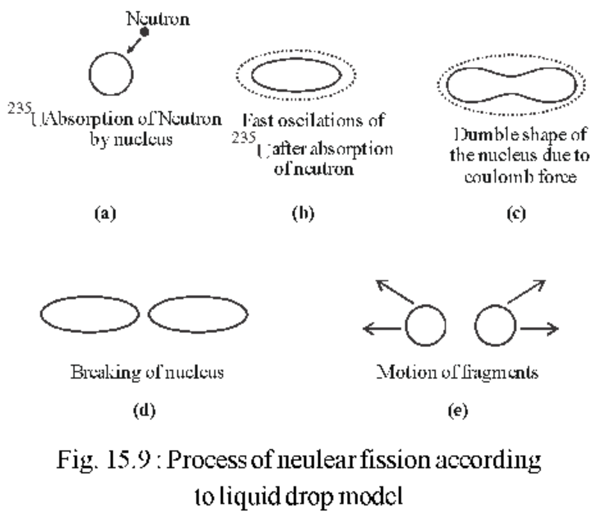

Fig. 15.9 : Process of neulear fission according to liquid drop model

(a) Absorption of a thermal neutron by ^{235}U nucleus

(b) Violent vibrations of compound nucleus formed after obsorption

(c) development of neck in the nucleus

(d) fission of nculeus

(e) formation of fission fragments and emission of fast neutrons

If the internal excitation energy of nucleus, after the absorption of thermal neutrons is less than the corresponding critical energy for breaking the nulceus the fission is not possible. That is why the fission of ^{235}U and ^{239}Pu is possible but ^{238}U and ^{244}Am are not fissonable by thermal neutrons. Such nuclei can be made to fission by fast neutrons.

Example 15.16 In process of nulear fission a ^{235}U nucleus absorbs a neutron to form a ^{236}U nucleus. Calculate the internal energy recieved by the nucleus in this process.

Given : $M\left({}^{235}U\right) = 235.0439\,u$

$$M\left({}^{236}U\right) = 236.0455\,u$$

$$M\left({}^{1}n\right) = 1.0086\,u$$

and take $1u = 931$ MeV

Solution : The given process can be described as

$$^{235}U + {}^{1}n \longrightarrow {}^{236}U$$

Sum of the masses of reactants before this reaction

$$M_i = M\left({}^{235}U\right) + M\left({}^{1}n\right)$$

$$= 235.0439 + 1.0086 = 236.0525\,u$$

and mass of the product

$$M_f = 236.0455\,u$$

Clearly $M_i > M_f$ implying a mass defect meaning that the decrease in rest mass energy appears in the form of internal energy given by

$$E = \left(\Delta Mc^2\right) = \{236.0525 - 236.0455\}\,uc^2$$

$$= \{.0070\} \times 931\text{MeV} = 6.51\text{MeV}$$

Example 15.17 For the fission process

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^{1}_{0}n$$

Calculate the energy released

Given $M\left({}^{235}_{92}U\right) = 235.0439\,u$

$$M\left({}^{1}_{0}n\right) = 1.00867\,u$$

$$M\left({}^{140}_{54}Xe\right) = 139.9054\,u$$

$$M\left({}^{94}_{38}Sr\right) = 93.9063\,u$$

Solution : For the said reaction Q value is

$$Q = \Big[M\left({}^{235}_{92}U\right) + M\left({}^{1}_{0}n\right) - M\left({}^{140}_{54}Xe\right) -$$

$$M\left({}^{94}_{30}Sr\right) - 3M\left({}^{1}_{0}n\right)uc^2$$

$$= [235.0439 + 1.00867 - 139.9054 -$$

$$93.9063 - 3(1.00867) \times 931\text{MeV}$$

$$= [0.22353] \times 931 \approx 208\,\text{MeV}$$

Example 15.18 Calculate the energy released in fission of 1 kg ^{235}U, assuming energy per fission to be 200 MeV.

Solution : One mole of uranium (atomic mass 235) meaning 0.235 kg mass contains number of atoms (nuclei) = Avogadro Number $N_A = 6.023 \times 10^{23}$

$$\text{number of nuclei in 1 kg of } {}^{235}U = \frac{6.02 \times 10^{23}}{0.235}$$

Energy obtained from fission of 1 kg uranium

(This energy is equivalent to energy obtained form the explosion of several thousand tons of TNT)

15.11 Controlled and Uncontrolled Chain Reactions

In the case of fission of a high mass nucleus like ^{235}U we have seen that on an average 2.5 neutrons are produced per fission. This means that if 100 uranium nuclei are under fission than we are obtaining nearly 250 neutrons more. These neutrons are called secondary neutrons and are fast. If these neutrons are slowed down to thermal energies then under appropriate conditions they can produce more fission in other ^{235}U nuclei present in the fissionable material. This further produces more neutrons which causes more fissions. This leads to a formation of chain of fission events and process is termed as a chain reaction. Such a chain reactions is depicted in Figure 15.10 where only two neutrons are shown to proceed the charge reaction.

