Sample Paper-03
Mathematics
Class – XI

Time allowed: 3 hours   Maximum Marks: **100**

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. Find the domain of the function $f(x) = \dfrac{1}{\sqrt{2-x^2}}$

2. If $A = \{y = \sin x, 0 \le x < \dfrac{\pi}{4}\}$ and $B = \{y = \cos x, 0 \le x < \dfrac{\pi}{4}\}$ then what is $(A \cap B)$

3. What is the maximum value of a if $a = 1 - \sin x$

4. Name the locus of points (M), the sum of whose distance from two given points is a constant

5. Check whether the three points (2, 0), (5, 3), (2, 6) are collinear.

6. Write the condition so that the equation $ax^2 + ay^2 + bx + cy + d = 0$ represents a circle.

Section B

7. Find the inverse of the function $f(x) = x^2 - x + 1, x > \dfrac{1}{2}$

8. Find the vertex, axis , Focus , Directrix and latus rectum of the parabola $8y^2 + 24x - 40y + 134 = 0$

9. Express $\dfrac{7-4i}{3+2i}$ in the form $a + ib$

10. Solve the inequality $(x-2)((x-3) > 0$

11. Find the general value of x if $\tan 5x = \dfrac{1}{\tan 2x}$

12. In a single throw of 2 dies what is the probability of getting a prime number on each die.

13. If $f(x) = x^3 - x$; $\phi(x) = \sin 2x$ Find the value $f[\phi(\frac{\pi}{12})]$

14. If $\tan A = \dfrac{m}{m+1}$ and $\tan B = \dfrac{1}{2m+1}$ prove that $\tan A + \tan B + \tan A \tan B = 1$

15. If $f : R \to R$ is defined as follows: $f(x) = \begin{cases} 1 & if \ x \in Q \\ -1 & if \ x \notin Q \end{cases}$ Find $f(\sqrt{3}), f(3), f(\sqrt{3}+1)$

16. Prove that the equation

$sin\theta = x + \dfrac{1}{x}$ is impossible if x is real

17. Find the domain of the function for which $f(x) = \phi(x);, if \ f(x) = 3x^2 + 1, and \phi(x) = 7x - 1$

18. Find the limit $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x}$

19. Solve $2\sin^2 x + 14\sin x \cos x + 50 \cos^2 x = 26$

Section C

20. Differentiate $\sin x$ from the first principle w.r.t. x

21. Find the sum of n terms of the series $12 + 16 + 23 + 33 + 46 \ldots\ldots$

22. Find the equation of a circle whose diameter is the line joining the points $(x_1, y_1) \ and \ (x_2, y_2)$

23. Calculate the mean deviation about the mean from the following data

x_i	5	7	9	10	12	15
f_i	14	6	2	2	2	4

24. How many numbers can be formed with the digits $1, 2, 3, 4, 3, 2, 1$ so that odd digits are in odd places and even digits are in even places.

25. Two engineers go for an interview for two vacancies in the same grade. The probability of

engineer 1 (E1) getting selected is $\dfrac{1}{3}$ and that of engineer 2 (E2) is $\dfrac{1}{5}$. Find the probability that

only one of them will be selected.

26. How many numbers are there between 1 and 1000(both included) that are not divisible by 2, 3, and 5?

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ANSWER

Section A

1. **Solution:**

   Domain of is in the open interval $(-2, 2)$

2. **Solution:**

   $(A \cap B) = \{\phi\}$

3. **Solution**

   Max value is 2

4. **Solution:** Ellipse

5. **Solution**

   Condition for colinearity is not satisfied here since

   $$\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} \qquad \neq 0$$

6. **Solution:**

   $$b^2 + c^2 - 4ad > 0$$

Section B

7. **Solution:**

   $$y = x^2 - x + 1$$

   $$y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

   $$y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

   $$x = \frac{1}{2} + \sqrt{y - \frac{3}{4}}$$

   $$f^{-1}(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$$

8. **Solution:**

Equation is $8y^2 + 24x - 40y + 134 = 0$

$= 4y^2 + 12x - 20y + 67 = 0$

This can be written as

$y^2 - 5y = -3x - \dfrac{67}{4}$

$\left(y - \dfrac{5}{2}\right)^2 = -3x - \dfrac{67}{4} + \dfrac{25}{4} - 3\left(x + \dfrac{7}{2}\right)$