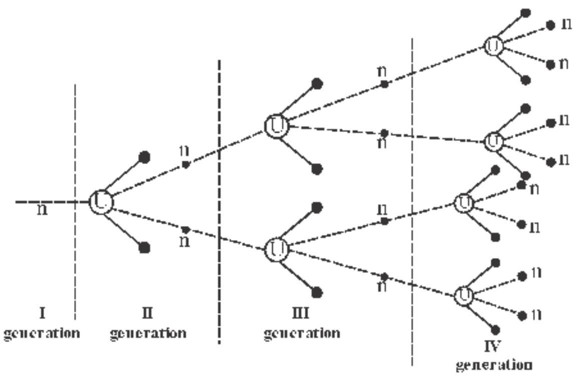

Figure 15.10 : A schematic representation of a chain reaction. Here shows fission fragments and n represents neutrons

For a chain reaction to be self sustained it is essential that at least one neutron produced during each fission, on the average, cause another fission. This condition is often expressed in terms of a parameter K called neutron multipication factor or reproduction factor and it represents the average number of neutrons obtained fission. It is also defined as follows

$$K = \frac{\text{number of neutrons present in a particular generation}}{\text{number of neutrons present at the begining of previous generation}} \quad \ldots (15.36)$$

For ^{235}U based chain reactions maximum possible value of K=2.5, however it is usually less than this, because (i) some of the neutrons may leak from the fissionable material and (ii) some of the neutrons may be absorbed by nuclei present in fissionable material without causing fission.

If $K < 1$ it is abvious that the chain reaction will not be substained because after each subsequent fission the number of neutrons will go on decreasing and ultimately chain reaction wil stop.

If $K > 1$ the rate of reaction will increase rapidly e.g if $K = 1.5$ and in some generation 100 neutrons are produced then in next generation 150 neutrons are obtained and so on. In such a case reaction is called uncontrolled and proceeds so rapdily producing a large amount of energy in the form of heat ultimately leading to explosion. In an atomic bomb such an uncontrolled reaction is required.

If $K = 1$ then reaction is self substained. For example if in certain generation 100 neutrons are obtained then in next generation again 100 neutrons are obtained. Once started the chain reaction will proceed at a constant rate. Such a reaction is called controlled chain reaction. In nuclear reactors $K \simeq 1$ is maintained.

In natural uranium two isotopes ^{235}U and ^{238}U are present in percentage abundance 99.3% and 0.7% respectively. ^{238}U is not fissionable by slow (thermal) neutrons, rather it captures thermal neutron to form ^{239}U which undergoes radioactive decay. ^{235}U is fissionable by thermal neutrons however because of its small percentage in natural uranium the fission probability is too small. So for increasing the probability of fission the content of ^{235}U is increased upto 3%. by artifical means. Such urnaium is called enriched. Even with enrich uranium there are following three difficulties in making a chain reaction 'go'.

(1) The neutron leakage problem : A certain percentage of neutrons obtained by fission will simply leak out of the fissionable material and lost to chain reaction. Some of the neutrons may also be absorbed by the shields enclosing the fissionable material and reaction will not proceed. If too many neutrons are leaked, the chain reaction will not proceed.

Leakage is a surface effect, its magnitude proportional to the square of dimension of fissionable material. For example if material is in the form of a sphere then leakage will be proportional to surface area $= 4\pi r^2$. Neutron production however, is a volume effect, proportional to the cube of a typical dimension (for spherical material volume $= 4/3\pi r^3$). The fraction of neutrons lost by leakage can be reduced by making the material volume large enough there by decreasing its surface to volume ratio ($= 3/r$ for a spherical material). Thus there must be a minimum size or minimum mass of the fissionable material for chain reaction to continue. This mass is known as critical mass.

(2) The neutron energy problem : Neutrons produced in fission are fast with kinetic energies about 2

MeV, but fission takes place due to slow neutrons. Therefore the fast neutrons must be slowed down to thermal energies (0.04 eV). This is done by using substances called moderators. A moderating substance should have the following properties (i) mass of its atom has to be small enough so that in collision of fast neutrons with atoms of the moderator there is an appreciable loss in the kinetic energy of neutrons (b) it does not absorb neutrons excessively to remove them from the fission chain. Water, heavy water ($D_2 = 0$) and graphite are commonly used moderators.

(3) The neutron Capture problem : The ^{238}U nuclei present in nuclear fuel are good absorbers of neutrons having energy in 1 - 100 eV range. As the 2 MeV fast neutrons produced in fission are slowed down in the moderator they must pass through the 1 - 100 eV energy interval in which they are highly likely probable to be captured by ^{238}U nuclei . To minimise this problem uranium fuel (usually in the form of rods) and the moderator are not intimatley mixed but are "clumped" remaining in close contact but arranged in such a manner that most of the neutrons comes into contact with fuel rods after moderation.

The controlled chain reaction is utilized in nuclear reactors.

15.11 Nuclear Reactor

In a nuclear reactor the energy obtained from nuclear fission is converted into electrical energy for power generation. In a nuclear reactor controlled chain reaction is used. Here we will discuss in brief about a nuclear reactor based on fission of by ^{235}U thermal neutrons. A simplified design of a nuclear reactor is shown in Figure 15.11. Uranium is taken in the form of cylinderical rods arranged in a regular pattern in the active reactor care. The volume of core is filled by moderatoring material like, water, heavy water or graphite etc.

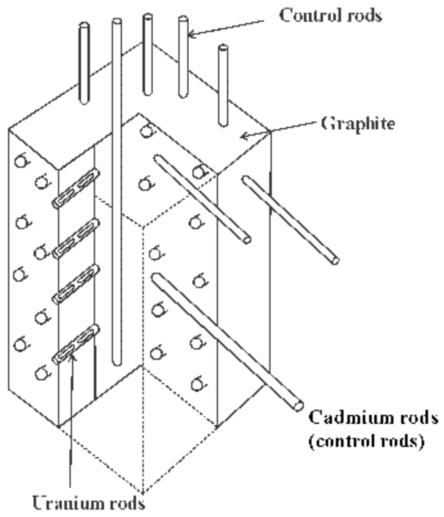

Fig. 15.11 : A simplified diagram of design of a nuclear reactor

When fission takes place in a urainum rod majority of the fast neutrons produced escape from rod and enters into the moderator. In moderator, these neutrons make collisions with moderstor atoms. After few collisions their energy decreases from 2 MeV to thermal energy range. The distances between rods is adjusted in such a manner that a neutron escaping from one rod is generally slowed down to thermal energies before reahing the other rod. This reduces the possibility of absorption of 1- 100 eV neutrons by ^{238}U present in rods. The geometry of core is designed such that the leakage of neutrons is limited to the condition that out of average of 2.5 neutrons produced per fission one neutron is avaiable for triggering next fission. In this condition multipication factor K = 1 and chain reaction proceeds at a constant rate. If by due to the leakage of neutrons is decreased then condition K > 1 is there which may lead to explosion. To control this cadmium rods are used which are inserted upto certain depth in the moderator. Cadmium is a good absorber of neutrons. In fact at the start of chain reaction K be kept greater than unity and then cadimium rods are quickly pushed into the moderator upto such depth that condition K = 1 is achieved. If necessity arises cadmium rods are pushed to full depth to achieve condition $K < 1$. In this condition reactor is shut off. This is done for safety or mainainence of nuclear reactor. Some liquid coolent like water at high pressure or molten sodium is circulated arround the reactor core to extract heat generated. The heat is used to produce steam from water. The steam so produced is used to run turbines and electric power generation.

The reactor region is surround by thick concrete walls so that the harmful radiations produced by highly radioactive fission products do no enter the environment. Thus safety of persons working in nuclear power plants and residents of neighbourhood is ensured.

In addition to power generation nuclear reactors are also used for producing radioactive isotopes used in various applications, and for obtaining neutron mean for research purpose. Although the nuclear power plants play on important role in electricity generation, the problem of disposal of hazadeous nuclear waste is very serious. There are a number of International rules and laws for safe operation of nuclear reactor which are to be adhered strictly. Even a small mistake in operation of a nuclear reactor may lead to destruction therefore a constant montioring is required. In 1986, at chernobyl in Ukraine (formely a part of USSR) the reactor core melted and large amount of radioactivity was released , which adversely affected of population of the adjoining area and East Europian countries. Indian atomic energy programme maintains a good safety record as per international norms.

Example 15.19 The energy obtained per fission of ^{235}U nucleus is about 200 MeV. If a nuclear reactor based on ^{235}U is producing 1000 kW power than how many ^{235}U nuclei are undergoining fission per second.

Solution : As per question, the energy generated per second = 1000×10^3 J

$$= \frac{10^6}{1.6\times10^{-19}} eV = 6.25\times10^{24} eV$$

So the number of fissions per second

$$= \frac{6.25\times10^{24}}{200\times10^6} = 3.12\times10^{16}$$

15.13 Nuclear Fusion

While disscussing the binding energy per nucleon curve we pointed out that large amount of energy can be released if light nuclei are combined to form nuclei of some what larger mass number, a process called fusion. The mass of final products is always less than sum of the masses of the reactants. The lost mass is converted into energy.

Fusion is a difficult process, for the two nuclei to fuse they must come close to one another with in the range of attractive nuclear force. This is a difficult task as this process is hindered by the mutual Coulomb repulsion that tends to prevent two positively charge nuclei from coming close together and fuse. To overcome the Coulomb barrier, the kinetic energies of the reacting nuclei most be large in the range 0.1 MeV to 1 MeV (see example 15.16). Such energies can be achieved through particle accelerators (like cyclotron). However in accelerators the probability that particles are scattered is much more than that of fusion. Also the energy input needed to accelerate one paticle (nucleus) for bomording on the other is higher than the energy released through fusion. So there is no hope of useful power generation through fusion in a bulk material using particle accelerators. The best hope for fusion to occur in bulk material is to raise the temperature of material in the form of gas to such values so that the particles accquire large speeds and come close enough during the collisions for fusion to take place. The process is called thermonuclear fusion and the temperature required is of the order of 10^9K.

As an example of fusion process, consider the fusion of two deuterons

$$^2_1H + ^2_1H \longrightarrow ^3_1H + ^1_1H + 4.0\,MeV$$

The $\left(^3_1H\right)$ nucleus obtained in the process again combines with one deutron to form a helium nuclei

$$^3_1H + ^2_1H \rightarrow ^2_4He + ^1_0n + 17.6\,MeV$$

On combining above two equations the resultant reaction is

$$3\left(^2_1H\right) \rightarrow ^4_2He + ^1_1H + ^1_0n + 21.6\,MeV$$

Thus the over all result is combining of three deutrons to form a helium nuclei and 21.6MeV energy released.

Although the energy obtained from fusion mentioned above is small compared to 200 MeV energy obtained in fission of a ^{235}U nucleus, however if we compare the energy obtained from fusion of 1 kg of deuterium with that obtained from 1 Kg of ^{235}U, then it is much more. (see example 15.17). In addition to this fusion products are not radioactive while that obtained in fission are highly radioactive so fusion promises to be a clean source of energy.