Let $Y = y - \dfrac{5}{2}$

$X = x + \dfrac{7}{2}$

$Y^2 = -3X$

This is of the form $y^2 = -4ax$

Latus rectum is $= 3$

Vertex $\left(-\dfrac{7}{2}, \dfrac{5}{2}\right)$

Axis $y = \dfrac{5}{2}$

Focus $\left(-\dfrac{7}{2} - \dfrac{3}{4}, \dfrac{5}{2}\right)$

Directrix : referred to New axis : $X = a = \dfrac{3}{4}$

Directrix referred to Old axis : $\dfrac{3}{4} = x + \dfrac{7}{2}$

$x = \dfrac{3}{4} - \dfrac{7}{2}$

$x = -\dfrac{11}{4}$

9. **Solution:**

$$\dfrac{7 - 4i}{3 + 2i} = \dfrac{7 - 4i}{3 + 2i} \times \dfrac{3 - 2i}{3 + 21}$$

$$\dfrac{13 - 26i}{13} = 1 - 2i$$

10. **Solution**

   Either both factors are negative or both factors are positive to have this in equality. if $x < 2$ both factors are negative and if $x > 3$ both factors are positive. Hence the solution is $x \in \{(-\infty, 2) \cup (3, \infty)\}$

11. **Solution**

$$\tan 5x = \cot 2x$$

$$\tan 5x = \tan(\frac{\pi}{2} - 2x)$$

$$5x = (\frac{\pi}{2} - 2x)$$

$$5x = n\pi + (\frac{\pi}{2} - 2x)$$

$$7x = n\pi + \frac{\pi}{2}$$

$$x = \frac{1}{7}(n\pi + \frac{\pi}{2})$$

12. **Solution**

Total number of occurrence = $6 \times 6 = 36$

On each die there are 3 prime numbers $\{2, 3, 5\}$

Hence total number of favorable cases $3 \times 3 = 9$

Probability of getting a prime in each die = $\frac{9}{36} = \frac{1}{4}$

13. **Solution:**

$$\phi(\frac{\pi}{12}) = \sin 2.(\frac{\pi}{12})$$

$$= \sin \frac{\pi}{6}$$

$$= \frac{1}{2}$$

$$f(x) = (\frac{1}{2})^3 - \frac{1}{2}$$

$$= \frac{1}{8} - \frac{1}{2}$$

$$= -\frac{3}{8}$$

14. **Solution:**

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \dfrac{m}{m+1} \cdot \dfrac{1}{2m+1}} = 1$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B + \tan A \tan B = 1$$

15. **Solution:**

$$f(\sqrt{3}) = -1$$

$$f(3) = 1$$

$$f(\sqrt{3+1}) = 1$$

16. **Solution:**

Use the inequality $AM \geq GM$

$$AM \text{ between } x, \frac{1}{x} = \frac{x + \frac{1}{x}}{2}$$

$$GM \text{ between } x, \frac{1}{x} = \sqrt{x.\frac{1}{x}} = 1$$

$$\frac{x + \frac{1}{x}}{2} \geq 1$$

$$x + \frac{1}{x} \geq 2$$

Since $-1 \leq \sin\theta \leq 1$

$\sin\theta = x + \dfrac{1}{x}$ is impossible

17. **Solution:**

$$f(x) = \phi(x)$$

$$f(x) = 3x^2 + 1$$

$$\phi(x) = 7x - 1$$

$$3x^2 + 1 = 7x - 1$$

$$3x^2 - 7x + 2 = 0$$

$$(x-2)(3x-1) = 0$$

$$x = 2, x = \frac{1}{3}$$

Hence $f(x)$ and $\phi(x)$ are equal when the domain is in the set $\left\{\dfrac{1}{3}, 2\right\}$

18. **Solution**

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

$$= \lim_{x \to 0} \frac{1 - (1 - 2\sin^2 \frac{x}{2})}{x}$$

$$= \lim_{x \to 0} \frac{\sin^2 \frac{x}{2}}{x}$$

$$= \lim_{x \to 0} \frac{\sin \frac{x}{2}}{2 \frac{x}{2}} \sin \frac{x}{2}$$

$$= \frac{1}{2} . 1.0$$

$$= 0$$

19. **Solution:**

$$2\sin^2 x + 14\sin x \cos x + 50\cos^2 x = 26$$

$$= 2\sin^2 x + 14\sin x \cos x + 50\cos^2 x = 26(\sin^2 x + \cos^2 x)$$

$$= -24\sin^2 x + 14\sin x \cos x + 24\cos^2 x = 0$$

$$= 24\sin^2 x - 14\sin x \cos x - 24\cos^2 x = 0$$

$$= 24\tan^2 x - 14\tan x - 24 = 0$$

$$\tan x = \frac{14 \pm \sqrt{196 + 2304}}{48}$$

$$\tan x = \frac{14 \pm \sqrt{2500}}{48}$$

$$\tan x = \frac{14 \pm 50}{48}$$

$$\tan x = \frac{64}{48}; or; -\frac{36}{48}$$

$$\tan x = \frac{4}{3} \ or -\frac{3}{4}$$

Section C

20. **Solution:**

$$y = \sin x$$

$$y + \Delta y = \sin(x + \Delta x)$$

$$\Delta y = \sin(x + \Delta x) - y$$

$$\Delta y = \sin(x + \Delta x) - \sin x$$

$$\Delta y = 2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}$$

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\dfrac{2x+\Delta x}{2}\sin\dfrac{\Delta x}{2}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cos\dfrac{2x+\Delta x}{2}\sin\dfrac{\Delta x}{2}}{\dfrac{\Delta x}{2}}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \cos x$$

$$\frac{dy}{dx} = \cos x$$

Note: As $\Delta x \to 0$; $\dfrac{\Delta x}{2}$ $also \to 0$

21. **Solution**:

The successive First order of difference is $4,7,10,13,\dots$ this is an AP.

The second order difference is(Difference of the first difference) $3,3,3,\dots$

Third order difference (Difference of second order differences) is all 0

n th term

$$T_n = T_1 + (n-1)\Delta T_1 + \frac{(n-1)(n-2)}{2!}\Delta T_2 + \frac{(n-1)(n-2)(n-3)}{3!}\Delta T_3$$

$$= 12 + 4(n-1) + 3\frac{(n-1)(n-2)}{2}$$

$$= \frac{3n^2 - n + 22}{2}$$

$$\text{Sum} = \frac{1}{2}\left(3\,\Sigma n^2 - \Sigma n + 22n\right)$$

$$= \frac{1}{2}\left(3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 22n\right)$$

$$= \frac{1}{2}\left(n^3 + n^2 + 22n\right)$$

22. **Solution**:

Let the point A be (x_1, y_1) and B be (x_2, y_2)

Let the point C be a point be (x, y) on the circle

Then AC and BC are perpendicular

Product of Slopes of line AC and BC =-1

$$\frac{y-y_1}{x-x_1} \cdot \frac{y-y_2}{x-x_2} = -1$$
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

23. **Solution**

x_i	f_i	$f_i x_i$	$|x_i -9|$	$f_i |x_i -9|$
5	14	70	4	56
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	4	60	6	24
	$N = \Sigma f_i = 26$	$\Sigma f_i \, x_i = 234$		$f_i\Sigma|x_i - 9| = 100$

$$Mean = \bar{X} = \frac{1}{N}(\Sigma f_i \, x_i) = \frac{234}{26} = 9$$

$$MeanDeviation = M.D = \frac{1}{N}(\Sigma f_i \, |x_i - 9|) = \frac{100}{26} = 3.84$$

24. **Solution:**

The odd digits $1,3,3,1$ can be arranged in their 4 places in $\dfrac{4!}{2!2!}$ ways

Even digits $2,4,2$ can be arranged in their 3 places in $\dfrac{3!}{2!}$

Hence the total number of arrangements $= \dfrac{4!}{2!2!} \times \dfrac{3!}{2!} = 6 \times 3 = 18$ ways

25. **Solution**

Probability of one of them getting selected $P(E_1 or E_2) = 1$- (Probability of both getting selected + Probability of none getting selected)

= $1-[P(E_1 \cap E_2)+P(E_1' \cap E_2')]$

= $1-(\dfrac{1}{3} \times \dfrac{1}{5} + \dfrac{2}{3} \times \dfrac{4}{5})$

$$= 1-(\frac{1}{15}+\frac{8}{15})$$

$$= 1-\frac{9}{15}=\frac{6}{15}=\frac{2}{5}$$

26. **Solution**

Let A denote the set of numbers that are divisible by 2, B set of numbers that are divisible by 3, C set of numbers that are divisible by 5, D set of numbers that are divisible by both 2 and 3, E set of numbers that are divisible by both 2 and 5, F set of numbers that are divisible by 3 and 5, G set of numbers that are divisible by all the three numbers

$$a+(n-1)d = T_n$$

$$n = \frac{T_n}{d} - \frac{a}{d} + 1$$

In this case $\frac{a}{d}=1$, Hence $n = integer\ part\ of\ \frac{T_n}{d}$

$$n(A) = [\frac{1000}{2}] = 500$$

$$n(B) = [\frac{1000}{3}] = 333$$

$$n(C) = [\frac{1000}{5}] = 200$$

$$n(D) = [\frac{1000}{2\times3}] = 166$$

$$n(E) = [\frac{1000}{2\times5}] = 100$$

$$n(F) = [\frac{1000}{3\times5}] = 66$$

$$n(G) = [\frac{1000}{2\times3\times5}] = 33$$

Numbers that are divisible by 2, 3, 5 are

$$n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cup B)-n(A\cup C)-n(B\cup C)+n(A\cap B\cap C)$$

$$= 500+333+200+1666+100+66+33$$

$$= 734$$

Numbers that are not divisible by 2, 3, 5 are

$$1000-734 = 266$$