Example 15.20 The deuteron $_1^2H$ has a charge + e and has a radius nearly 2fm. Two such deuterons are fired at each other with the same initial kinetic energy K. What must be the value of K if the two particles are brought to rest by their mutual Coulomb repulsion when the two are just touching. Also calculate the temperature corresponding to this kinteic energy?

Solution : Because, the two deuetrons are momentarily at rest when they just touch other, their total kinetic energy has been converted into electrostatic potential energy. If we treat them as point charges separated by a distance 2R (R = radius of each nucleus) then from conservation of energy

$$2K = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi \in_0}\frac{e^2}{2R}$$

or $$2K = \frac{1}{4\pi \in_0}\frac{e^2}{4R}$$

$$= \frac{\left(9\times10^9\ N.m/C^2\right)\left(1.6\times10^{\ 19}C\right)^2}{4\left(2\times10^{-15}\ m\right)}$$

$$= 2.7\times10^{-14}\ J \simeq 170\,keV$$

If T is the temperture corresponding to K them

$$K_{av} = \frac{3}{2}kT$$

or $$T = \frac{2}{3}\frac{K_{av}}{k}$$

$$= \frac{2}{3}\frac{170\times10^3 eV}{\left(8.62\times10^{\ 5}eV/K\right)}$$

$$= 1.31\times10^9\ K$$

Example 15.21 From fusion of 3 deutrons approximately 21.6 MeV energy is released. Calculate the energy released from the fusion of 1 Kg of deuterium.

Solution : One mole of deuterium (0.002 kg) contains 6.02×10^{23} (Avogedro number) nuclei, hence number of nuclei in 1 kg of deuterium is

$$= \frac{6.02\times10^{23}}{0.002} = 3.01\times10^{26}$$

Since 3 deuteron fuses to give 21.6 MeV of energy, so energy corresponding to one deuteron

$$= \frac{21.6}{3} = 7.2\,MeV$$

$\therefore$ Energy released in fusion of 1 Kg deuterium is

$$= 3.01\times10^{26}\times7.2\,MeV$$

$$= 21.67\times10^{26}\ MeV$$

$$= 21.67\times10^{27}\times1.6\times10^{\ 19}\ J$$

$$= 34.67\times10^{13}\ J$$

In example 15.14 we have seen that the energy obtained from fission of 1 kg of uranium is about 8.19×10^{13} J so energy obtained from fusion is 4 times more. In example 15.16 we have determined the temperature required for initiating a fusion reaction. At such high temperature electrons completely detatched from atoms and the matter is in the form of completely ionised state called plasma consisting of nuclei and electrons. As we shall see in next subsection the thermonuclear fusion is the source of energy in stars. First fusion reaction was performed on earth in 1952 when a thermonuclear (Hydrogen) bomb exploded. To achieve temperature high enough for the fusion to take place an atomic bomb was exploded prior to the hydrogen bomb. Hydrogen bomb is an example of uncontrolled thermonuclear fusion and is very destructive.

The fuel used for fusion on earth is deuterium which is available in natural water and with oceans as almost unlimited source of water we are sure of fuel supply for several thousand years, so if controlled thermonuclear is possible on earth most of our energy demaded can be fulfilled. Fusion reactors are not designed and functional till date. There are a number of difficulties involved for harressing fusion energy for power generation. On of the difficulty is to obtain high temperature required for initiation of fusion. At present pulsed lasers are used to produce temperature of 10^8 K in laboratories. However, major problem is of confiment of plasma. For fusion reaction to occur in large number it is necessary to confine plasma at very high temperatures and high particle density. It is abvious that at such high temperatures plasma is not be confined to a solid container is the biggest problem in achieving the controlled fusion reaction on earth. Scientist are working on two techniques called magnetic confinment and inertial confinment. A device called toka mak has been designed in which magentic confinement is utilized and power levels of 1 MW for 1 second has been achieved . It is expected that by the mid of this century the controlled fusion reactors shell be made available for power production.

15.13.1 Thermonuclear Fusion in Sun and Stars

The sun radiates at the rate of 3.9×10^{26} W and has been doing so for about 4.5×10^{8} years. Prior to 1930, it was assumed that energy generation in sun was due to burning of carbon (coal) and oxygen in its interior part. However, the sun whose mass is 2.0×10^{30} kg was doing so it would have last only for a few thousand years. Another point of view was that , as due to cooling of core the pressure in the core could be reduced and it would have shrinked under the action of its strong gravitational forces. Thus gravitational energy could be transformed into internal energy resulting in increase of temperature of core making sun to radiate continuously. Calculations shows however, that the sun could have radiated from this cause for 10^8 years. The present composition of the sun's core is about 35% of hydrogen by mass, about 64% of helium and 1% of other elements. Due to absence of heavy elements fission cannot be the source of energy generation in sun.

In 1939 American Scientist Bethe proposed that energy generation in sun and other stars is due to thermonuclear fusion in which hydrogen nuclei are fusing to form helium nuclei. This process known is proton cycle is represented by the following set of reactions

$${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^- + \gamma + 0.42\,MeV \qquad \ldots(i)$$

$$e^- + e^- \rightarrow \gamma + \gamma + 1.02\,MeV \qquad \ldots(ii)$$

$${}_1^2H + {}_1^1H \rightarrow {}_2^3He + \gamma + 5.49\,MeV \qquad \ldots(iii)$$

$${}_2^3He + {}_2^3He \rightarrow {}_2^4He + {}_1^1H + {}_1^1H + 12.86\,MeV \qquad \ldots(iv)$$

For the fourth of the above reactions to take place it is essential that first three reactions should take place twice each so that two ${}_2^3He$ nuclei needed for the fourth reaction are made avaiable. Thus considering first three reaction twice each and fourth reaction once, the overall effect is

$$4\left({}_1^1H\right) + 2e^- \rightarrow {}_2^4He + 2\nu + 6\gamma + 26.7\,MeV$$

If in above equation, we add two electrons to each side, then above equation assumes the following from

$$4\left({}_1^1H + e^-\right) \rightarrow \left({}_2^4He + 2e^-\right) + 2\nu + 6\gamma + 26.7\,MeV \qquad \ldots(15.36)$$

Note that the quantities in the parantheses represents atoms of hydrogen and helium. The energy released in the reaction is 26.7 MeV, this can be varified as mass defect

$$Q = (m_i - m_f)c^2 = \left[4M_{{}_1^1H} - M_{{}_2^4He}\right]c^2$$

Where $M\left({}_1^1H\right)$ and $M\left({}_2^4He\right)$ are atomic masses of hydrogen and helium having values 1.007825u and 4.002603u respectively

$$\therefore \quad Q = \left[4(1.007825)u - 4.002603u\right]\times[931.5\,MeV/u]$$

$$= 26.7\,MeV$$

As γ rays are massless and neutrinos are of negligibly small rest mass so these masses are not included in calculations.

The temperature of inner core of sun is estimated to be nearly 1.5×10^{7} K. A little while ago we have seen that fusion requires temperature arround 10^{9}K then why fusion takes place in sun. Answer to this puzzle lies in that we have calculated the kinetic energy of nuclei from relation K = 3/2 ^{KT} ineffect corresponding to the energy kinetic energy of nuclei. Because of large mass of sun there exist nuclei in large number having energies far greater than the average energy, such nuclei are responsible for fusion in sun.

According to present estimate hydrogen in sufficient amount is available in the sun for the fusion to continue for next 5×10^{9} year. After that sun core will consit of helium only. Due to gravitational forces sun's core will contract and its temperature will rise. Due to this outer envelope of the sun will expand to much possibly large enough to encompass the orbit of sun. In terminology of astronomy then sun will become a red giant.

Important Points

1. Nucleus, consits of protons (+ve charge +e) and neutrons (neutral) collectively knwon as nucleons. A nuclide is represented by ${}_{Z}^{A}X$ where X is chemical symbol of element corresponding to nucleus, A is mass number (nucleon number) and Z is proton (Atomic) number, neutron number N = A - Z. Ordinary hydrogen nucleus consits of a single proton only.

2. Nuclei are almost spherical with radii given as

   $R = R_0A^{1/3}$ where $R_0 = 1.2\,\text{fm}$

   accordingly nuclear volume $V = \dfrac{4}{3}\pi R_0^3 A$

   i.e $V \propto A$.

   This means that the density of nucleus is independent of its mass number. Its value is about $2.3\times10^{17}\ \text{kg}/\text{m}^3$ which is same nearly for all nuclei.

3. Nuclear masses are expressed in unified atomic mass unit (u)

   $$1u = \frac{\text{mass of } {}^{12}\text{C atom}}{12} = 1.66054\times10^{-27}\ \text{kg}$$

   also $\quad 1u = 931.5\,\text{MeV}/c^2$

4. Mass of a nucleus M is less than the sum of masses of its constituents nucleons $\sum m$.

   Mass defect is given by $\Delta M = \sum m - M$.

   Energy equivalent to mass defect is called binding energy ΔE_b i.e $\Delta E_b = \Delta Mc^2$. For a nucleus consisting Z protons and N neutrons

   $\Delta E_b = \left[Zm_p + Nm_n - M\right]c^2$

   where m_p and m_n are masses of proton and neutron respectively and M is mass of nucleus. IF in place of nuclear masses, atomic masses are used

   $\Delta E_b = \left[ZM_H + Nm_n - {}_{Z}^{A}M\right]c^2$

   Where M_H = mass of hydrogen atom, and ${}_{Z}^{A}M$ is mass of atom of the corresponding nucleus. Binding energy is a measure of nuclear stability. If we are able to separate a nucleus into its nucleons then this much of energy is to be supplied to the nucleus.

5. The binding energy per nucleon ΔE_{bn} for a nucleus is the quantity obtained on dividing its binding energy E_b by its mass number A

$\Delta E_{bn} = \Delta E_b / A$

A higher value of ΔE_{bn} indicates more stability of nucleus. A graph plotted between ΔE_{bn} and A suggests that middle mass nuclei are relatively more stable than either light mass or heavy mass nuclei. This curve also suggests the possibility of energy release by fission of high mass nucleus into middle mass nuclei or fusion of light nuclei into a middle mass nuclei.

6. Nuclear force: Nuclear force is a very strong attractive force which bounds nucleons together inside a nucleus. This is a very short range force (range ~ few fm), however, in this range it is 50 ~ 60 times greater than electrostatic force of repulsion between protons.

Nuclear forces are charge independent. For a given separation nuclear forces between n-p, p-p and n-n are very nearly same. Nuclear forces are non central they have the property of saturation. For separation between nucleons less than 1 fm the nuclear forces becomes repulsive.

7. Nuclei of heavy mass elements like uranium , thorium, radium, etc decay spontaneously by emitting α, βor γ radiations. This process is called radioactivity. This a nuclear phenomenon which is independent of external parameters like pressure, temperate, phase change and chemical combinations.

8. α particles are nuclei of helium (^4_2He) and are positively charged. β rays are stream of electron (e^-)or positron (e^-) . γ rays (neutral) are very high energy photons with wavelength smaller than that for X rays.

9. Radioactive decay law : Radioative is a random process and obeys laws of probaility (statistics). According to Ruther ford-Soddy decay law, the rate of radioative decay at a given instant is proportional to the number of active nuclei present at that instant i.e $-\frac{dN}{dt} = \lambda N$. The constant of proportionality λ is called as decay constant. Accordingly the number of active nuclei at time t is given by

$N = N_0 e^{-\lambda t}$

Where N_0 is number of active nuclei at t = 0. Thus number of active nuclei in a radioactive material decays exponentially with time.

10. $\lambda = \frac{dN/dt}{N}$ is rate of radioatice decay per atom, or decay probability per unit time. At $t = 1/\lambda$ the number of radioactive nuculei becomes 1/e of initial number N_0.

11. Activity R : This is number of decays per unit time

$R = \left|\frac{dN}{dt}\right| = \lambda N$ and $R = R_0 e^{\lambda t}$, $R_0 = \lambda N_0$

R_0 is initial activity. Activity too decreases exponentially with time. Its SI unit is (Bq) and 1 Bq = 1 disintegration/s. The traditional unit for radioactivity is curie (Ci) which is activity of 1g of radium

$1Ci = 3.7 \times 10^{10}$ disintegration/s $= 3.7 \times 10^{10}$ Bq

12. Half life : The time interval in which the number of active nuclei in a radioactive sample (or activity of sample) reduces to half its initial value N_0 (or initial activity R_0) is called as half life

$$T = \frac{\ell n2}{\lambda} = \frac{0.693}{\lambda}$$

the number of active atoms present at time t in terms of half life is given by

$$N = \frac{N_0}{(2)^{t/T}}$$

13. Average half life : This is the time (t) in which the number of activite atoms N and activity R both decay to 1/e of their initial values.

$$\tau = \frac{1}{\lambda} = \frac{T}{\ell n2}$$

14. In α decay a heavy mass nucleus (parent) X changes into another nucleus Y by emitting an α particle

$${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$$

The atomic number of Y is 2 less than that of X while mass number is 4 less than A. The disintegration energy for the process is

$$Q = \left(M_x - M_y - M_\alpha\right)c^2$$

A major part of this is in the form of the kinetic energy of α particle. α particle energy spectrum is a set of discrete energies suggesting quantization of nuclear energy levels.

15. In β^- decay the parent nucleus X changes into daughter nucleus Y by emitting an electron and an antineutrino

$${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \nu$$

The mass number of daughter Y is smae as that of X but atomic number increases by 1. Actually in β^- decay a neutron inside the nucleus changes into a proton, electron and an antineutrino

$${}_{0}^{1}n \rightarrow {}_{1}^{1}p + {}_{-1}^{0}e^- + \nu$$

proton remians inside the nucleus while eletron and antineutrino are emitted. For β^- decay disintegration energy is given by

$$Q = \left(M_x - M_y\right)c^2$$

electron and anti neutrino share this energy in varibale proportions.

16. In β^+ decay parent nucleus X, changes into daughter nucleus Y by emitting a positron $\left(e^+\right)$ and a neutrino (v).

$${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e + \nu$$

The mass number of daughter Y is same as that of X but atomic number decreases by 1. In β^- decay a proton inside nucleus transforms into a neutron, an electron and a neutrino

$${}_{1}^{1}p \rightarrow {}_{0}^{1}n + {}_{1}^{0}e + \nu$$

Above transformation is not possible out side nucleus. For β^+ decay the disintegration energy is given by

$$Q = \left(M_x - M_y - 2m_e\right)c^2$$

again this energy is shared between positron and neutrino.

17. In certain nuclei, electron capture process takes place instead of a β decay. In this process the nucleus capture an atomic electron belonging to some inner shell of atom which combines with a proton in nucleus to form a neutron.

$$^{A}_{Z}X + ^{0}_{-1}e \rightarrow ^{A}_{Z-1}Y + \nu$$

18. For β decay (both β^+ and β^-) energy spectrum of β particles is continuous which varies from zero to a maximum value Q. This is because of sharing of energy between β particle and neutrino (anti neutrino).

19. Neutrino and antineurino both are neutral particles of negligible rest mass. Neutrino hypothesis was given by Pauli for explanation of energy spactrum, momentum and angular momentum conservation in β decay.

20. γ decay : Often, after α or β decay the daughter nucleus is formed in one of its excited state, such a nucleus returns to a lower energy state or ground state by emitting photons having energy equal to the energy difference between final and initial states. Such photons are called γ ray photons

$$^{A}_{X}Z^* \rightarrow ^{A}_{X}Z + \gamma$$

In γ ray both the mass number and atomic number are uncharged. The γ ray energy spectrum is discrete.

21. In nuclear fission a high mass nuclei breaks into two middle mass nuclei and energy is released in the process. The fission in ^{235}U can be triggered by a thermal neutron. The energy realeased in fission

Q = (final binding energy ΔE_{bf}) - (Initial binding energy ΔE_{bi}). About 200 MeV is released per fission of ^{235}U. If the energy of fast neutrons emitted in fission is moderated to thermal energy then such neutrons can fission more ^{235}U nuclei. This leads to a chain reaction. Chain reaction may be controlled or uncontrolled. A chain reaction is called controlled if from neutrons obtain from fission of a nucleus only one is available for next fission. Acontrolled chain reaction is used in nuclear reactors for power generation. Uncontrolled chain reaction is destructive and used in atomic bomb.

22. Nculear reactor is a device in which a self sustained controlled chain reaction is employed for power generation. Usually enriched uranium 235 is used as nuclear fuel. Fast neutrons obtained in fission are slowed down to thermal energies by collisions form atoms of moderating substances like water, heavy water or graphite. Chain reaction is controlled by cadmium rods, cadmium is a good absorber of neutrons. Energy generated in form of heat in nuclear reactor is extracted by coolents like water, molten sodium etc. To shield environment from harmful nuclear radiations produced in a reactor it is surrounded by thick concrete walls.

Questions For Practice

23. In process of a nuclear fusion two light nuclei combines to form a nucleus of relatively high mass number and energy is released. For thermonuclear fusion very high temperature ~10^9 K is required. Thermonuclear fusion is the source of energy generation in stars. Fusion reactor is still under develoment.

Multiple Choice Questions

1. The radius of $^{64}_{30}Zn$ is about (in fm)

(a) 1.2 (b) 2.4
(c) 4.8 (d) 3.7

2. If the mass of $^{7}_{3}Li$ isotope is 7.016005 u and masses of H atom and neutron are respectively 1.007825 u and 1.008665 u then the binding of Li nucleus is

(a) 5.6 MeV (b) 8.8 MeV
(c) 0.42 MeV (d) 39.2 MeV

3. If at some given instant there are 1.024×10^{24} active atoms in some radioactive sample, then number remaining after eight half lives will be

(a) 1.024×10^{20} (b) 4.0×10^{17}
(c) 6.4×10^{18} (d) 1.28×10^{19}

4. It is found that in an archaeological specimen of wood the activity of ^{14}C was 10 dis/minute per gram of specimen while the activity in fresh wood is found to be 14.14dis/minute per gram. If the half life of ^{14}C is 5700 year the age of the sample approximately is

(a) 2850 years (b) 4030 years
(c) 5700 years (d) 8060 years

5. $^{238}_{92}U$ after a series of decays transforms to stable end product $^{206}_{82}Pb$. The number of α and β particles emitted in the process are

(a) 8, 8 (b) 6, 6
(c) 6, 8 (d) 8, 6

6. For deuteron the binding energy per nucleon is 1.115 MeV then for this nucleus the mass defect is

(a) 2.23 u (b) 0.0024 u
(c) 0.027 u (d) more information is required

7. Two proton are separated by 10 Å. Let F_n and F_e be the nuclear force and electrostatic force between them, so

(a) $F_n >> F_e$ (b) $F_e >> F_n$
(c) $F_n = F_e$ (d) F_n is slightly more than F_e

8. For a deuteron and an alpha paticle binding energies per nucleon are x_1 and x_2 respectively then the energy released Q in the fusion reaction is

$^{2}_{1}H + ^{2}_{1}H + \rightarrow ^{4}_{2}He + Q$

(a) $4(x_1 + x_2)$ (b) $4(x_1 + x_1)$
(c) $2(x_1 + x_2)$ (d) $2(x_2 - x_1)$

9. Of the following given below the nucleus having highest binding energy per nucleon is

(a) $^{238}_{92}U$ (b) $^{4}_{2}He$
(c) $^{16}_{8}O$ (d) $^{56}_{26}Fe$

10. In nuclear reactor of 40% efficiency 10^{14} dis/sec takes places. If energy per fission is 250 MeV then power output of the reactor is

(a) 2 kW (b) 4 kW
(c) 81.6 kW (d) 3.2 kW

11. Origin of β^- electrons emitted during decay is

(a) from inner orbits of an atom
(b) from free electron present in atom in nucleus
(c) from disintegration of a neutron in nucleus
(d) from a photon emitted from the nucleus

12. In an average life

(a) half of the nuclei decay
(b) more than half of the nuclei decay
(c) less then half of the nuclei decay
(d) all the nuclei decay

13. On increasing the mass number which of the nuclear properties is not changed

(a) mass (b) volume
(c) binding energy (d) density

14. Which of the following is electromagentic wave

(a) α rays (b) β rays
(c) γ rays (d) cathode rays

15. ^{23}Ne after energy absorption decays into two α particles and an unknown nucleus. The unknown nucleus is

(a) Oxygen (b) Boron

(c) Silicon (d) Carbon

Very Short Answer Questions

1. What is the number of protons and neutrons in $^{22}_{15}X$ nucleus.
2. Write energy equivalent (in MeV) of 1 μ mass.
3. A nucleus after β decay converts into its isotope or isober which?
4. For which, α or β rays the energy spectrum is discrete.
5. On what type of chain reaction the working of a nuclear reactor is based?
6. Write name of any one material used as moderator in nuclear reactors.
7. Write relation between half life (T) and decay constant (λ) for a radioactive substence.
8. What is the SI unit for activity.
9. After four half lives how much percentage of a radioactive substance remains?
10. Which nuclear reaction is responisble for energy generation in sun?
11. A radioactive element having mass number 218 and atomic number 84 emits β^- particles. What are the mass number and atomic number after decay?
12. Does there a loss in mass number after γ decay.
13. From which it is easier to take out a nucleon, iron or lead?
14. A nucleus undergoes fission into two enequal parts. Which of the two (lighter or heavier) parts will have more kinetic energy?
15. If the nucleons of a nucleus are well separated from each other total mass increases. From where this mass comes.

Short Answer Type Questions

1. An hydrogen molecule has two protons and two electrons. In discussing behaviour of the hydrogen molecule the nuclear force between these protons is always ignored, why?
2. A student claims that a hevier form of hydrogen decays by α emission. What will be your reaction?
3. Defined unified atomic mass unit (u).
4. Explain the meaning of nuclear mass defect.
5. Define Radioactivity.
6. Mention the Ruther Ford-Soddy decay law.
7. Give definitions of half life and mean life of a radioactive substance and write relation between then.
8. What is α decay? What is the type of α particle energy spectrum?
9. β ray energy spectrum is continuous? What is the meaning of this?
10. The neutrino hypothesis is helpful in describing which conservation laws in β decay process?
11. Write any two properties of the nulear forces?
12. What do you mean by binding energy per nucleon? How it is related with nuclear stability?
13. Define nuclear fission.
14. What is meant by critical mass in reference to nuclear chain reaction.
15. Heavy water is a good moderator in nuclear reactors? Why?

Essay Type Questions

1. Describe composition of the nucleus and discuss nuclear forces?
2. Explain mass effect and binding energy? Explain the main conculsions which can be drawn from the binding energy per nucleon versus mass number diagram.
3. Write law of Radioactive decay. Using the exponential decay law derive expressions for half life and mean life of a radioactive element.
4. What is meant by nuclear fission? Why a nuclear fission chain is not self substained? Explain what is to be done to make a chain reaction sustained.
5. Draw a simple diagram of a nuclear reactor and explain its working.
6. Explain β decay. Discuss the neutrino hypotheseis of β decay.

7. Discuss α decay from a radioactive nucleus. Explain that the α ray energy spectrum consists of a set of discrete energies.

8. How does the proton-proton cycle proceeds in fusion. Why such thermonuclear reactions can not be performed in laboratory.

Answer

Mutliple Choice Questions -

1. (C)	2. (D)	3. (B)	4. (A)
5. (D)	6. (B)	7. (B)	8. (B)
9. (D)	10. (C)	11. (C)	12. (D)
13. (D)	14. (C)	15. (D)	

Very Short Answer Questions

1. 15, 17
2. 931.5 MeV
3. Isobaric
4. α particle
5. controlled
6. graphite/Heavy water/water
7. $T = \frac{\ell n 2}{\lambda} = \frac{0.693}{\lambda}$
8. 1 Bq =/ disintegration/sec
9. 6.25%
10. Thermonuclear fusion
11. 218, 85
12. No
13. Lead nucleus
14. Lighter nucleus
15. from binding energy of the nucleus

Numerial Questions

1. Radius of a nucleus of mass number 16 is 3×10^{15} m. What is the radius of a nucleus of mass number 128.

(Ans : 6×10^{-15} m)

2. Calculate binding energy for $^{56}_{26}$Fe nucleus. [Given, atomic mass of $^{56}_{26}$Fe $= 55.9349u$, mass of neutron $= 1.00867u$, mass of proton $= 1.00783u$ and $1u = 931\,MeV/c^2$.

(Ans : 492 MeV)

3. The half life of a radioactive substance X is 3s. Initially a specimen of this substance contains 8000 atoms. Calculate (i) its decay constant (ii) time t at which 1000 atoms remain active in this sepcimen.

(Ans : 0.231 s ^{1}, 9 s)

4. A radioactive nucleus undergoes decay as follows $X \xrightarrow{\alpha} X_1 \xrightarrow{\beta^-} X_2 \xrightarrow{\alpha} X_3 \xrightarrow{\gamma} X_4$ If the mass number of X is 180 and atomic number is 72 calculate the mass number and atomic number of X_4.

(Ans : 172,69)

5. 200 MeV energy is obtained per fission of U-235. If a reactor with U- 235 as fuel generates 1000 kW power then calculate the number of disintegrations per second for nuclei in this reactor.

(Ans: 3.12×10^{16})

6. For the fusion reaction $^2_1H + ^2_1H \rightarrow ^3_2He + ^1_0n$ masses of deuteron, helium and neutrons are respectively 2.015 u, 0.017 u and 1.009 u. If 1 kg of deuterium undergoes complete fusion then calculate amount of energy released.

(Ans : 208 MeV)

7. Calculate the Q value for reaction $^{235}_{92}U + ^1_0n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^1_0n + Q$ Given

mass of $^{235}_{92}U$ = 235.0435 u

mass of $^{14}_{54}Xe$ = 139.9054 u

mass of $^{94}_{38}Sr$ = 93.9063 u

mass of ^1_0n = 1.00867 u

and 14 = 931 MeV/C^2

(Ans : 208 MeV)

8. Calculate the mass of ^{227}Th for 1mci activity, its half life is 1.9 y.

(Ans : 1.206×10^{6} g)

9. In some experiment the activity of a given radioactivite was found to be 6400 dis/min. On repeating the experiment after 6 days the activity

was found to be 400 dis/min. Determine the half life of element.

(Ans : 1.5 days)

10. An α particles is emitted by a $^{226}_{88}Re$ nucleus. If the energy of α particles is 4.662 MeV then what is the total energy released in the process.

(Ans : 4.746 MeV)

11. A nucleus ^{176}X, β transforms to ^{176}Y after β decay. If the atomic masses of X and Y are respectively 175. 9426944 and 175.941420 μ then determine the maximum energy of emitted β particles.

(Ans : 1.182 MeV)